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Reidar Hahn

# Non-elliptical cavities

Special thanks to R. Laxdal (TRIUMF) and A. Facco (INFN & FRIB)



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# Outline

- Why non-elliptical cavities?
- Applications and trends
- Cavity concepts
- Examples of TEM cavities
- Basic principles of acceleration – figures of merit, beam dynamics for low beta
- Cavity beta/frequency choice
- Other cavity types
- Design issues
- Fabrication
- Cryomodule
- On-going developments



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## Useful relations

Protons/H- ( $A = 1, Q = 1$ )

$$p = \gamma\beta m_0 c = \frac{\gamma\beta E_0}{c}$$

$$E = W + E_0 = \gamma E_0$$

$$W = (\gamma - 1)E_0$$

$$F_\xi = |e|\xi$$

$$W = |e|V_{\text{eff}} \cos \varphi$$

Heavy Ions ( $A, Q$ )

$$p = \gamma\beta A m_0 c = \frac{\gamma\beta A E_0}{c}$$

$$E = W + A E_0 = \gamma A E_0$$

$$W/A = (\gamma - 1)E_0$$

$$F_\xi = Q e \xi$$

$$W/A = (Q/A) V_{\text{eff}} \cos \varphi$$





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# Particle velocity vs. kinetic energy

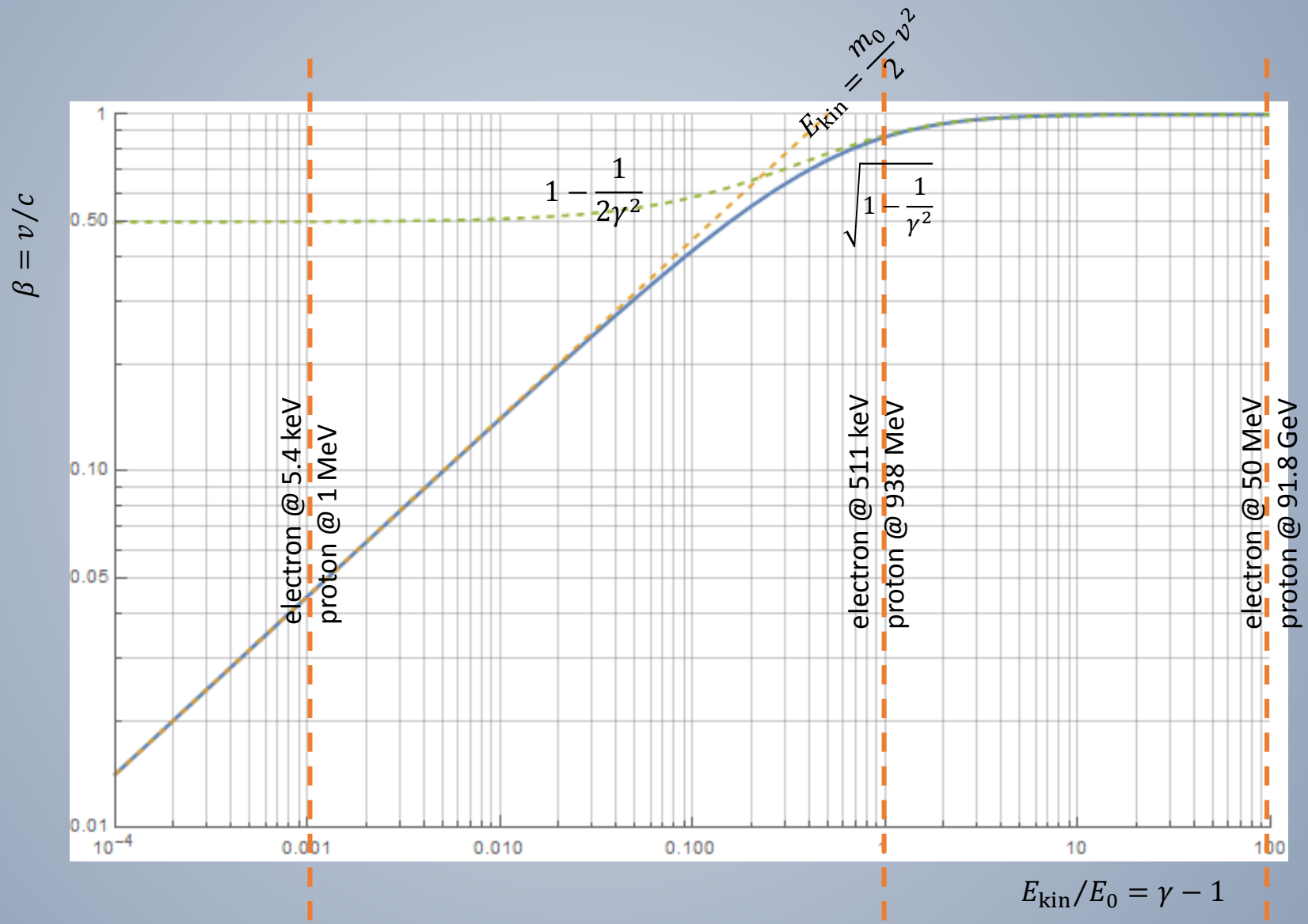




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# Accelerating electrons vs. accelerating ions

Example: a 300kV DC bias is enough to get electrons going at a relativistic speed (ie  $E_0=511\text{keV}$  so  $\gamma=1.58$ ,  $v/c=\beta=0.78$ ) – for protons a 300kV bias only produces  $v/c=\beta=0.025$  – for  $A=30$   $v/c=\beta=0.005$

- Electron –  $0.511\text{MeV}/c^2$

- 300kV -  $\gamma=1.58$ ,  $v/c=\beta=0.78$
- 550MeV -  $\gamma=1011$ ,  $v/c=\beta=1$



8 gm



ARIEL 300kV e-gun

- Protons –  $938\text{ MeV}/c^2$

- 300kV -  $\gamma=1.003$ ,  $v/c=\beta=0.025$
- 550MeV -  $\gamma=1.58$ ,  $v/c=\beta=0.78$



160 kgm



TRIUMF 500MeV cyclotron

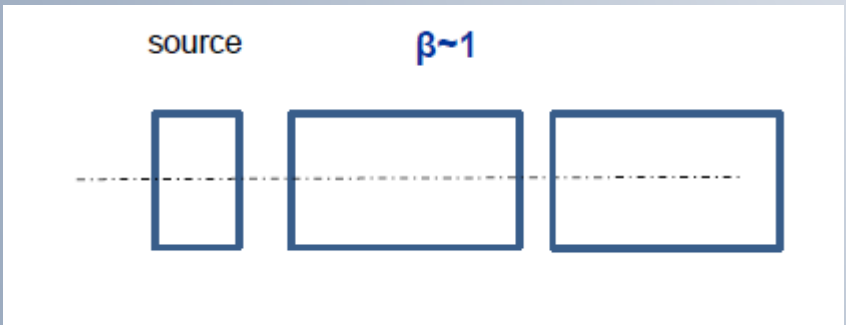


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# Accelerating electrons vs. accelerating ions

## Electrons

Common building blocks – all designed for  $\beta = 1$ .



## Ions

Various building blocks – different technologies, each optimized for a certain velocity range

