Recent Developments on the Kalman Filter and Track Fitting Routines

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June 28, 2017

Contents

Recent History

Some Recent History

- The most recent MAUS version held a little surprise for those unsuspecting analysists...
  The standard P-Value distributions that we had come to trust had changed!
- A small bug fix from back in February was never completely merged (*Sorry!*)
- Recent analyses were becoming reliant on P-Values to apply cuts to data, which obviously can't be allowed to change so easily.

- The efficiency of the pattern recognition stage was questioned late last year, which lead to a phase of improvements from Adam
- This year, the tracker software review has provided the momentum to see the fitting routines reach an optimal performance
- Adam has been working to test different fit models, code structures and ideas in order to improve the performance.

The Bug

There's more than one way to calculate a residual!

Normally, we calculate a residual as the signed difference between a fitted and a measured value, and calculate the covariance matrix in the usual fashion, e.g.

$$r = m - x$$
$$\mathbf{R} = \mathbf{M} + \mathbf{X}$$

BUT

if the measurement is already used in the fit and it's covariance matrix has been statistically included, the fitted value is correlated with the measurement.

The residual and covariance matrices are correctly calculated as:

$$r = m - x$$
$$\mathbf{R} = \mathbf{M} - \mathbf{X}$$

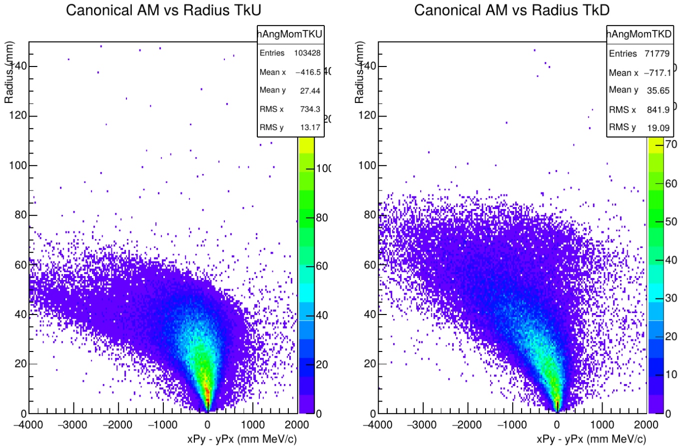
Pattern Recogntion Improvements

A Brief Summary

- At present the track finding efficiency is looking very good $\sim 99\%$
- The Angular momentum singularity appears to have disappeared
- Identified the longitudinal fit as the next key enefficiency
- Now have several options for pattern recognition, some are in the official MAUS release if you know where to look:
  1. A minuit powered circle fit - rather than matrix inversion
  2. A full minuit powered helix ift
  3. A semi analytical helix fit

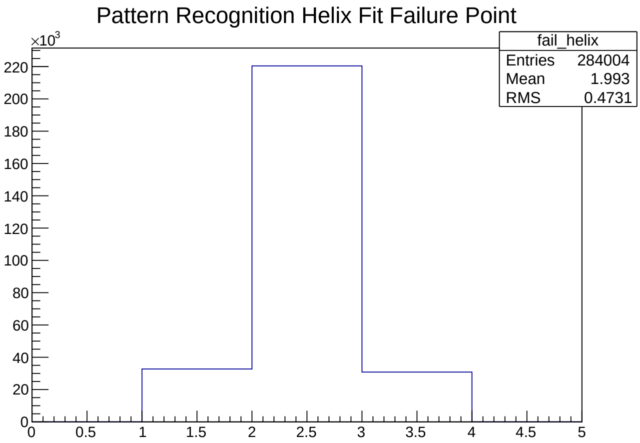
Canonical Angular Momentum Deficiency

Recent analyses have suggested that the bug is largely fixed.
Although an inefficiency may still be present.



Where We're Going Next

The key stumbling point appears to be the number of turns calculation (bin 2), as opposed to the circle fit (bin 1) or longitudinal fit (bin 3).



For more information...

- Additional plots are attached to this presentation showing data from both the Minuit based fit and the Matrix Inversion fit.
- Some details need to be ironed out (e.g. units on the chi-squared, etc), but efficiency numbers are available at the end.
- If there are burning questions or suggestions Adam is the point of contact, equally you could try to extract an invite to the tracker software review meetings.
- Further improvements are expected in the next few versions of MAUS.

