Novel Application of Density Estimation in MICE

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CM48

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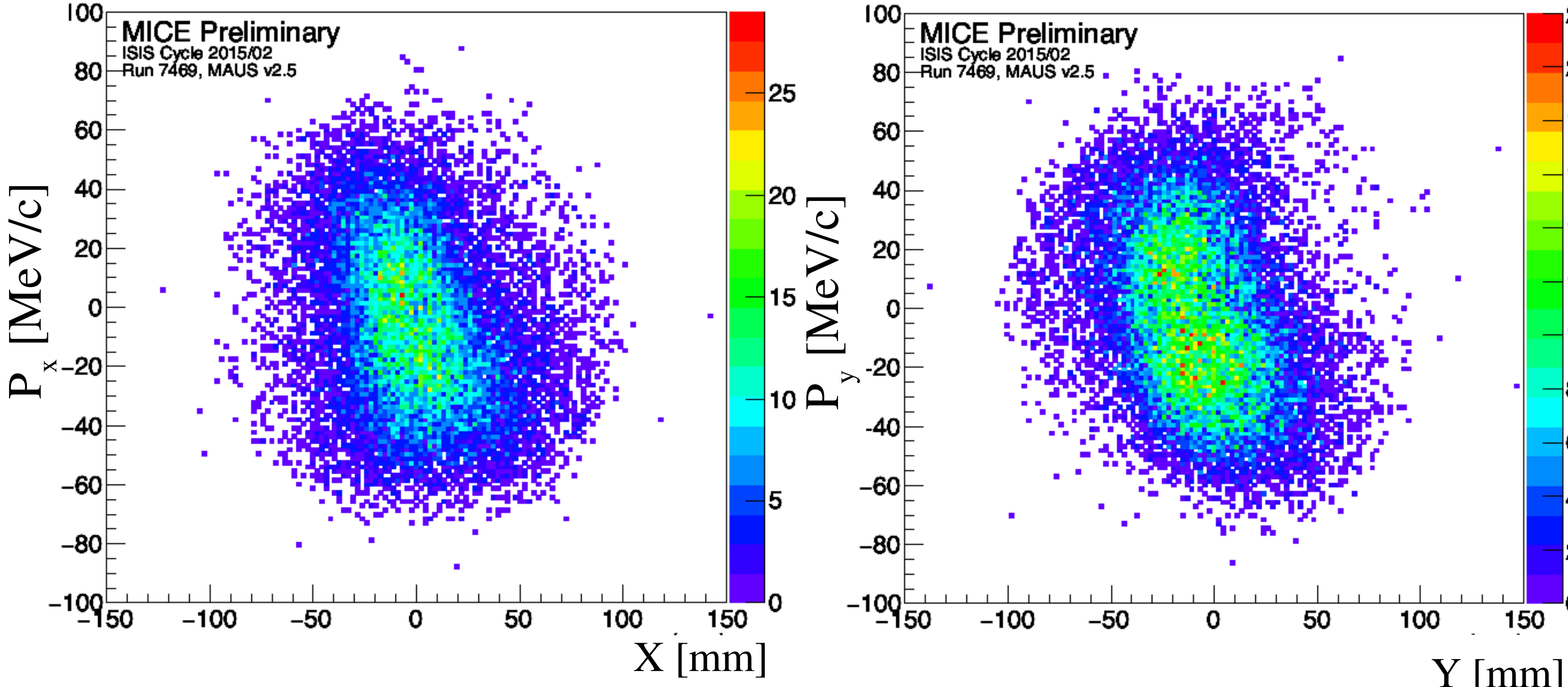
ILLINOIS INSTITUTE OF TECHNOLOGY