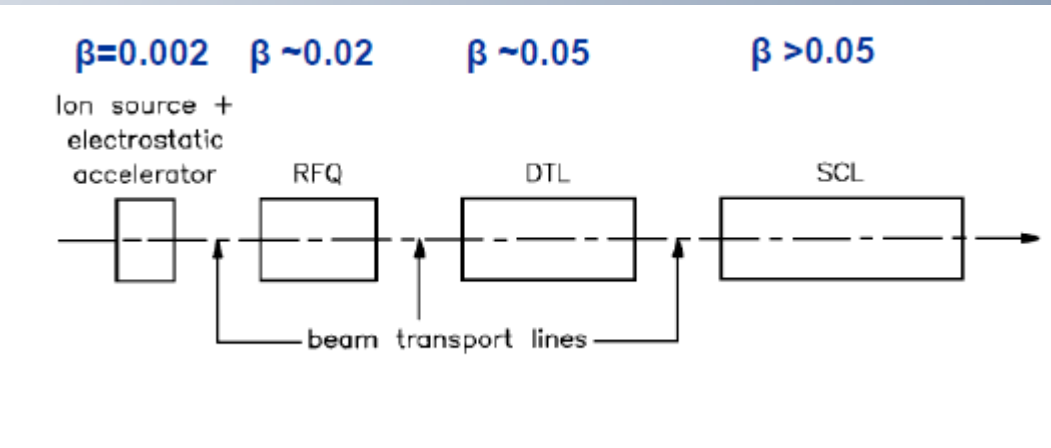


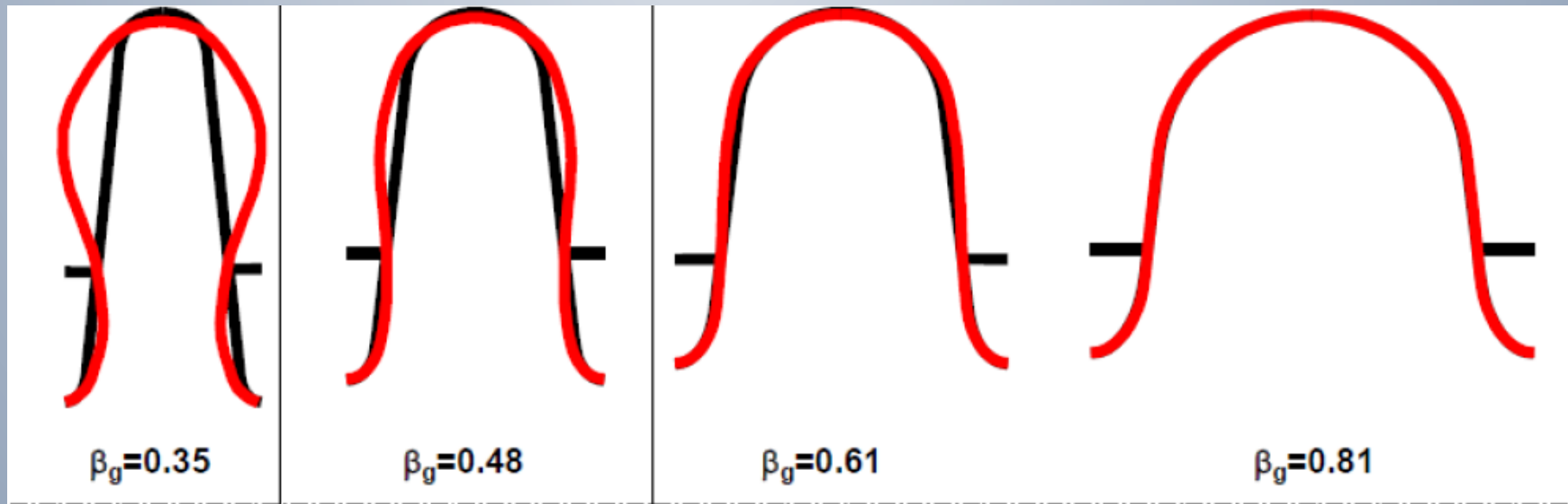




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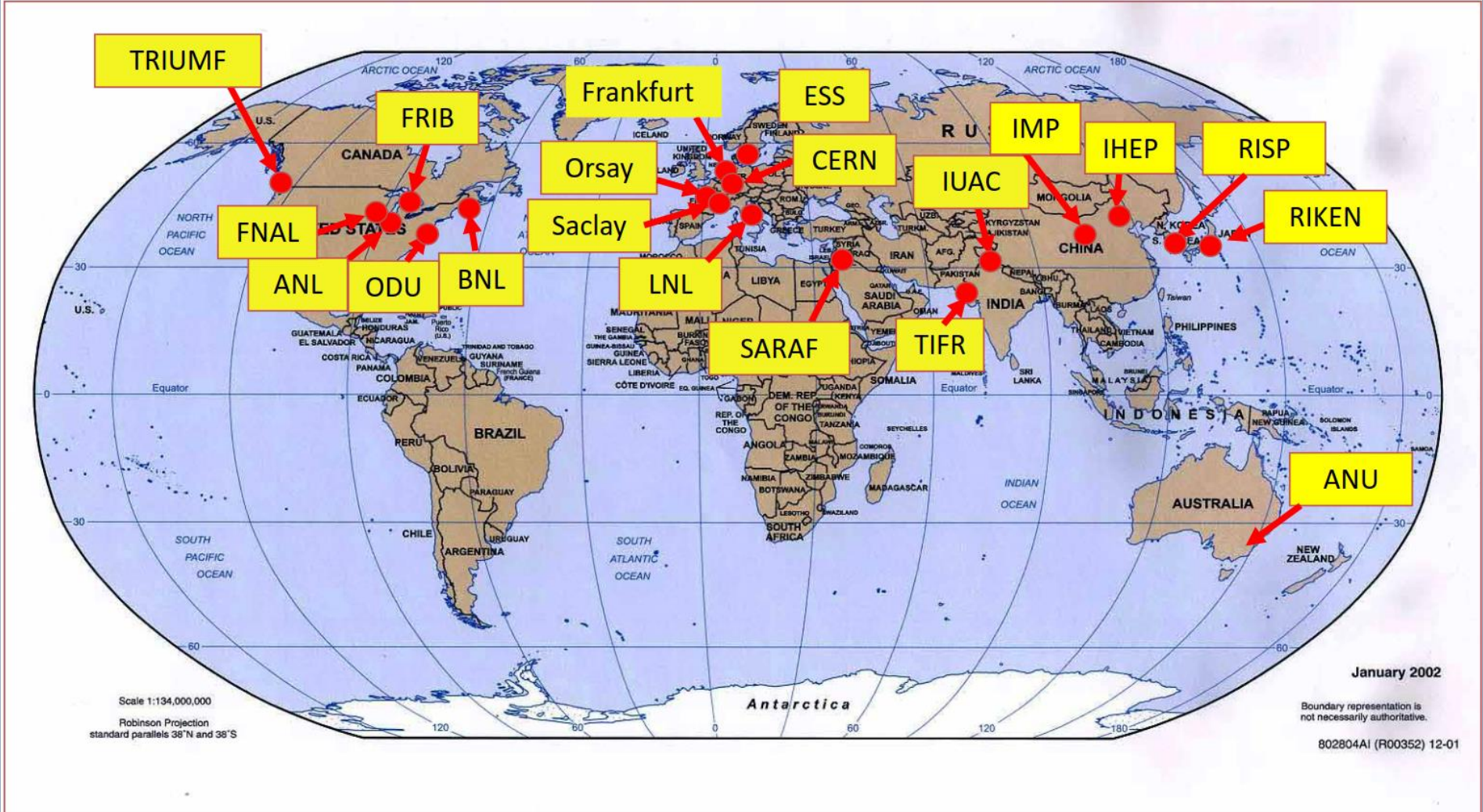
# Limitations of elliptical cavities

- Elliptical cavities have been designed starting at  $\beta \geq 0.5$  for CW applications, for  $\beta \geq 0.6$  for pulsed (SNS, ESS).
- The  $\pi$ -mode requires cell-to-cell distance of  $\beta\lambda/2$ , but outer diameter  $\approx 0.9 \lambda$ , i.e. at low  $\beta$  the cavity looks more like bellows, sensitive to LFD!





# Non-elliptical SRF Community around the world







# Resonator types for low beta acceleration

- Quarter wave resonator (QWR)  $\beta \approx 0.04 \dots 0.2$
- Half wave resonator (HWR)  $\beta \approx 0.1 \dots 0.5$
- Single spoke resonator (SSR)  $\beta \approx 0.15 \dots 0.7$
- Multi-spoke resonator (MSR)  $\beta \approx 0.06 \dots 1$
- For comparison: Elliptical cavities  $\beta \approx 0.5 \dots 1$

