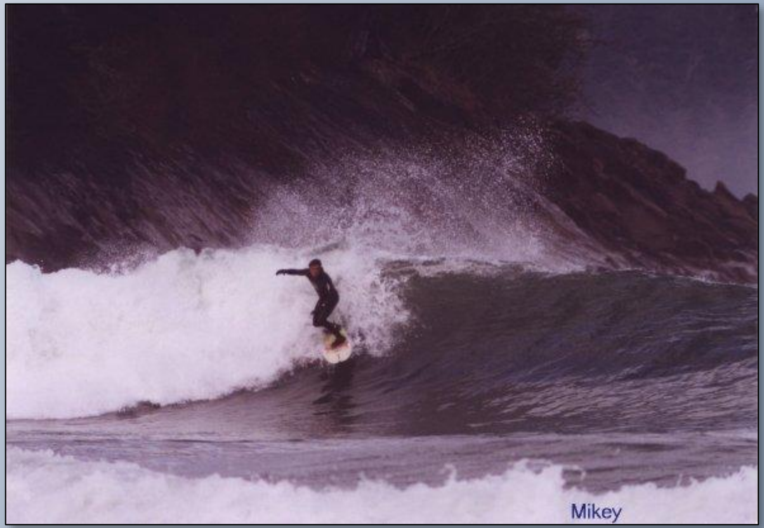
RF Basics and TM Cavities

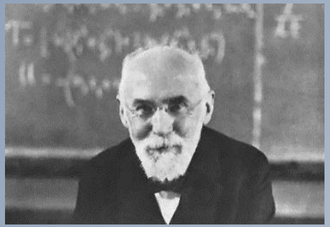
Erk JENSEN, CERN

DC versus RF

DC accelerator

RF accelerator





Hendrik A. Lorentz
1853 – 1928

Lorentz force

- A charged particle moving with velocity $\vec{v} = \frac{\vec{p}}{m\,\gamma}$ through an electromagnetic field in vacuum experiences the Lorentz force $\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$.
- The total energy of this particle is $W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma\, mc^2$, the kinetic energy is $W_{kin} = mc^2(\gamma - 1)$.
- The role of acceleration is to increase W.
- Change of W (by differentiation):

$$W dW = c^2 \vec{p} \cdot d\vec{p} = q c^2 \vec{p} \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right) dt = q c^2 \vec{p} \cdot \vec{E} dt$$

$$dW = q \vec{v} \cdot \vec{E} dt$$

Note: **Only the electric field can change the particle energy!**

James Clerk Maxwell
1831 – 1879

Maxwell's equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = \mu_0 \vec{J} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$

1. Why not DC?

DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$, which is solved by $\vec{E} = -\nabla\Phi$

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2. Circular machine: DC acceleration impossible since $\oint \vec{E} \cdot \mathrm{d}\vec{s} = 0$

With time-varying fields:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}, \quad \oint \vec{E} \cdot \mathrm{d}\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{A}.$$


Photo:
Reidar Hahn

Maxwell's equations in vacuum (continued)

Source-free:

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

curl (rot, $\nabla \times$) of 3rd equation and $\frac{\partial}{\partial t}$ of 1st equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0.$$

Using the vector identity $\nabla \times \nabla \times \vec{E} = \nabla\nabla \cdot \vec{E} - \nabla^2\vec{E}$ and the 4th Maxwell equation, this yields:

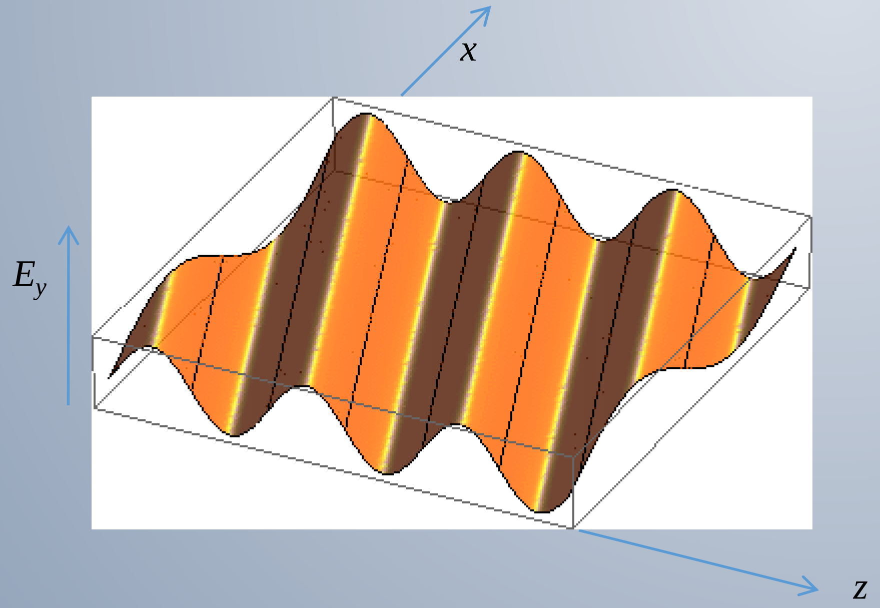
$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0,$$

i.e. the 4-dimensional Laplace equation.

From waveguide to cavity

Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}(z\cos\varphi + x\sin\varphi)$$



Wave vector $\vec{k}$:

the direction of $\vec{k}$ is the direction of propagation,

the length of $\vec{k}$ is the phase shift per unit length.

$\vec{k}$ behaves like a vector.

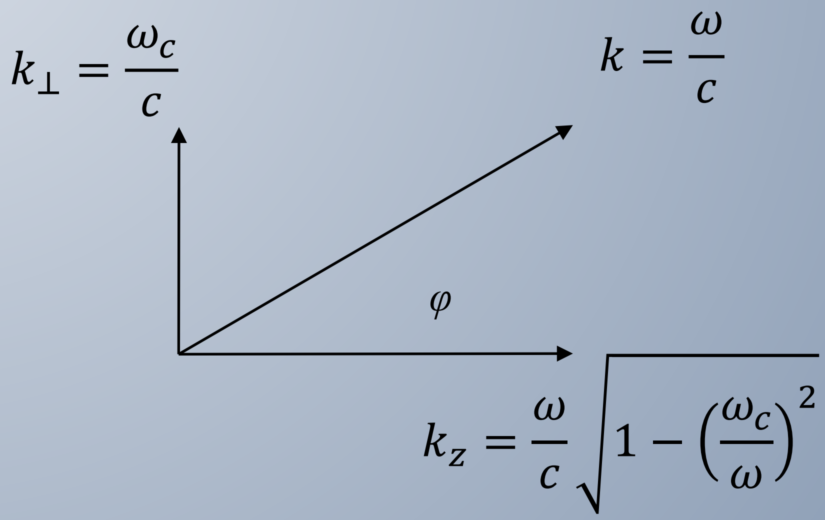
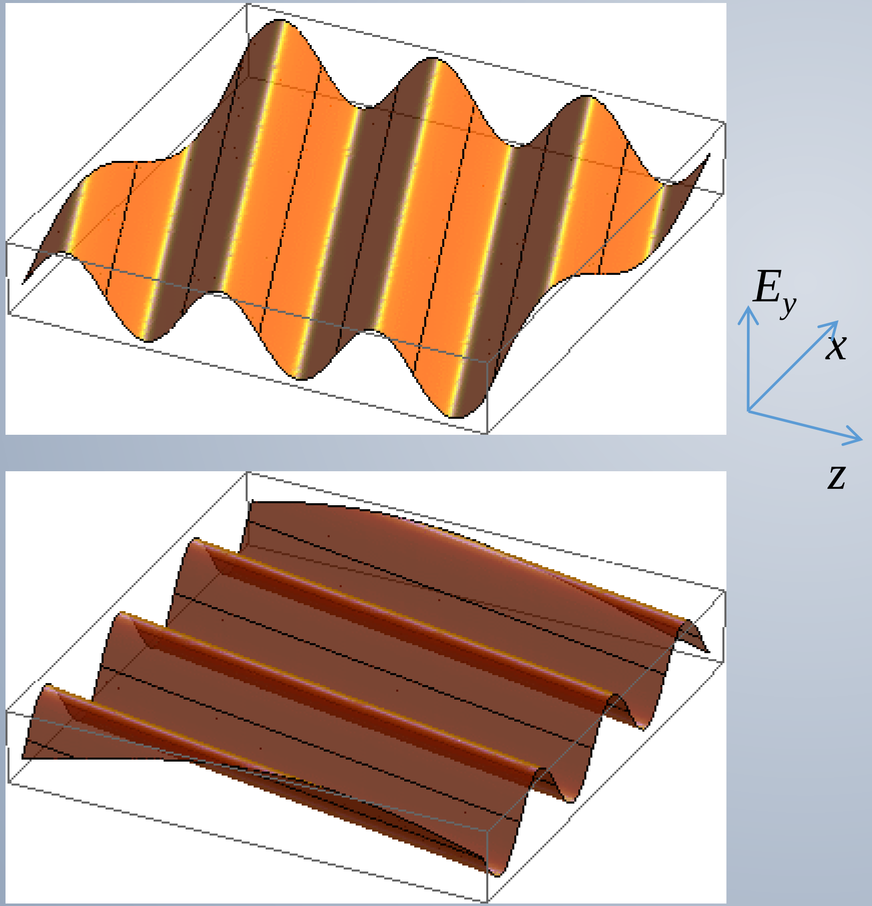
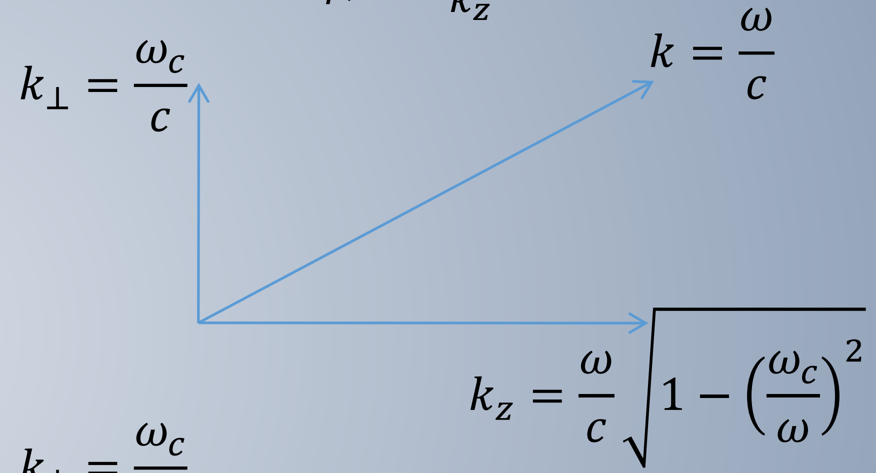


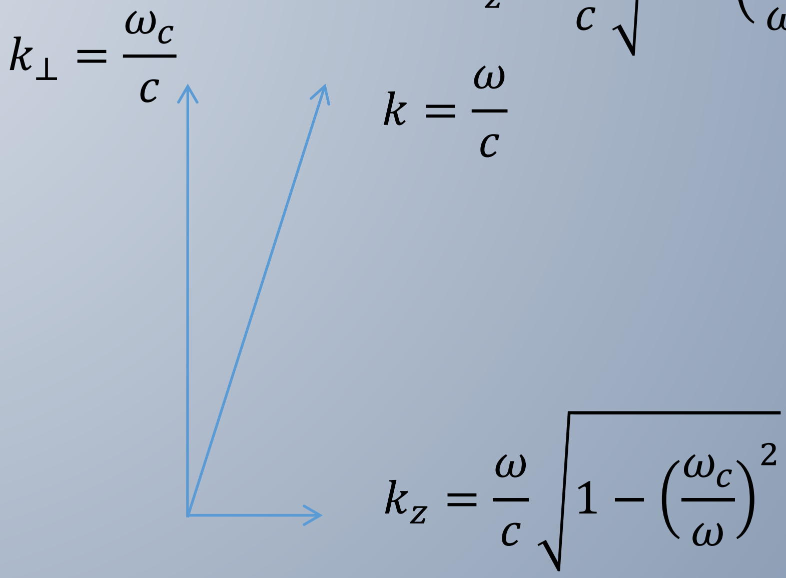
Photo: Reidar Hahn

Wave length, phase velocity

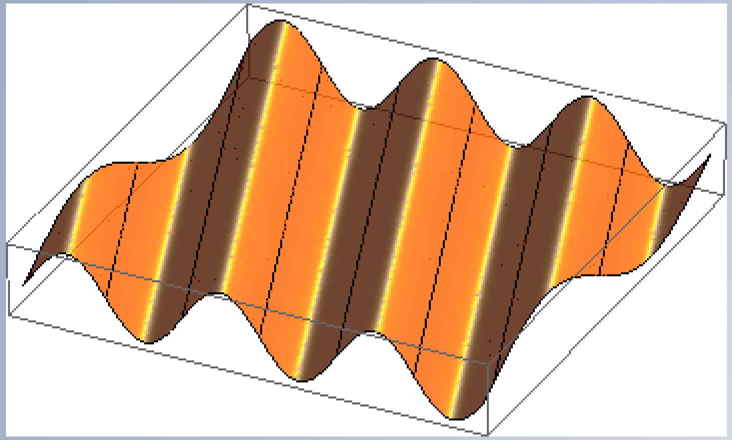
- The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\lambda_z$.

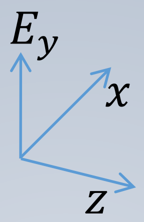






Superposition of 2 homogeneous plane waves





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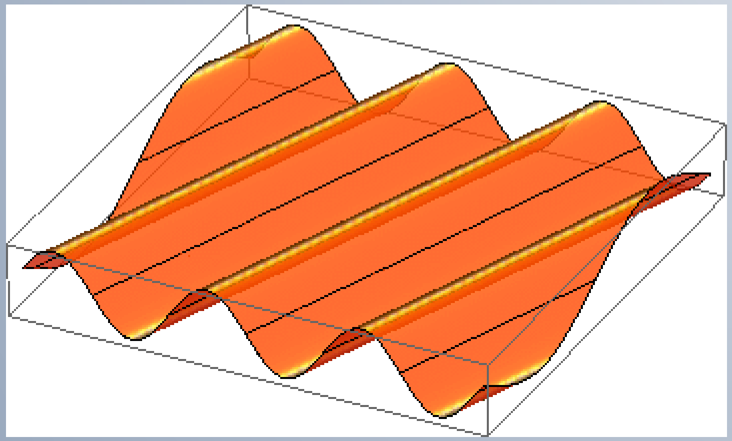

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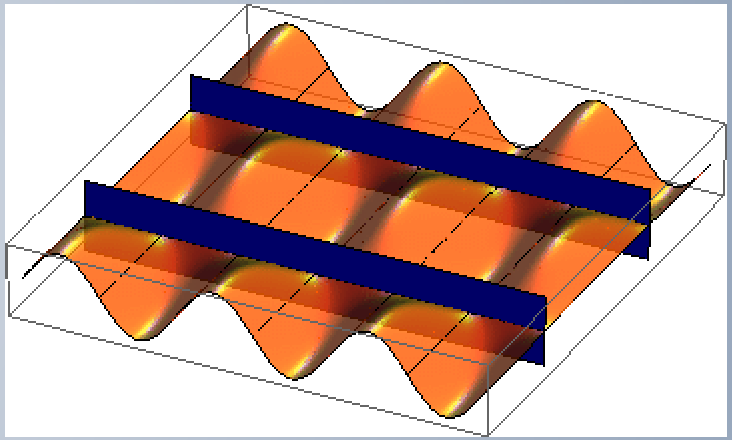


Metallic walls may be inserted where $E_y \equiv 0$

without perturbing the fields.

Note the standing wave in x-direction!

This way one gets a hollow rectangular waveguide.

Rectangular waveguide

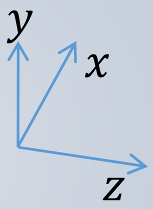
Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

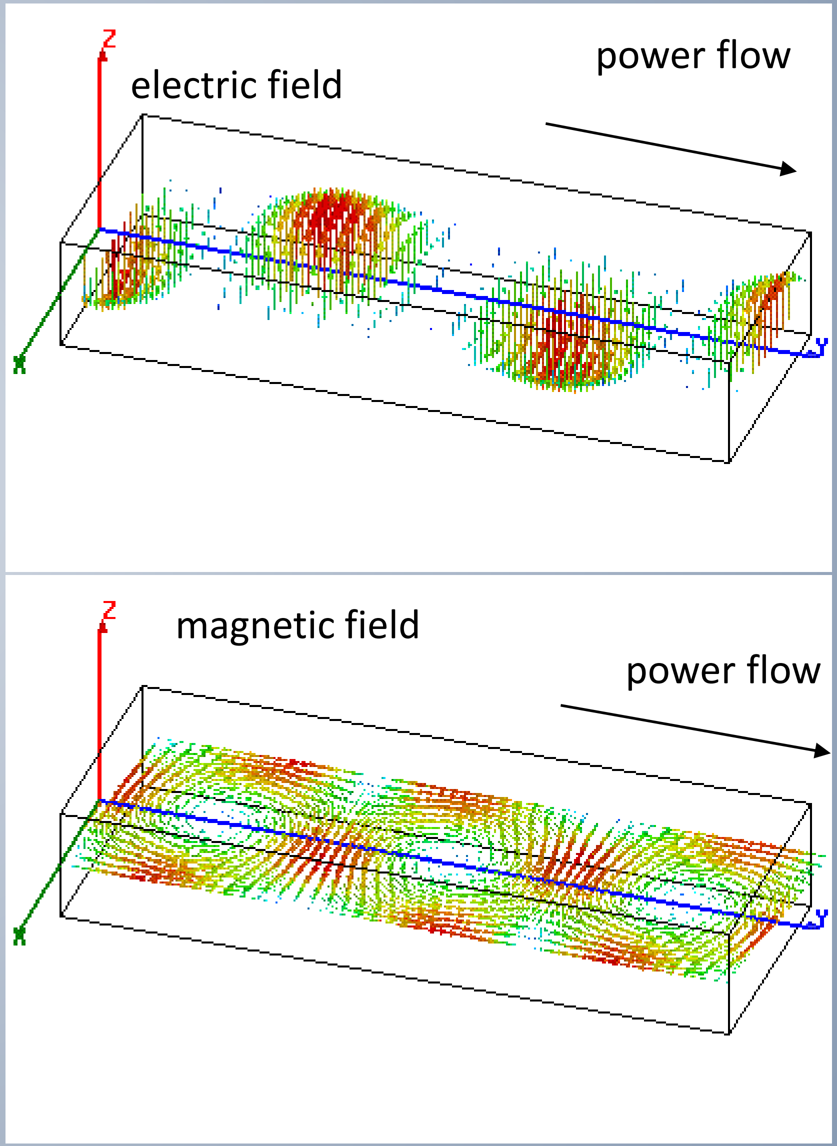
Example 1: "S-band": 2.6 GHz ... 3.95 GHz,

Waveguide type WR284 (2.84" wide), dimensions:
72.14 mm x 34.04 mm.
cut-off: $f_c = 2.078$ GHz.

Example 2: "L-band" : 1.13 GHz ... 1.73 GHz,

Waveguide type WR650 (6.5" wide), dimensions:
165.1 mm x 82.55 mm.
cut-off: $f_c = 0.908$ GHz.



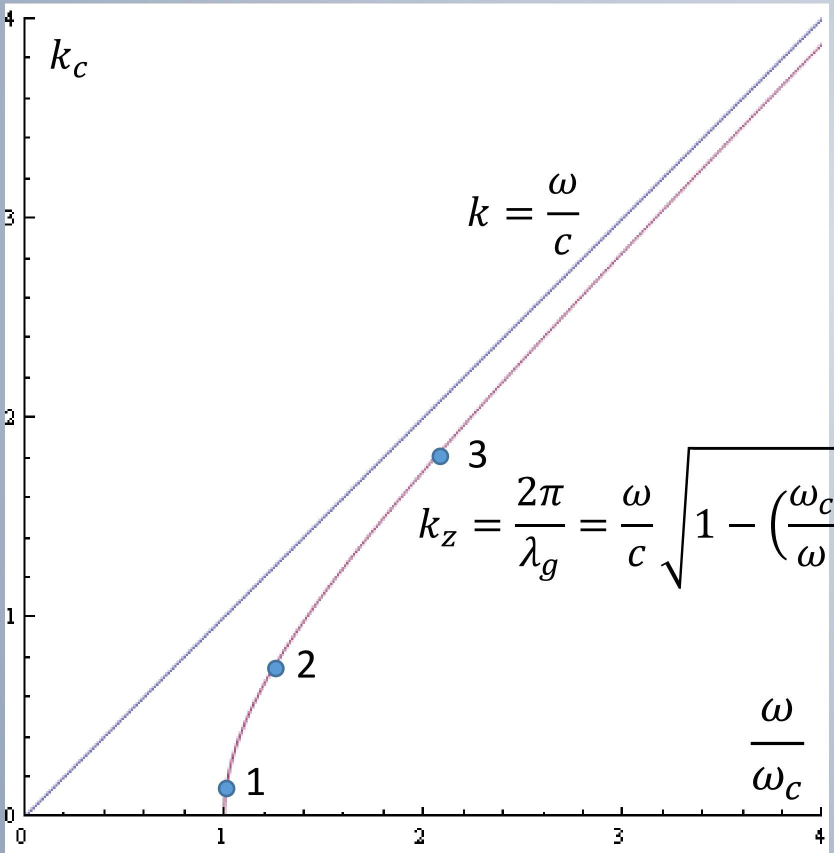


Both these pictures correspond to operation at 1.5 f_c.

power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint \vec{E} \times \vec{H}^* \cdot \mathrm{d}\vec{A}\right\}$

Waveguide dispersion

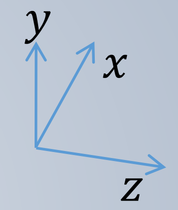
What happens with different waveguide dimensions (different width a)?



$$k_z = \frac{2\pi}{\lambda_g} = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

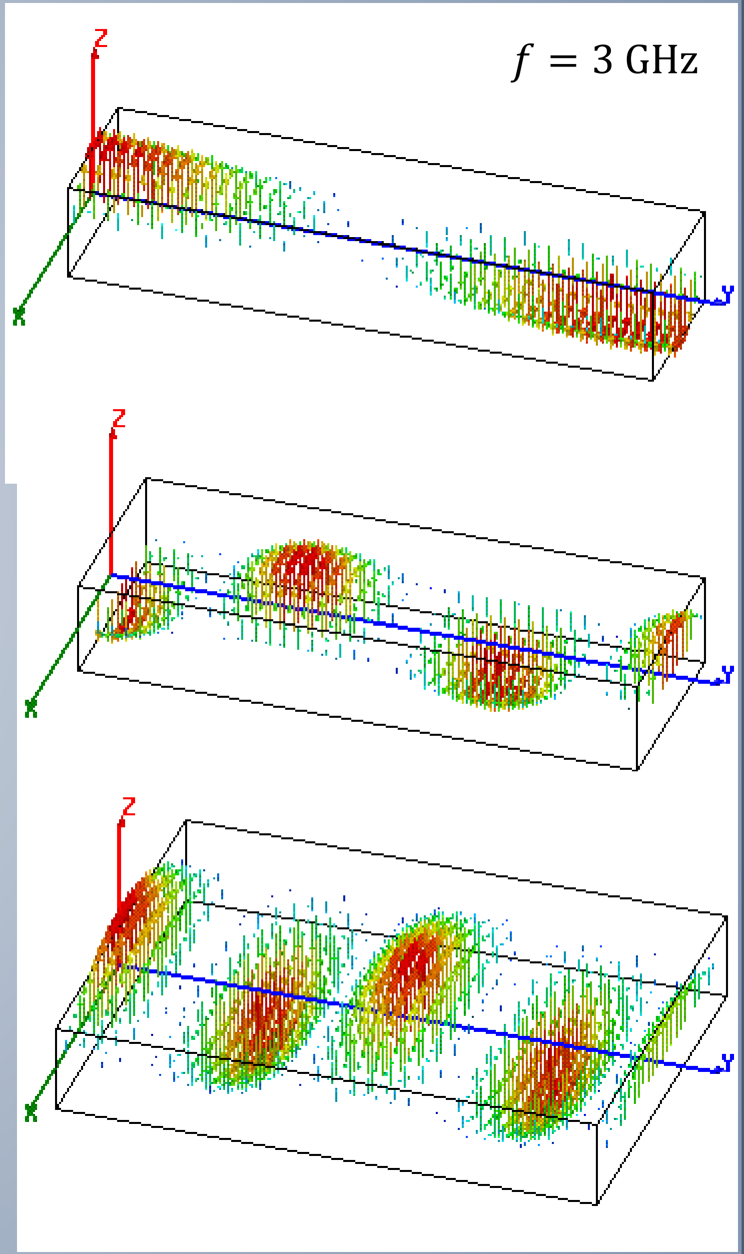
cutoff: $f_c = \frac{c}{2a}$

1:
a = 52 mm,
$f/f_c = 1.04$



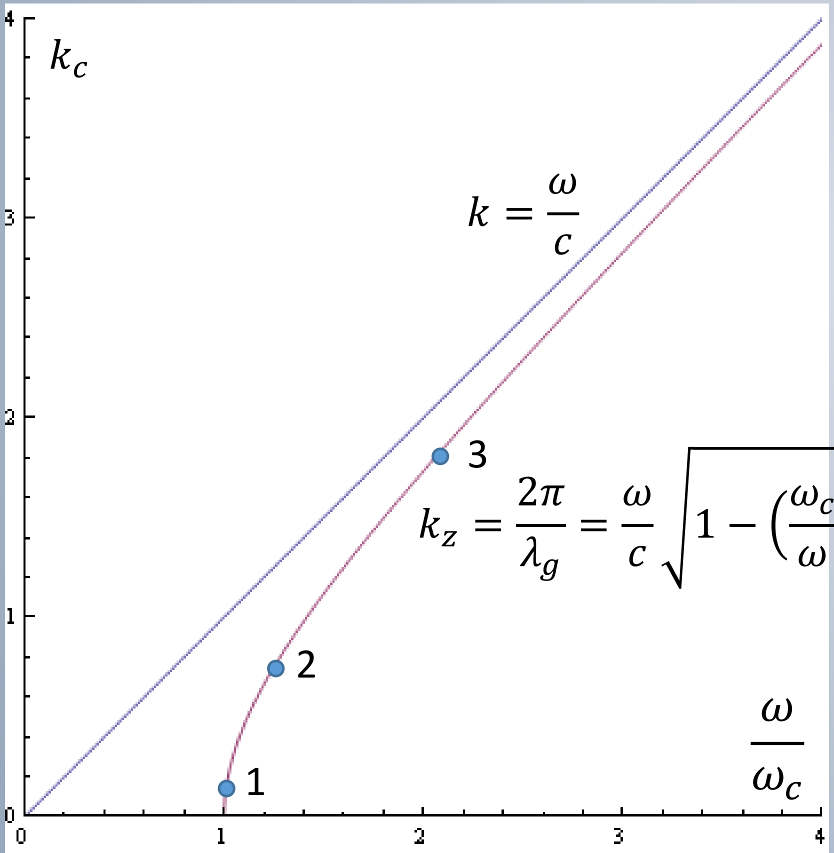
2:
a = 72.14 mm,
$f/f_c = 1.44$

3:
a = 144.3 mm,
$f/f_c = 2.88$



Phase velocity $v_{\varphi,z}$

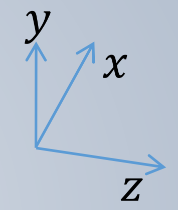
The phase velocity $v_{\varphi,z}$ is the speed at which the crest (or zero-crossing) travels in z-direction.
Note on the 3 animations on the right that, at constant f, $v_{\varphi,z} \propto \lambda_g$. Note also that at $f = f_c$, $v_{\varphi,z} = \infty$!
With $v \to \infty$, $v_{\varphi,z} \to c$!



$$k_z = \frac{2\pi}{\lambda_g} = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \frac{\omega}{v_{\varphi,z}}$$

cutoff: $f_c = \frac{c}{2a}$

1:
a = 52 mm,
$f/f_c = 1.04$



2:
a = 72.14 mm,
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3:
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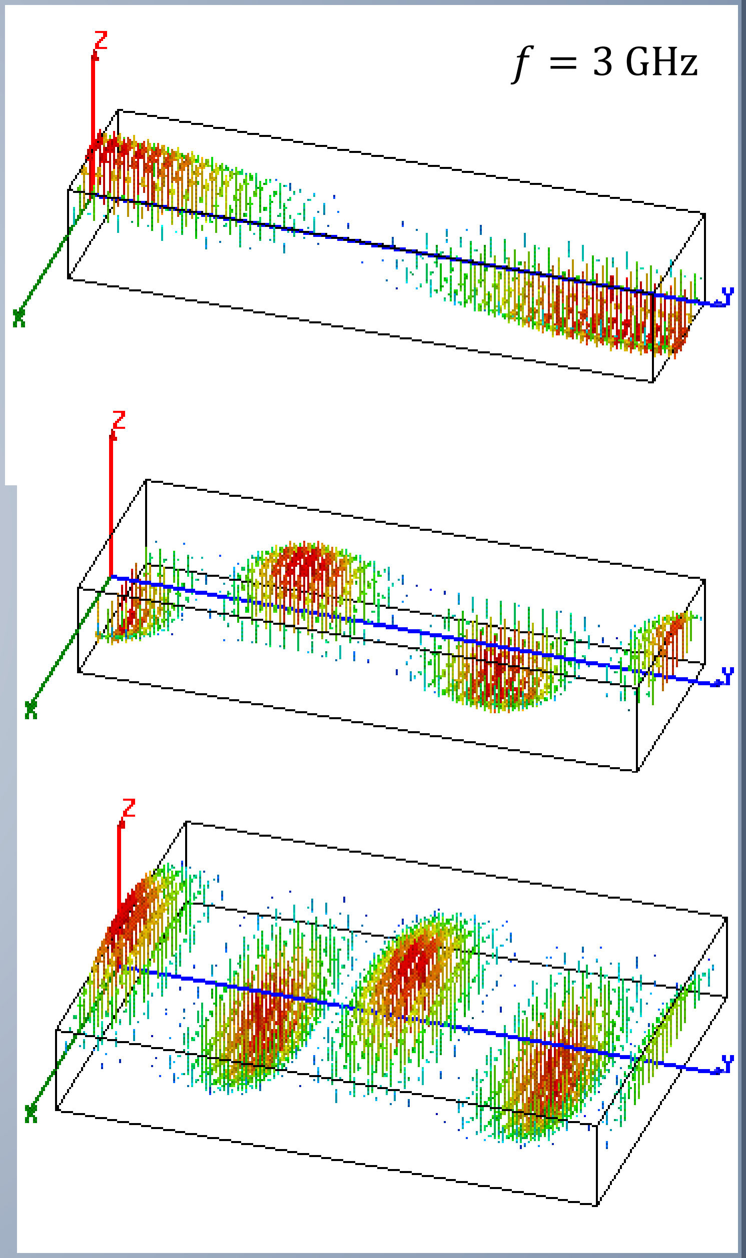



