Magnetic Field Analysis of the MICE Cooling Channel Magnets

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Introduction

Aim to produce a field model of the cooling channel magnets, that matches the data as close possible.

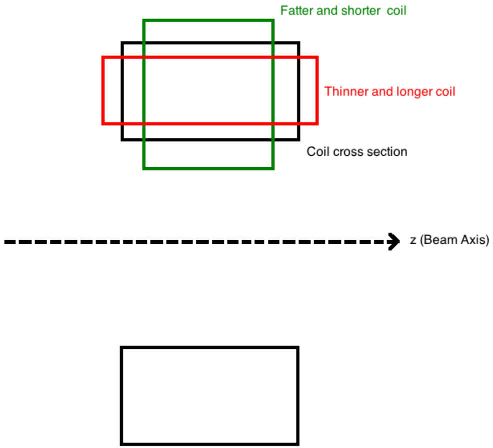
To do this most of the field is represented by the so-called ‘geometrical fit’, which takes into account the coil geometry and rotations. Then the residual field is fitted with a Fourier-Bessel series to account for abnormalities in the field due to things like welds, magnetised iron etc.

The magnetic fields will then be represented by:

$$B_{Model} = B_{Geo} + B_{FB} (\approx B_{Data})$$

Geometrical Fit

The geometrical fit mixes two axial field maps that bracket the aspect ratio of the coil.



The two bracketing fields are produced using a closed current loop model that uses elliptical integrals. This only produces axial fields so $B_\phi = 0$. This is not true for the actual coils since they won't be perfectly aligned with the centre of the bore.

The fit takes 7 parameters:

- Mixing
- Coordinate scaling
- Field scaling
- Rotation angle about y axis (θ_y)
- Rotation angle about x axis (θ_x)
- Offset from y axis
- Offset from x axis

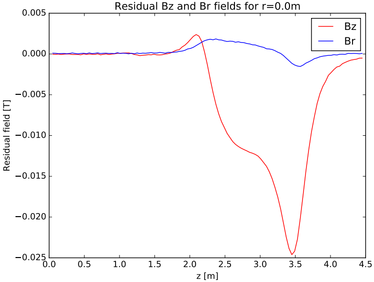
The geometrical fitting procedure is:

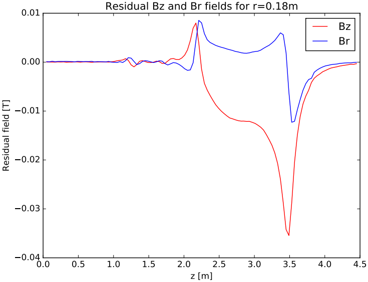
- Mix two bracket fields in proportion: fitField $=$ prop $\times$ field1 $+ (1 -$ prop$) \times$ field2
- Scale coordinates: lengthFactor $\times r$ and lengthFactor $\times z$
- Scale field: fieldFactor $\times(B_r, B_{phi}, B_z)$
- Rotate and apply offsets to the mixed + scaled field using simple matrix multiplication
- Interpolate rotated field back onto the mapper data's coordinates
- Calculate χ^2 between model and mapper data

Where χ^2 is given by:

$$\chi^2 = \sum_{r,\phi,z} \sum_{\text{points}} \left(\frac{B_{x,p}^{\text{Data}} - B_{x,p}^{\text{Model}}}{\sigma} \right)^2$$

SSD Centre Coil geometrical fit residual field.





Fourier Bessel Fit

Now have the residual field between the geometrical fit model and the data.

This residual field *should* obey Maxwell's equations in vacuum as there are no sources of current within the mapped region. Hence the scaler field obeys the Laplace equation:

$$\nabla^2 \Phi = 0$$

Which has the solution:

$$\Phi(r, \phi, z) = R(r)P(\phi)Z(z)$$

And from this scaler field you can work out each component of the field using:

$$B_i(r, \phi, z) = \frac{\partial \Phi(r, \phi, z)}{\partial i}$$

Where $i = r, \phi, z$.

Then you do some separation of variables and some maths[1]...

[1] Omitted in the interest of brevity

Fourier-Bessel series giving B_r field components

$$
\begin{aligned}
B_r(r,\phi,z) \;=\; & \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} A_{nl}\mathrm{I}'_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\alpha_{nl})\sin\left(\frac{l\pi}{z_{\max}}z\right) \\
+ & \sum_{n=0}^{\infty}\sum_{l=1}^{\infty} B_{nl}\mathrm{I}'_n\left(\frac{l\pi}{z_{\max}}r\right)\cos(n\phi+\beta_{nl})\cos\left(\frac{l\pi}{z_{\max}}z\right) \\
+ & \sum_{n=0}^{\infty} A_{n0}nr^{n-1}\cos(n\phi+\alpha_{n0})z \\
+ & \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\mathrm{J}'_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\gamma_{nm})\sinh\left(\frac{\zeta_{nm}}{r_{\max}}z\right) \\
+ & \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm}\mathrm{J}'_n\left(\frac{\zeta_{nm}}{r_{\max}}r\right)\cos(n\phi+\delta_{nm})\cosh\left(\frac{\zeta_{nm}}{r_{\max}}z\right) \\
+ & \sum_{n=0}^{\infty} E_n nr^{n-1}\cos(n\phi+\varepsilon_n)
\end{aligned}
$$

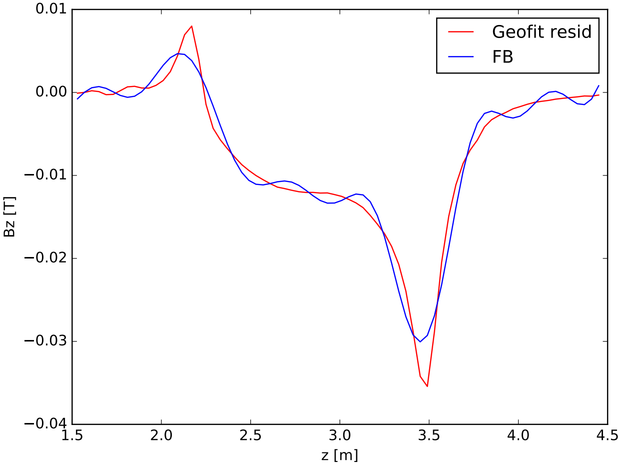
Use different method to find each set of coefficients/phases owing to the properties of the Bessel functions:

- Fourier-Bessel terms – Fit to B_z on the cylindrical surface (outermost probe measurement) since hyperbolic terms are 0.
- Hyperbolic terms – After subtracting the Fourier-Bessel terms from the residual B_z, the hyperbolic terms are found by fitting the the ends of the cylinder using data from all mapper probes.
- Multipole terms – Doesn't effect B_z. After subtracting the FB and hyperbolic terms, the multipole coeffiecients/phases are found from the residual B_r averaged over z.

Some results

$n = 0, 1$ and $l = 0, 1, 2, 3, 4, 5, 6, 7$

SSD Centre Coil



Future work

- Find optimum amount of terms
- Find Hyperbolic and multipole terms
- Add with geometrical fit model and compare with data

Any questions?