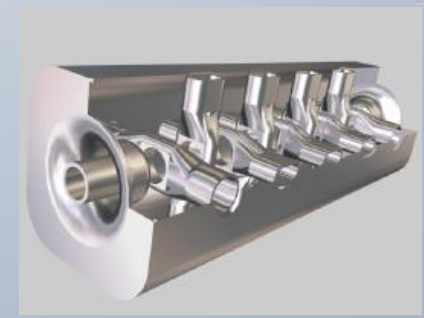
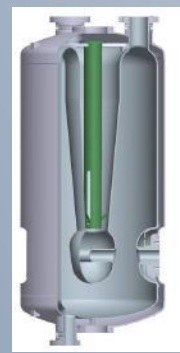
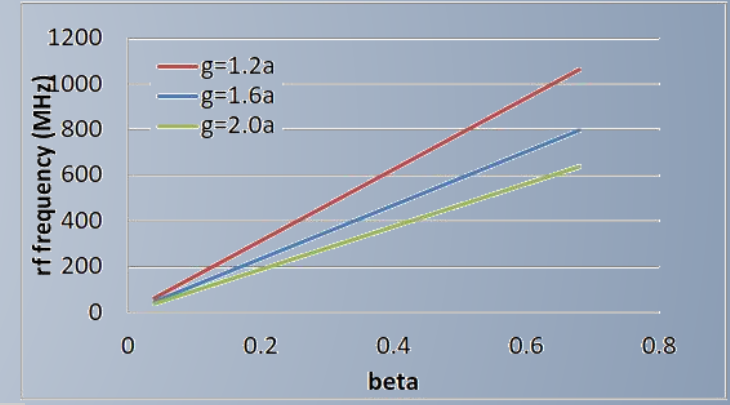
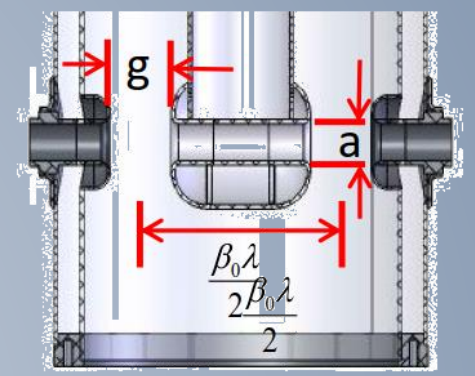




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# Coaxial resonator

- Consider a coaxial geometry with grounded end plates, an inner conductor with radius  $a$  and an outer conductor with radius  $b$ .
- A standing wave occurs with  $E_r$  vanishing on the end walls at  $z = 0$  and  $z = d$ .

- The remaining non-zero field components are

$$B_\theta = \frac{\mu_0 I_0}{\pi r} \cos\left(\frac{p\pi z}{d}\right),$$

$$E_r = -j2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0}{2\pi r} \sin\left(\frac{p\pi z}{d}\right),$$

where  $\omega = \frac{p\pi c}{d}$ ,  $p = 1, 2, 3, \dots$

- Peak voltage:

$$\widehat{V}(z) = \int_a^b E_r(z) dr = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I_0}{\pi} \ln \frac{b}{a} \sin\left(\frac{p\pi z}{d}\right)$$

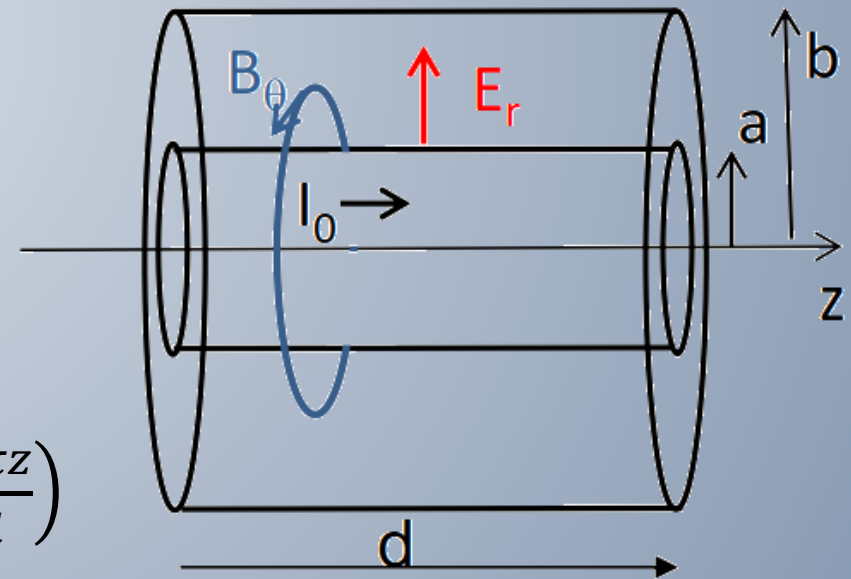






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# Quarter-wave resonator (QWR)

- The most popular coaxial TEM mode cavity is the quarter wave resonator – capacitively loaded  $\lambda/4$  transmission line
- The inner conductor is open at one end with a resonant length of  $(1 + 2p) \lambda/4$ ,  $p = 0,1,2, \dots$
- For acceleration,  $p = 0$  is chosen.
- The maximum voltage builds up on the open tip – the maximum current at the root.
- A beam tube is arranged near the end of the tip.

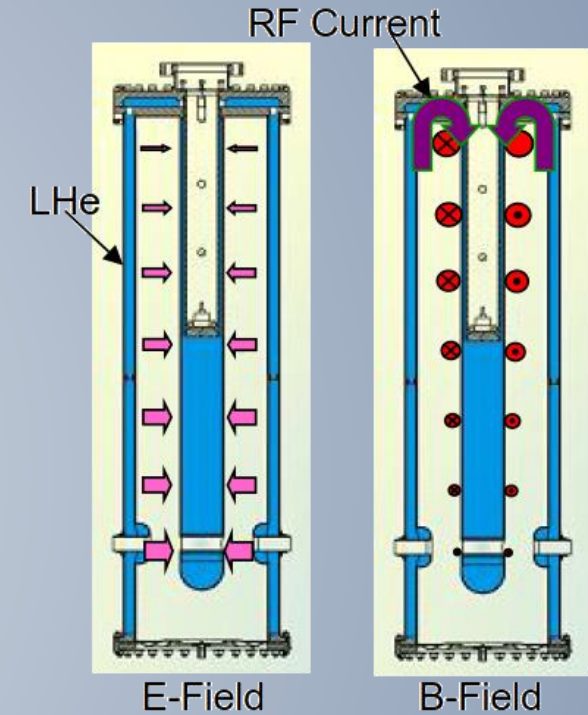




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# Half-wave resonator (HWR)

- In the HWR the beam port is at the centre of the inner conductor of a coaxial resonator, coincident with the maximum voltage for  $p = 1$ .
- Magnetic fields loop around the inner conductor with peak fields at the shorted ends.
- For acceleration,  $p = 1$  is chosen.

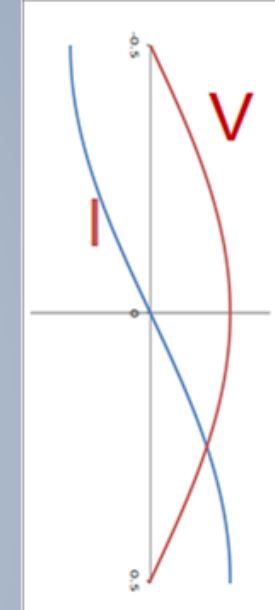
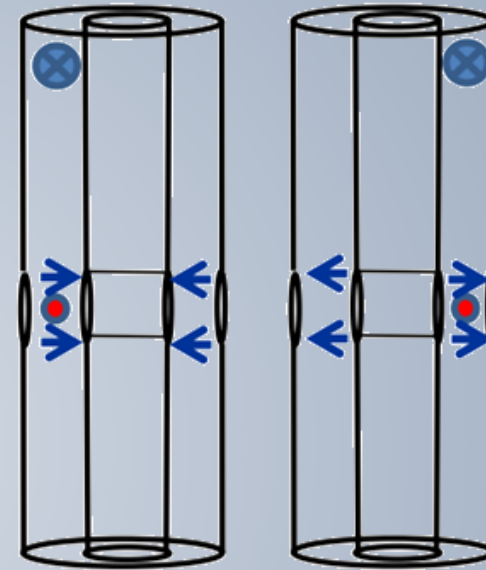






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# QWR vs. HWR

- QWR is the cavity of choice for low beta applications where a low frequency is needed
  - requires  $\sim 50\%$  less structure compared to HWR for the same frequency – rf power loss is  $\sim 50\%$  of HWR for same frequency and  $\beta_0$ .
  - allows low frequency choice giving larger longitudinal acceptance.
  - $R/Q$  twice that of HWR.
  - Asymmetric field pattern introduces vertical steering especially for light ions that increases with velocity – avoid use for  $\beta_0 > 0.2$ .
  - Less mechanically stable than HWR due to unsupported end (microphonics).
- HWR is chosen in mid velocity range ( $\beta_0 > 0.2$ ) or where steering must be eliminated (i.e. high intensity light ion applications)
  - produces twice rf losses for the same  $\beta_0$  and  $\lambda$ .
  - is 2x longer for the same frequency.
  - Pluses are the symmetric field pattern and increased mechanical rigidity.