Photo: Reidar Hahn

Summary waveguide dispersion and phase velocity:

In a **general** cylindrical waveguide:

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_\perp^2} = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

Propagation in z-direction: $\propto e^{j(\omega t - k_z z)}$

$Z_0 = \frac{\omega\mu}{k_z}$ for TE, $Z_0 = \frac{k_z}{\omega\varepsilon}$ for TE

$$k_z = \frac{2\pi}{\lambda_g}$$

Example: TE10-mode in a rectangular waveguide of width a:

$$k_\perp = \frac{\pi}{a}$$

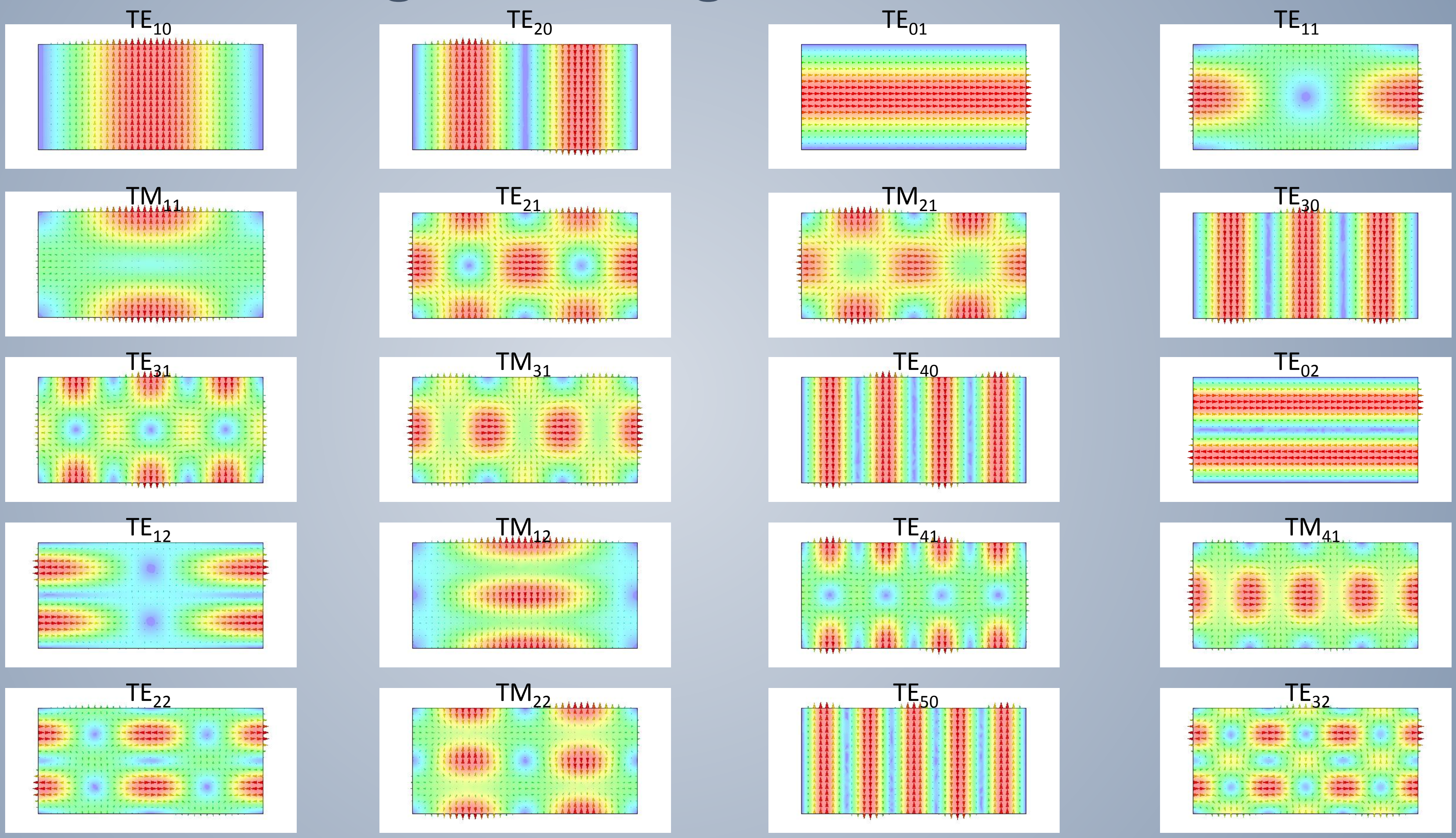
$$\gamma = \mathrm{j}\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{\omega\mu}{k_z}$$

$$\lambda_{\text{cutoff}} = 2a.$$

In a hollow waveguide: phase velocity $v_\varphi > c$, group velocity $v_{gr} < c$, $v_{gr} \cdot v_\varphi = c^2$.

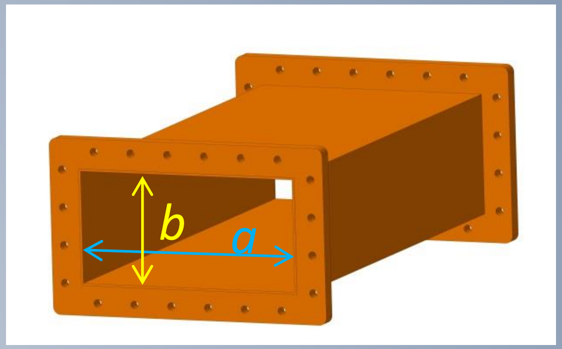
Rectangular waveguide modes

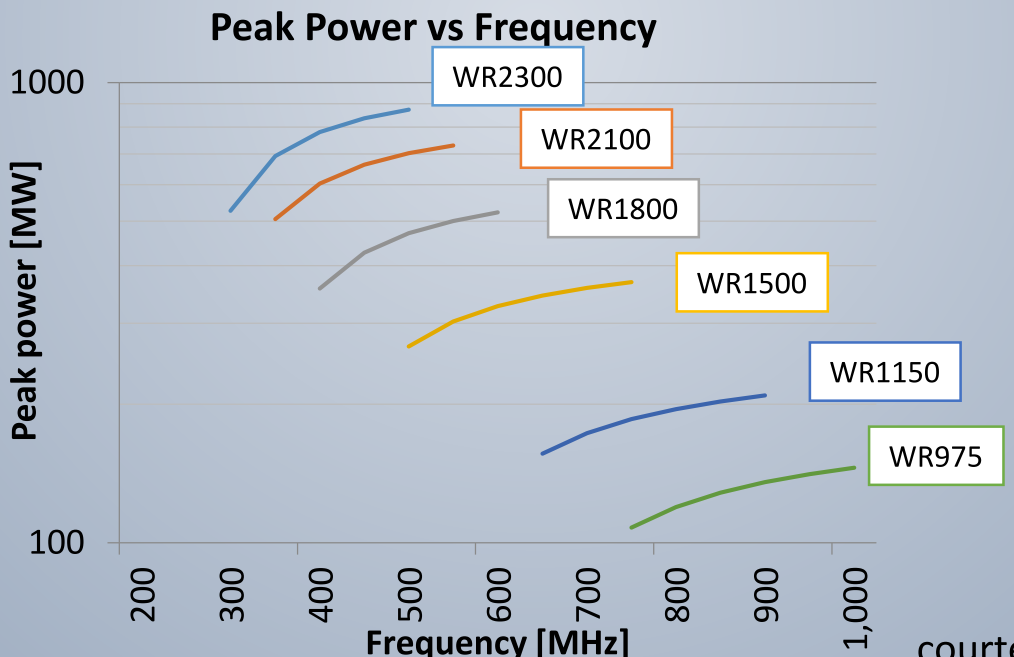


plotted: *E*-field

Some more standard rectangular Waveguides

Waveguide name			Recommended frequency band of operation (GHz)	Cutoff frequency of lowest order mode (GHz)	Cutoff frequency of next mode (GHz)	Inner dimensions of waveguide opening (inch)
EIA	RCSC	IEC				
WR2300	WG0.0	R3	0.32 — 0.45	0.257	0.513	23.0 × 11.5
WR1150	WG3	R8	0.63 — 0.97	0.513	1.026	11.50 × 5.75
WR340	WG9A	R26	2.2 — 3.3	1.736	3.471	3.40 × 1.70
WR75	WG17	R120	10 — 15	7.869	15.737	0.75 × 0.375
WR10	WG27	R900	75 — 110	59.015	118.03	0.10 × 0.05
WR3	WG32	R2600	220 — 330	173.571	347.143	0.034 × 0.017

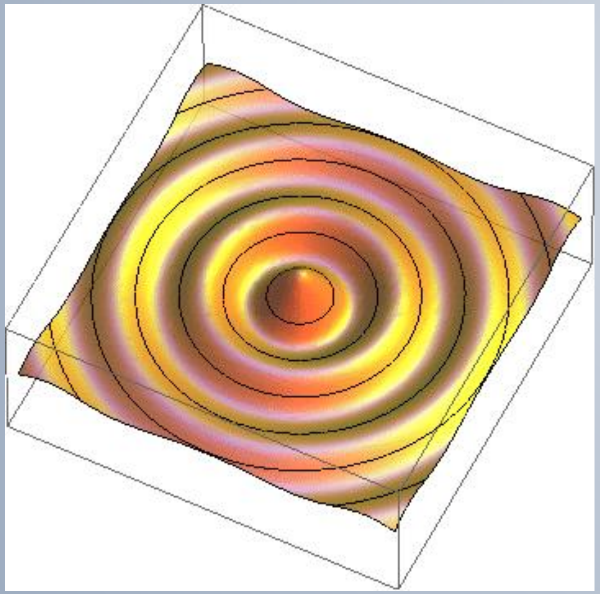




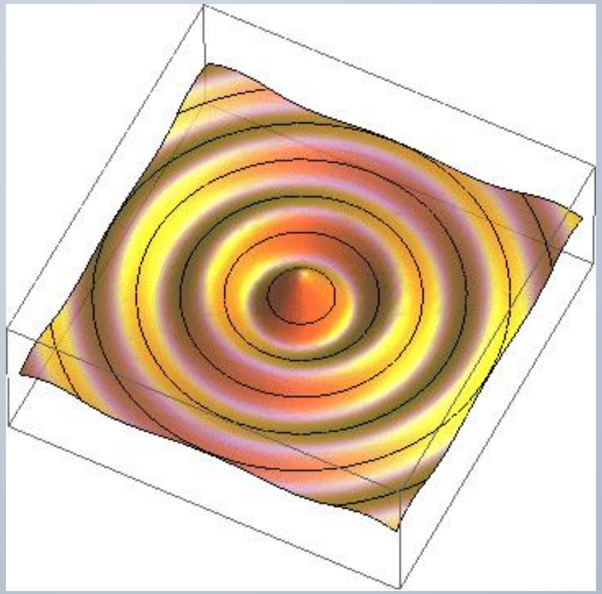
courtesy: Eric Montesinos/CERN

Radial waves

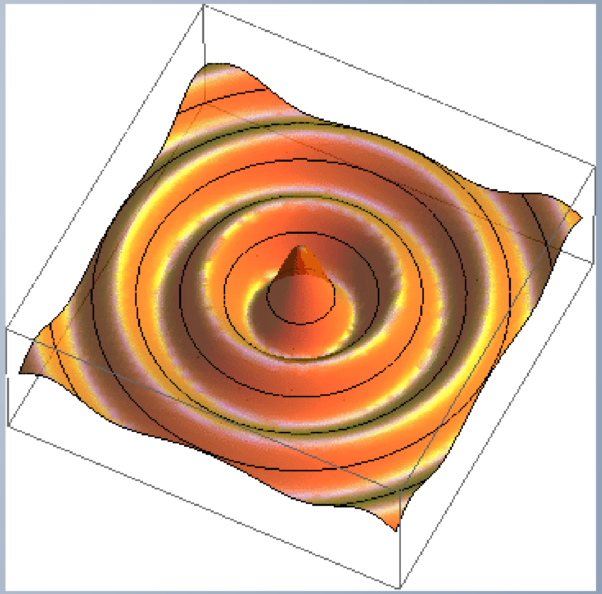
Also radial waves may be interpreted as superposition of plane waves.
The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$$



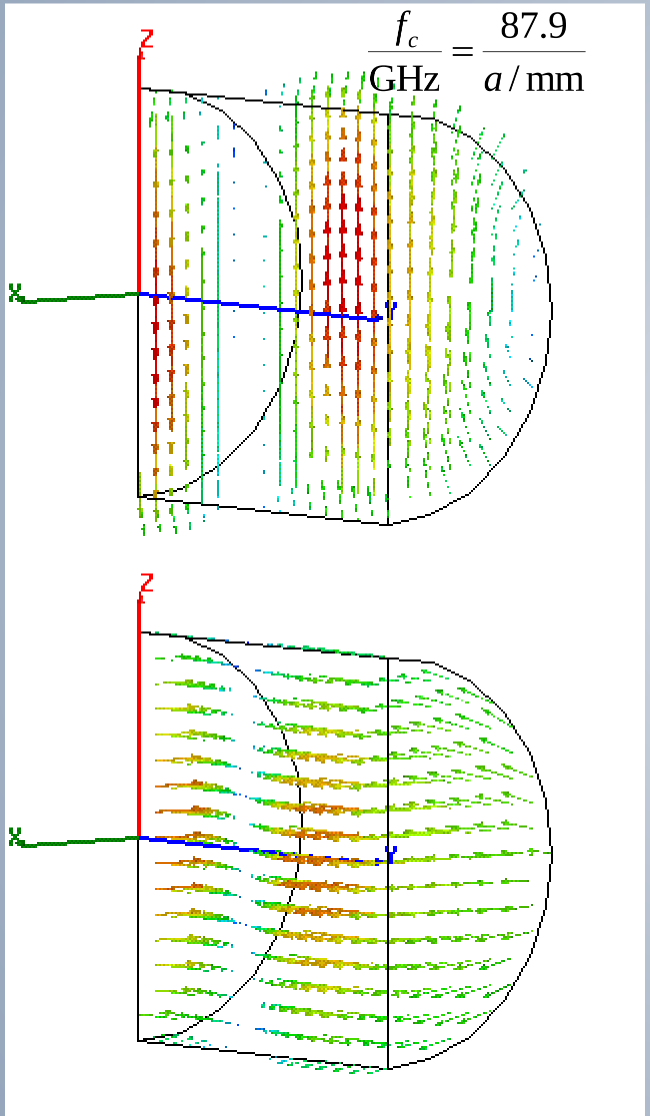
$$E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$$

Round waveguide

$f/f_c = 1.44$

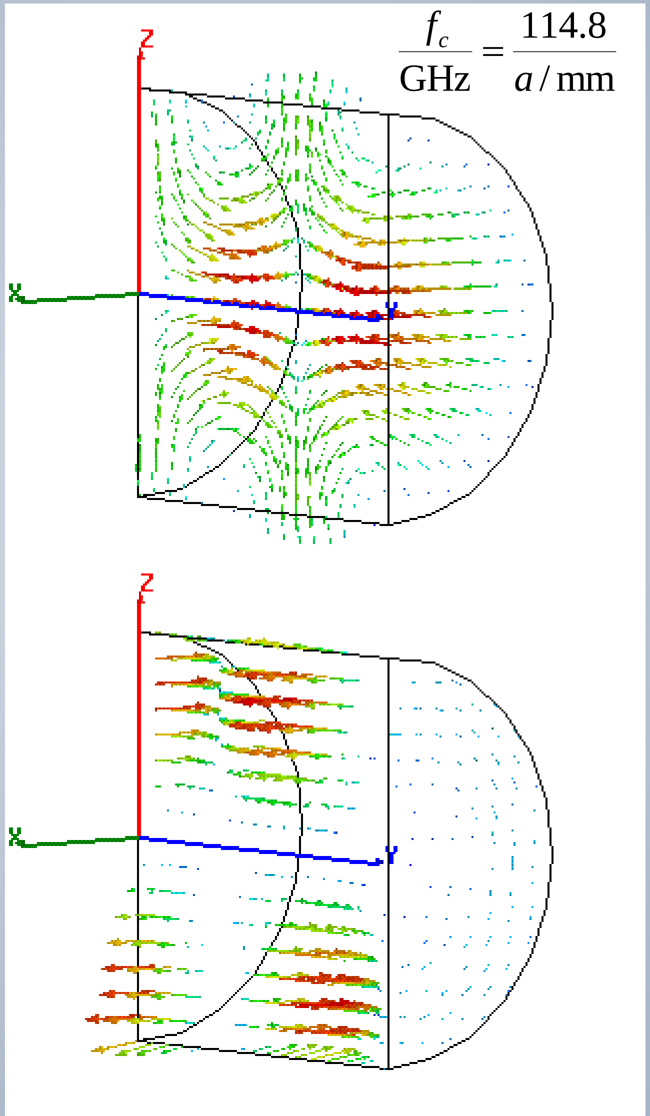
TE_{11} – fundamental

$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$$



TM_{01} – axial field

$$\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$$



TE_{01} – low loss

$$\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$

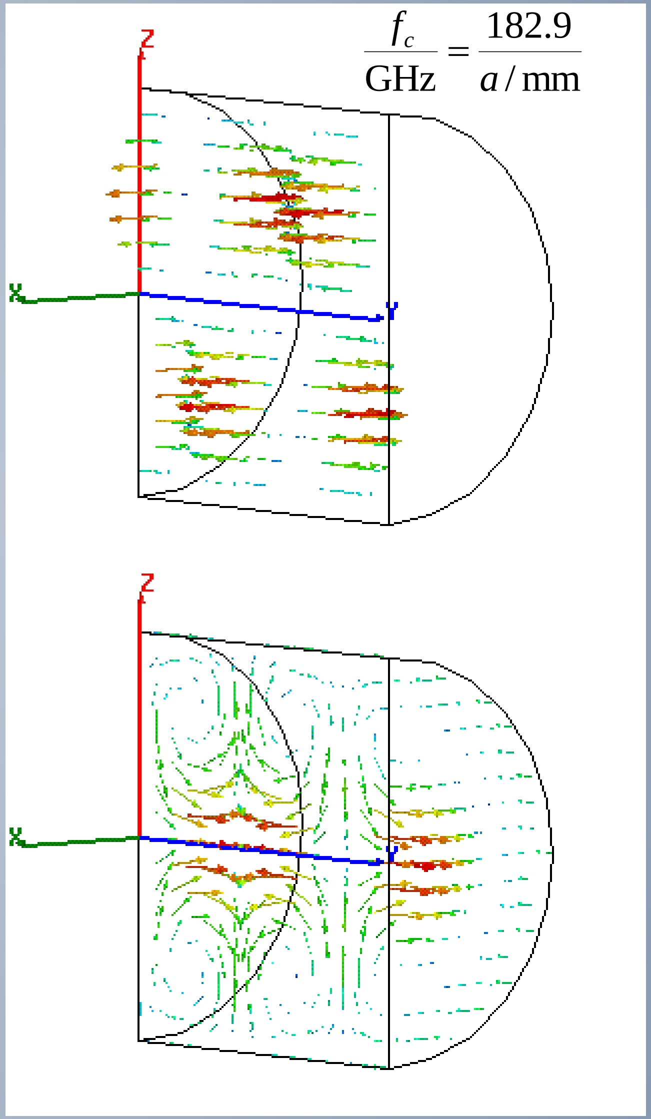
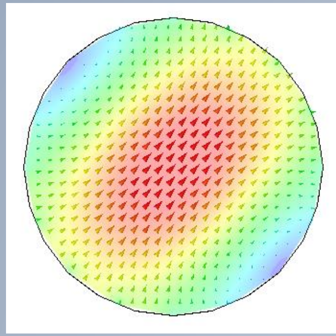
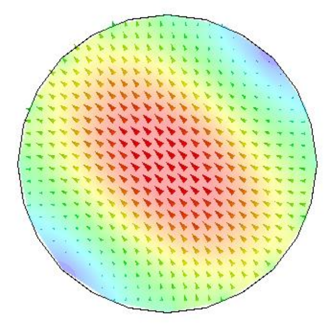
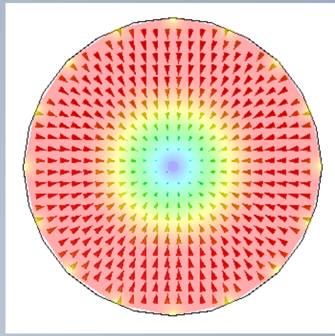


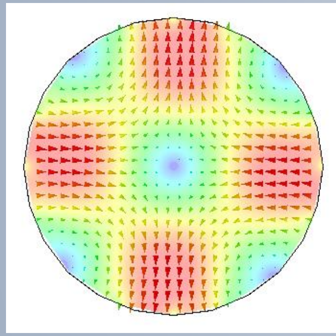

Photo: Reidar Hahn

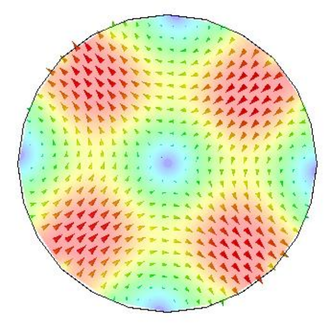
Circular waveguide modes

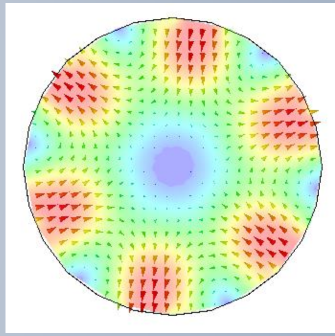

TE_{11}

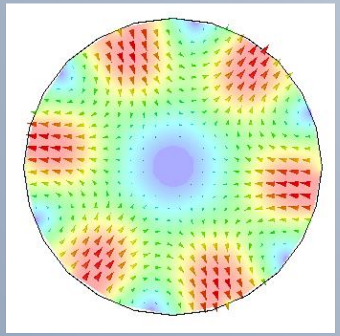

TE_{11}

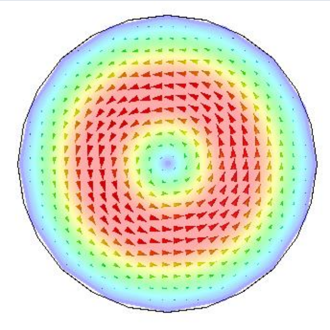

TM_{01}

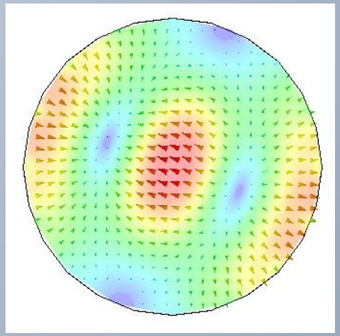

TE_{21}


TE_{21}


TE_{31}


TE_{31}


TE_{01}


TM_{11}

plotted: *E*-field


Photo: Reidar Hahn

General waveguide equations:

Transverse wave equation (membrane equation): $\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0.$

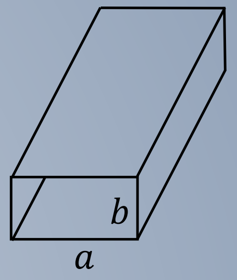
	TE (or H-) modes	***TM (or E-) modes***
Boundary condition:	$\vec{n} \cdot \nabla T = 0$	$T = 0$
longitudinal wave equations (transmission line equations):	$\frac{\mathrm{d}U(z)}{\mathrm{d}z} + jk_z Z_0 I(z) = 0$ $\frac{\mathrm{d}I(z)}{\mathrm{d}z} + \frac{jk_z}{Z_0} U(z) = 0$	
Propagation constant:	$k_z = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$	
Characteristic impedance:	$Z_0 = \frac{\omega\mu}{k_z}$	$Z_0 = \frac{k_z}{\omega\varepsilon}$
Ortho-normal eigenvectors:	$\vec{e} = \vec{u}_z \times \nabla T$	$\vec{e} = -\nabla T$
Transverse fields:	$\vec{E} = U(z)\vec{e}$ $\vec{H} = I(z)\,\vec{u}_z \times \vec{e}$	
Longitudinal fields:	$H_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T\,U(z)}{\mathrm{j}\omega\mu}$	$E_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T\,I(z)}{\mathrm{j}\omega\varepsilon}$


Photo: Reidar Hahn

Special cases: rectangular and round waveguide

Rectangular waveguide: transverse eigenfunctions

TE (H-) modes:	$T_{mn}^{(H)} = \frac{1}{\pi}\sqrt{\frac{a\,b\,\varepsilon_m\varepsilon_n}{(mb)^2+(na)^2}}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$
TM (E-) modes:	$T_{mn}^{(E)} = \frac{2}{\pi}\sqrt{\frac{a\,b}{(mb)^2+(na)^2}}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$



$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Round waveguide: transverse eigenfunctions

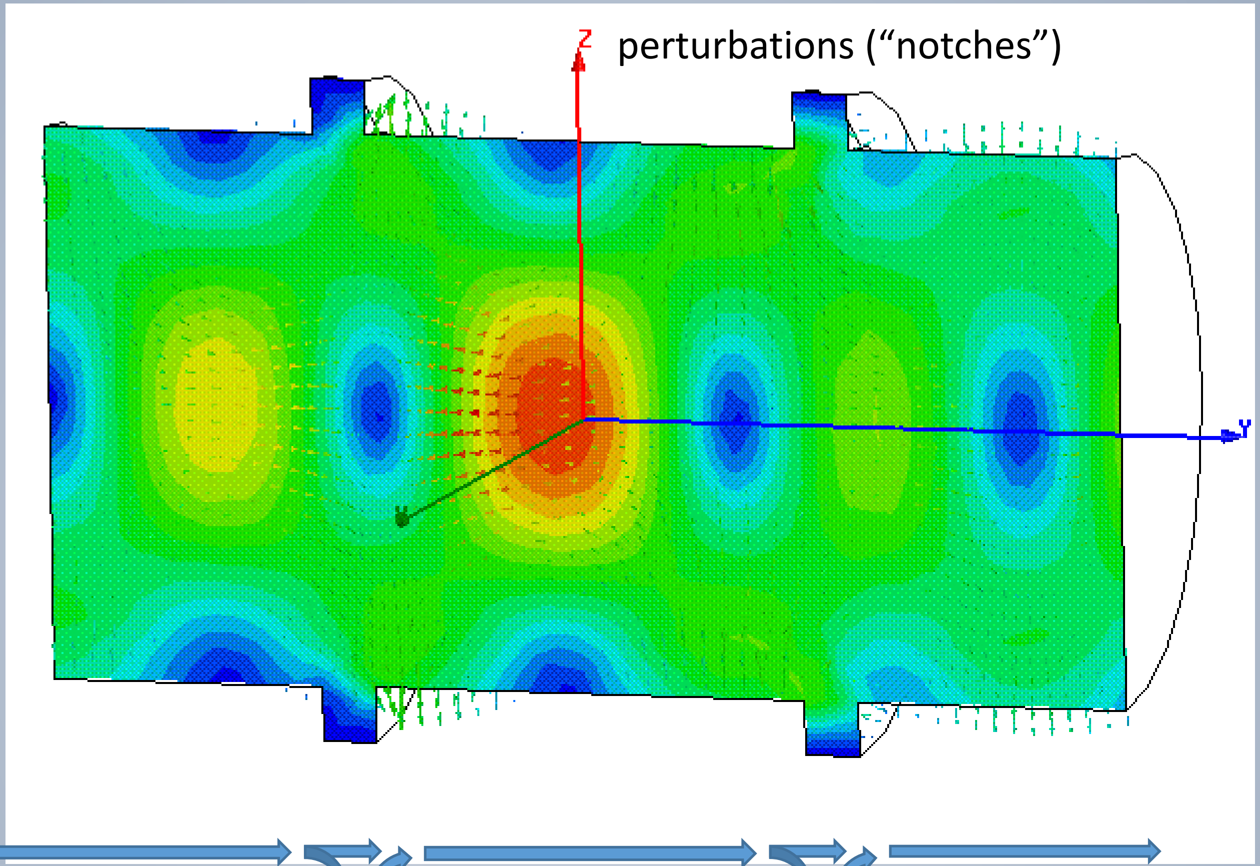
TE (H-) modes:	$T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi(\chi_{mn}'^2 - m^2)}}\frac{J_m\left(\chi'_{mn}\frac{\rho}{a}\right)}{J_m(\chi'_{mn})}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$
TM (E-) modes:	$T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}}\frac{J_m\left(\chi_{mn}\frac{\rho}{a}\right)}{J_{m-1}(\chi_{mn})}\begin{Bmatrix}\sin(m\varphi)\\ \cos(m\varphi)\end{Bmatrix}$