Testing Kalman

Testing Set Up

- Construct a third party program that links to MAUS and pulls out the fitting routines, without the huge over-heads of running the MAUS framework and Geant4 Models.
- Construct measurement and propagation models that represent any conceivable linear system - Starting with simple models and progressing to realistic ones.
- The fitting framework is capable of fitting any dimension of state-space with any dimension of measurements - so we test different models with different dimensions.

Models Under Test

1. 0th Order
2. 1st Order
3. Simplified MAUS Straight and Helical
4. Modified MAUS Straight and Helical

In order to analyse each model, we will be looking at Smoothed Residuals, Chi-Squared Residuals, Pulls, Chi-Squared values and P-Values.

0th Order Validation

Perform analytical calculations to see if the calculated distributions are in agreement with the MC model.

This system is composed of 1D measurements, a 1D state space (temperature measurements) with modelled process noise and measurement noise.

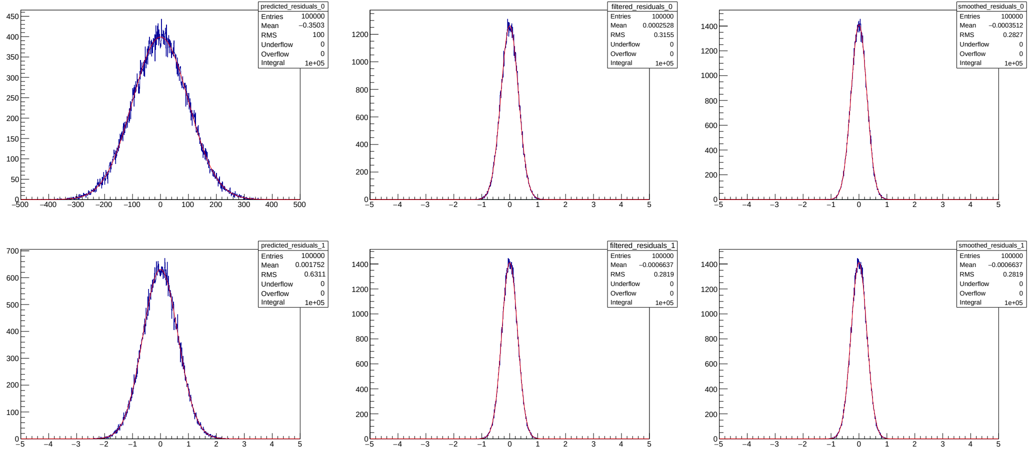
The fit therefore involves :

- 2 measurements
- 1 prediction calculation
- 2 filtering calculations
- 1 smoothing calculation
- 2 pulls, 2 smoothed residuals and 2 filtered residuals.

The variances calculated by the fit should describe the expected residuals.

0th Order Validation

Predicted, Filtered and Smoothed Residuals. Exactly as calculated!



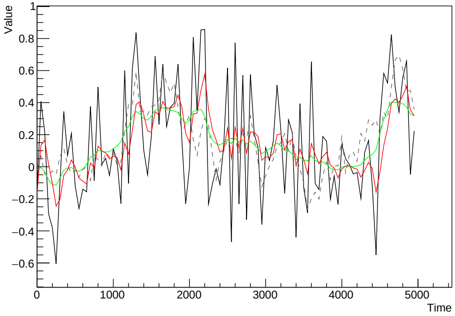
1st Order Validation

We can be confident that the basic formulae are behaving as expected.

Now we test the statistical measures. This is where it starts to get interesting!

The 1D Model is composed of:

- Any number of 1D measurements
- 1D Temperature system model (as previous)
- Variable process noise (temperature stability)
- Variable measurement noise (measument accuracy)

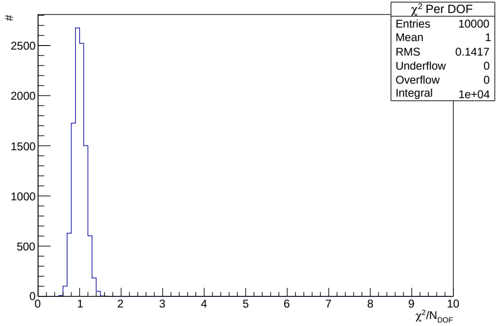


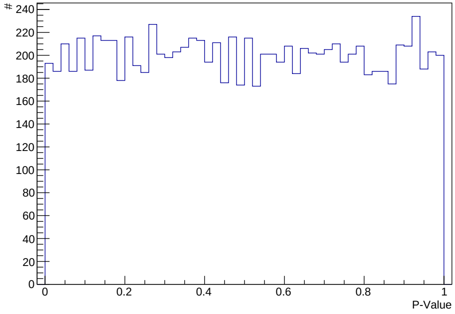
1st Order Validation
No Noise

When there is no process noise, we can calculate a "classical" chi-squared value.