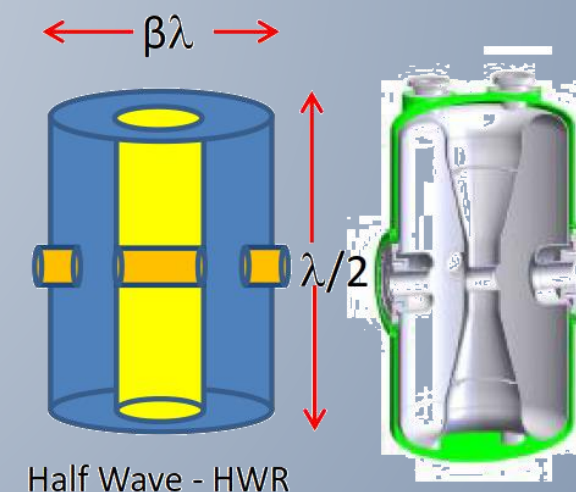
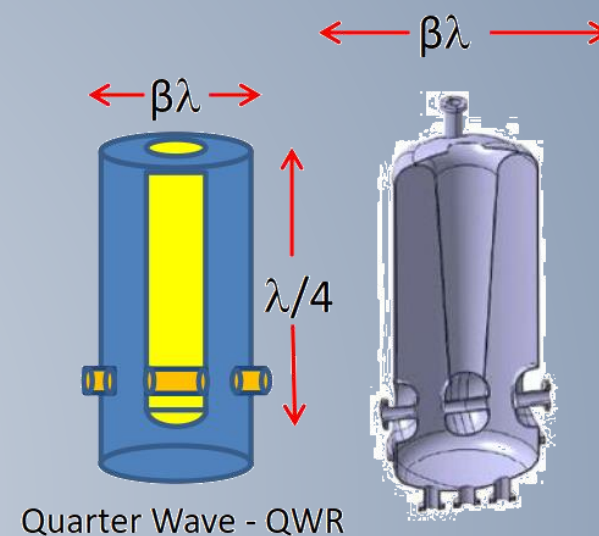
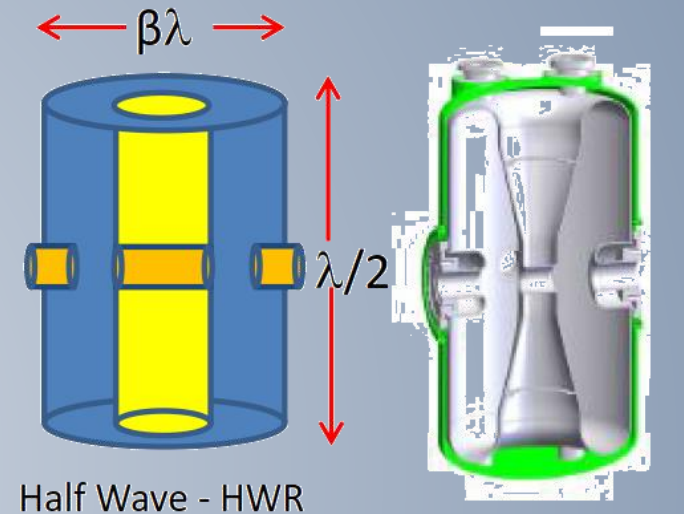




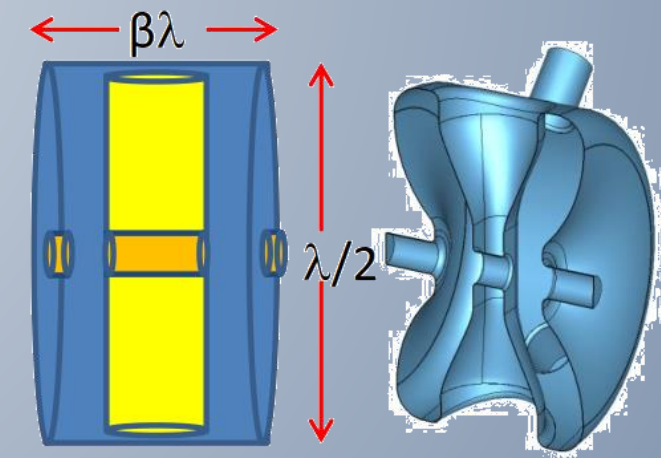
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# HWR vs. Single Spoke Resonator (SSR)

- A single spoke resonator (SSR) is another variant of the half-wave TEM mode cavity class.
- In HWR the outer conductor is coaxial with the inner conductor (with diameter  $\beta_0\lambda$ ) while in the spoke cavities the outer cylinder is co-axial with the beam tube with diameter  $\lambda/2$ . It means that for  $\beta_0 < 0.5$  the SSR has a larger overall physical envelop than the HWR for the same frequency.
- Thus for low beta applications ( $0.1 < \beta < 0.25$ ) HWRs are chosen at  $\approx 160$  MHz, while SSRs are preferred at  $\approx 320$  MHz.
- The spoke geometry allows an extension along the beam path to provide multiple spokes in a single resonator giving higher effective voltage, but with a narrower transit time acceptance.



Half Wave - HWR



Single spoke - SSR

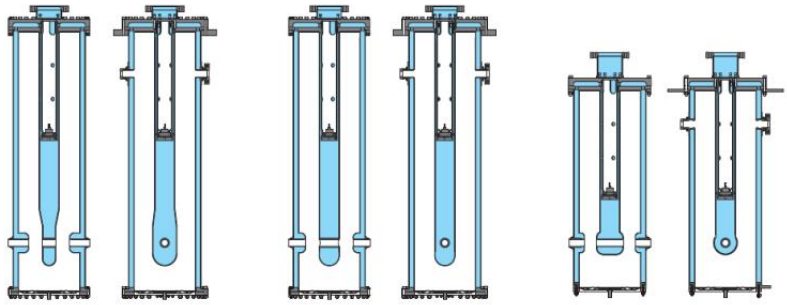




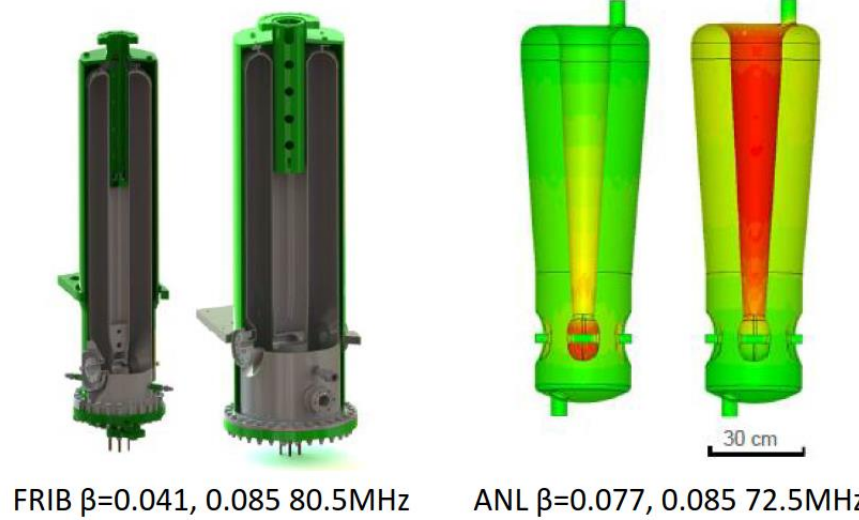
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# Cavity types – QWRs