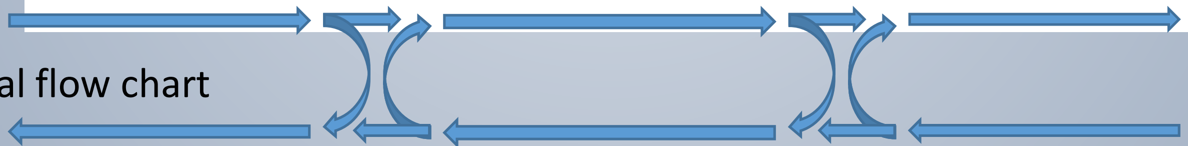
$\emptyset = 2a$

$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

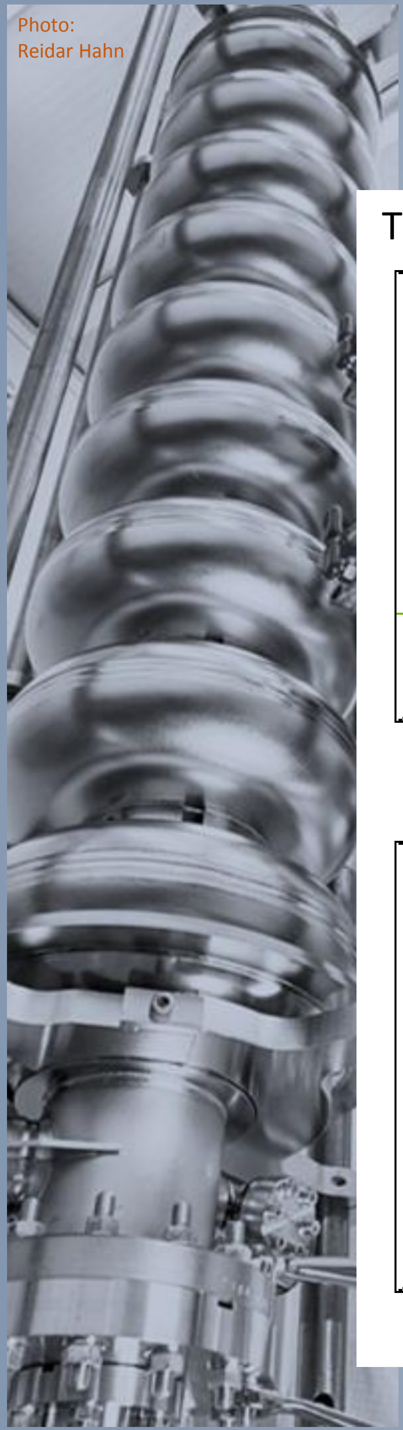
where in both cases $\varepsilon_i = \begin{cases}1 & \text{if} \quad i = 0\\ 2 & \text{if} \quad i \neq 0\end{cases}$

Waveguide perturbed by notches





Reflections from notches lead to a superimposed standing wave pattern.
“Trapped mode”



Short-circuited waveguide

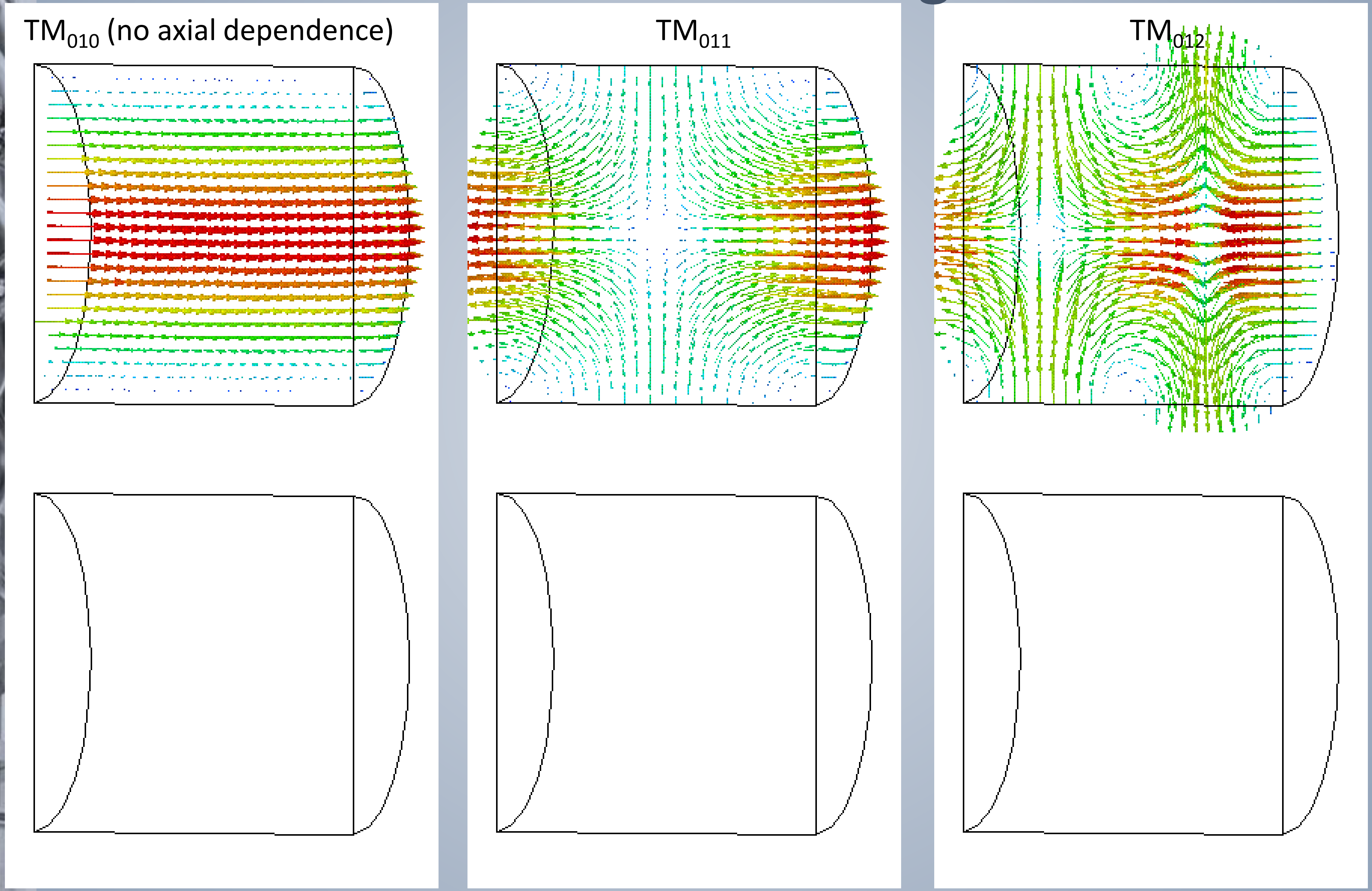
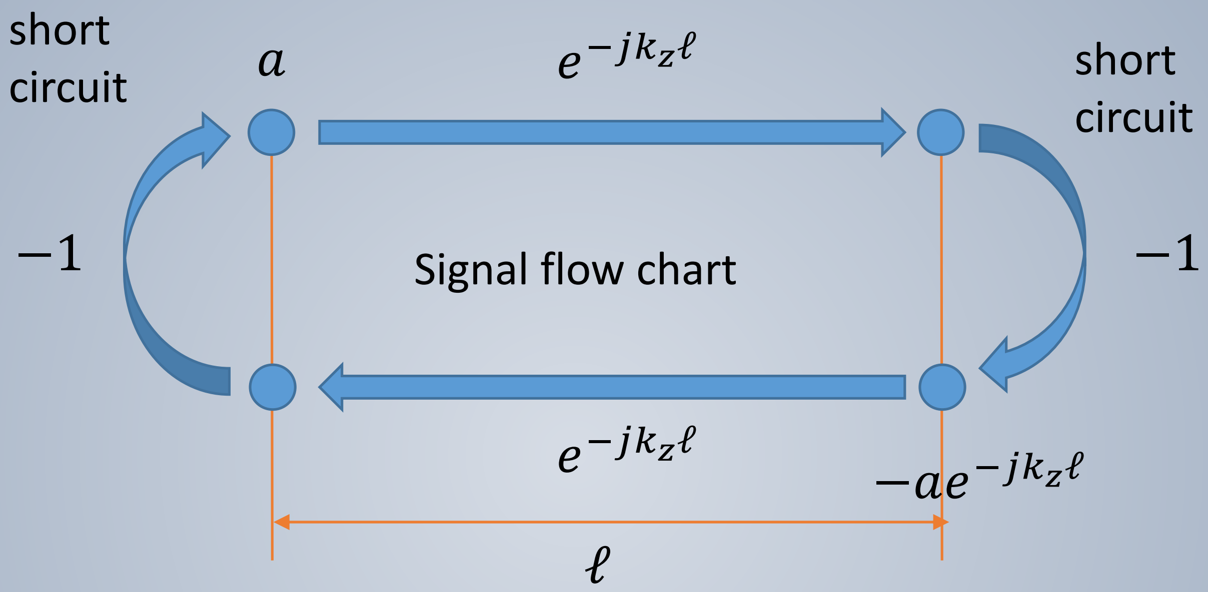



Photo: Reidar Hahn

Single WG mode between two shorts



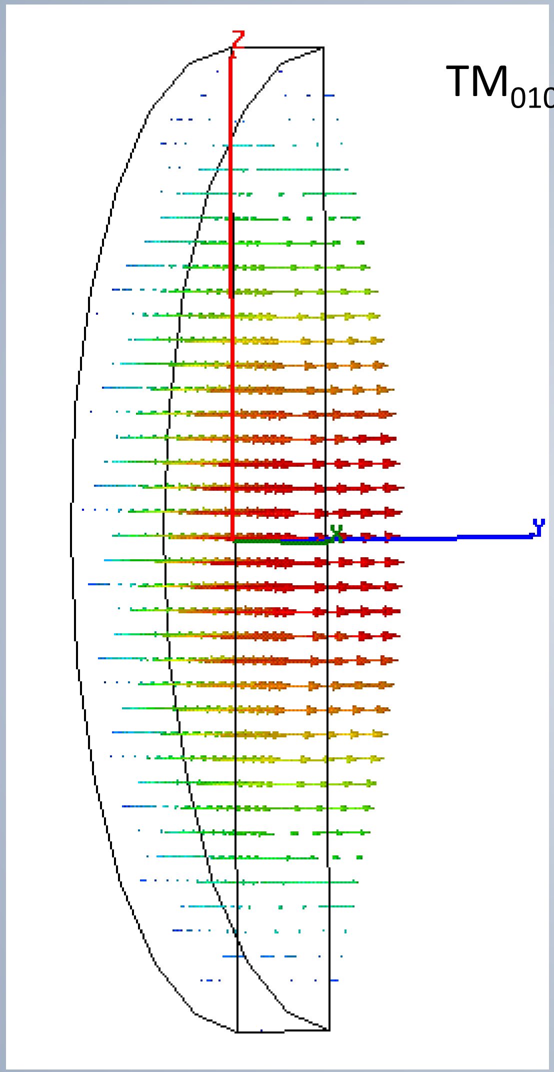
Eigenvalue equation for field amplitude a:

$$a = e^{-jk_z 2\ell} a$$

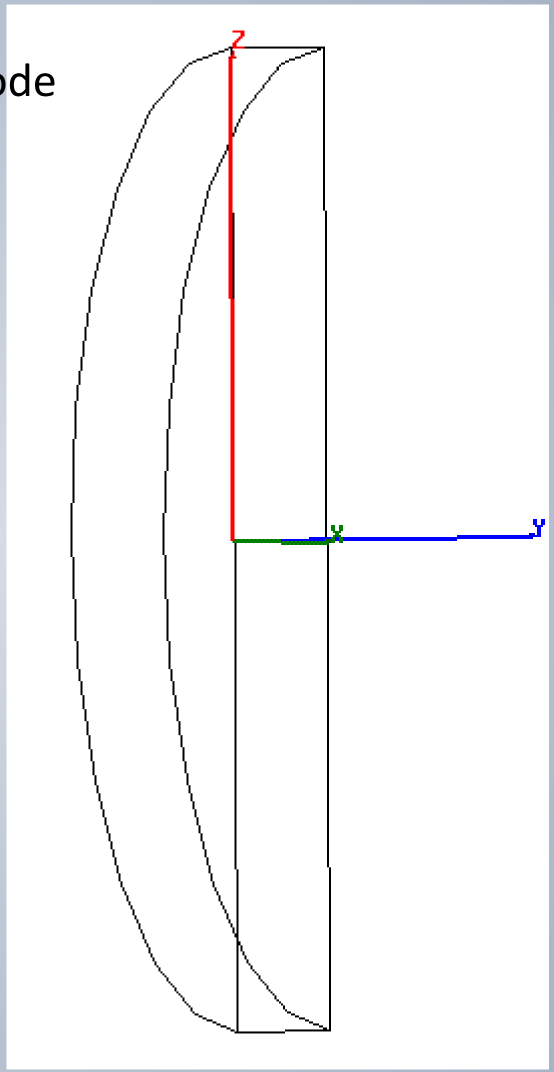
Non-vanishing solutions exist for $2k_z\ell = 2\pi m$:

With $k_z = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c\,\frac{m}{2\ell}\right)^2$.

Simple pillbox (only 1/2 shown)



TM$_{010}$-mode

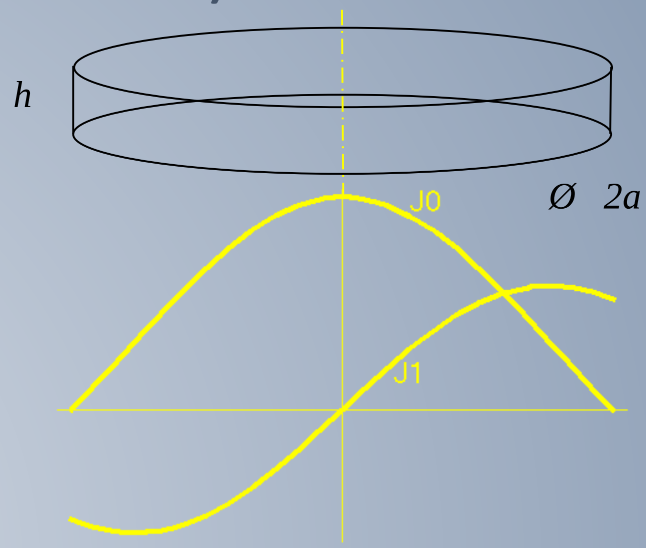


electric field (purely axial) magnetic field (purely azimuthal)

Pillbox cavity field (w/o beam tube)

$$T(\rho,\varphi)=\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01}\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

with $\chi_{01}=2.40483\ldots$



The only non-vanishing field components :

$$E_z=\frac{1}{j\omega\varepsilon}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi=\mu_0\sqrt{\frac{1}{\pi}}\frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$\omega_0\big|_{\text{pillbox}}=\frac{\chi_{01}c}{a},\quad \eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\ \Omega$$

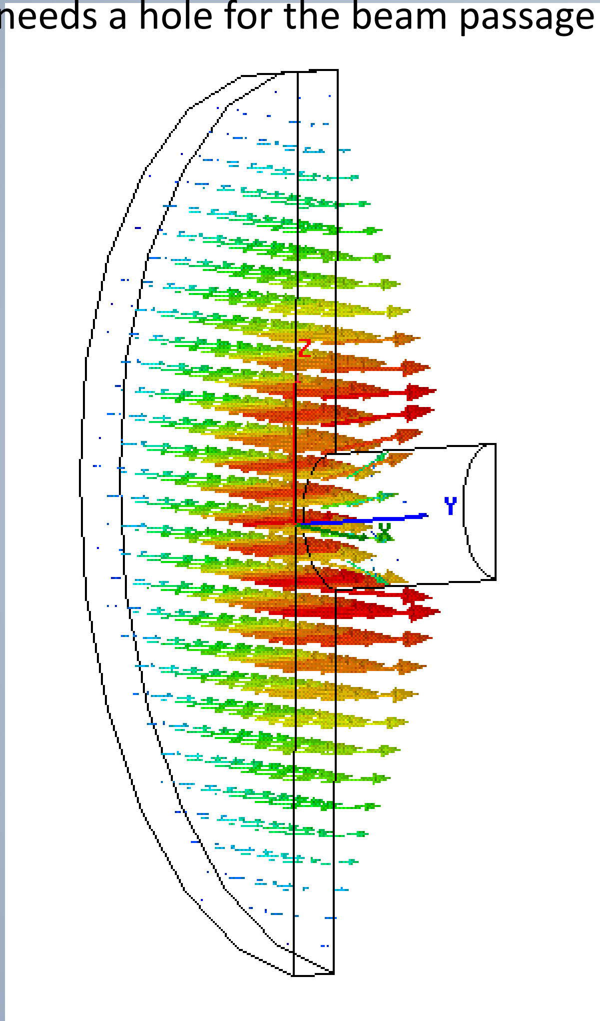
$$Q\,\Big|_{\text{pillbox}}=\frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1+\frac{a}{h}\right)}$$

$$\frac{R}{Q}\bigg|_{\text{pillbox}}=\frac{4\eta}{\chi_{01}^3\pi J_1^2(\chi_{01})}\frac{\sin^2\left(\frac{\chi_{01}}{2}\frac{h}{a}\right)}{h/a}$$

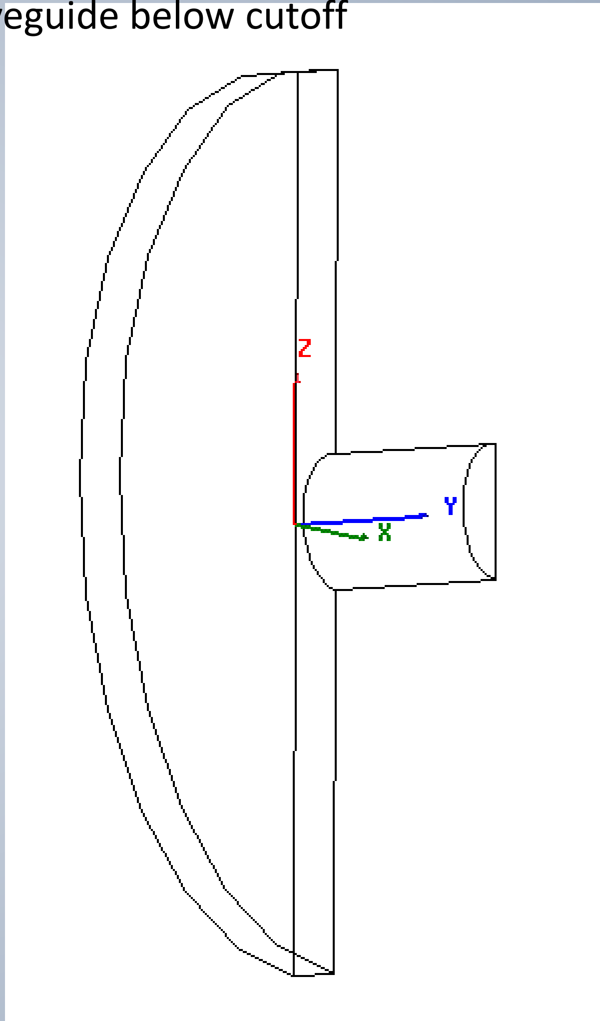
Pillbox with beam pipe

TM_{010}-mode (only 1/4 shown)

One needs a hole for the beam passage – circular waveguide below cutoff



electric field

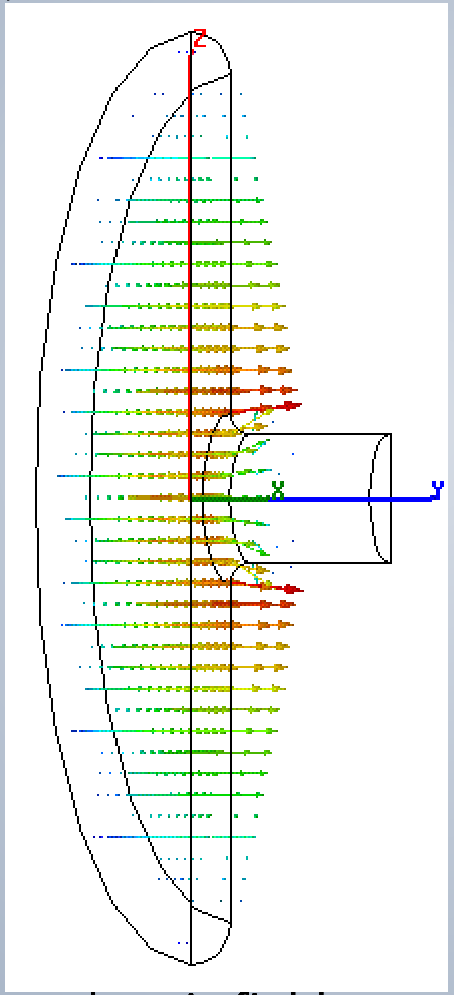


magnetic field

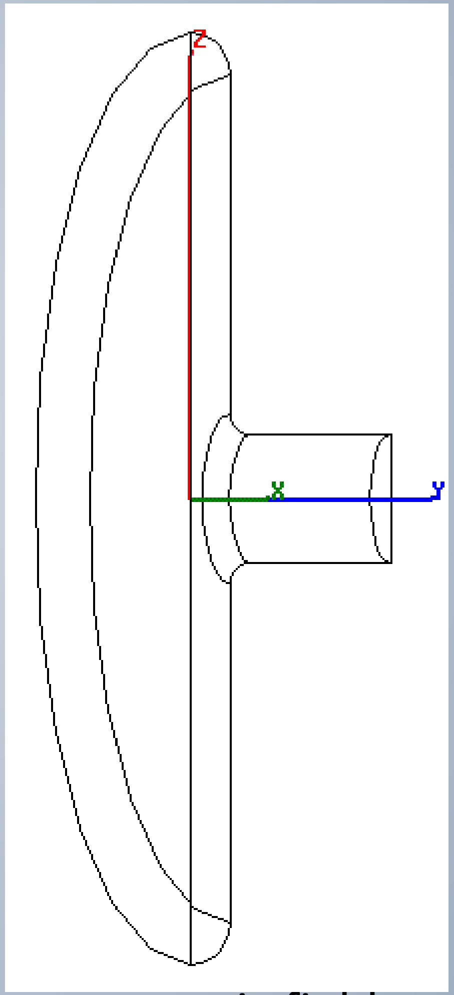
A more practical pillbox cavity

TM_{010}-mode (only 1/4 shown)

Rounding of sharp edges (to reduce field enhancement!)



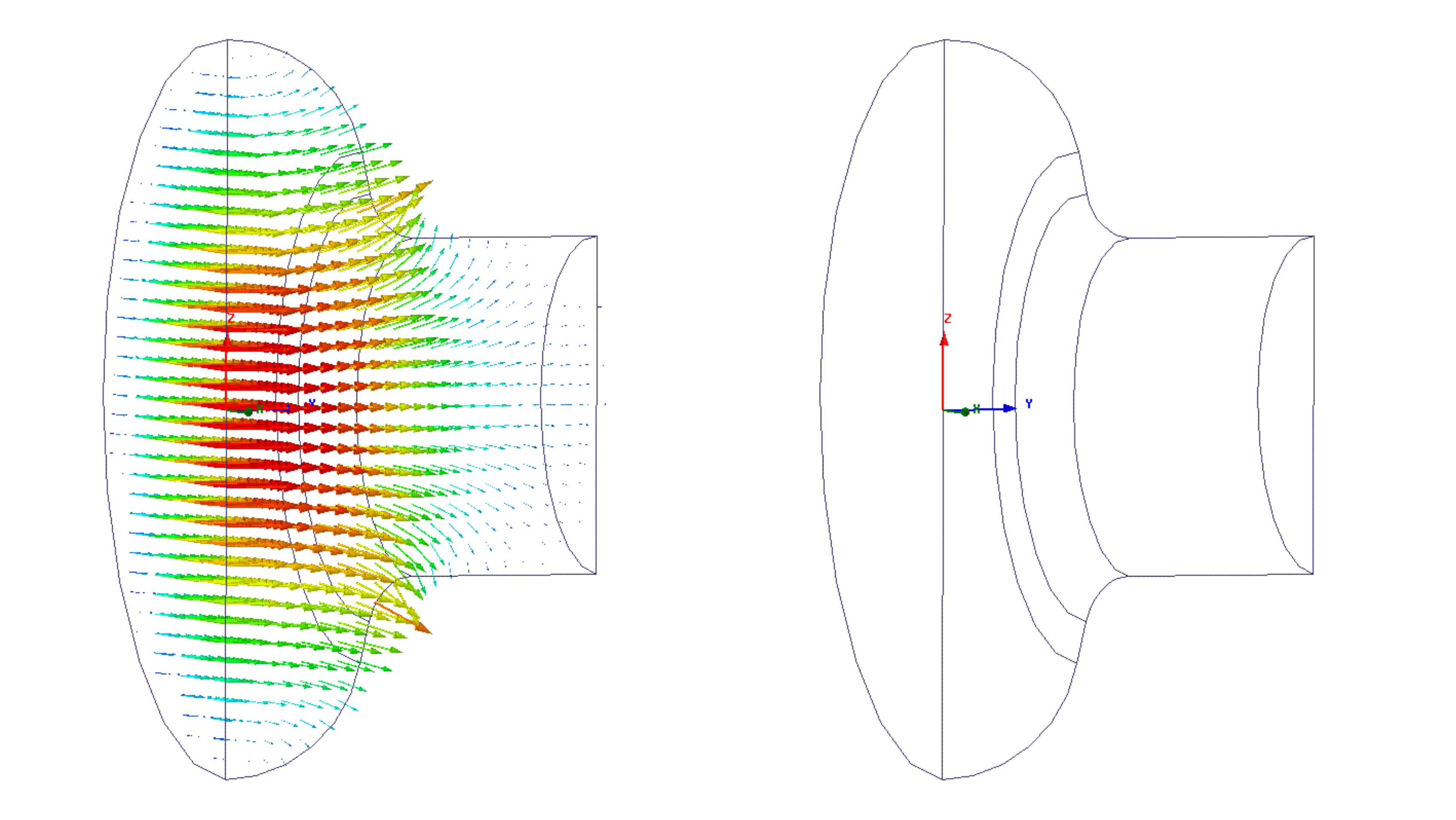
electric field



magnetic field

A (real) elliptical cavity

TM$_{010}$-mode (only 1/4 shown)



electric field

magnetic field

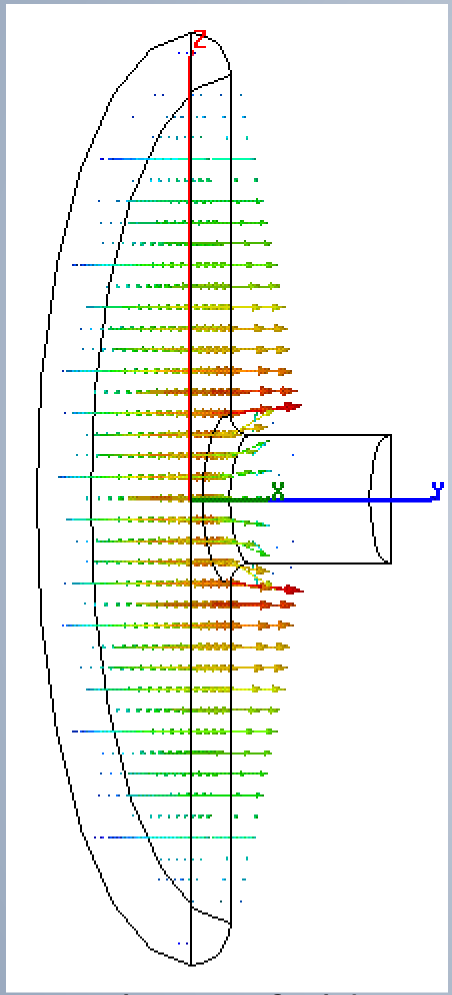

Photo:
Reidar Hahn

Choice of frequency

- Size:
  - Linear dimensions scale as f^{-1}, volume as f^{-3}.
  - amount of material, mass, stiffness, tolerances, …
  - Outer radius of elliptical cavity $\sim 0.45\ \lambda$.
- Beam interaction:
  - r/Q increases with f – but also for HOMs!
  - short bunches are easier with higher f.
- Technology:
  - superconducting: BCS resistance $\propto f^2$.
  - Power sources available?
  - Max. accelerating voltage?