Fermilab

MICE

Motivation

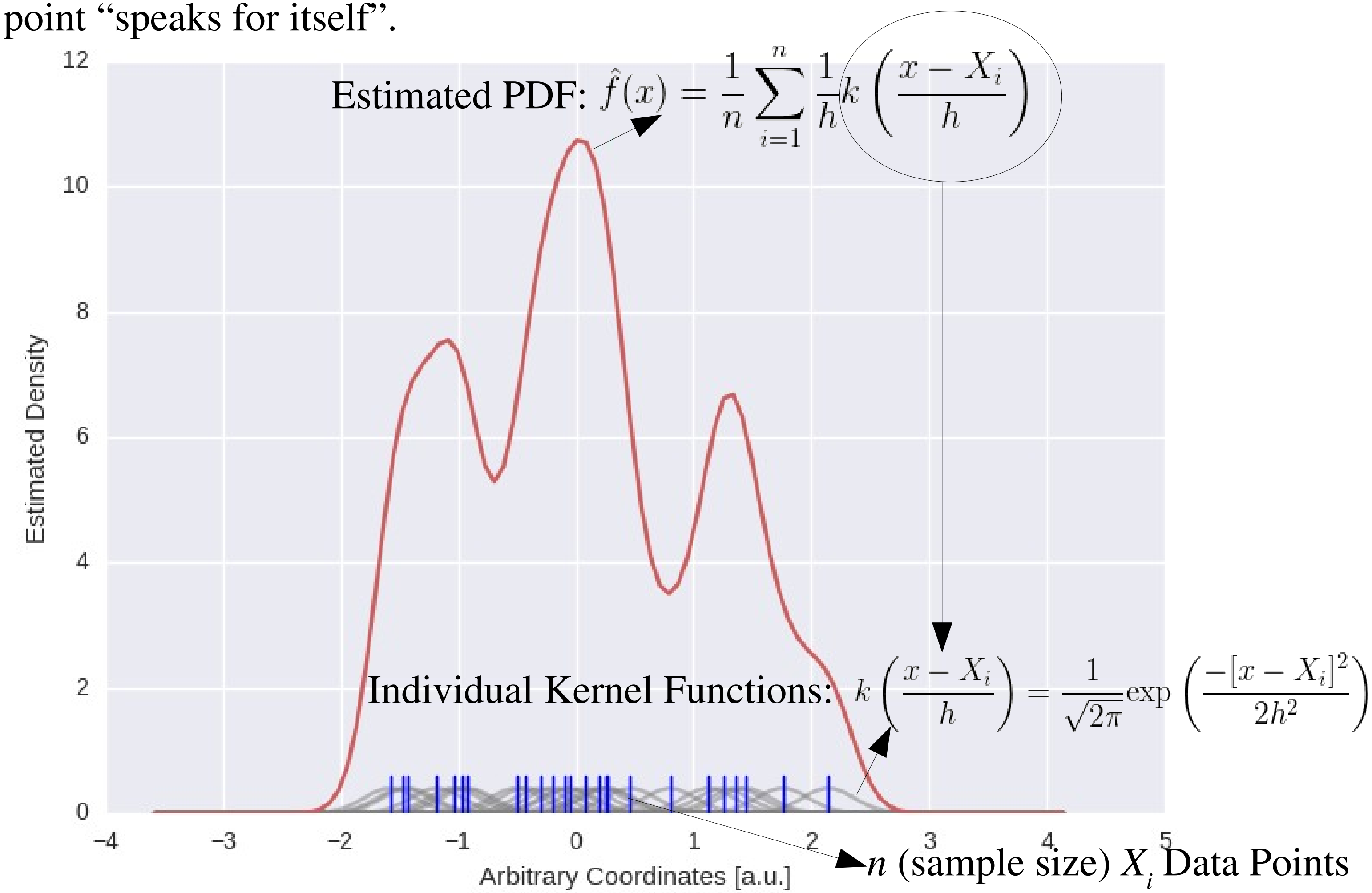
Measured Muon Beam Phase Space



- Real-life particle beam is non-Gaussian:
  - ★ Chromatic, non-linear effects cause heating.
  - ★ Beam loss affects cooling.
- Density Estimation (DE) techniques:
  - ★ Estimates probability density function (PDF) or phase-space density with no assumptions about the underlying distribution.