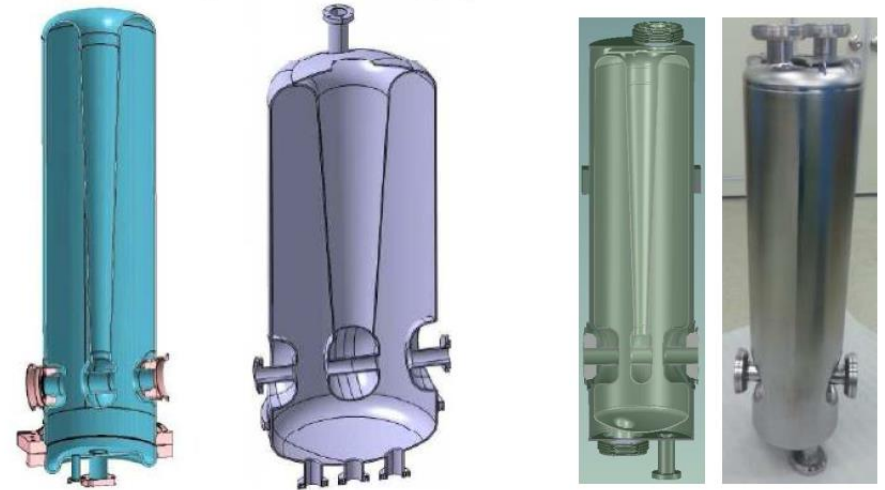
## TRIUMF ISAC-II Resonators



SCB low  $\beta$  (5.7%) 106.08 MHz  
 SCB medium  $\beta$  (7.1%) 106.08 MHz  
 SCC high  $\beta$  (11%) 141.44 MHz



FRIB  $\beta=0.041, 0.085$  80.5MHz  
 ANL  $\beta=0.077, 0.085$  72.5MHz



Spiral-2  $\beta=0.007, 0.12$  88.05MHz  
 RAON  $\beta=0.047, 81.25$  MHz

Typical range:  
 $0.04 < \beta < 0.2$   
 $50 \text{ MHz} \leq f \leq 160 \text{ MHz}$

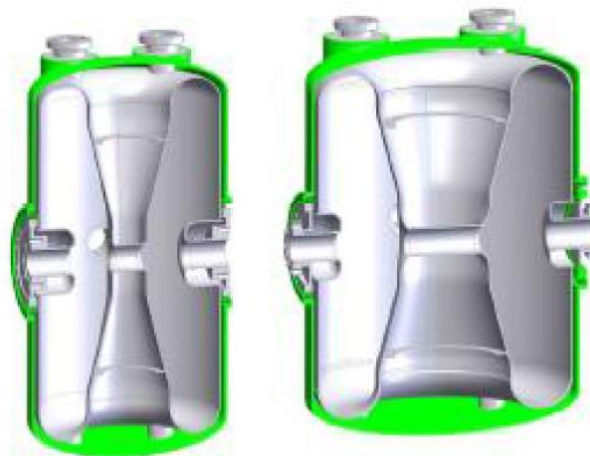


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# Cavity types – HWRs



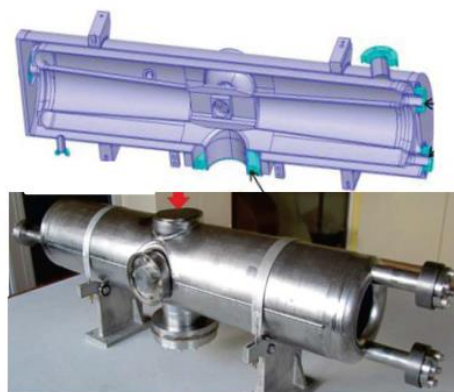
IMP  $\beta=0.10$ ,  $f=162.5\text{MHz}$



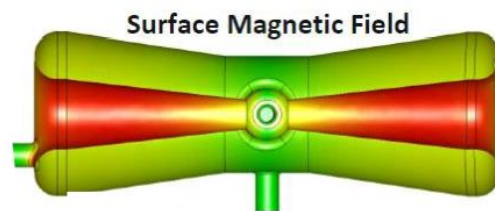
FRIB  $\beta=0.29, 0.53$   $f=322\text{MHz}$



FRIB  $\beta=0.29, 0.53$   $f=322\text{MHz}$



IFMIF  $\beta=0.11$ ,  $f=175\text{MHz}$



ANL  $\beta=0.112$ ,  $f=162.5\text{MHz}$

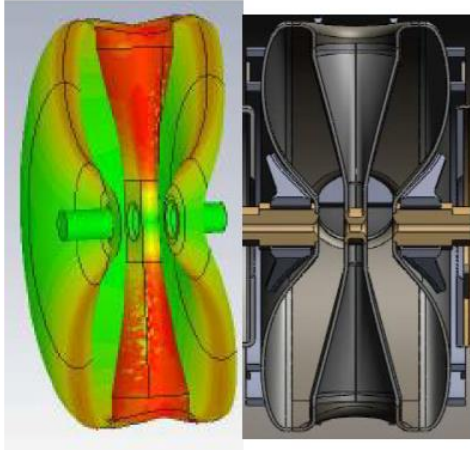
Typical range:  
 $0.1 < \beta < 0.5$   
 $140 \text{ MHz} \leq f \leq 325 \text{ MHz}$





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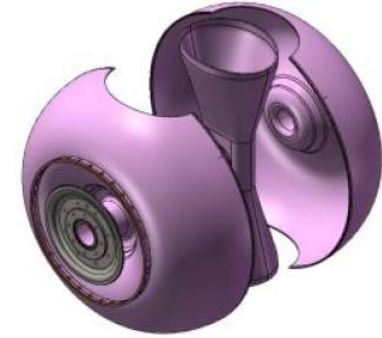
# Cavity types – SSRs



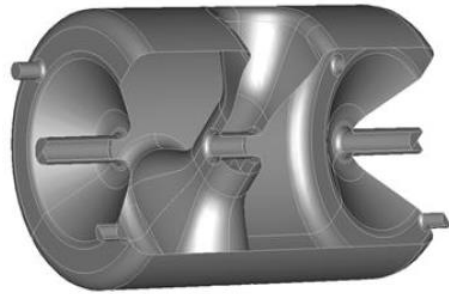
IHEP  $\beta=0.12$ ,  $f=325\text{MHz}$



FNAL  $\beta=0.215$ ,  $f=325\text{MHz}$



TRIUMF/RISP  $\beta=0.3$ ,  $f=325\text{MHz}$



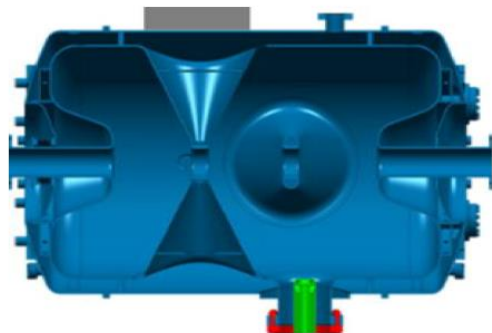
325 MHz,  $\beta_0 = 0.82$   
Single-Spoke Cavity



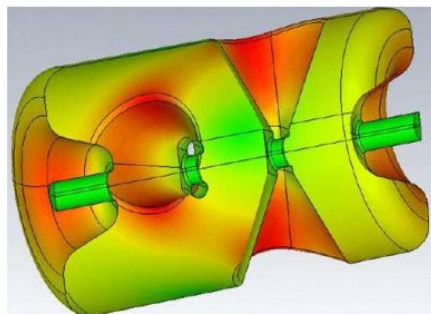
Typical range:  
 $0.15 < \beta < 0.7$   
 $320 \text{ MHz} \leq f \leq 700 \text{ MHz}$



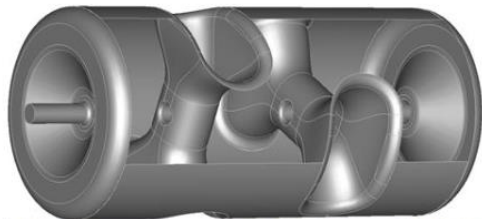
# Cavity types – multi-cell



ESS/IPN  $\beta=0.50$ ,  $f=352\text{MHz}$



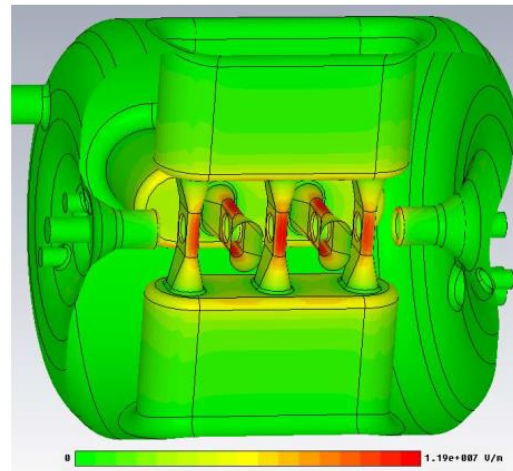
IAP 360 MHz,  $\beta_0 \sim 0.1$   
19 gap CH resonator