Residual between data and smoothed estimates, weighted by the meausrement errors.

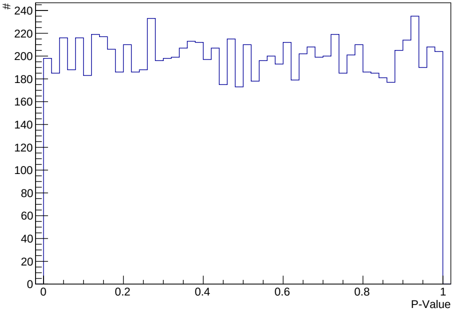
100 measurements, 1D fit, so 99 degrees of freedom.



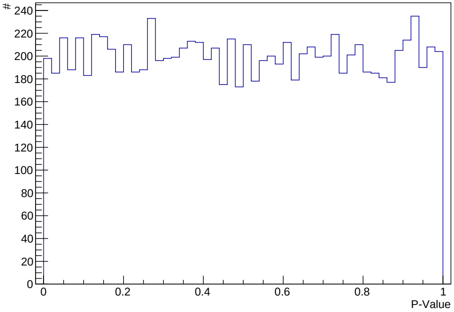


1st Order Validation
No Noise

Equally we can calculate a P-Value from the pulls or from the smoothed estimates.



Pull P-Values

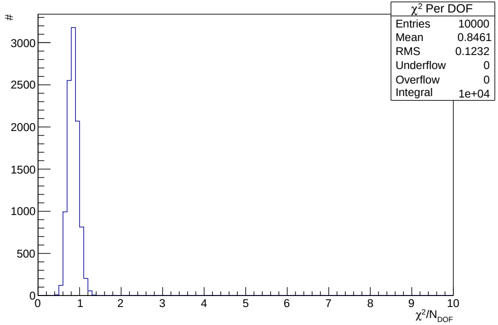


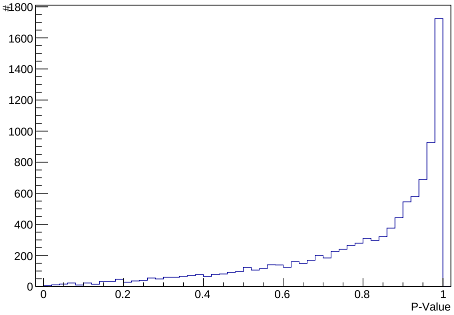
Smoothed P-Values

1st Order Validation
With Noise

Now that we allow noise into the system, *and* we allow the fit to vary to compensate, the “classical” Chi-Squared calculation is no longer applicable.

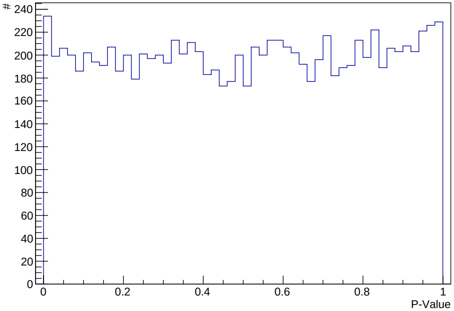
We allow the fit to be pulled more by the measurements to compensate for the true information varying. Hence the chi-square actually decreases.



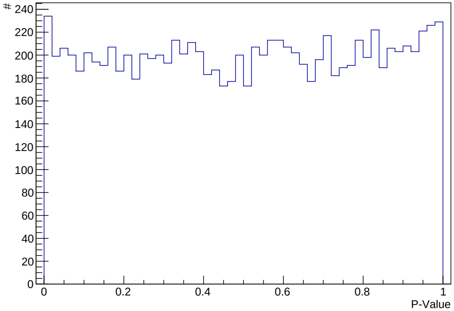


1st Order Validation
With Noise

Again we can calculate a P-Value from the pulls or from the smoothed estimates.



Pull P-Values



Smoothed P-Values

1st Order Validation

What statistical measusure should we use?

I would recommend measures that are based on the smoothed residuals. This is the default in the MAUS data structure - so no need to worry.

P-Values claculated from pulls should be almost identical unless there is something strange in the system e.g. a measurment plane that behaves differently and isn't accounted for.