Reminder – Density Estimation (DE)

- Estimates the unknown Probability Density Function (PDF) using smooth weight functions (kernels) of certain widths, h.
- Each data point "speaks for itself".

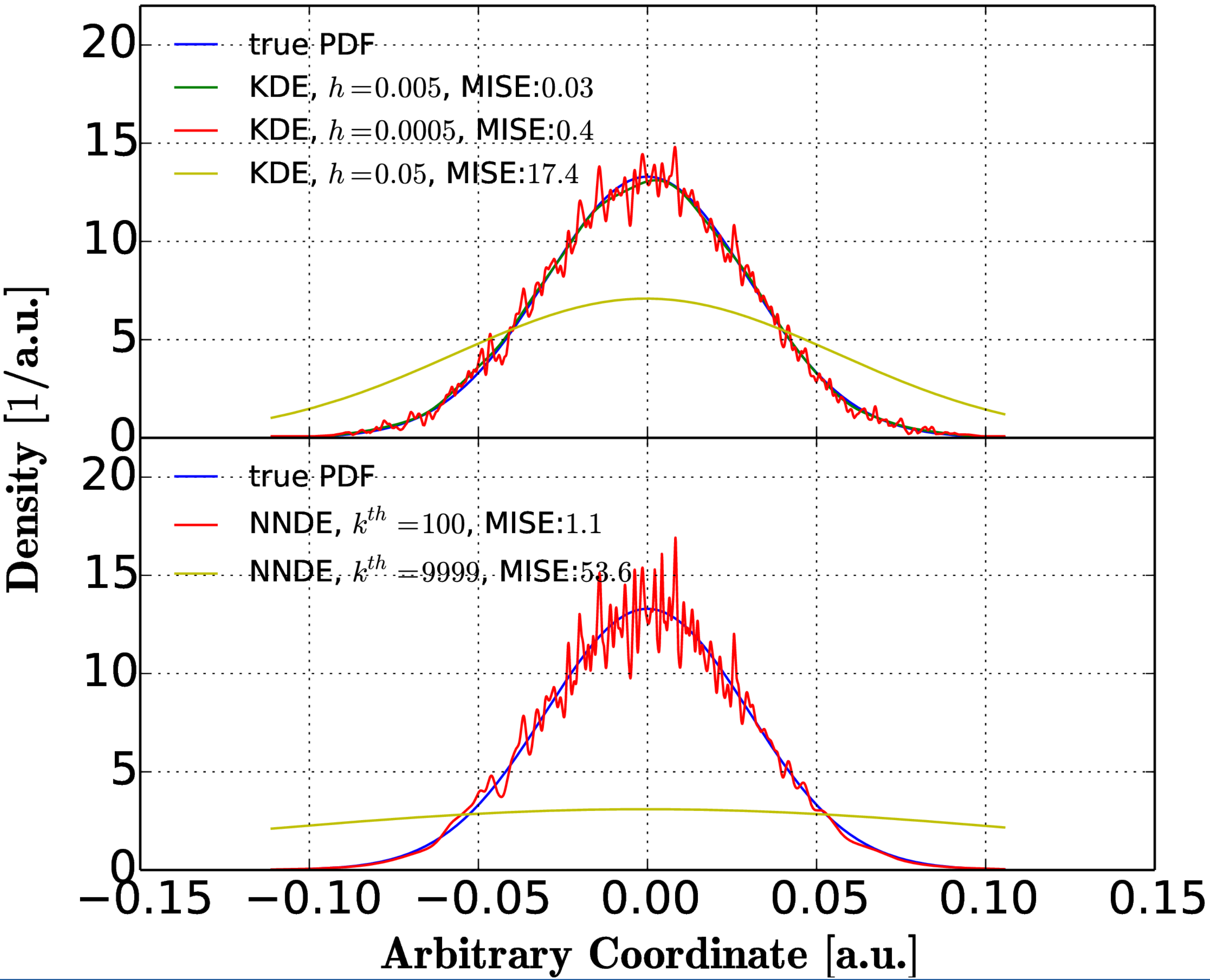


- Two types covered:
  - ★ Kernel Density Estimation (KDE): shown above.
  - ★ Kernel-based Nearest Neighbor (NNDE): kernel widths are the distance between each data point and its near neighbor.