Characterizing a cavity

Acceleration voltage and R/Q



electric field

- I define

$$V_{acc} = \int_{-\infty}^{\infty} E_z e^{j\frac{\omega}{\beta c}z}\, dz\,.$$

- The exponential factor accounts for the variation of the field while particles with velocity βc are traversing the cavity gap.
- With this definition, V_{acc} is generally complex – this becomes important with more than one gap (cell).
- For the time being we are only interested in $|V_{acc}|$.
- The square of the acceleration voltage $|V_{acc}|^2$ is proportional to the stored energy W; the proportionality constant defines the quantity called "R-upon-Q":

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}.$$

› **Attention – different definitions are used in literature!**

Transit time factor

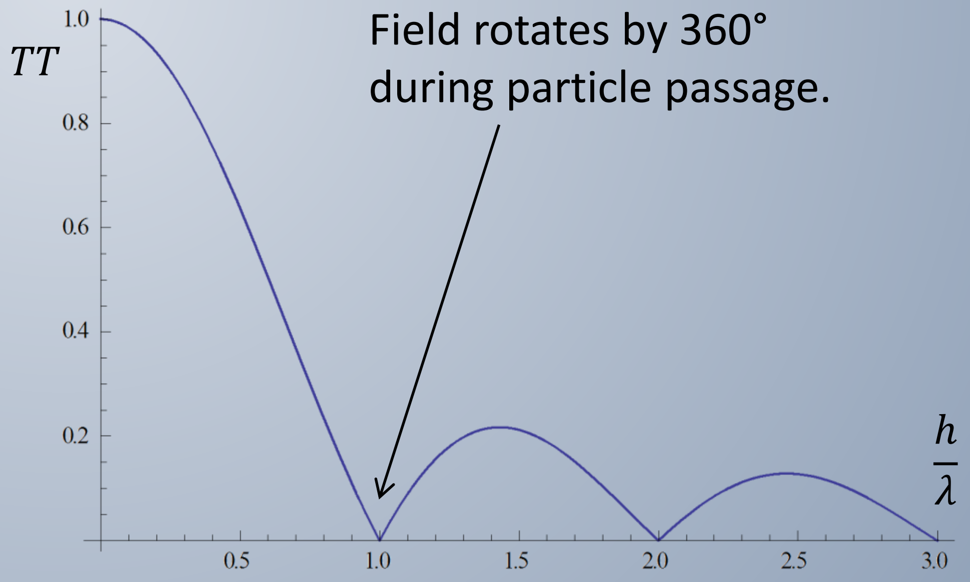
- The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see:

$$TT = \frac{|V_{acc}|}{\left|\int E_z\, dz\right|} = \frac{\left|\int E_z e^{j\frac{\omega}{\beta c}z}\, dz\right|}{\left|\int E_z\, dz\right|}.$$

- The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) h is:

$$TT = \frac{\sin\left(\frac{\chi_{01} h}{2a}\right)}{\frac{\chi_{01} h}{2a}}$$

(remember: $\omega_0 = \frac{2\pi c}{\lambda} = \frac{\chi_{01} c}{a}$)



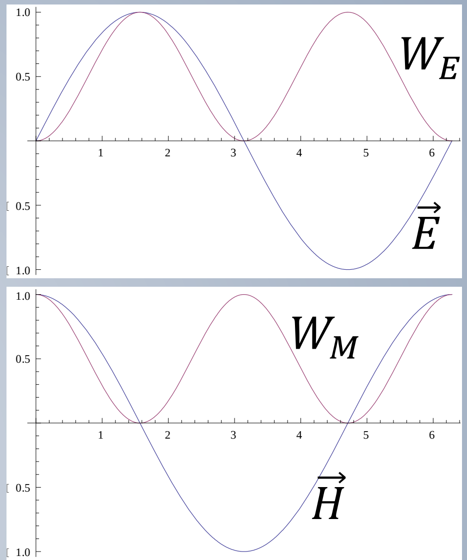
Stored energy

- The energy stored in the electric field is

$$W_E = \iiint\limits_{\text{cavity}} \frac{\varepsilon}{2}\left|\vec{E}\right|^2 dV\,.$$

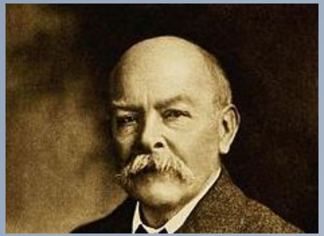
- The energy stored in the magnetic field is

$$W_M = \iiint\limits_{\text{cavity}} \frac{\mu}{2}\left|\vec{H}\right|^2 dV\,.$$



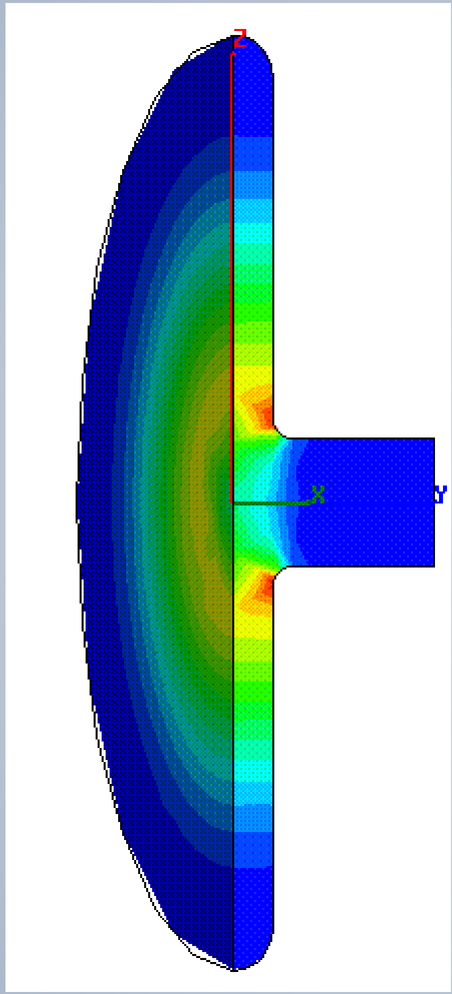
- Since $\vec{E}$ and $\vec{H}$ are $90°$ out of phase, the stored energy continuously swaps from electric energy to magnetic energy.
- On average, electric and magnetic energy must be equal.
- In steady state, the Poynting vector describes this energy flux.
- In steady state, the total energy stored (constant) is

$$W = \iiint\limits_{cavity} \left(\frac{\varepsilon}{2}\left|\vec{E}\right|^2 + \frac{\mu}{2}\left|\vec{H}\right|^2\right) dV.$$

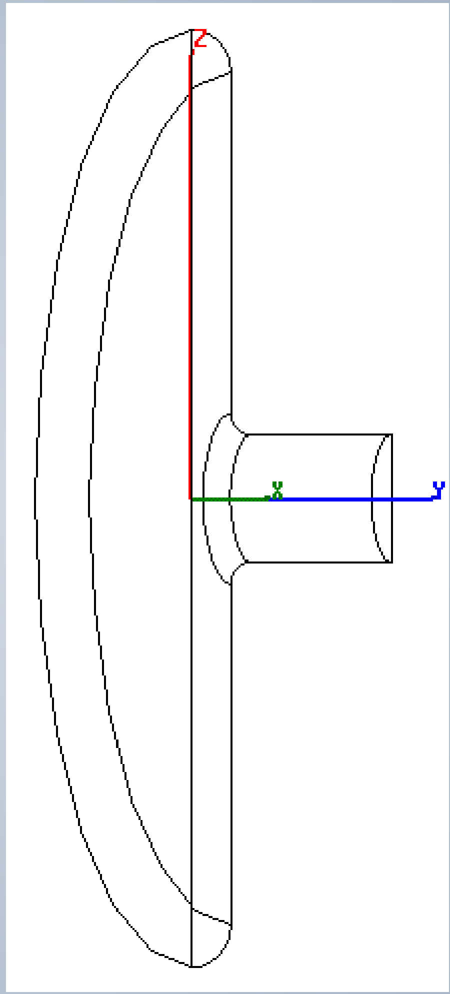


John Henry Poynting
1852 – 1914

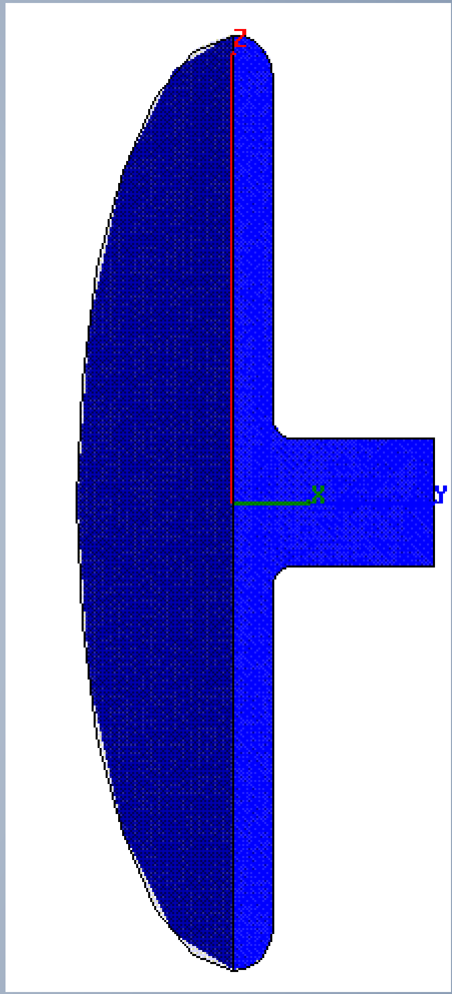
Stored energy and Poynting vector



electric field energy



Poynting vector



magnetic field energy

Wall losses & Q_0

- The losses P_{loss} are proportional to the stored energy W.
- The tangential $\vec{H}$ on the surface is linked to a surface current $\vec{J}_A = \vec{n} \times \vec{H}$ (flowing in the skin depth $\delta = \sqrt{2\,/\,(\omega\mu\sigma)}$).
- This surface current $\vec{J}_A$ sees a surface resistance R_s, resulting in a local power density $R_s|H_t|^2$ flowing into the wall.
- R_s is related to skin depth δ as $\delta\sigma R_s = 1$.
  - Cu at 300 K has $\sigma \approx 5.8 \cdot 10^7 \text{S/m}$, leading to $R_s \approx 8\ \text{m}\Omega$ at 1 GHz, scaling with $\sqrt{\omega}$.
  - Nb at 2 K has a typical $R_s \approx 10\ \text{n}\Omega$ at 1 GHz, scaling with ω^2.
- The total wall losses result from $P_{\text{loss}} = \iint\limits_{wall} R_s|H_t|^2\, dA$.
- The cavity Q_0 (caused by wall losses) is defined as $Q_0 = \frac{\omega_0 W}{P_{\text{loss}}}$.
- Typical Q_0 values:
  - Cu at 300 K (normal-conducting): $\mathcal{O}(10^3 \ldots 10^5)$, **should** improve at cryogenic T by a factor $\sqrt{RRR}$.
  - Nb at 2 K (superconducting): $\mathcal{O}(10^9 \ldots 10^{11})$

No! Anomalous skin effect!
improves only by a factor ≈ 10!

Shunt impedance

- Also the power loss P_{loss} is also proportional to the square of the acceleration voltage $|V_{acc}|^2$; the proportionality constant defines the “shunt impedance”

$$R = \frac{|V_{acc}|^2}{2\,P_{\text{loss}}}.$$

› **Attention, also here different definitions are used!**

- Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.
- Now the previously introduced term “R-upon-Q” makes sense:

$$\left(\frac{R}{Q}\right) = R/Q$$

Geometric factor

- With

$$Q_0 = \frac{\omega_0 W}{\iint\limits_{wall} R_s |H_t|^2 \, dA},$$

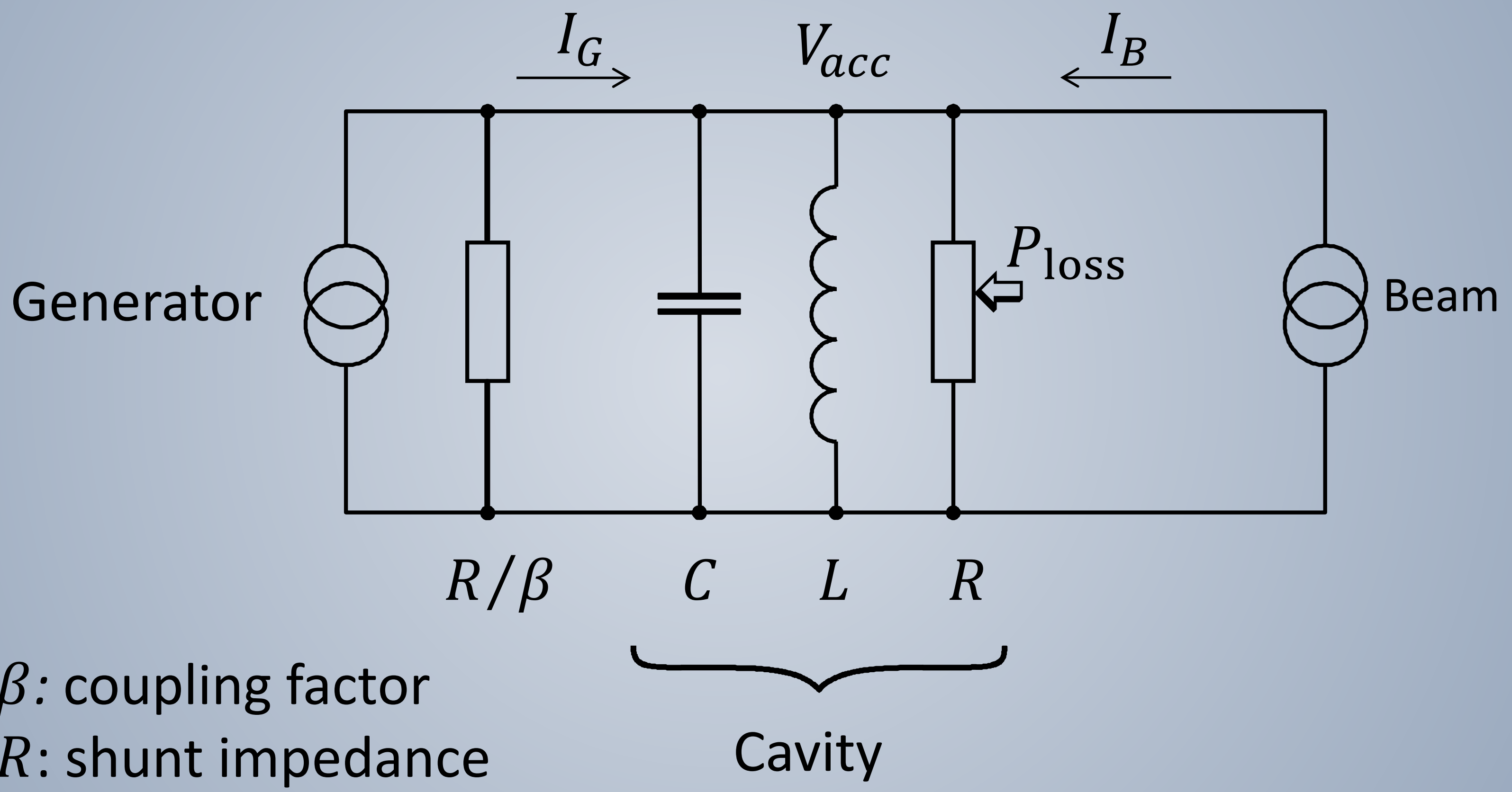
and assuming an average surface resistance R_s, one can introduce the "geometric factor" G as

$$G = Q_0 \cdot R_s = \frac{\omega_0 W}{\iint\limits_{wall} |H_t|^2 \, dA}.$$

- G has dimension Ohm, depends only on the cavity geometry (as the name suggests) and typically is $\mathcal{O}(100\ \Omega)$.
- Note that $R_s \cdot R = G \cdot (R/Q)$ (dimension Ω^2, purely geometric)
- G is only used for SC cavities.

Cavity resonator – equivalent circuit

Simplification: single mode



β: coupling factor

R: shunt impedance

$\sqrt{L/C} = \frac{R}{Q}$: R-upon-Q

Photo: Reidar Hahn

Power coupling - Loaded Q

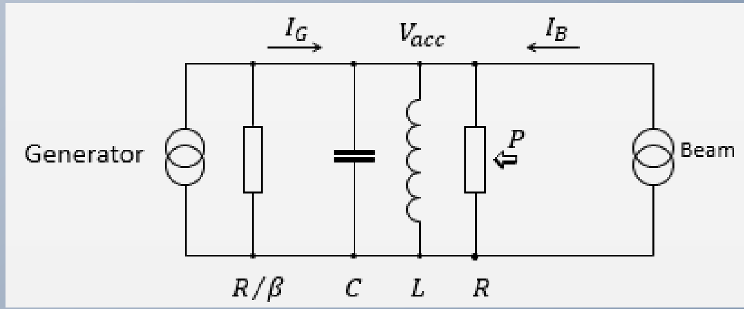
- Note that the generator inner impedance also loads the cavity – for very large Q_0 more than the cavity wall losses.
- To calculate the loaded Q (Q_L), losses have to be added:

$$\frac{1}{Q_L} = \frac{P_{\text{loss}} + P_{\text{ext}} + \cdots}{\omega_0\, W} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} + \frac{1}{\dots}.$$

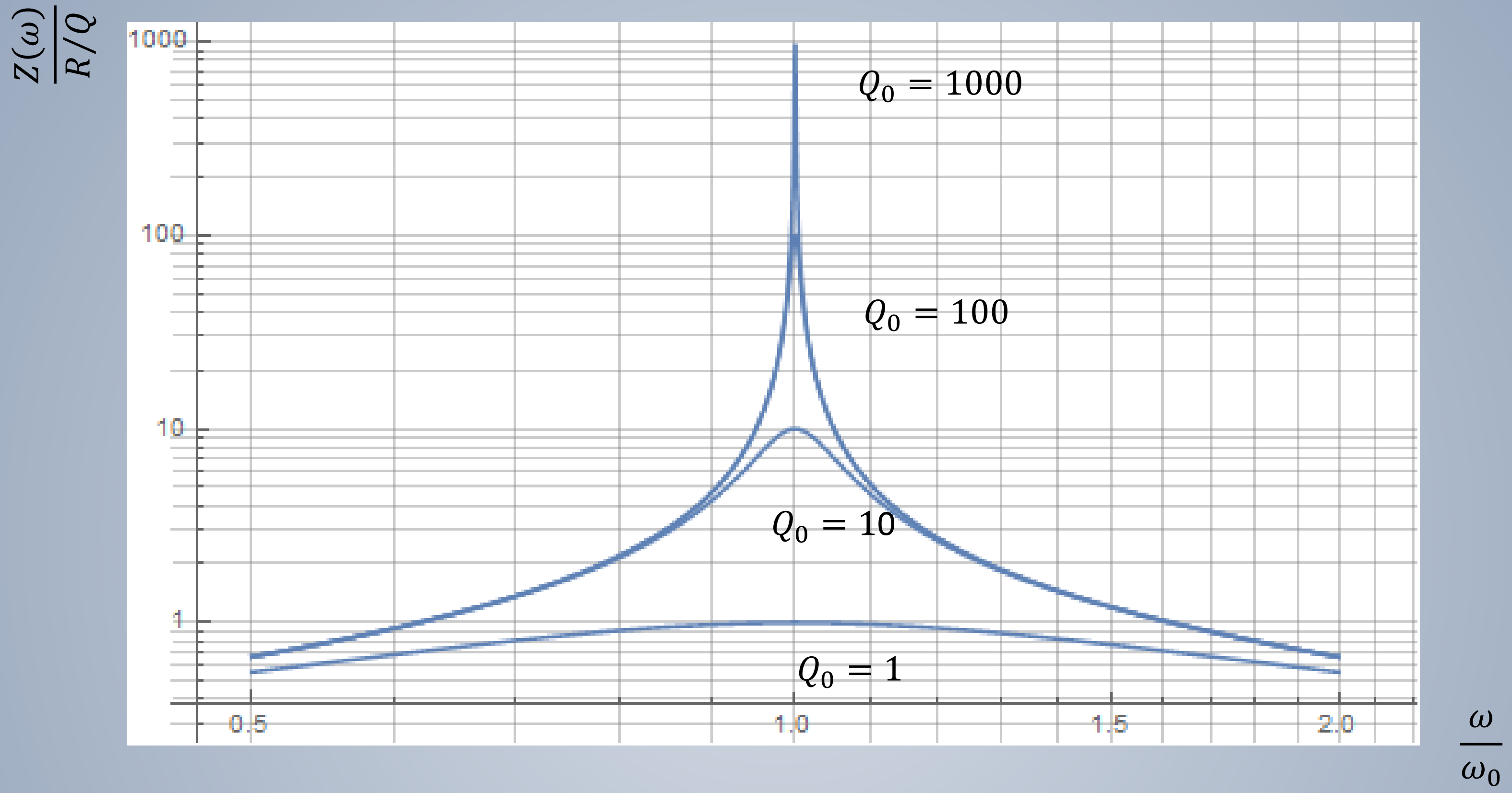
- The coupling factor β is the ratio $P_{\text{ext}}/P_{\text{loss}}$.
- With β, the loaded Q can be written

$$Q_L = \frac{Q_0}{1+\beta}.$$

- For NC cavities, often $\beta = 1$ is chosen (power amplifier matched to empty cavity); for SC cavities, $\beta = \mathcal{O}(10^4 \dots 10^6)$.



Resonance

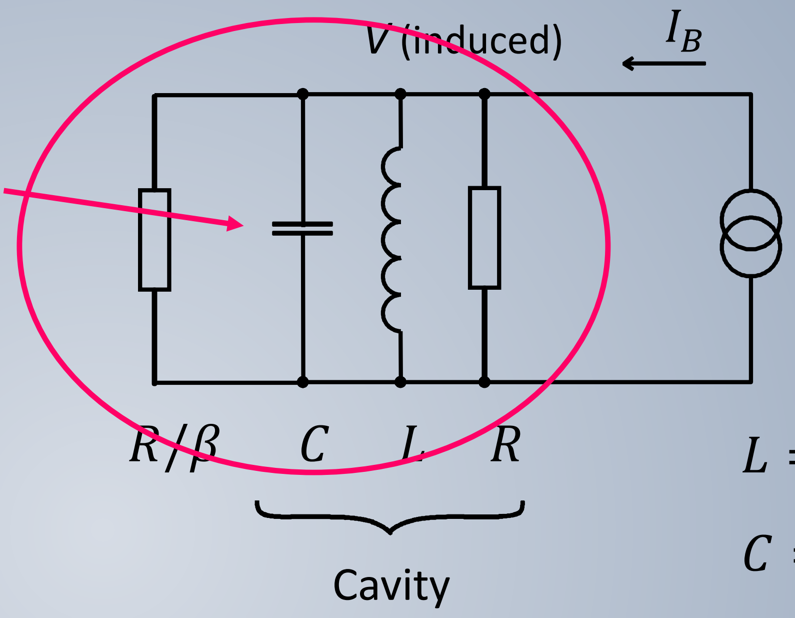


- While a high Q_0 results in small wall losses, so less power is needed for the same voltage.
- On the other hand the bandwidth becomes very narrow.
- Note: a 1 GHz cavity with a Q_0 of 10^{10} has a natural bandwidth of 0.1 Hz!
- … to make this manageable, Q_{ext} is chosen much smaller!

Loss factor

$$k_{\text{loss}} = \frac{\omega_0}{2}\left(\frac{R}{Q}\right) = \frac{|V_{acc}|^2}{4\,W} = \frac{1}{2C}$$

Impedance seen by the beam



$L = R/(Q_0\,\omega_0)$

$C = Q_0/(R\,\omega_0)$

Energy deposited by a single charge q: $k_{\text{loss}} q^2$

Voltage induced by a single charge q: $2\,k_{\text{loss}} q$

$$\frac{V_{acc}}{2\,k_{\text{loss}}\,q}$$

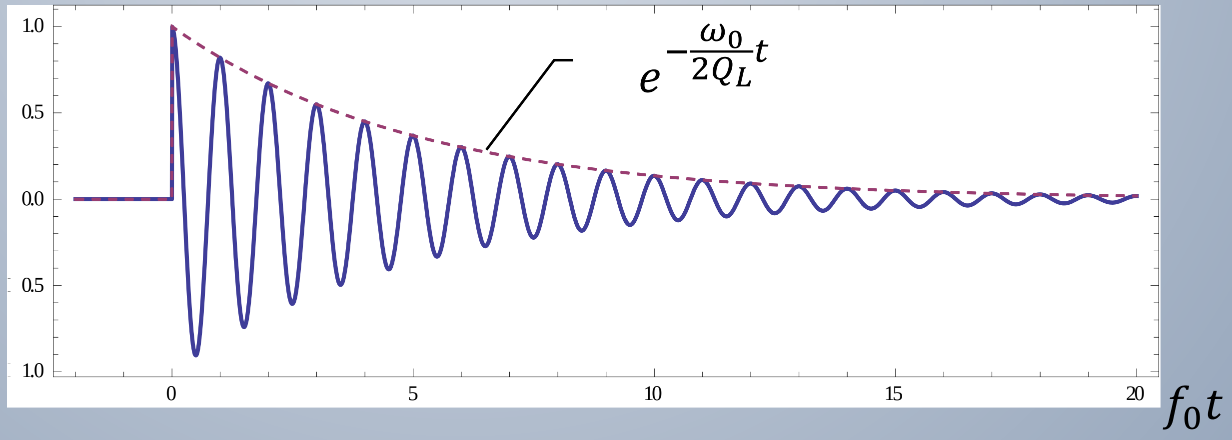


Photo: Reidar Hahn

Summary: relations V_{acc}, W and P_{loss}

Attention – different definitions are used in literature !

V_{acc}
Accelerating voltage

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}$$

$$k_{\text{loss}} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{|V_{acc}|^2}{4W}$$

$$R = \frac{|V_{acc}|^2}{2P_{\text{loss}}} = \frac{R}{Q}Q_0$$

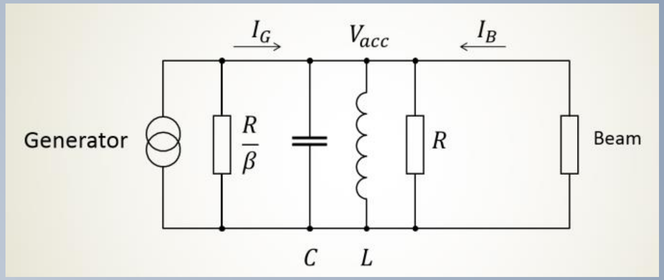
W
Energy stored

P_{loss}
wall losses

$$Q_0 = \frac{\omega_0 W}{P_{\text{loss}}}$$


Photo: Reidar Hahn

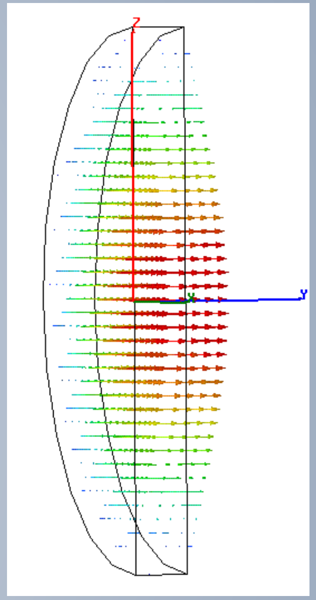
Beam loading

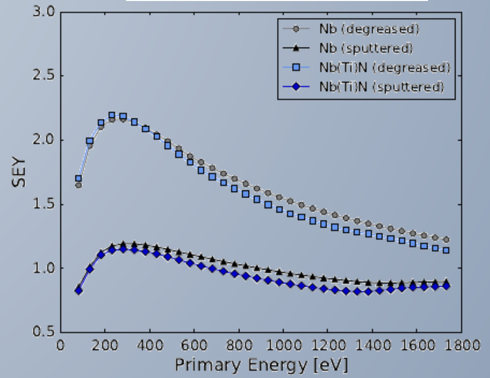


- The beam current "loads" the cavity, in the equivalent circuit this appears as an impedance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity ($\beta = 1$), beam loading will (normally) cause the accelerating voltage to decrease.
- The power absorbed by the beam is $-\frac{1}{2}\Re\{V_{acc} I_B^*\}$.
- For high power transfer efficiency RF → beam, beam loading must be high!
- For SC cavities (very large β), the generator is typically matched to the beam impedance!
- Variation in the beam current leads to **transient beam loading**, which requires special care!
- Often the "impedance" the beam presents is strongly reactive – this leads to a detuning of the cavity.

Multipactor

- The words "multipactor", "to multipact" and "multipacting" are artificially composed of "multiple" "impact".
- Multipactor describes a resonant RF phenomenon in vacuum:
  - Consider a free electron in a simple cavity – it gets accelerated by the electric field towards the wall
  - when it impacts the wall, secondary electrons will be emitted, described by the secondary emission yield (SEY)
  - in certain impact energy ranges, more than one electron is emitted for one electron impacting! So the number of electrons can increase
  - When the time for an electron from emission to impact takes exactly ½ of the RF period, resonance occurs – with the SEY>1, this leads to an avalanche increase of electrons, effectively taking all RF power at this field level, depleting the stored energy and limiting the field!
- For this simple "2-point MP", this resonance condition is reached at $\frac{1}{4\pi}\frac{e}{m}V = (fd)^2$ or $\frac{V}{112\,\mathrm{V}} = \left(\frac{f}{\mathrm{MHz}}\frac{d}{\mathrm{m}}\right)^2$. There exist other resonant bands.

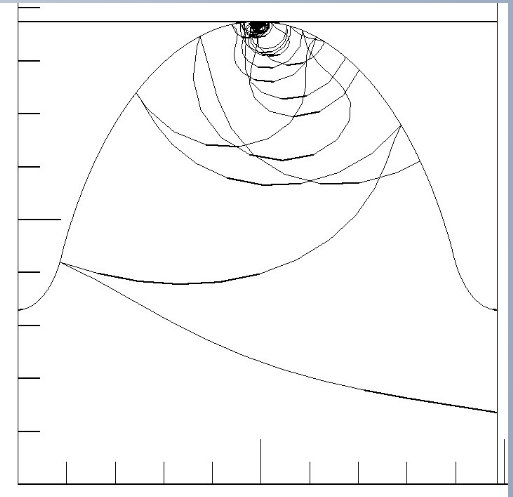




courtesy: Sarah Aull/CERN

Multipactor (contd.)

- Unfortunately, good metallic conductors (Cu, Ag, Nb) all have SEY>1!
- 1-point MP occurs when the electron impact where they were emitted
- Electron trajectories can be complex since both $\vec{E}$ and $\vec{B}$ influence them; computer simulations allow to determine the MP bands (barriers)
- To reduce or suppress MP, a combination of the following may be considered:
  - Use materials with low SEY
  - Optimize the shape of your cavity (→ elliptical cavity)
  - Conditioning (surface altered by exposure to RF fields)
  - Coating (Ti, TiN, NEG, amorphous C …)
  - Clearing electrode (for a superimposed DC electric field)
  - Rough surfaces

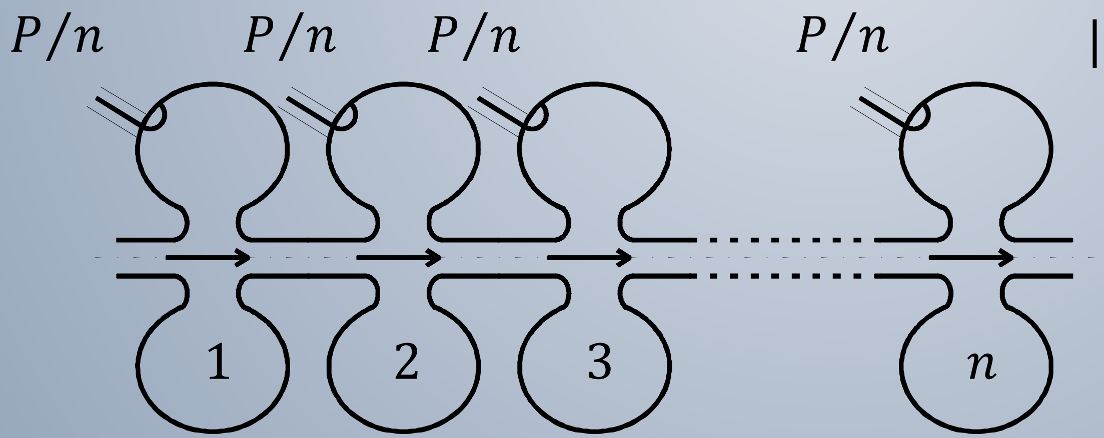


Many gaps


Photo: Reidar Hahn

What do you gain with many gaps?