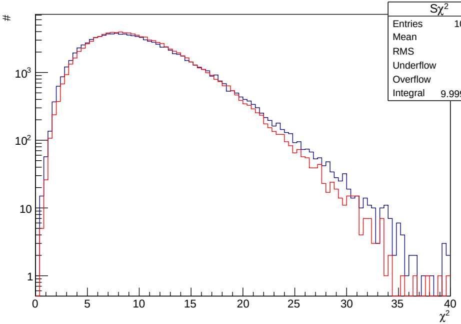
However, the P-Value distributions themselves are still not ideal!

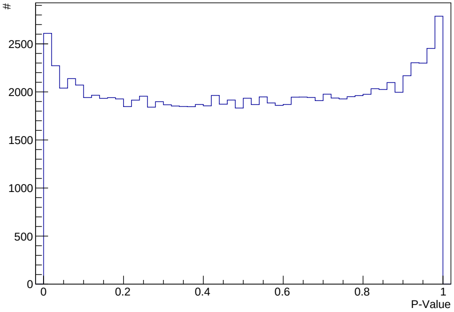
1st Order Validation

If we now reduce the system to 10 measurements and moderate process noise, we see disagreement between the theoretical and the reconstructed Chi-Squareds and P-Values.

The effect is very small and I have no idea where it is coming from.

But note the effect on the P-Value distribution - that could affect analyses!





MAUS Performance

Realistic and MAUS-like Models

This is unfortnately not as advanced as I would have liked.

Wanted to have simplified track fitting - perfect noise, idealised gaussian measurements, etc. For standard straight and helical tracks.

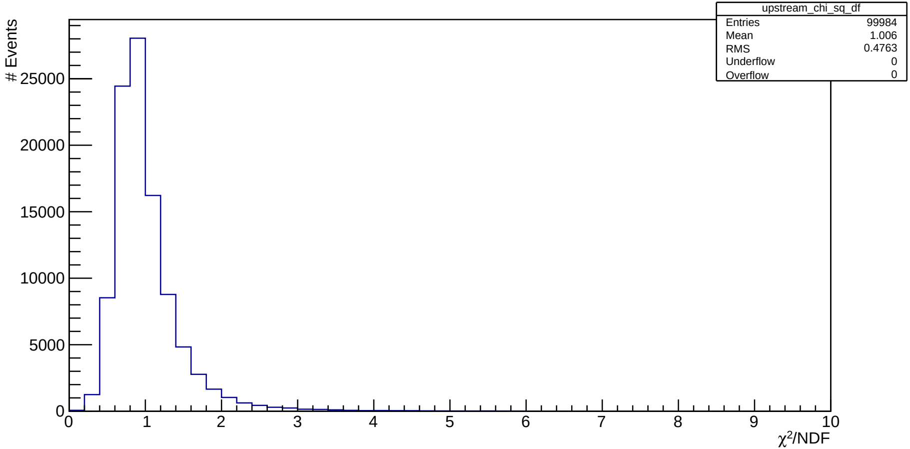
Then introduce non-linear features, etc. and view how the P-Values and Chi-Squared distributions change.

The models are mostly written, but I have not yet had the time to perform an anlysis similar to the one above.

However I can demonstrate the distributions that are currently in MAUS V2.9.1!
Using an ideal MC model in order to validate the reconstruction.
They will look slightly different to data!