500 MHz,  $\beta_0 = 1$   
Double-Spoke Cavity



ANL  $\beta=0.63$ ,  $f=345\text{MHz}$



IMP CH  $\beta=0.067$ ,  $f=162.5\text{MHz}$

CH: Crossbar H-mode





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# Accelerating cavity velocity/frequency chart

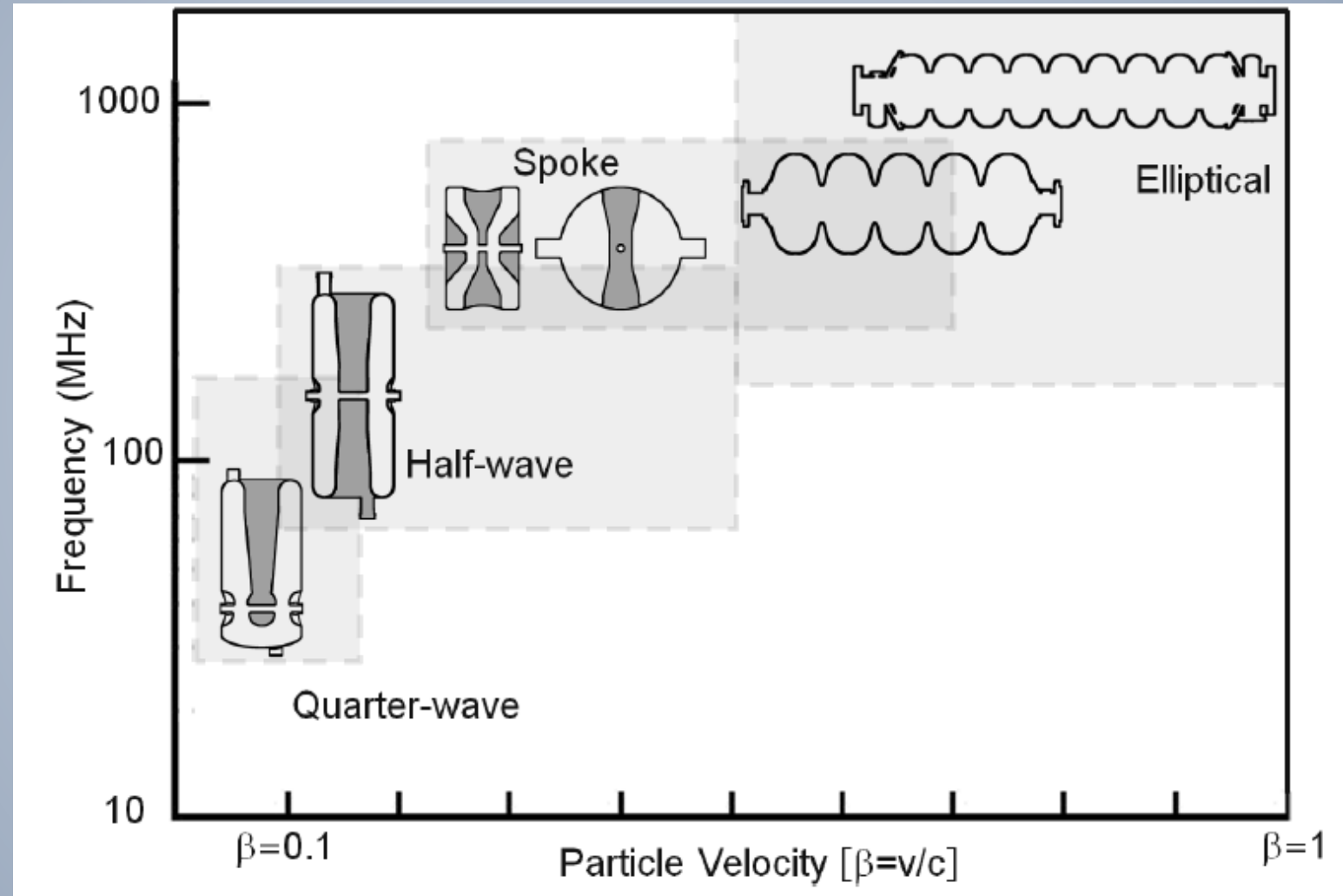
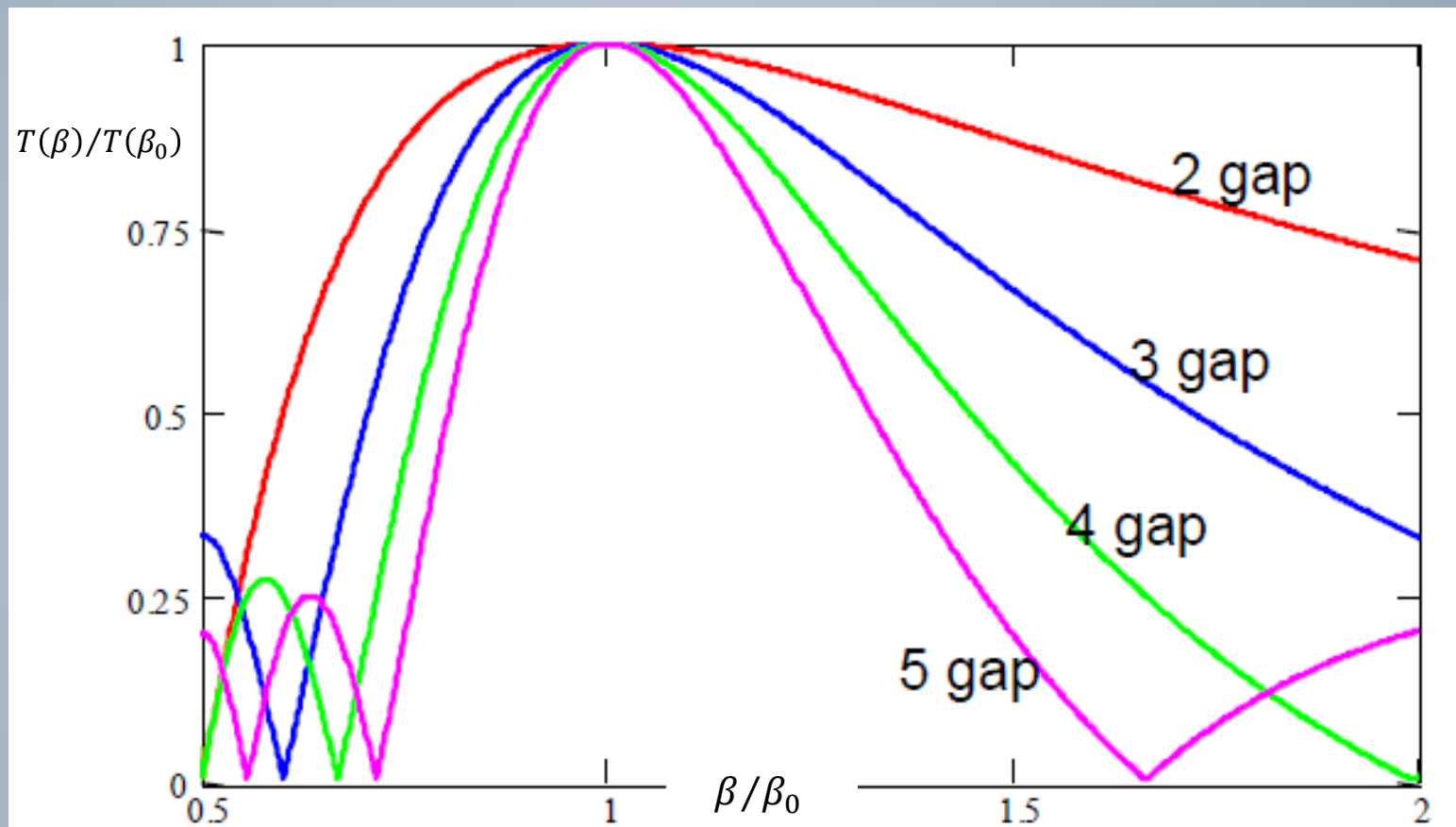




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# Transit Time factor vs. $\beta$ for multiple gaps



*Normalized transit time factor curves vs. normalized velocity, for cavities with different number of gap*





# High- $\beta$ spoke cavities

- High velocity spoke cavities with  $\beta > 0.8$  are being designed as alternative to elliptical cavities
- Features:
  - relatively compact
    - between 20% and 50% smaller (radially) for low- $\beta$  cavities
    - for high  $\beta$  diameter close to TM counterparts
  - allows low frequency at reasonable size
  - mechanically stable – high shunt impedance