- The R/Q of a single gap cavity is limited to some $100\ \Omega$.
  Now consider to distribute the available power to n identical cavities: each will receive P/n, thus produce an accelerating voltage of $\sqrt{2RP/n}$. (Attention: phase important!)
  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR .

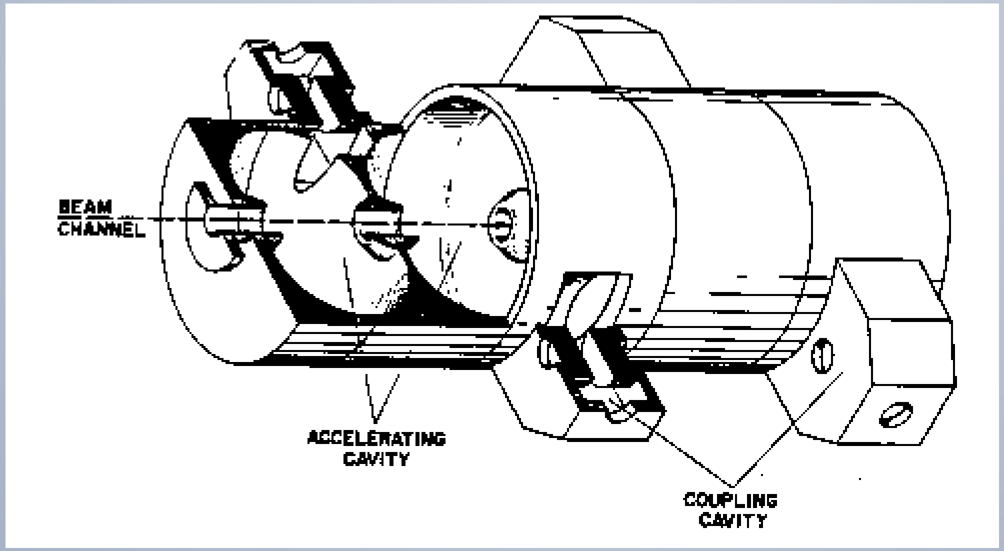


$$|V_{acc}| = n\sqrt{2R\frac{P}{n}} = \sqrt{2(nR)P}$$

Standing wave multi-cell cavity

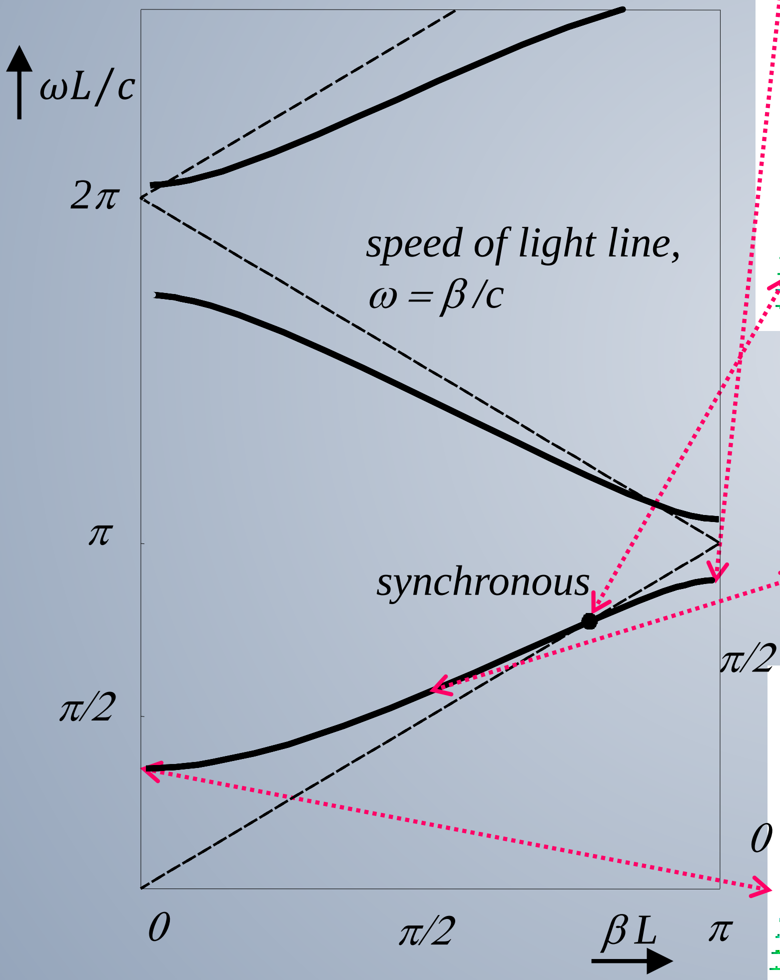
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).

- Coupled cavity accelerating structure (side coupled)



- The phase relation between gaps is important!

Brillouin diagram; Travelling wave structure



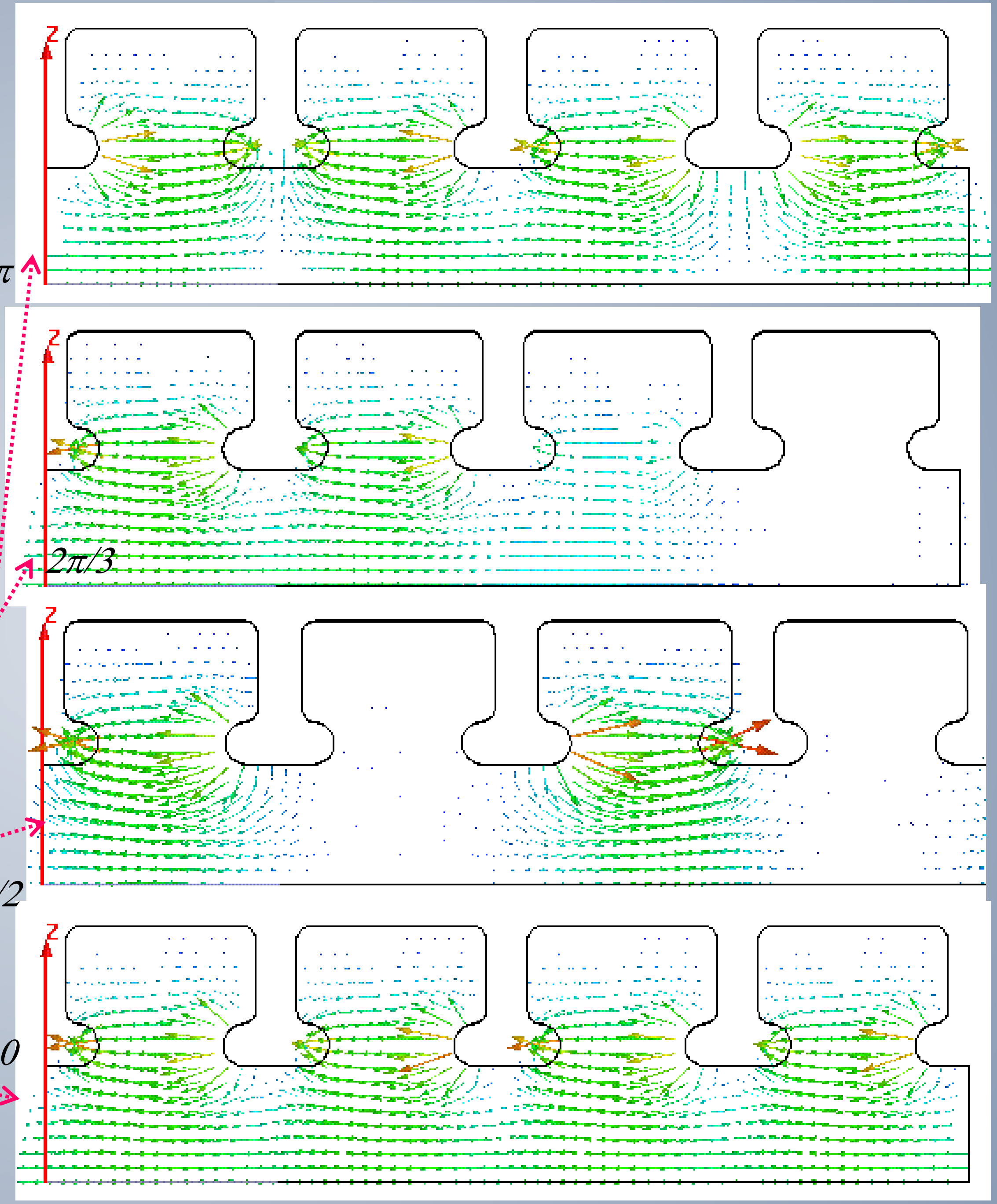
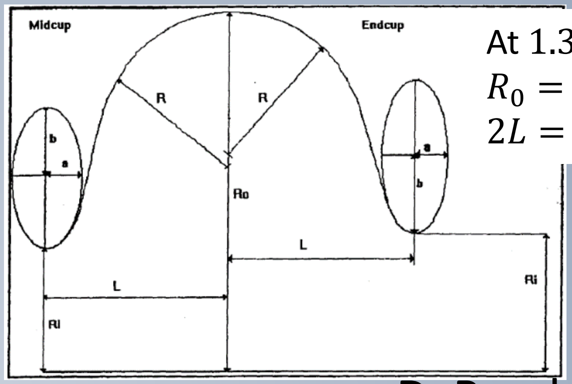



Photo: Reidar Hahn

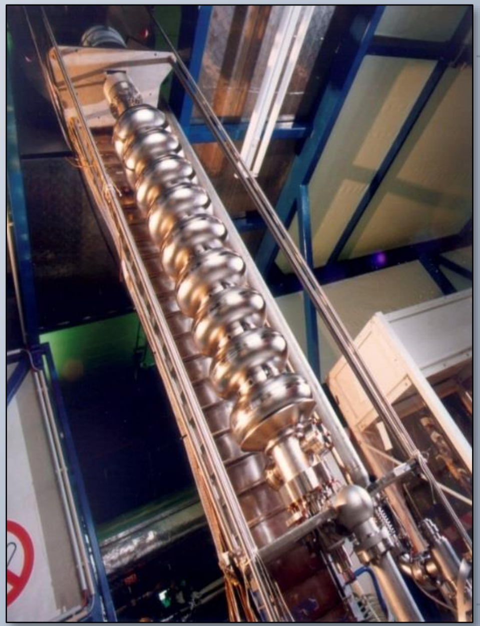
The elliptical cavity

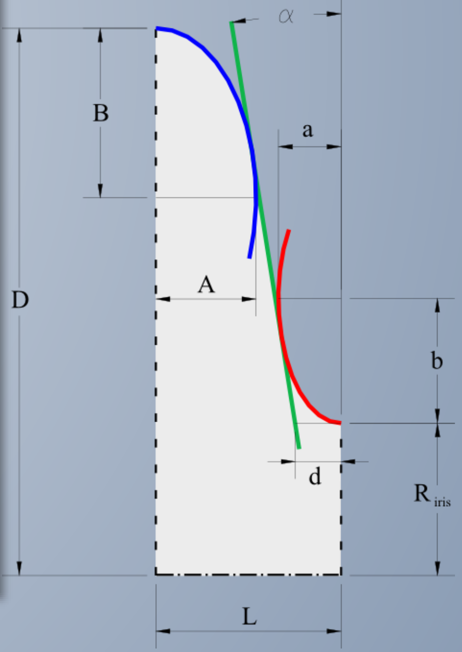
- The elliptical shape was found as optimum compromise between
  - maximum gradient ($E_{acc}/E_{\text{surface}}$)
  - suppression of multipactor
  - mode purity
  - machinability
- A multi-cell elliptical cavity is typically operated in π-mode, i.e. cell length is exactly $\beta\lambda/2$.
- It has become de facto standard, used for ions and leptons! E.g.:
  - ILC/X-FEL: 1.3 GHz, 9-cell cavity
  - SNS (805 MHz)
  - SPL/ESS (704 MHz)
  - LHC (400 MHz)



At 1.3 GHz:
$R_0 = 103.3$ mm,
$2L = 115.3$ mm.

D. Proch, 1993 *)





*): http://accelconf.web.cern.ch/AccelConf/SRF93/papers/srf93g01.pdf

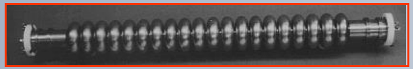

Photo: Reidar Hahn

Elliptical cavity – the *de facto* standard for SRF

FERMI 3.9 GHz

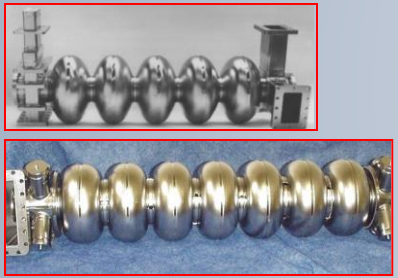
TESLA/ILC 1.3 GHz

S-DALINAC 3 GHz



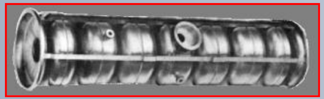
SNS $\beta = 0.61, 0.81,$ 0.805 GHz

CEBAF 1.5 GHz



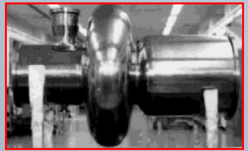
HERA 0.5 GHz

HEPL 1.3 GHz



CESR 0.5 GHz

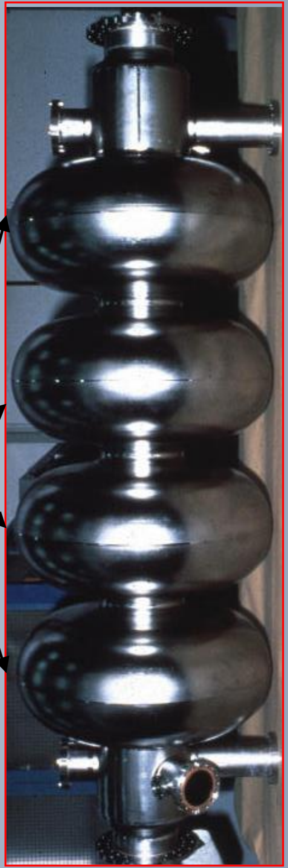
KEK-B 0.5 GHz



TRISTAN 0.5 GHz

LEP 0.352 GHz

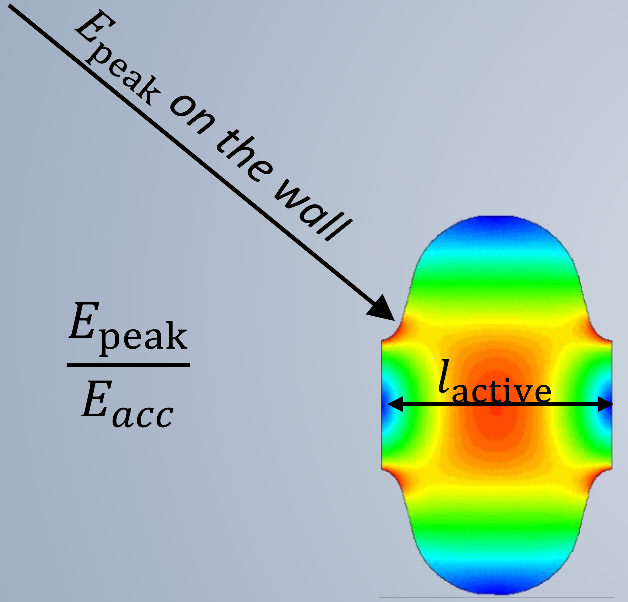
cells



Practical RF parameters 1

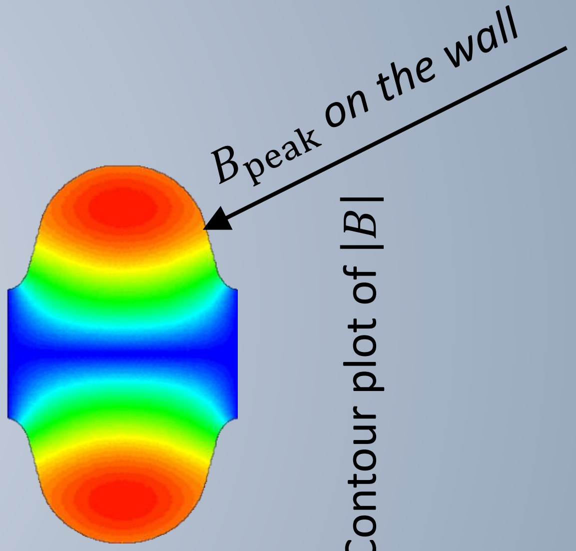
- Average accelerating gradient: $E_{acc} = \frac{\sqrt{\omega W (R/Q)}}{l_\mathrm{active}}$

$$\frac{E_\mathrm{peak}}{E_{acc}}$$



Contour plot of $|E|$

$$\frac{B_\mathrm{peak}}{E_{acc}}$$



Contour plot of $|B|$

The ratio shows sensitivity of the shape to the **field emission** of electrons.

The ratio shows limit in E_{acc} due to the breakdown of superconductivity (**quench**, Nb: $\approx$ 190 mT).

courtesy: Jacek Sekutovicz/DESY

Practical RF parameters 2

$$G \cdot (R/Q)$$

- Both G and R/Q are purely geometric parameters.
- Like the shunt impedance R, the product $G \cdot (R/Q)$ is a measure of the power loss for given acceleration voltage V_{acc} and surface resistance R_s.

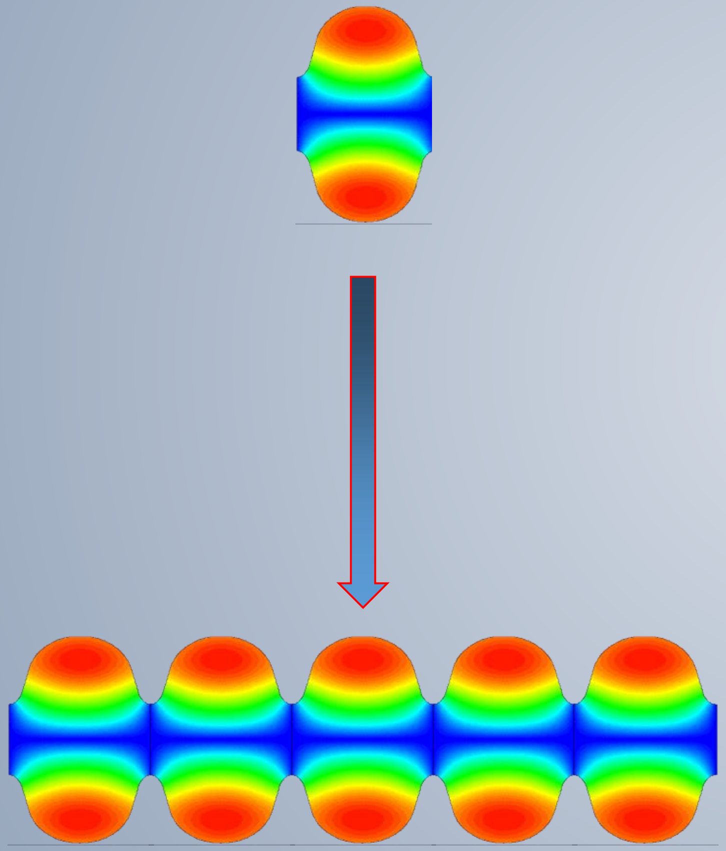
Minimize R_s:
operation at lower T,
better surface cleanliness,
lower residual resistance

$$P_{\text{loss}} = \frac{|V_{acc}|^2 R_s}{2\,\underbrace{G \cdot (R/Q)}}$$

Optimize geometry maximizing $G \cdot (R/Q)$.

courtesy: Jacek Sekutovicz/DESY

Single-cell versus multi-cell cavities



- Advantages of single-cell cavities:
  - It is easier to manage HOM damping
  - There is no field flatness problem.
  - Input coupler transfers less power
  - They are easy for cleaning and preparation

- Advantages of multi-cell cavities:
  - much more acceleration per meter!
  - better use of the power ($R \rightarrow n\ R$)
  - more cost-effective for most applications

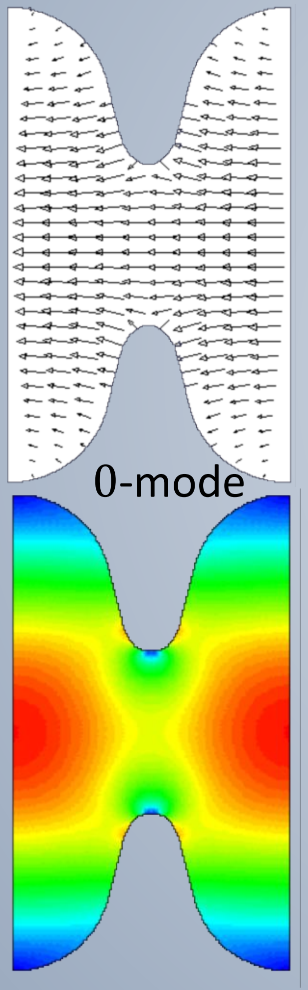
courtesy: Jacek Sekutovicz/DESY

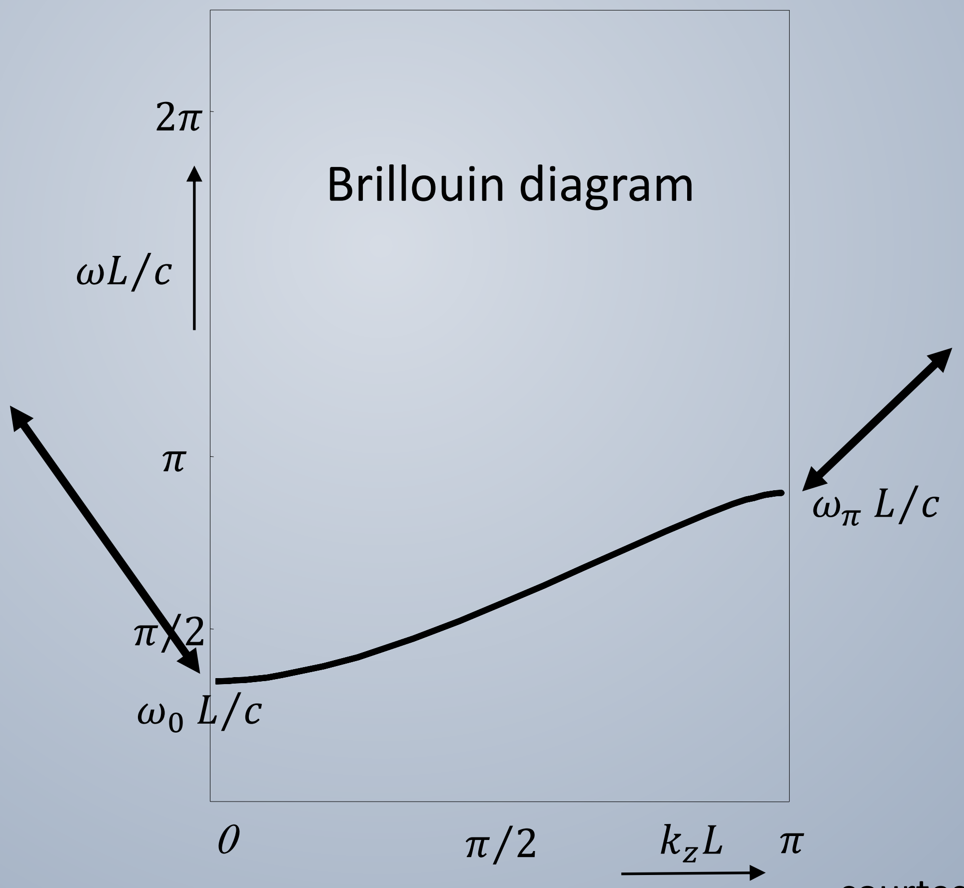
Photo:
Reidar Hahn

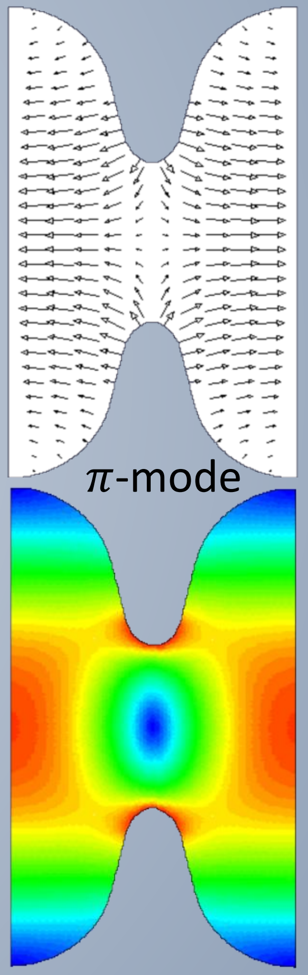
Practical RF parameters 3

- **Cell-to-cell coupling** k_{cc} will determine the width of the passbands in multi-cell cavities.

$$k_{cc} = 2\frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0}$$



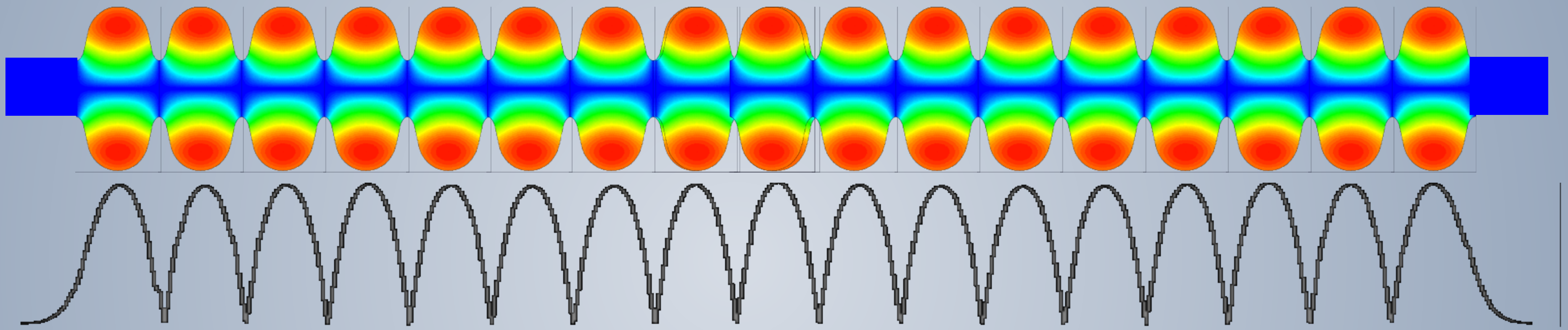




courtesy: Jacek Sekutovicz/DESY

Field flatness

- Field amplitude variation from cell to cell in a multi-cell structure
- Should be small for maximum acceleration.



- Field flatness sensitivity factor a_{ff} for a structure made of N cells:

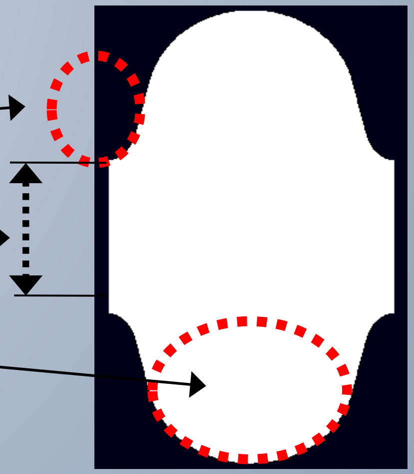
$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i}$$

a_{ff} is related to the cell-to-cell coupling as $a_{ff} = \frac{N^2}{k_{cc}}$ and describes the sensitivity of the field flatness on the errors in individual cells.

courtesy: Jacek Sekutovicz/DESY

Criteria for Cavity Design (1)

- Here: Inner cells of multi-cell structures
- Parameters for optimization:
  - Fundamental mode: $\frac{R}{Q}$, G, $\frac{E_{\text{peak}}}{E_{acc}}$, $\frac{B_{\text{peak}}}{E_{acc}}$, k_{cc}.
  - Higher order modes: $k_{\perp}$, k_z.
- The elliptical cavity design has distinct advantages:
  - easy to clean (rinse)
  - little susceptible to MP – can be conditioned …
- Geometric parameters for optimization:
  - iris ellipse half axes: a, b:
  - iris aperture radius: r_i,
  - equator ellipse half axes: A, B
- Problem: 7 parameters to optimize, only 5 to play with – some compromise has to be found!



courtesy: Jacek Sekutovicz/DESY

Criteria for Cavity Design (2)

Criterion	RF parameter	Improves if	examples
high gradient operation	E_{peak}/E_{acc} B_{peak}/E_{acc} $\downarrow$	r_i $\downarrow$	TESLA, CEBAF 12 GeV HG
low cryogenic losses	$\frac{R}{Q} \cdot G$ $\uparrow$	r_i $\downarrow$	CEBAF LL
High I_{beam}	$k_{\perp}, k_z$ $\downarrow$	r_i $\uparrow$	B-factory RHIC cooling LHeC

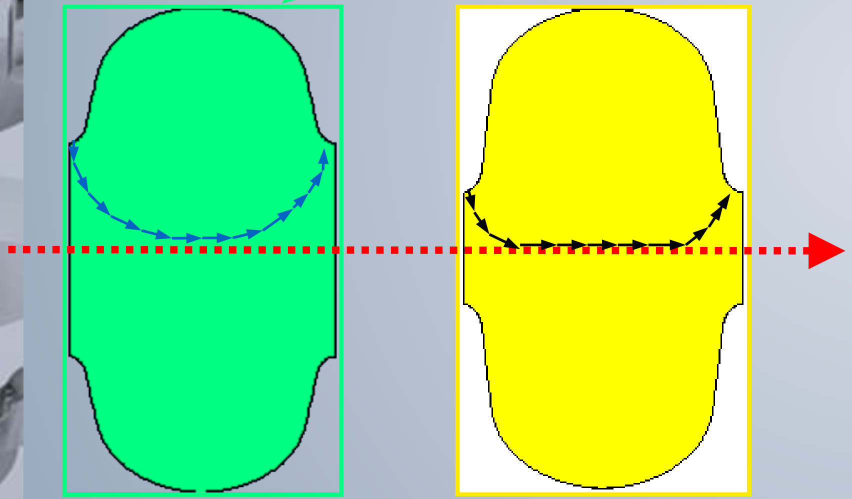
We see here that r_i is a very “powerful” variable to trim the RF-parameters of a cavity.