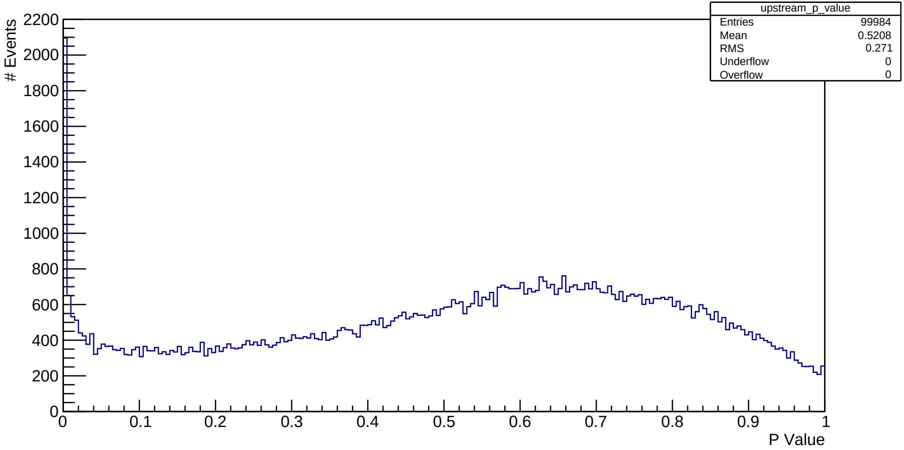
Straight Track Fitting

Chi-Squared Per Degree of Freedom



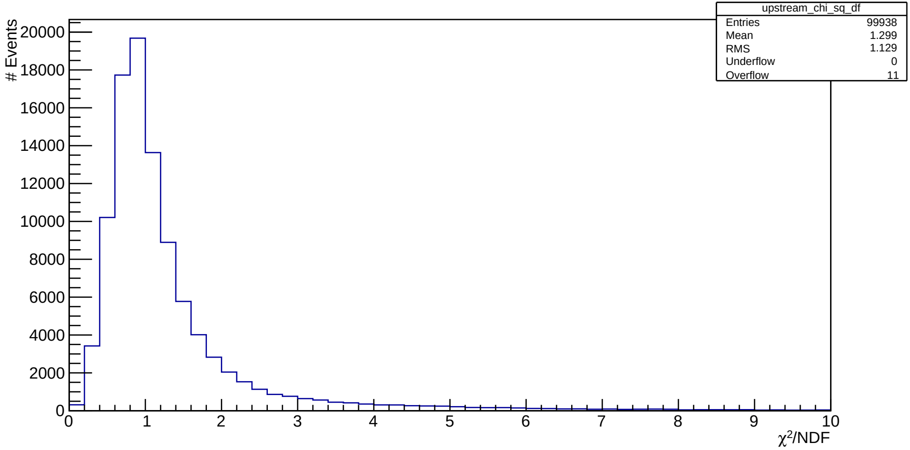
Straight Track Fitting

P-Value Distribution



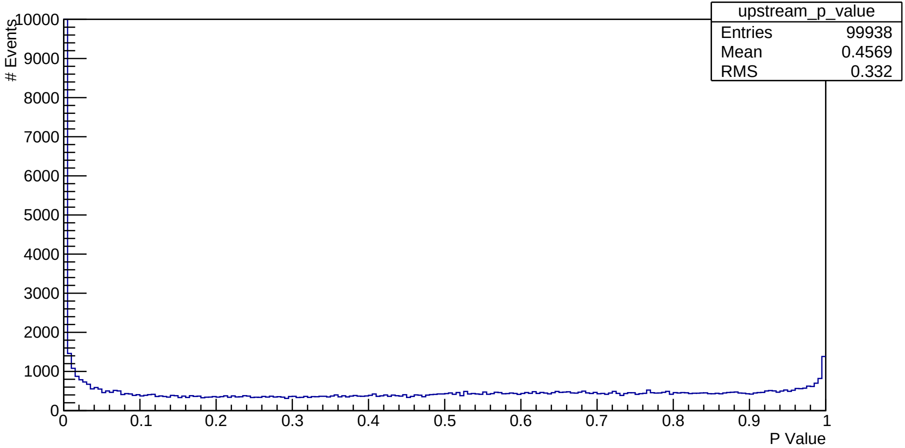
Helical Track Fitting

Chi-Squared Per Degree of Freedom



Helical Track Fitting

P-Value Distribution



Caveats

This represents the halfway point between where I am currently and the requirements for complete understanding.

- This is using Geant4's definition of reality.
- I have assumed that the tracker is perfectly aligned to the field.
- Noise and dead-channels are not modelled.
- Standard MAUS Solenoid fields have been used.

Previous experience suggets that data should be very similar to the MAUS MC, but we need to check.

Conclusions

- The fitting routines perform well, almost in perfect agreement with analytical calculations.
- There are still some odd features to understand, but they do not seem to affect performance, only the shape of the P-Value distribution.
- A selection of simple and increasingly complex models have be examined, some in great detail.
- The understanding of how ideal P-Values migrate into our real P-Values is still required.
- First indication from this study is that the fit is performing very well in MC, hopefully data will see a similar performance.
- The effects of field misalignmen and non-uniformity are still to come and will conclude the study.

For the Analysts

From a track fitting perspective. It is advised that future analyses should exmine the Chi-Squared per degree of freedom for each track. Not the P-Value.

The P-Value uniformity should show you how the model is performing, but the whole tail of the Chi-Squared distribution corresponds to the first bin in the P-Value distribution, so it is too sensitive to be useful for our analyses.

The analysis of field uniformity and field alignment dependence is next on the list.

Thank you.
Any Questions?