325 MHz  $\beta=0.82$  Single Spoke Cavity

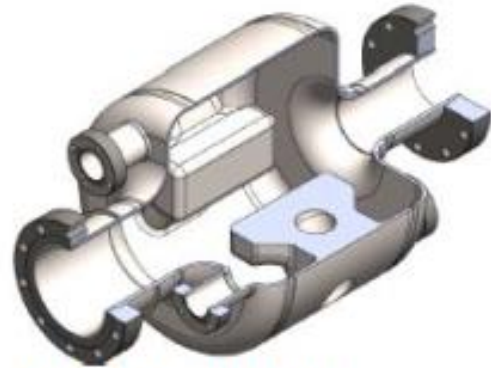


500 MHz  $\beta=1.0$  Double Spoke Cavity



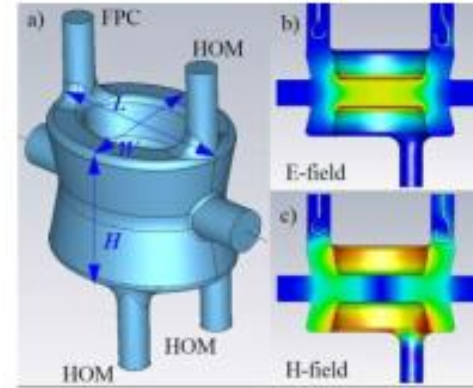
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# Deflecting mode cavities

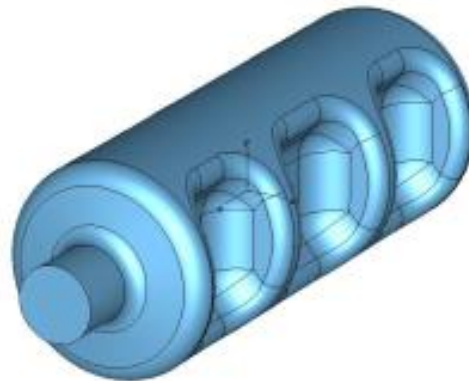


TRIUMF 650MHz

double quarter wave (DQW) – 400MHz – BNL/CERN



RFD – multi-cell – 953MHz – ODU



RF Dipole (RFD) – 400MHz – ODU/CERN

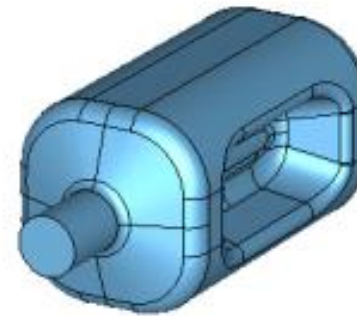


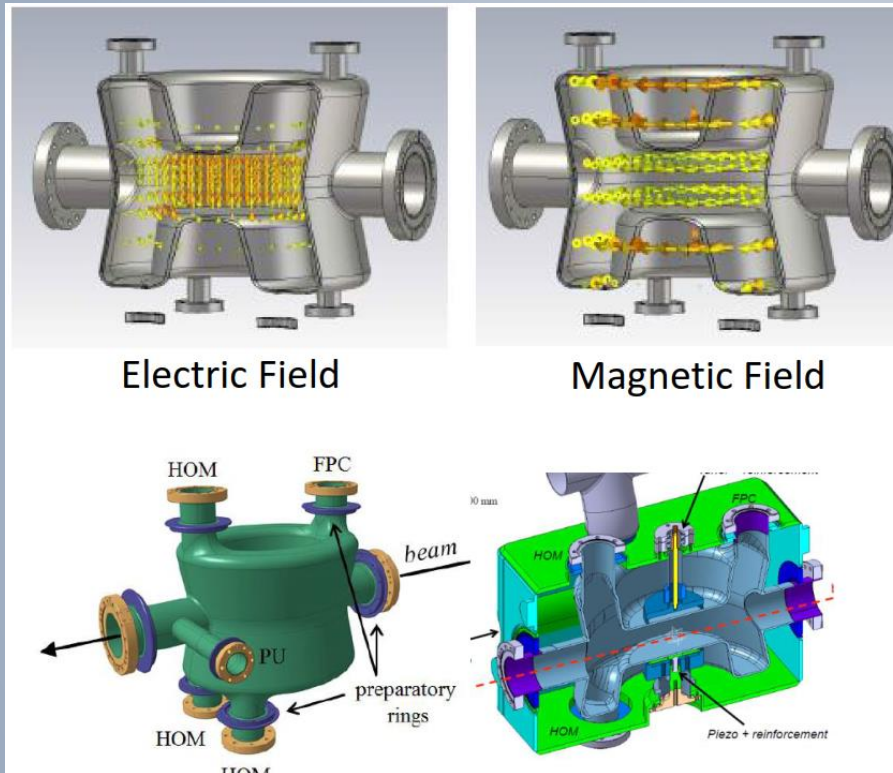




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# Prototype HL-LHC Crab Cavity prototypes

“DQW” (vertical deflection)



“RFD” (vertical deflection)

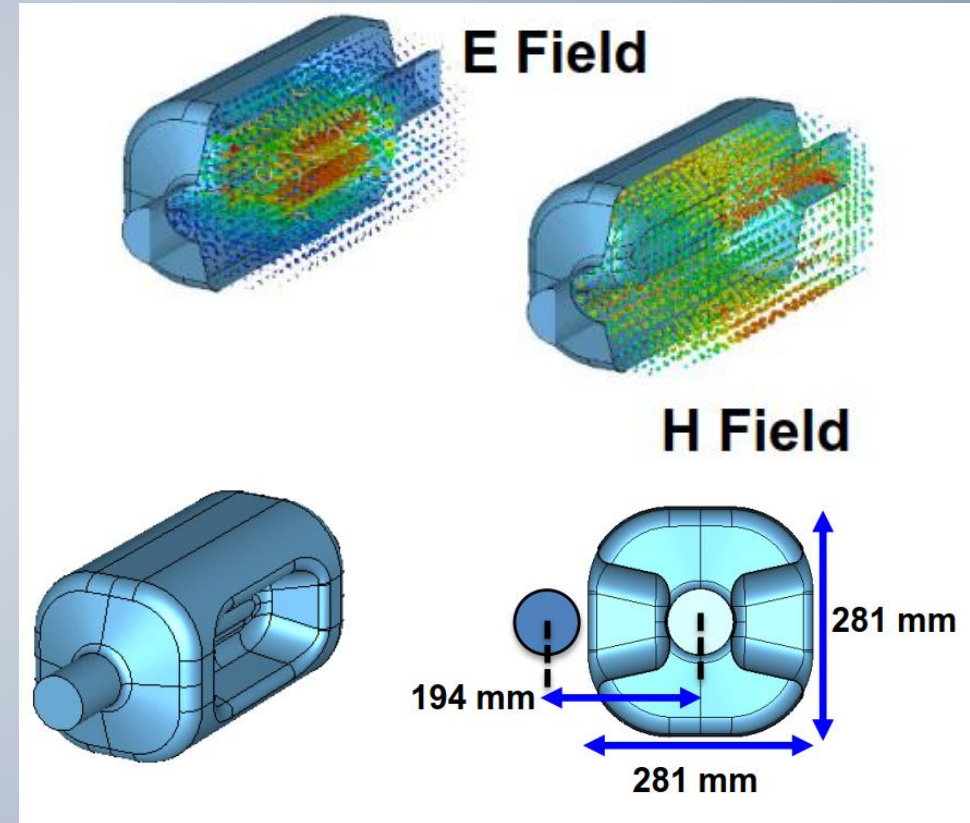
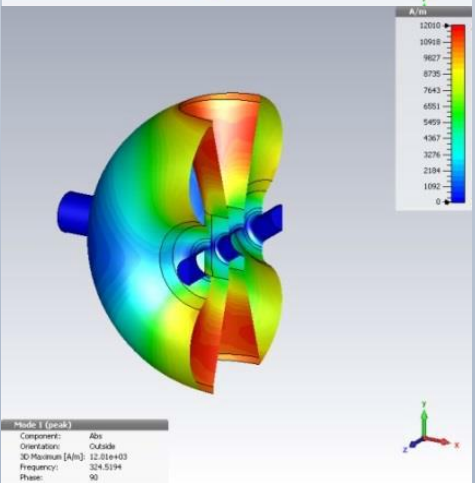
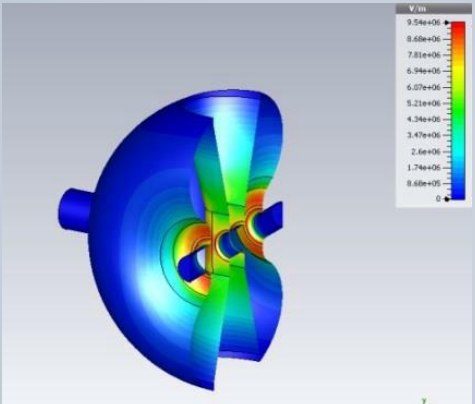
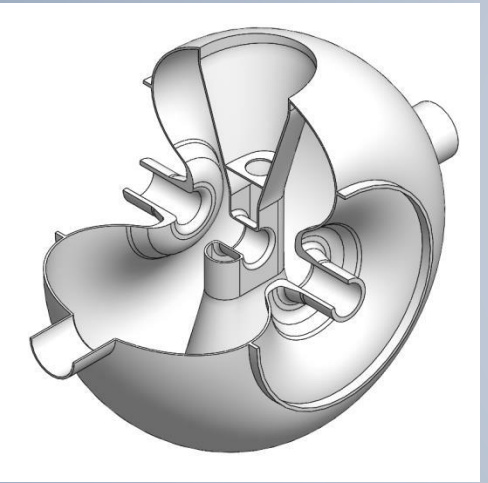