Of course it has to fit the aperture required for the beam!

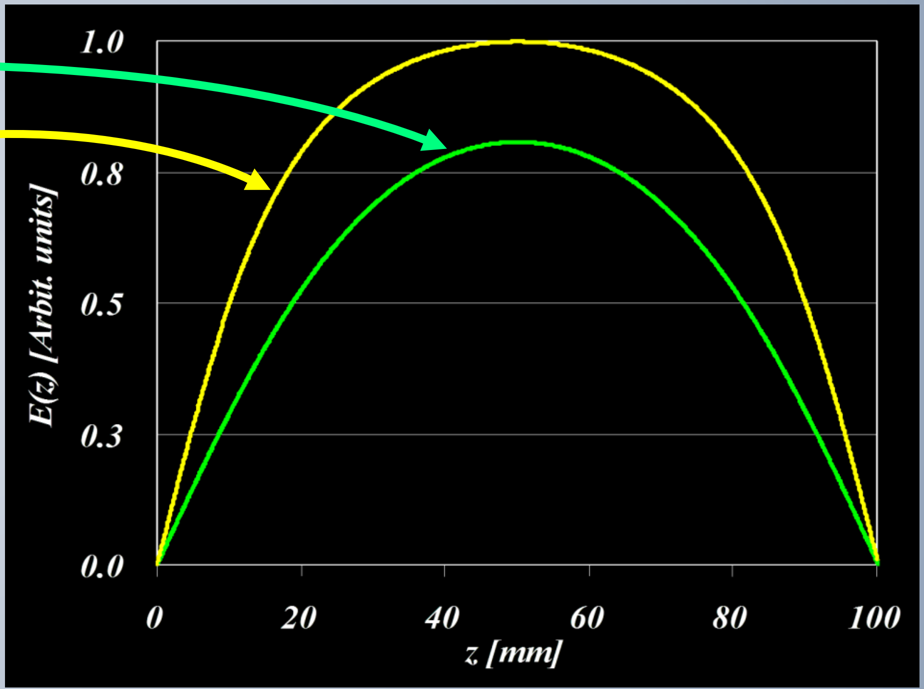
courtesy: Jacek Sekutovicz/DESY

Effect of r_i

- Smaller r_i allows to concentrate E_z where it is needed for acceleration



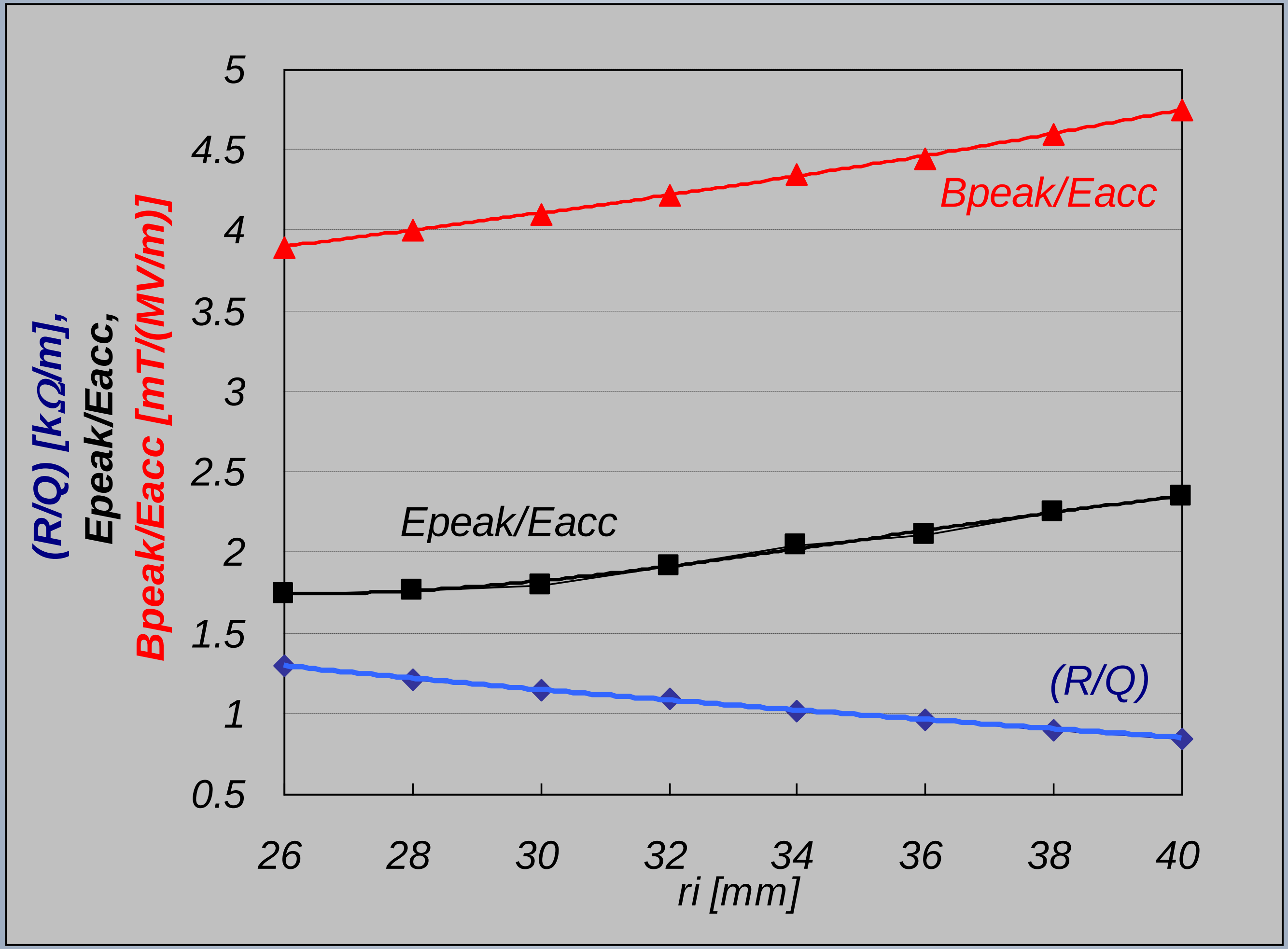
$r_i = 40\ \mathrm{mm}$ $\qquad$ $r_i = 20\ \mathrm{mm}$



$E_z(z)$ for small and big iris radius

courtesy: Jacek Sekutovicz/DESY

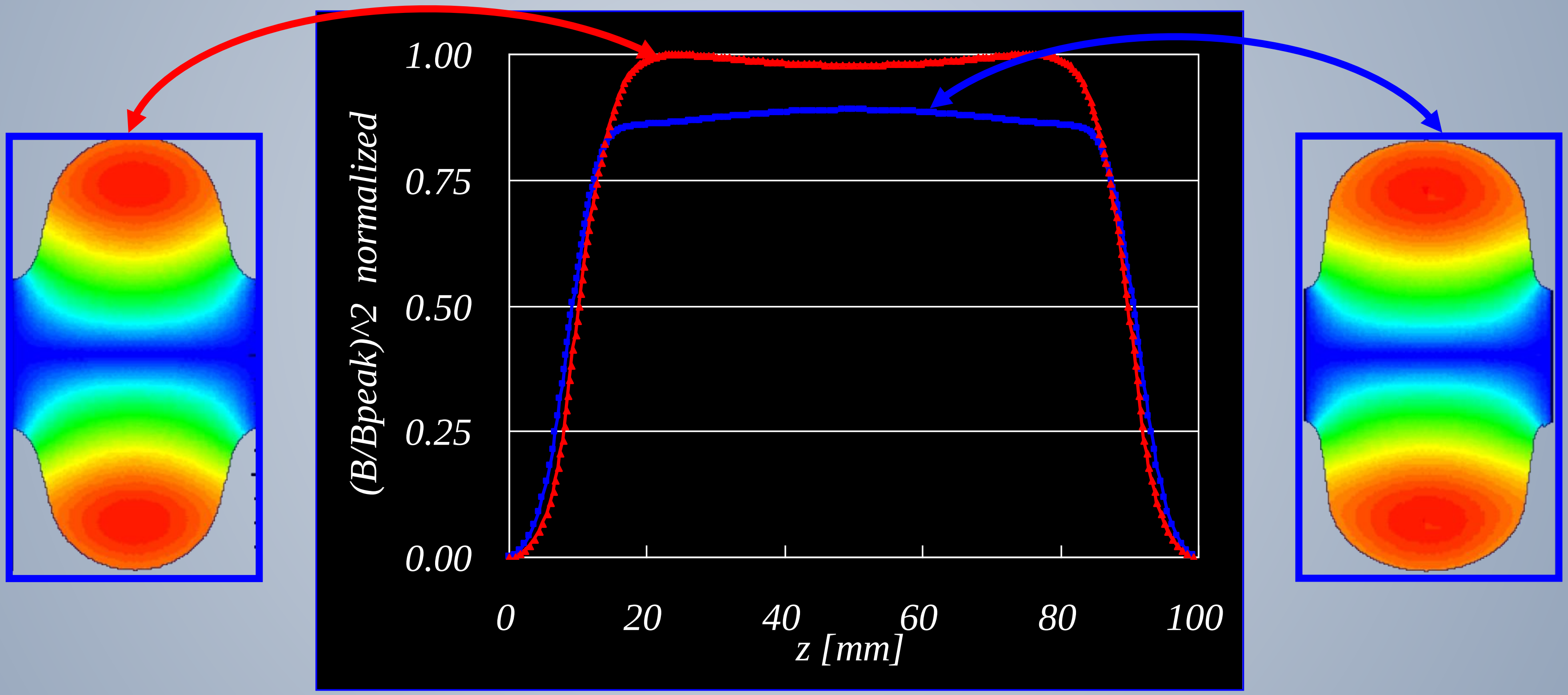
Example: cell optimization at 1.5 GHz



A. Mosnier, E. Haebel, SRF Workshop 1991

Equator shape optimization

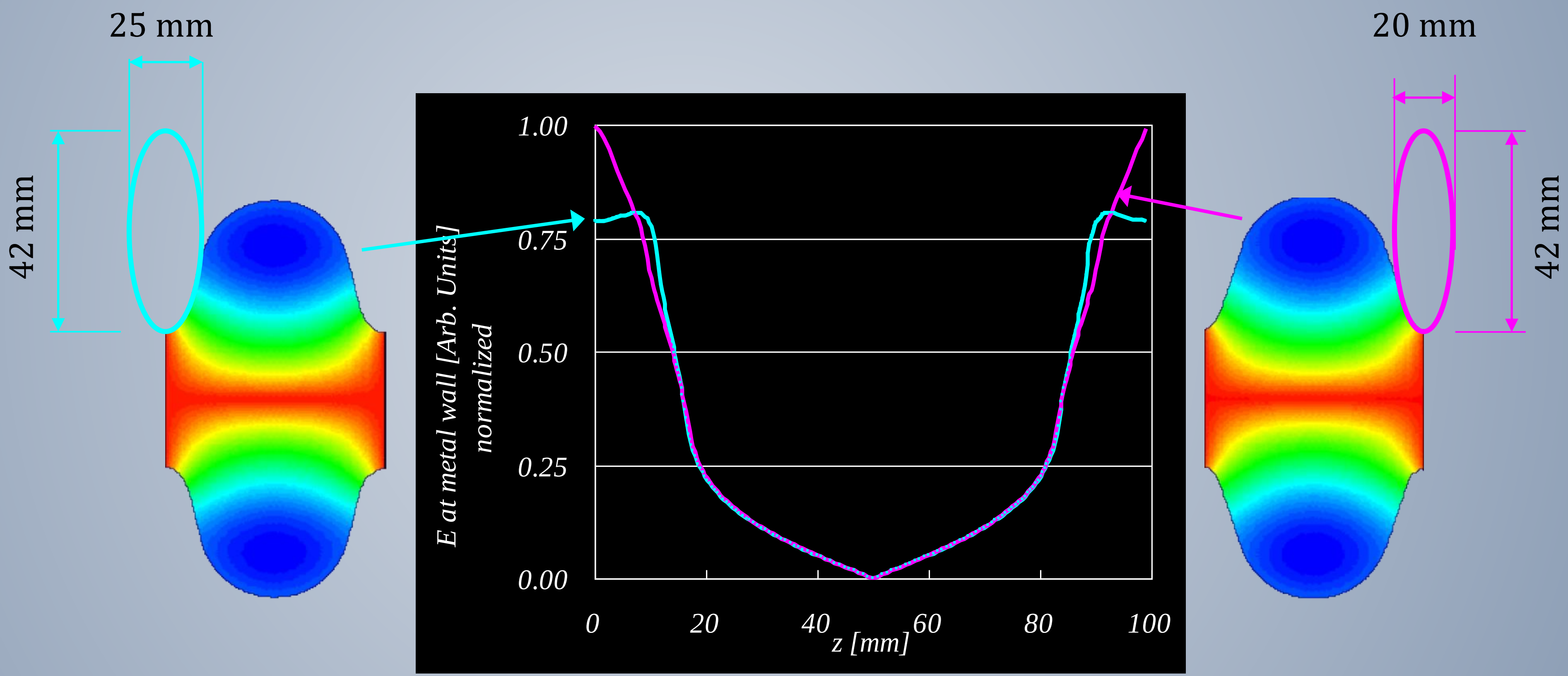
- B_{peak}/E_{acc} (and G) change when changing the equator shape.



courtesy: Jacek Sekutovicz/DESY

Iris shape optimization

- E_{peak}/E_{acc} changes with the iris shape

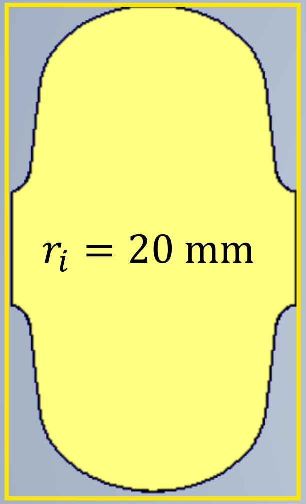


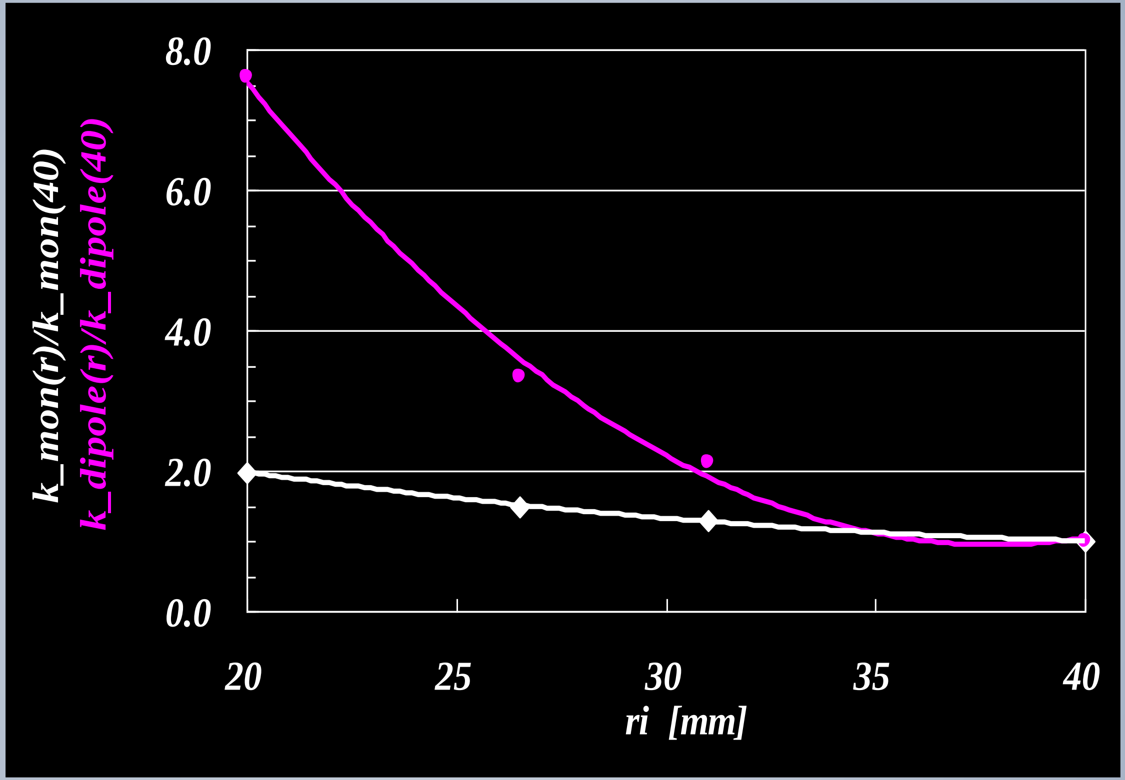
Both cells have the same: f_0, R/Q, and r_i.

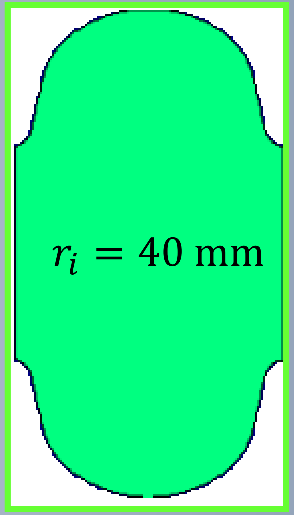
courtesy: Jacek Sekutovicz/DESY

Minimizing HOM excitation

HOMs loss factors ($k_{\text{loss},\perp}$, k_{loss})





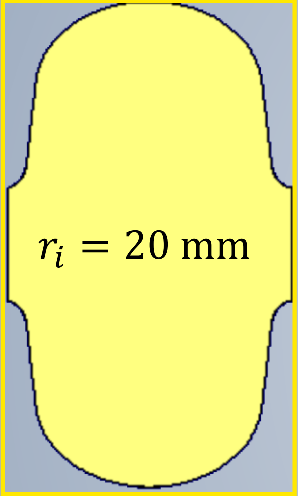


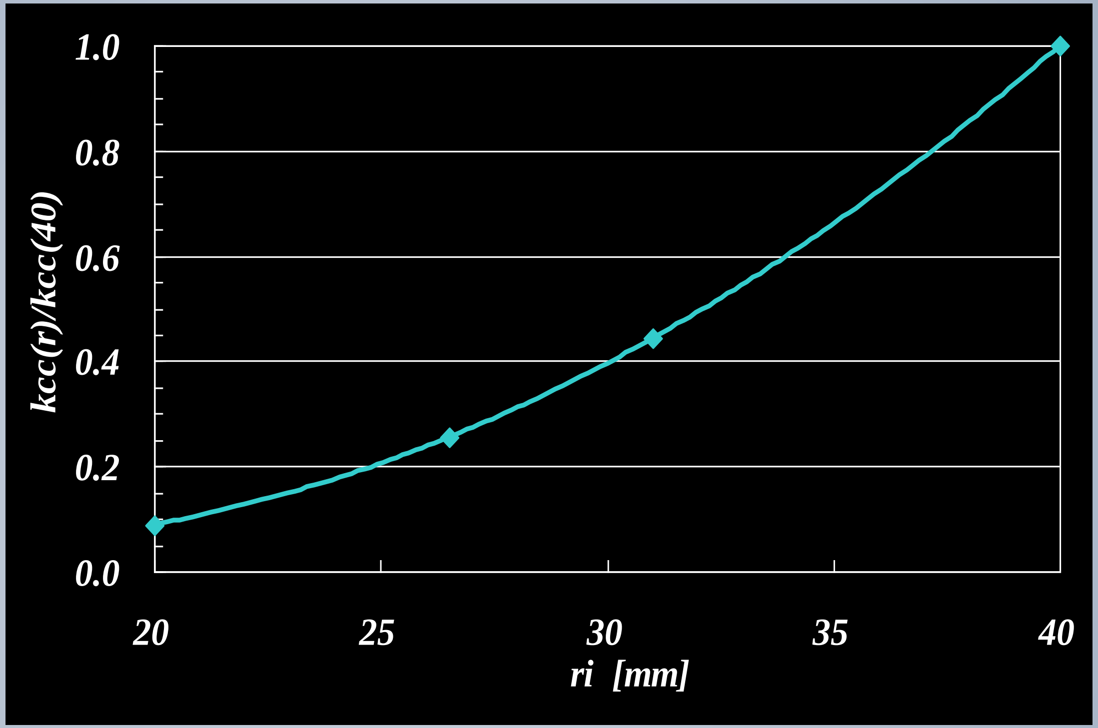
$R/Q = 152\ \Omega$
$B_{\text{peak}}/E_{acc} = 3.5\ \text{mT}/(\text{MV/m})$
$E_{\text{peak}}/E_{acc} = 1.9$

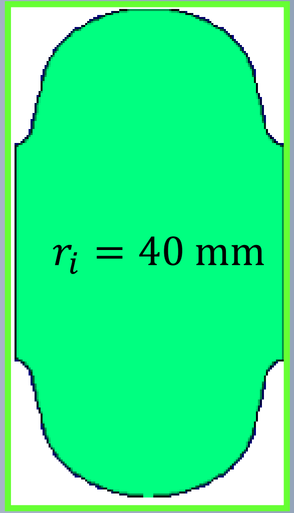
$R/Q = 86\ \Omega$
$B_{\text{peak}}/E_{acc} = 4.6\ \text{mT}/(\text{MV/m})$
$E_{\text{peak}}/E_{acc} = 3.2$

courtesy: Jacek Sekutovicz/DESY

Cell-to-cell coupling k_{cc}







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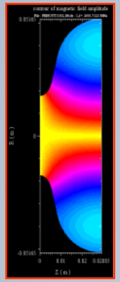
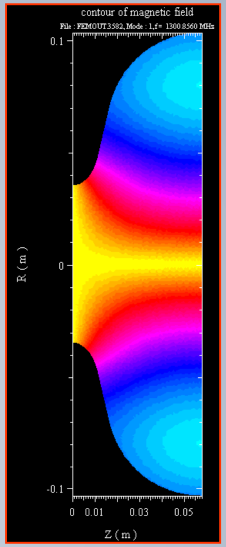
$B_{\mathrm{peak}}/E_{acc} = 4.6\,\mathrm{mT}/(\mathrm{MV/m})$
$E_{\mathrm{peak}}/E_{acc} = 3.2$

courtesy: Jacek Sekutovicz/DESY

Scaling the frequency



$\times\, \mathbf{2}$ $=$



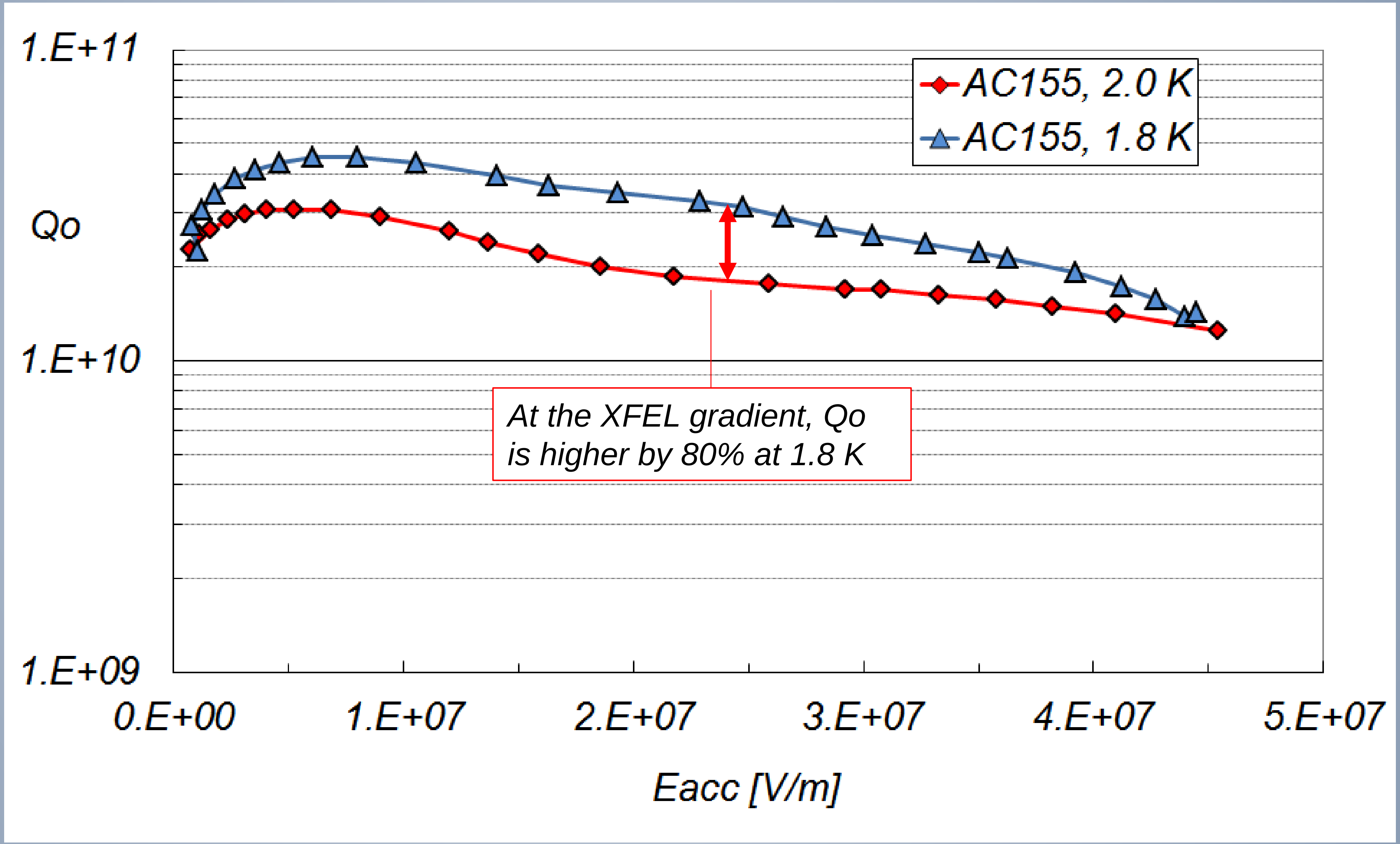
f_π	[MHz]	2600
R/Q	[Ω]	57
r/Q	[Ω/m]	2000
G	[Ω]	271

f_π	[MHz]	1300
R/Q	[Ω]	57
r/Q	[Ω/m]	1000
G	[Ω]	271

$r/Q = (R/Q)/l \propto f$ (or $(R/Q)/\lambda = \text{const}$)

courtesy: Jacek Sekutovicz/DESY

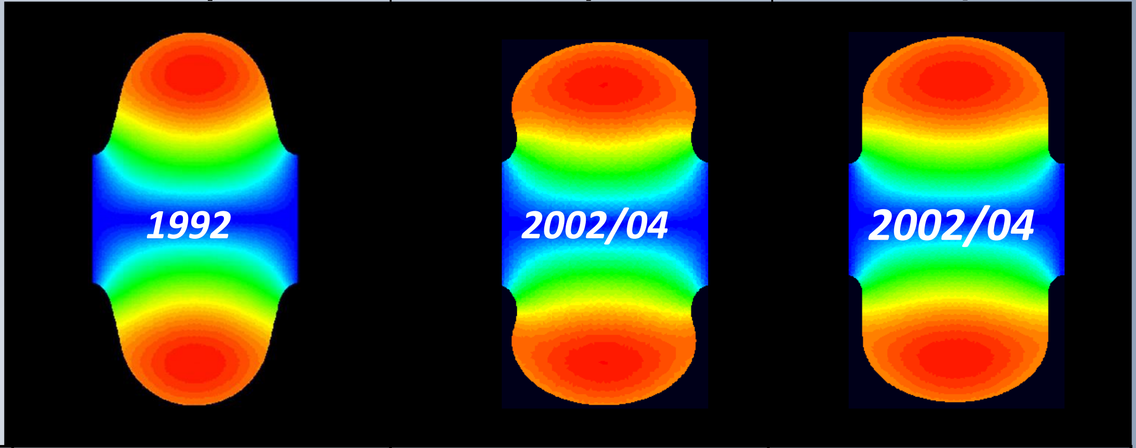
Operating temperature



courtesy: Jacek Sekutovicz/DESY

Historic evolution of inner cell geometry

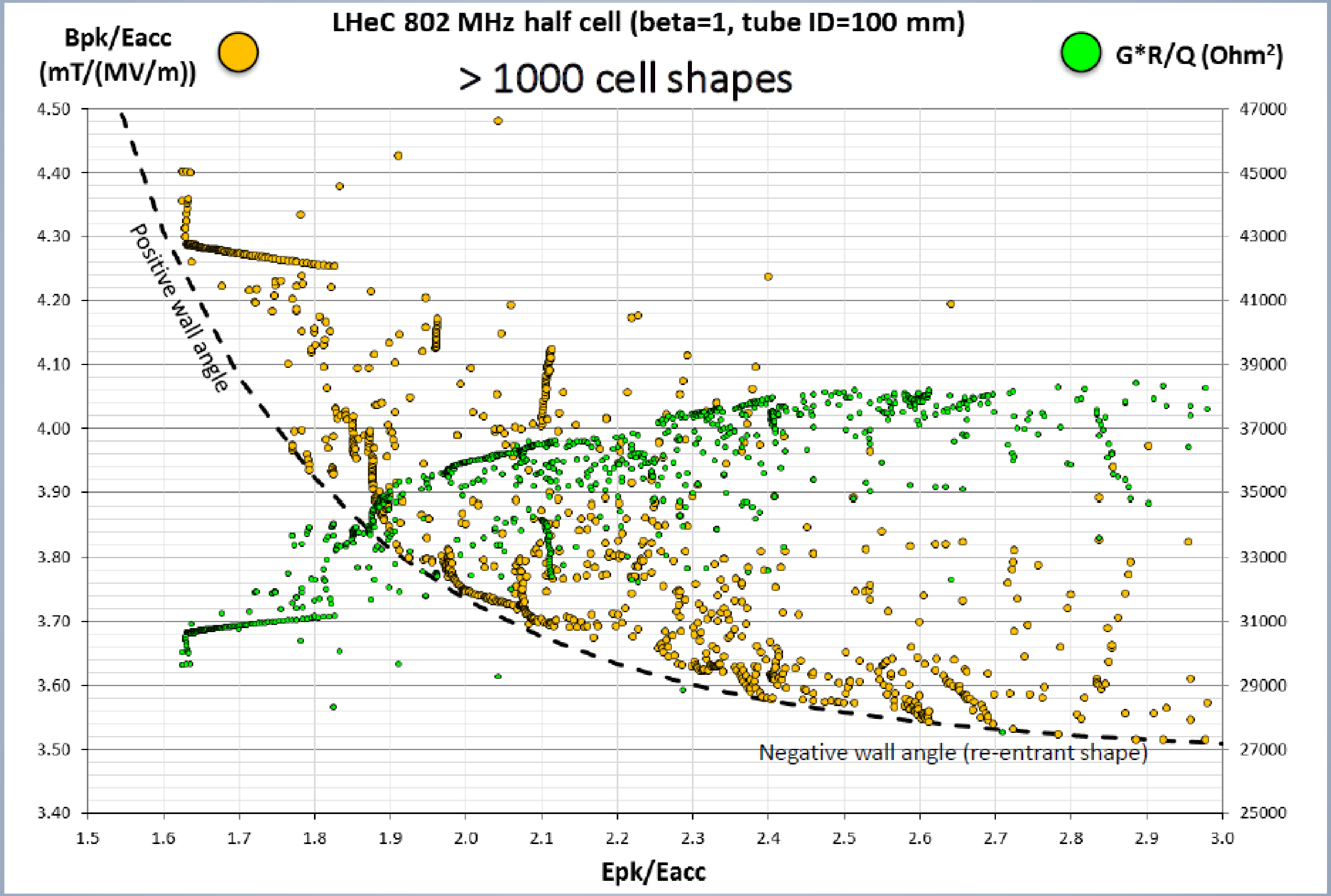
Example: ILC



		TESLA, optimized E_{peak}/E_{acc}	Re-entrant, optimized B_{peak}/E_{acc}	Low-loss (LL) optimized B_{peak}/E_{acc}
r_i	[mm]	35	30	30
k_{cc}	[%]	1.9	1.56	1.52
$E_{\mathrm{peak}}/E_{acc}$	-	1.98	2.30	2.36
$B_{\mathrm{peak}}/E_{acc}$	[mT/(MV/m)]	4.15	3.57	3.61
R/Q	[Ω]	113.8	135	133.7
G	[Ω]	271	284.3	284
$R/Q \cdot G$	[Ω*Ω]	30840	38380	37970
$k_{\mathrm{loss},\perp}\ (\sigma_z = 1\ \mathrm{mm})$	[V/pC/cm²]	0.23	0.38	0.38
$k_{\mathrm{loss}}\ (\sigma_z = 1\ \mathrm{mm})$	[V/pC]	1.46	1.75	1.72

courtesy: Jacek Sekutovicz/DESY

Cavity optimization example



courtesy: Frank Marhauser/JLAB

Higher order modes

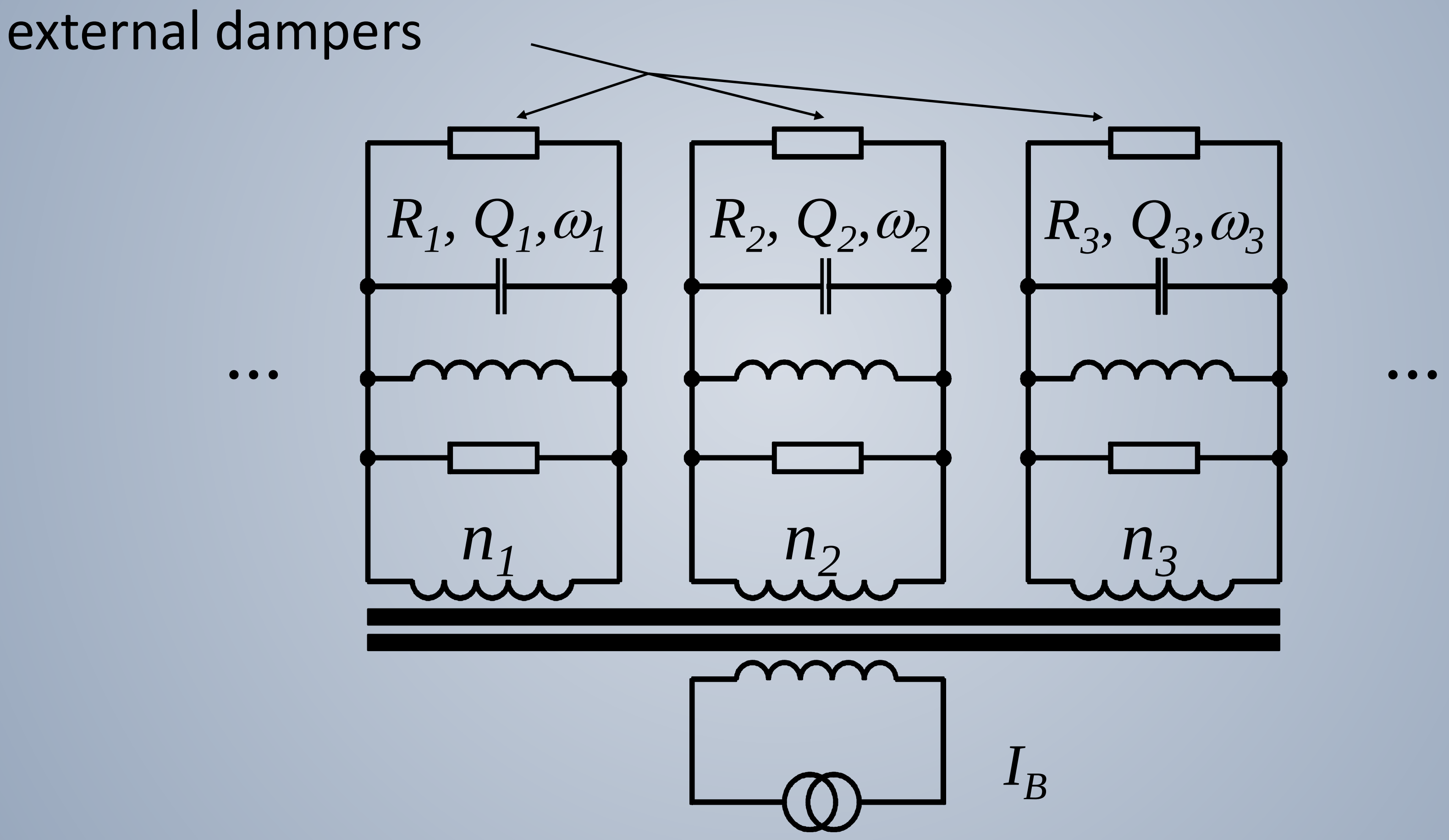
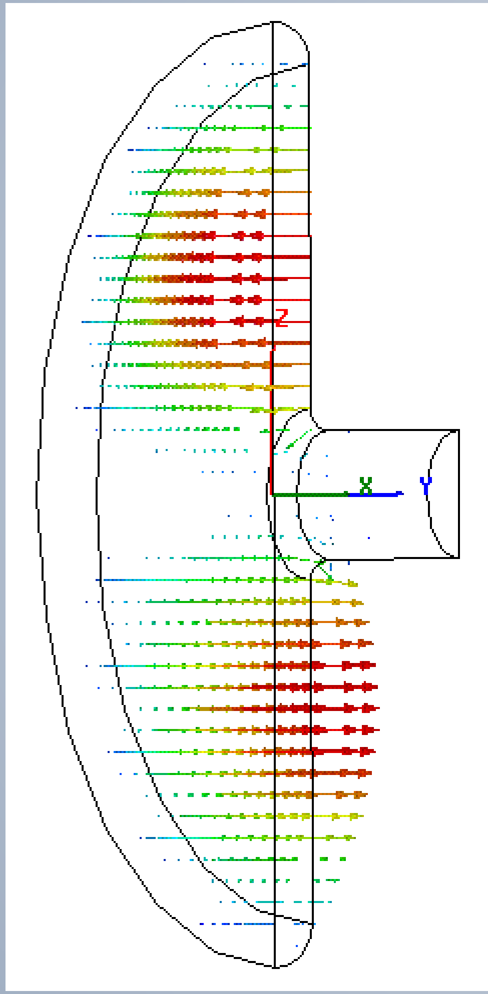


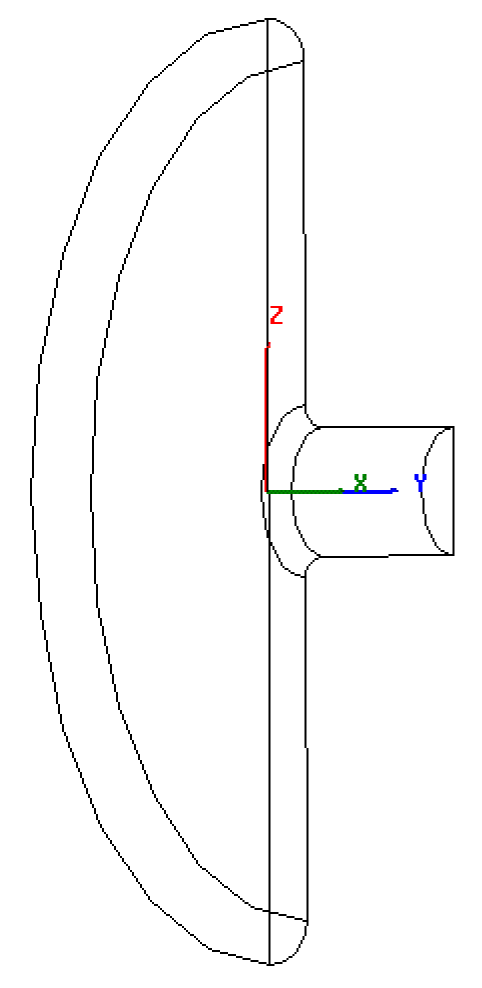
Photo:
Reidar Hahn

Pillbox: 1st dipole mode

TM_{110}-mode (only 1/4 shown)



electric field



magnetic field

Wolfgang Panofsky
1919 – 2007

Panofsky-Wenzel theorem

For particles moving virtually at $v = c$, the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

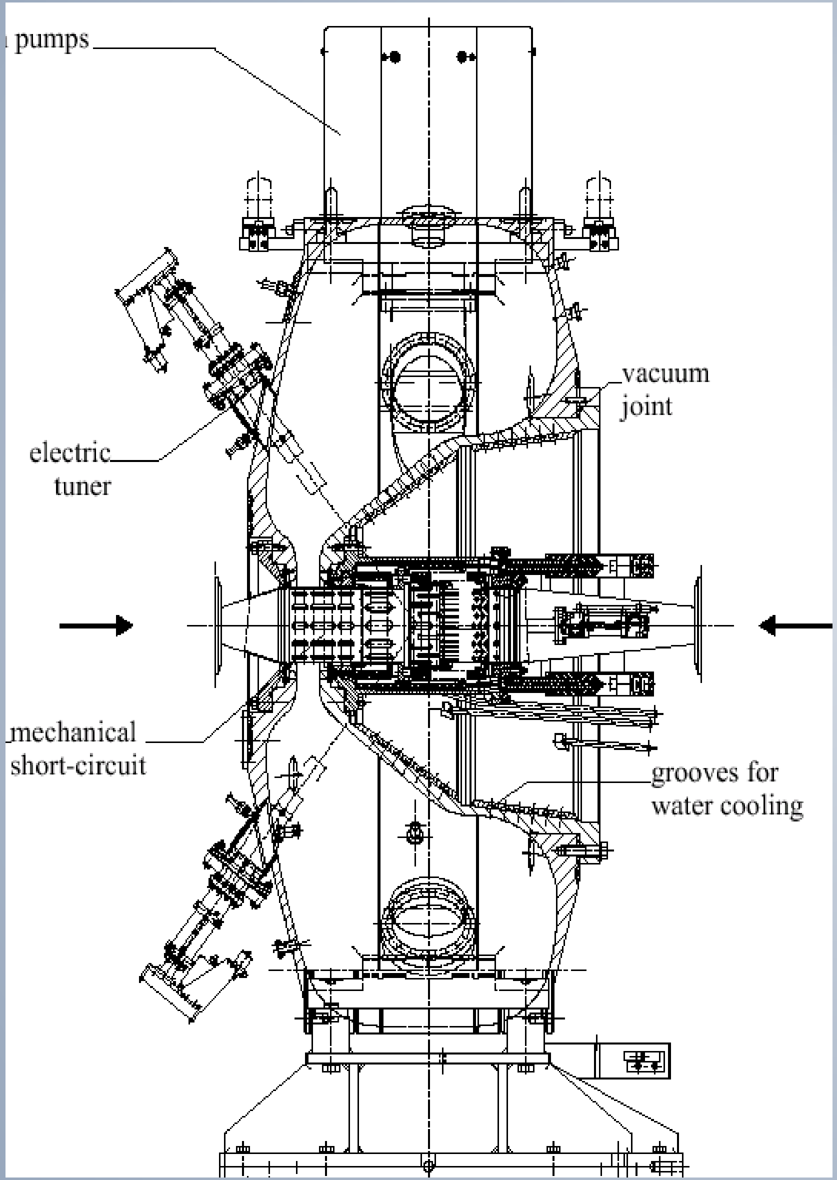
$$j\frac{\omega}{c}\vec{F}_{\perp} = \nabla_{\perp}F_z$$

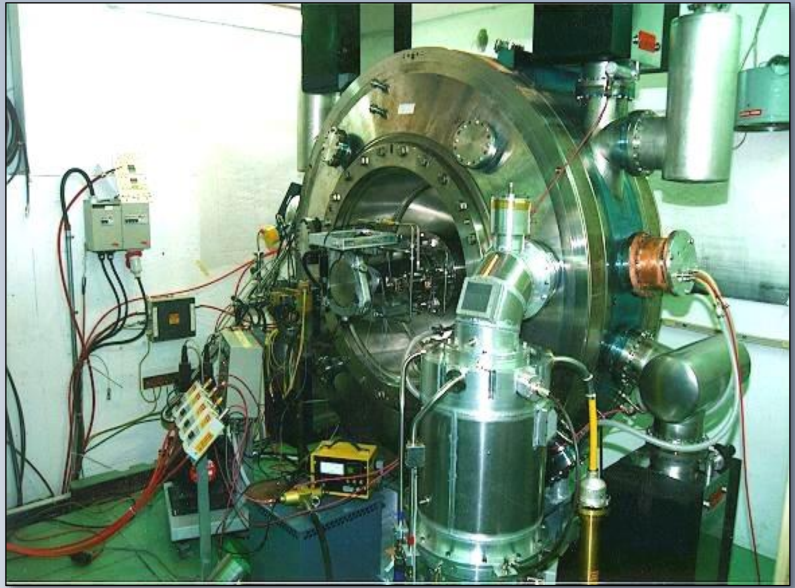
Pure TE modes: No net transverse force!

Transverse modes are characterized by
- the transverse impedance in $\boldsymbol{\omega}$-domain
- the transverse loss factor (kick factor) in $\boldsymbol{t}$-domain!

W.K.H. Panofsky, W.A. Wenzel: "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", RSI **27**, 1957]

CERN/PS 80 MHz cavity (for LHC)



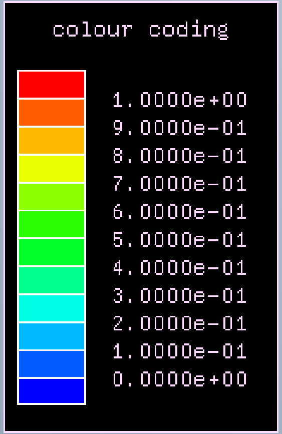


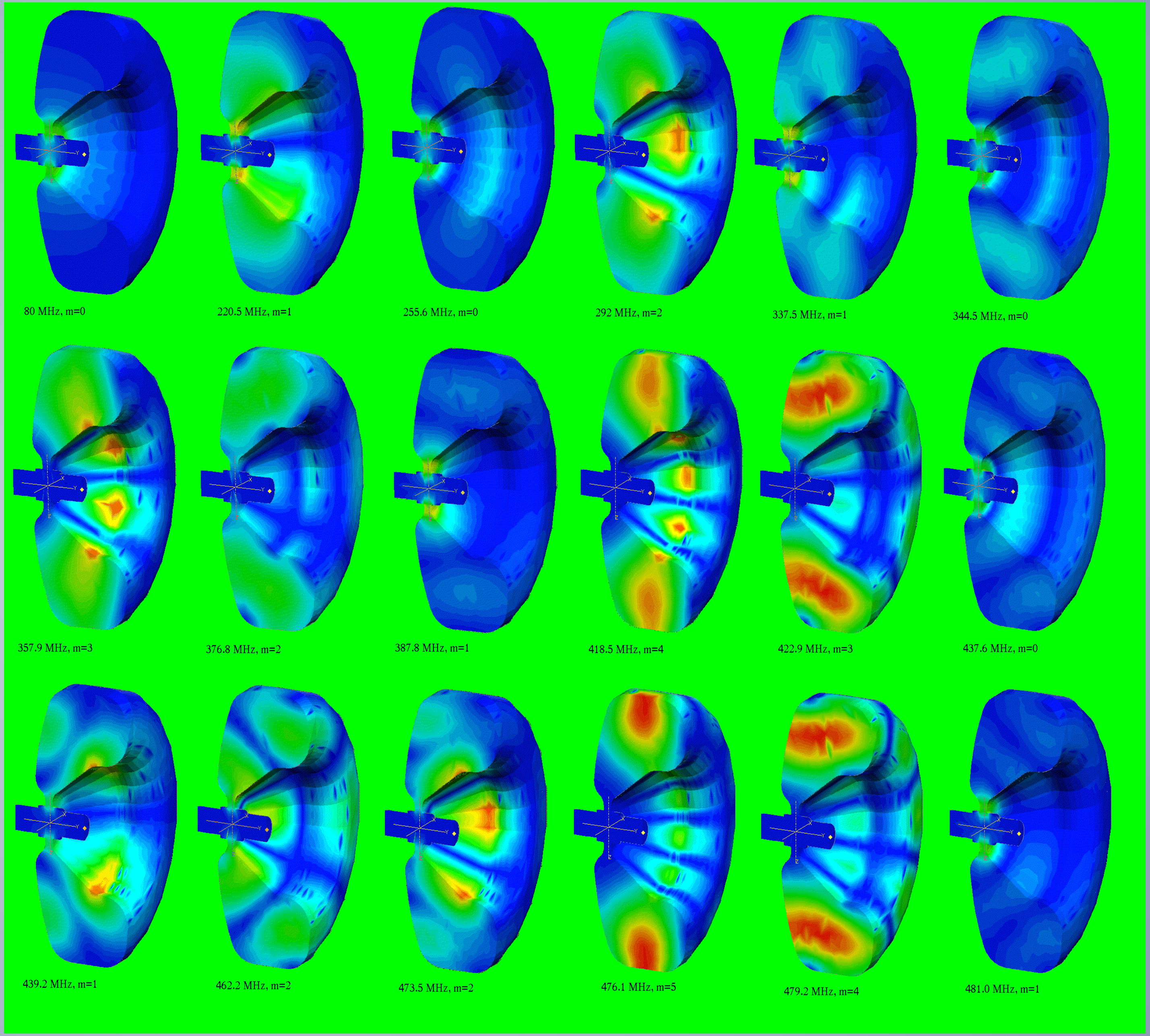
Higher order modes

Example shown: 80 MHz cavity PS for LHC.

Color-coded:

$$\left|\vec{E}\right|$$





Higher order modes (measured spectrum)

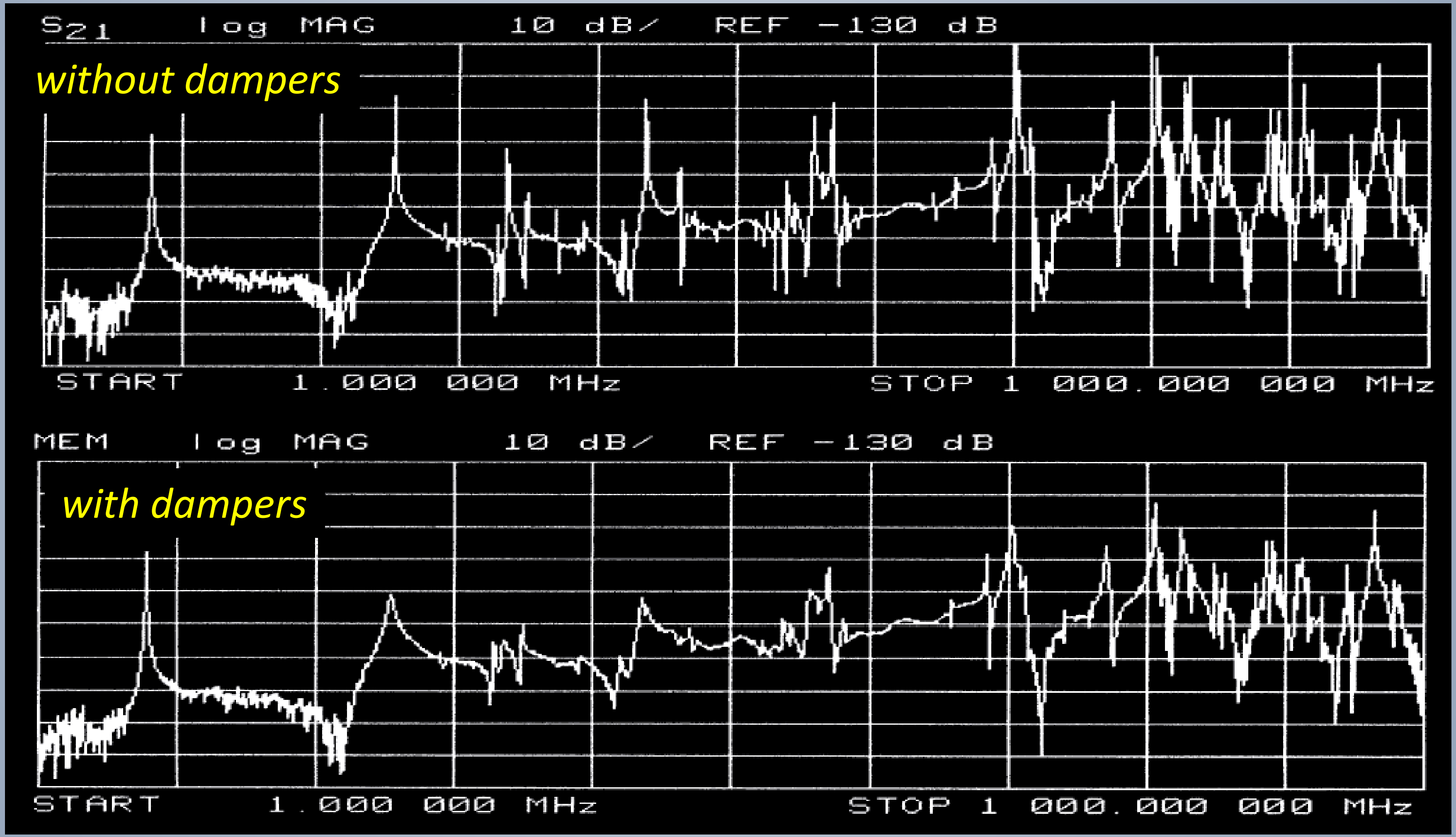
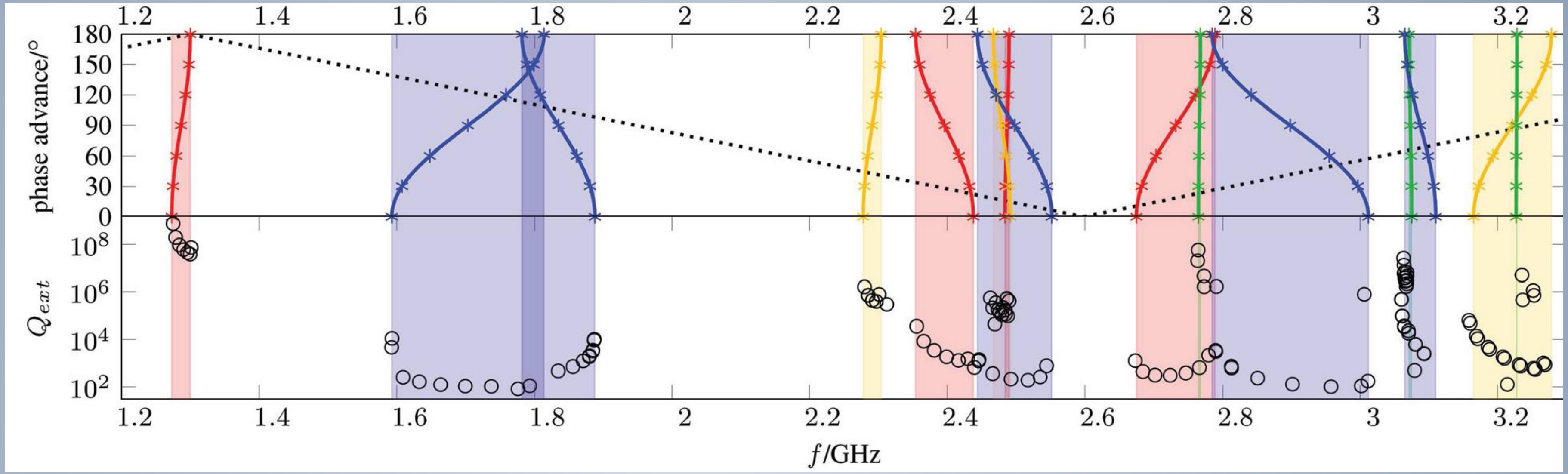


Photo:
Reidar Hahn

7-cell 1.3 GHz structure for bERLinPro

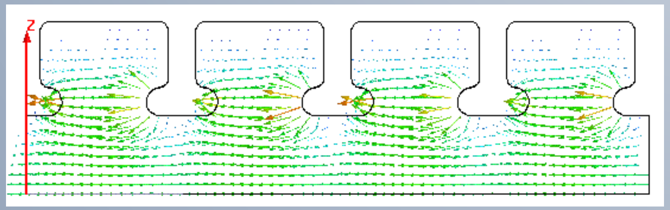


Band diagram (top) and Q-factors (bottom)

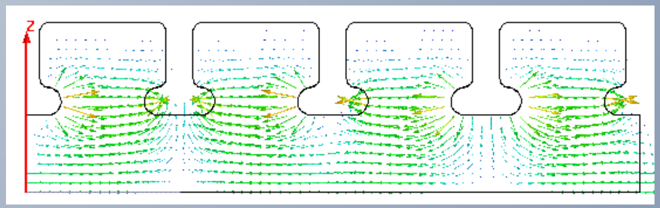
Galek et al.: IPAC2013

Reminder:

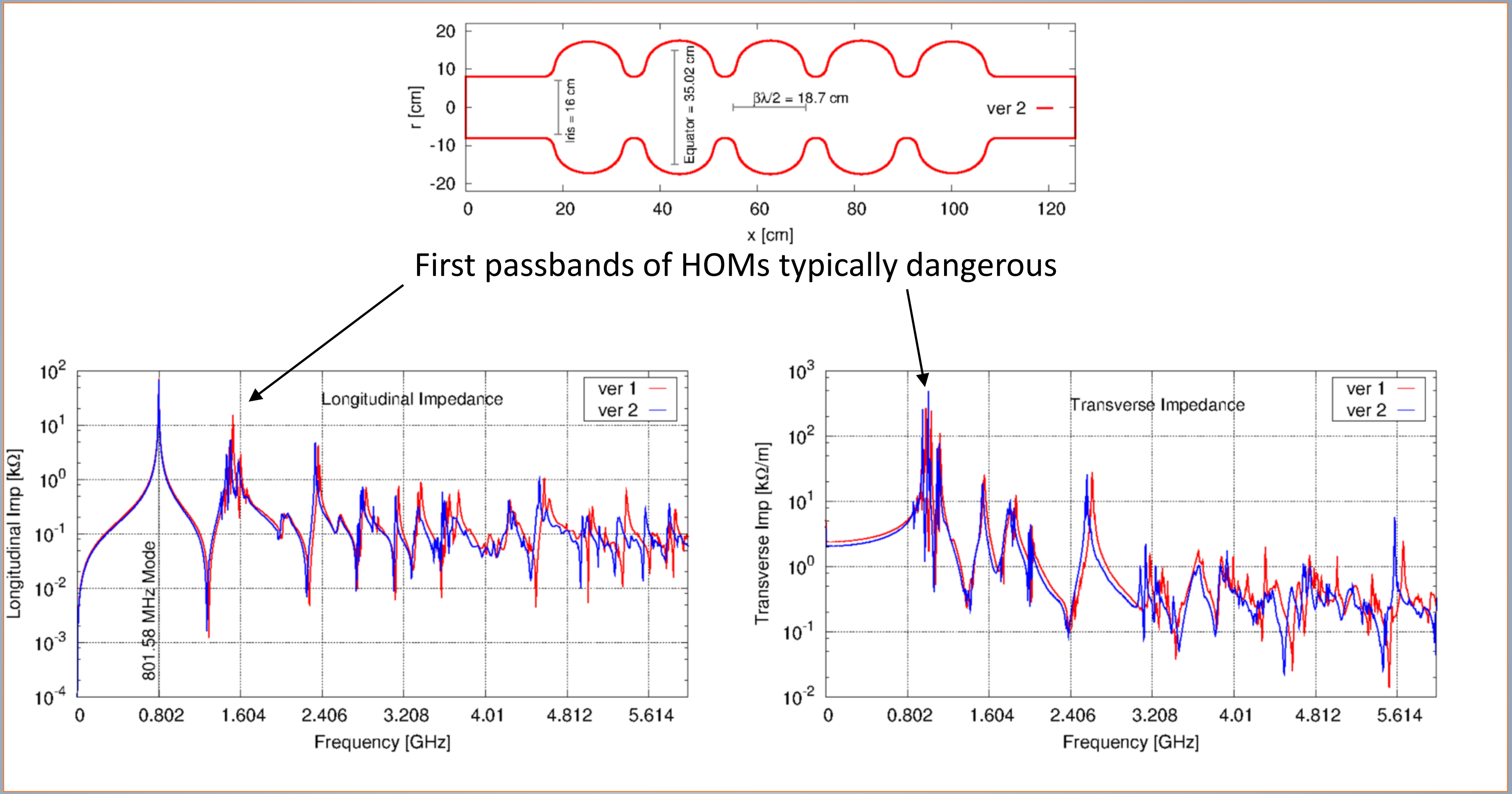
0-mode



π-mode



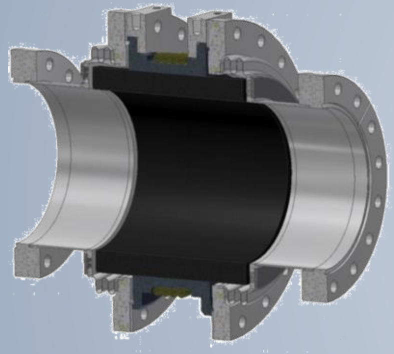
HOMs: Example 5-cell cavity



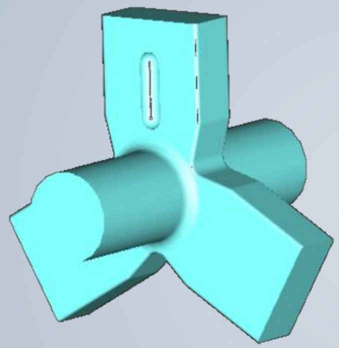
courtesy: Rama Calaga/CERN

HOM dampers

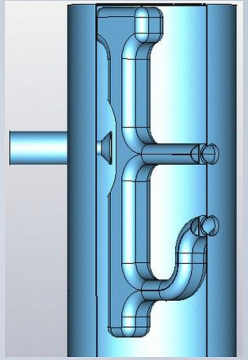
- Ferrite absorbers: broadband damper, room temperature
- Waveguides: better suited for higher frequencies (size!)
- Notch filters: narrow-band; target specific mode



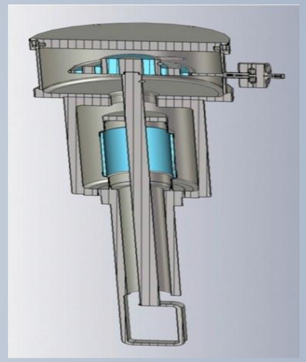
ferrite absorber



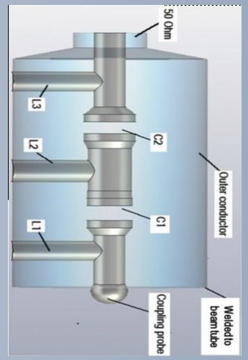
waveguides



notch filter



bandpass filter



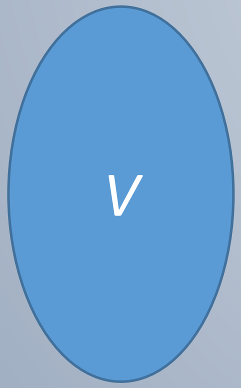
double notch

- Multi-cell cavities require broadband dampers
- More tomorrow by Eric Montesinos!