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# Example: TRIUMF/RISP 325 MHz SSR for $\beta = 0.3$ (balloon cavity)

- Geometry optimized to minimize peak field ratios ( $E_p/E_{acc}$ ,  $B_{peak}/E_{acc}$ ). Achieved:  $E_{peak} = 35 \text{ MV/m}$  and  $B_{peak} = 55.3 \text{ mT}$  for 2.5 MV.



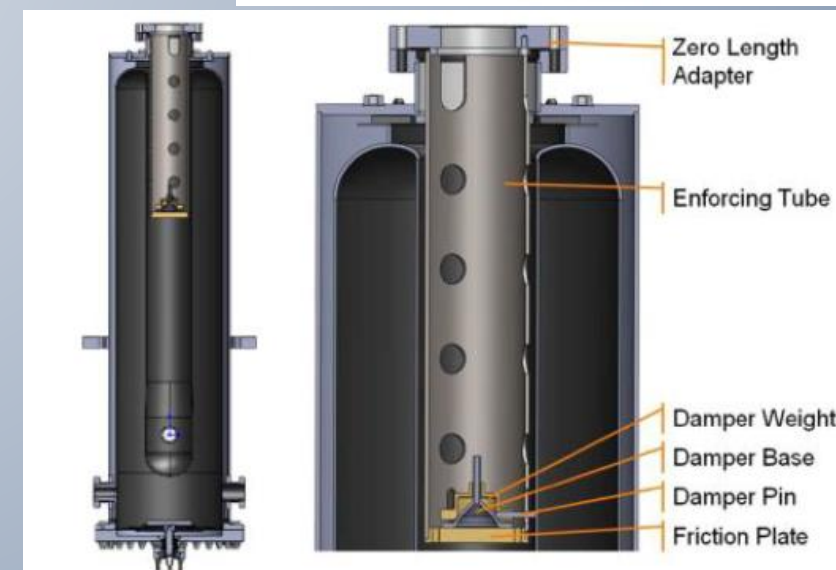
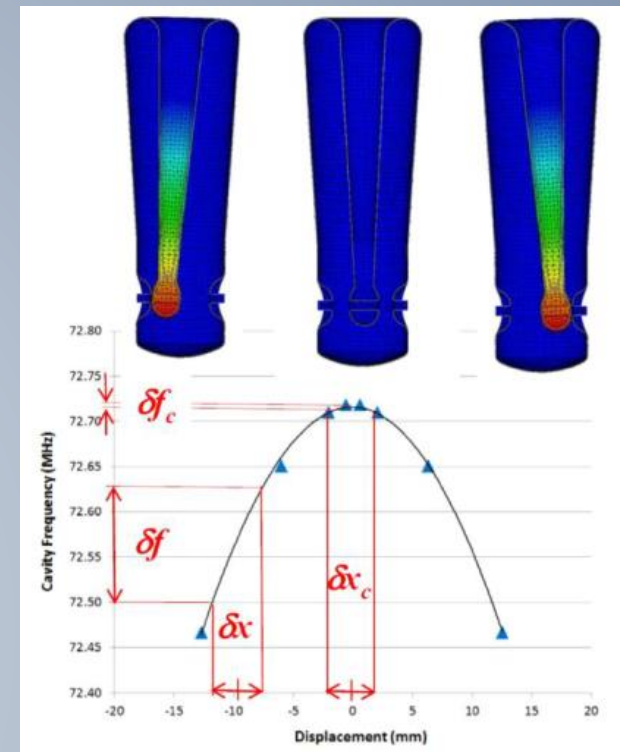
| Parameters               | Values | Units     |
|--------------------------|--------|-----------|
| RF Frequency             | 325    | MHz       |
| $E_{peak}/E_{acc}$       | 3.84   |           |
| $B_{peak}/E_{acc}$       | 6.07   | mT/(Mv/m) |
| G                        | 93     | $\Omega$  |
| R/Q                      | 233    | $\Omega$  |
| $L_{eff} (\beta\lambda)$ | 0.277  | m         |





# Microphonics

- Driven by mechanical vibration in the environment.
- QWRs are particularly problematic due to the pendulum action of the inner conductor, which can have very low mechanical frequencies ((50 ... 100) Hz)
  - need to reduce the RMS detuning to  $\ll 10\%$  of the available BW to avoid nuisance
  - the other option is to increase the BW (lower  $Q_L$ , costs power)
- Mitigation:
  - stiffening during design/manufacture
  - centering the inner conductor by plastic deformation so that  $df/dx = 0$ .
  - adding passive dampers
  - reduce environmental noise

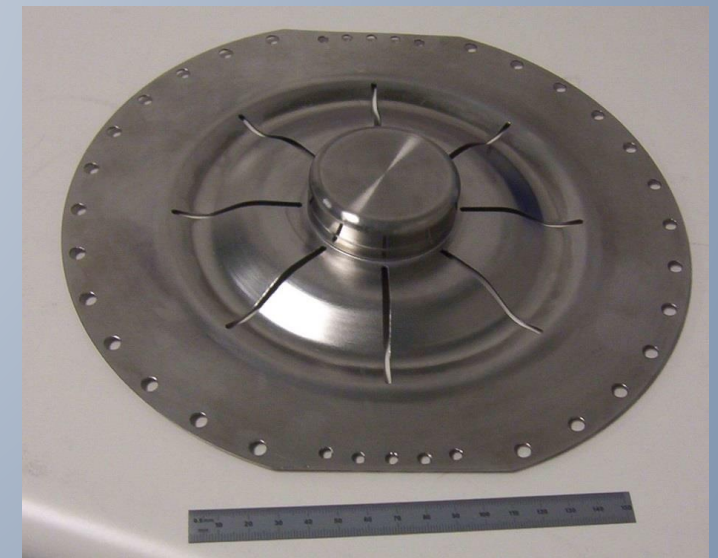
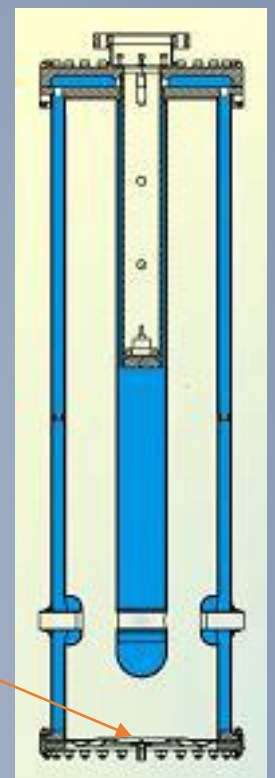




# Frequency compensation/tuning

- For QWRs a tuning plate at the open end near the beam tube is generally used.
- For QWRs with removable tuning plate, a Nb puck can be welded to it – this reduces the cavity  $f_0$  by increasing the equivalent  $C$ .
- This puck can be trimmed after final fabrication

tuning plate







# End of Non-elliptical cavities

Thank you very much!