Tuners

Small boundary perturbation

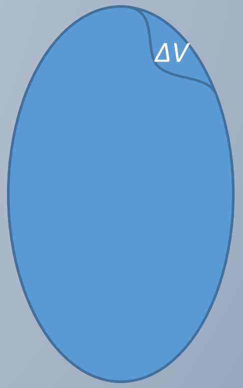
- Perturbation calculation is used to understand the basics for cavity tuning – it is used to analyse the sensitivity to (small) surface geometry perturbations.
  - This is relevant to understand the effect of fabrication tolerances.
  - Intentional surface deformation or introduced obstacles can be used to tune the cavity.
- The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).
- The result of this calculation leads to a convenient expression for the (de)tuning:



unperturbed: ω_0

$$\frac{\omega-\omega_0}{\omega}=\frac{\iiint_{\Delta V}\left(\mu_0|H_0|^2-\varepsilon|E_0|^2\right)dV}{\iiint_{V}\left(\mu_0|H_0|^2+\varepsilon|E_0|^2\right)dV}$$

Slater's Theorem

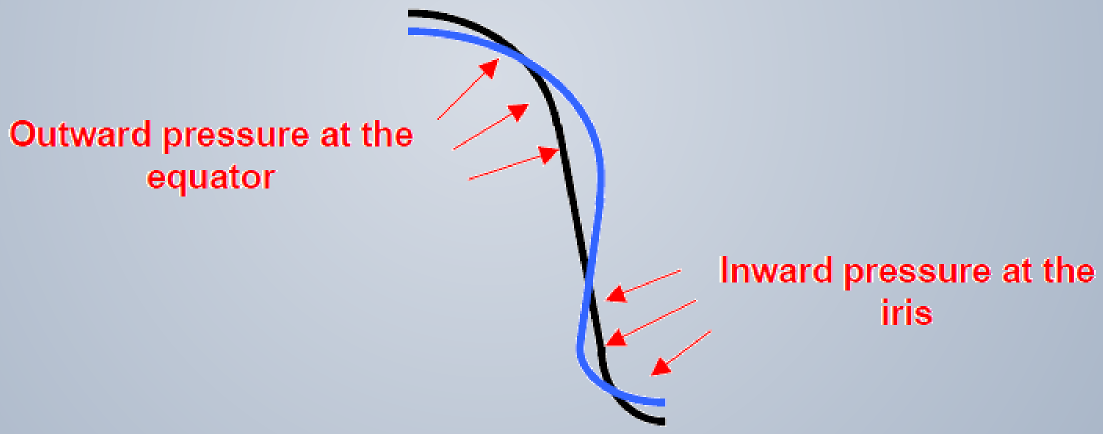


perturbed: ω

John C. Slater
1900 – 1976

Lorentz force detuning (“*LFD*”)

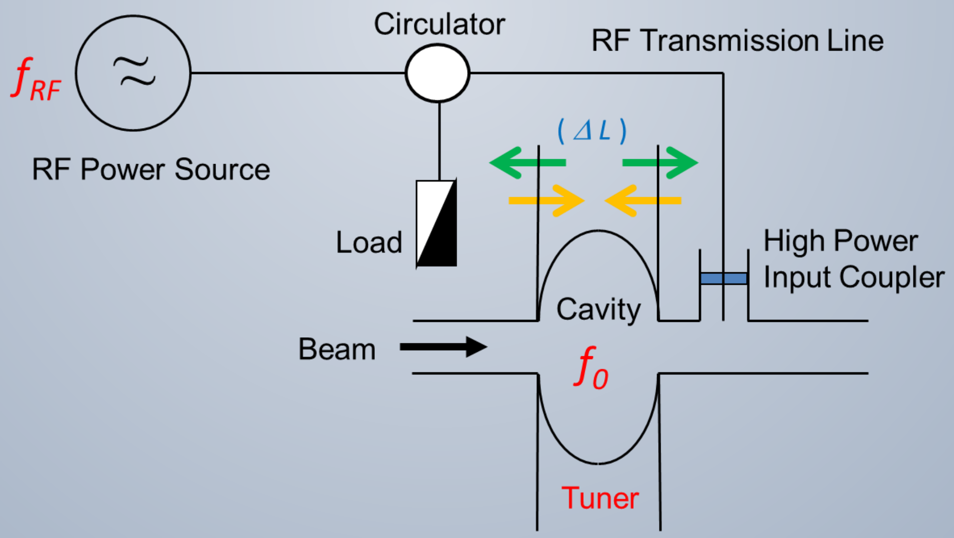
- The presence of electromagnetic fields inside the cavity lead to a mechanical pressure on the cavity.
- Radiation pressure: $P = \frac{1}{4}(\mu_0|H|^2 - \varepsilon_0|E|^2)$
- Deformation of the cavity shape:



- Frequency shift: $\Delta f = K\ L\ |E_{acc}|^2$; typical: $K\ L \approx -(1 \ldots 10)\mathrm{Hz}/\left(\frac{\mathrm{MV}}{\mathrm{m}}\right)^2$
- This requires good stiffness – and the possibility to tune rapidly!

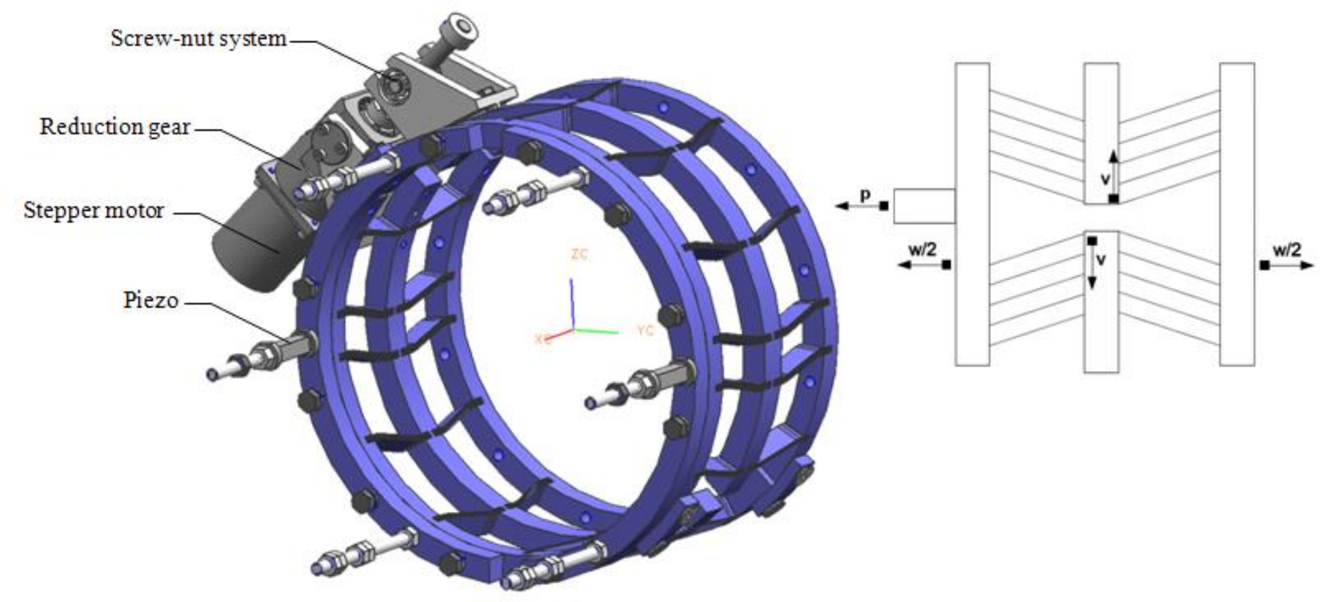
Tuner principle

- Slow tuners:
  - compensate for mechanical tolerances,
  - realized with stepper motor drives
- Fast tuners:
  - compensate Lorentz-force detuning and reactive beam loading
  - realized with piezo crystal (lead zirconate titanate – PZT)
- Tuning of SC cavities is often realized by deforming the cavity:



courtesy: Eiji Kako/KEK

Blade tuner

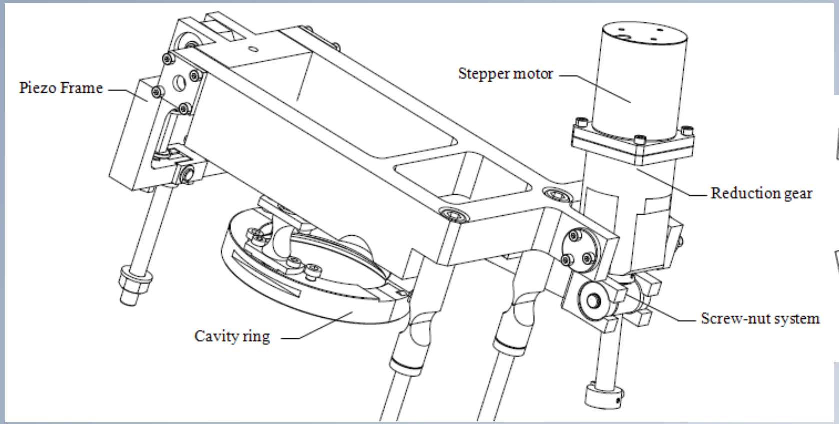


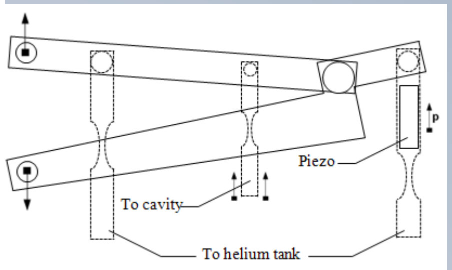
- Developed by INFN Milano
- Azimuthal motion transferred to longitudinal strain
- Zero backlash
- CuBe threaded shaft used for a screw nut system
- Stepping motor and gear combination driver
- Two piezo actuators for fast action
- All components in cold location

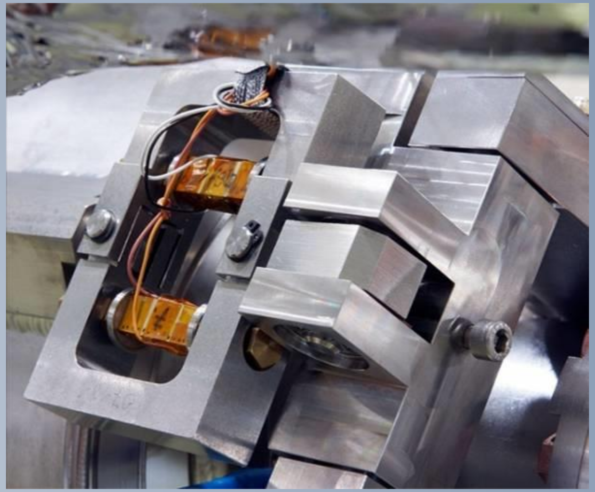
courtesy: Eiji Kako/KEK

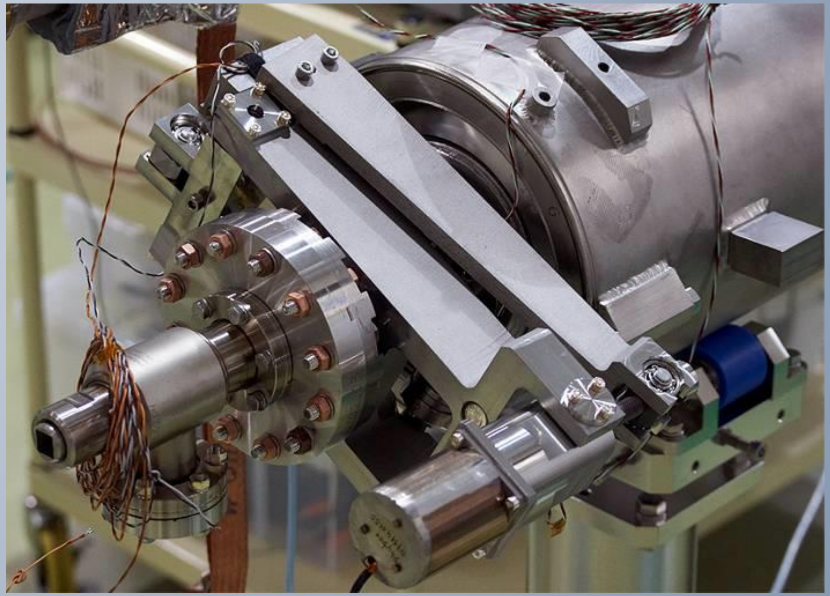
Photo: Reidar Hahn

"Saclay" lever-arm tuner





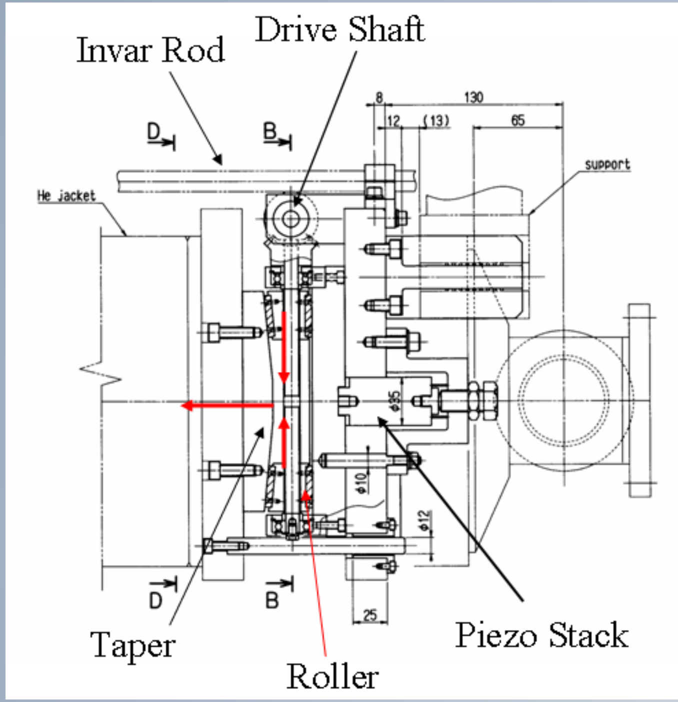


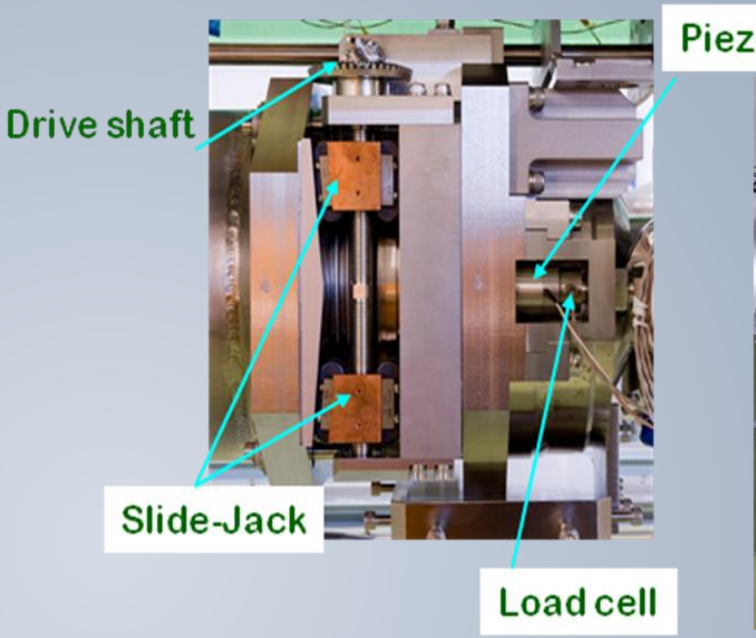


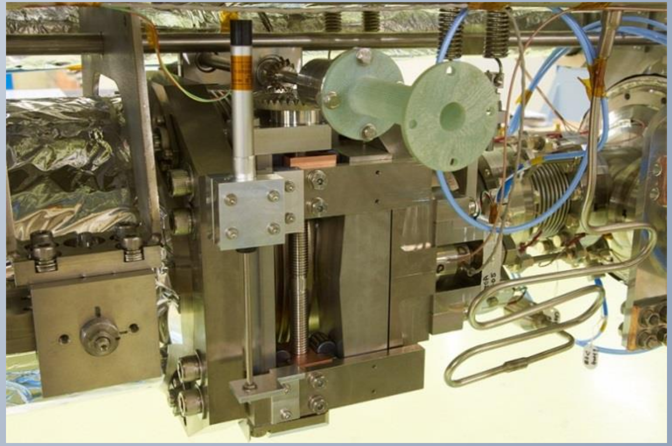
- Developed by DESY based on the Saclay design
- Double lever system (leverage 1.25)
- Cold stepping motor and gear combination
- Screw nut system
- Two piezo actuators for fast action in a preloaded frame
- All components in cold location

courtesy: Eiji Kako/KEK

Slide-jack tuner







- Developed by KEK for STF cryomodule
- Slide-jack mechanism
- Single high voltage piezo actuator for fast action
- Warm stepping motor for easy maintenance
- Access port for replacing piezo

courtesy: Eiji Kako/KEK

End of RF Basics and TM Cavities

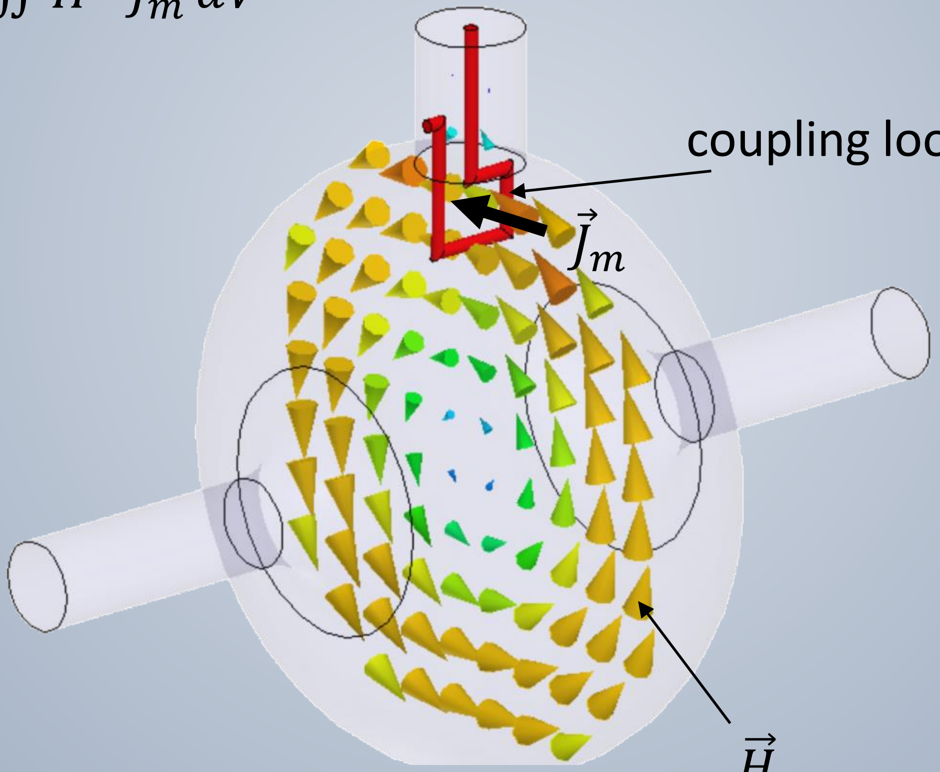
Thank you very much!

FPC

(Fundamental Power Coupler)

Magnetic (loop) coupling

- The magnetic field of the cavity main mode is intercepted by a coupling loop
- The coupling can be adjusted by changing the size or the orientation of the loop.
- Coupling: $\propto \iiint \vec{H} \cdot \vec{J}_m \, dV$

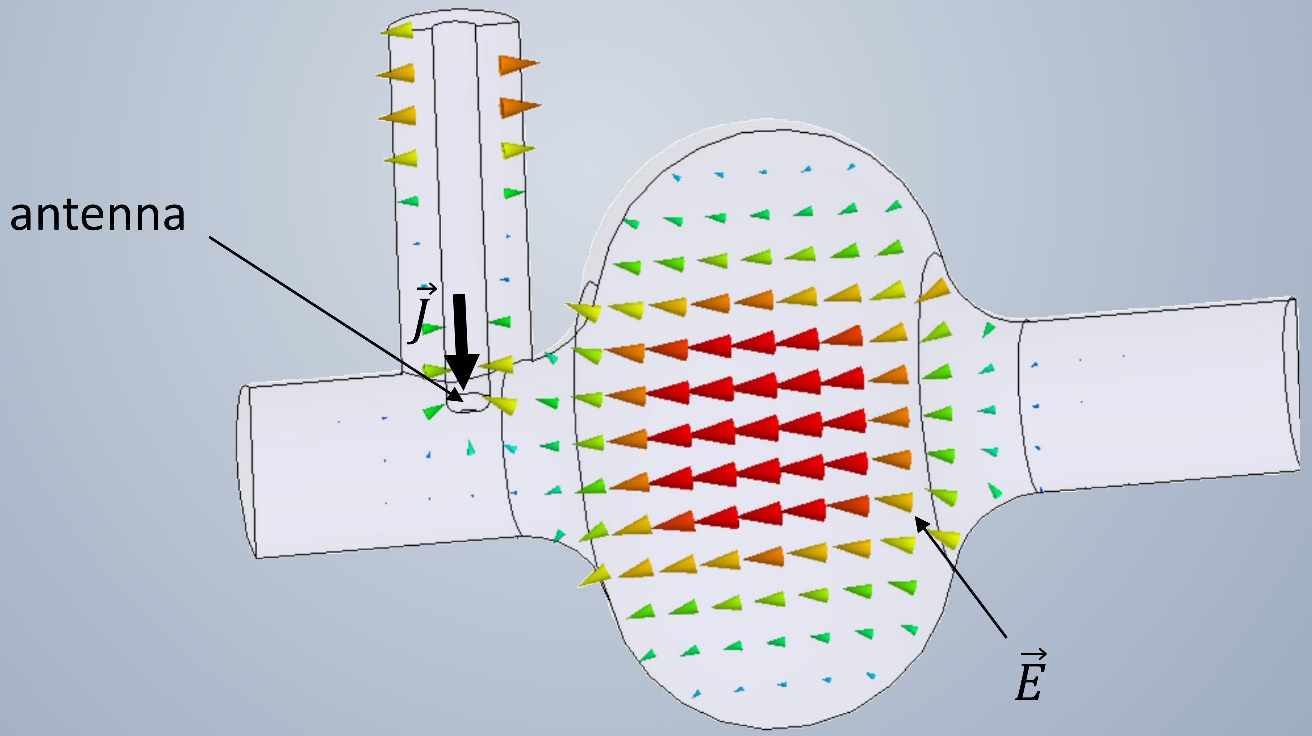


courtesy: David Alesini/INFN

Photo:
Reidar Hahn

Electric (antenna) coupling

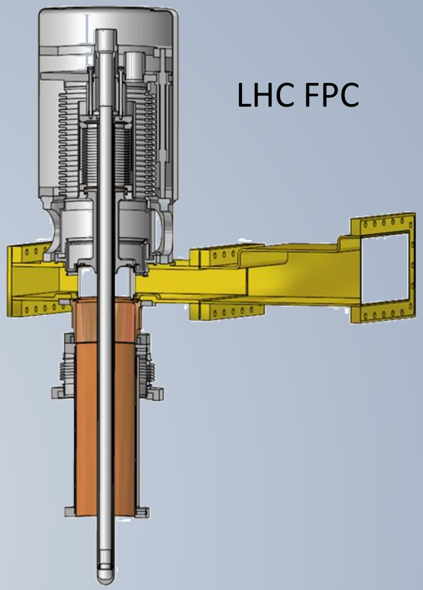
- The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.
- The coupling can be adjusted by varying the penetration.
- Coupling $\propto \iiint \vec{E} \cdot \vec{J} \, dV$

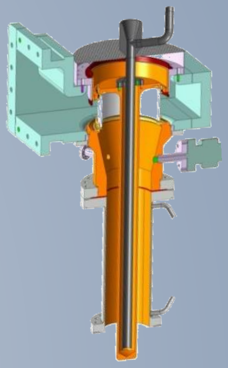


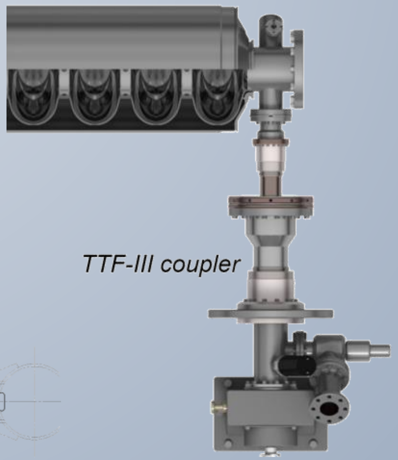
courtesy: David Alesini/INFN

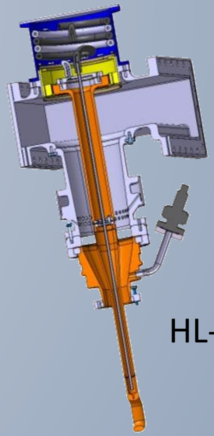
Fundamental Power Coupler – FPC

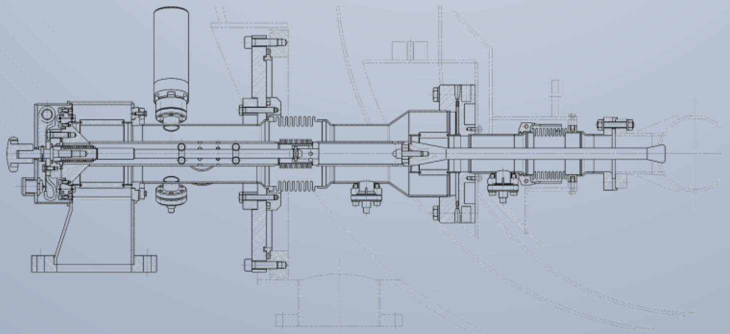
- The **Fundamental Power Coupler** is the connecting part between the RF transmission line and the RF cavity
- It is a specific piece of transmission line that also has to provide the cavity vacuum barrier.
- FPCs are amongst the most critical parts of the RF cavity system in an accelerator!
- A good RF design, a good mechanical design and a high quality fabrication are essential for an efficient and reliable operation.


LHC FPC


SPL FPC


TTF-III coupler


HL-LHC FPC



courtesy: Eric Montesinos/CERN