Error Analysis

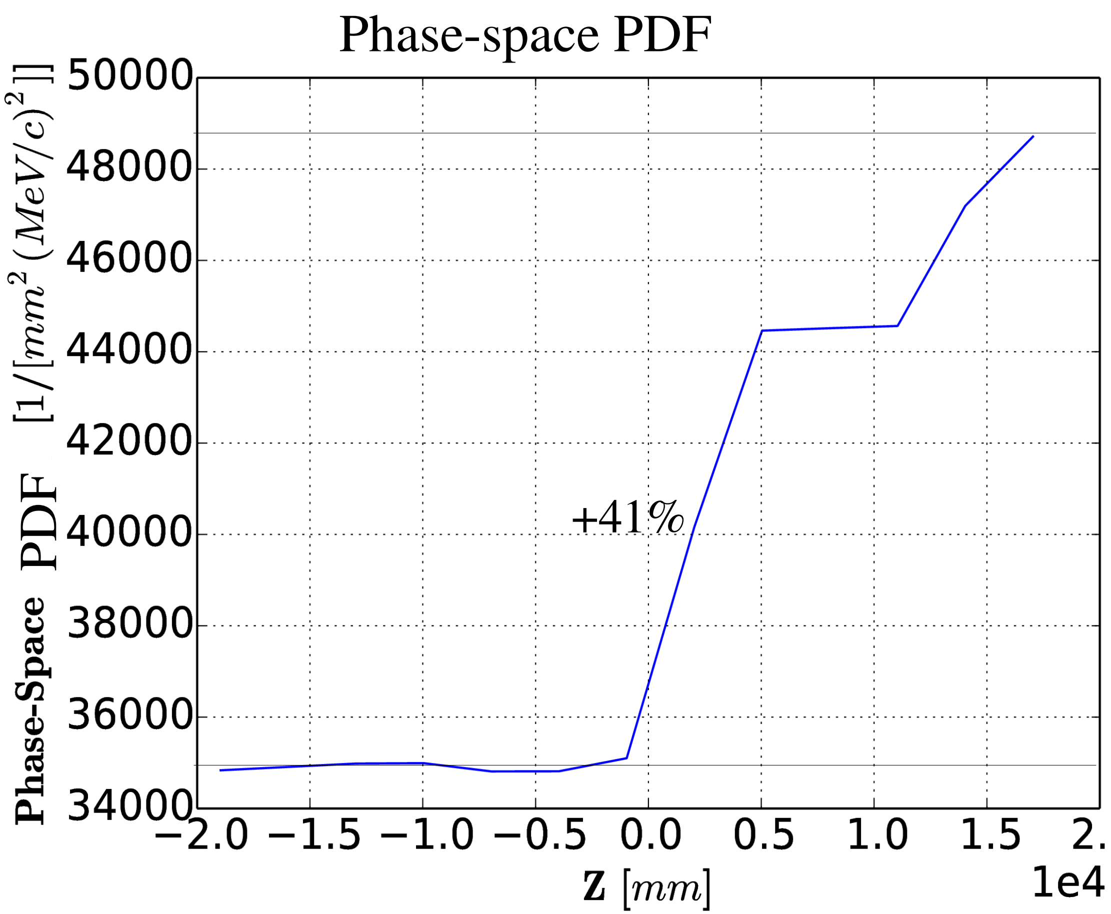
- Common measure of error: mean integrated square error (MISE),

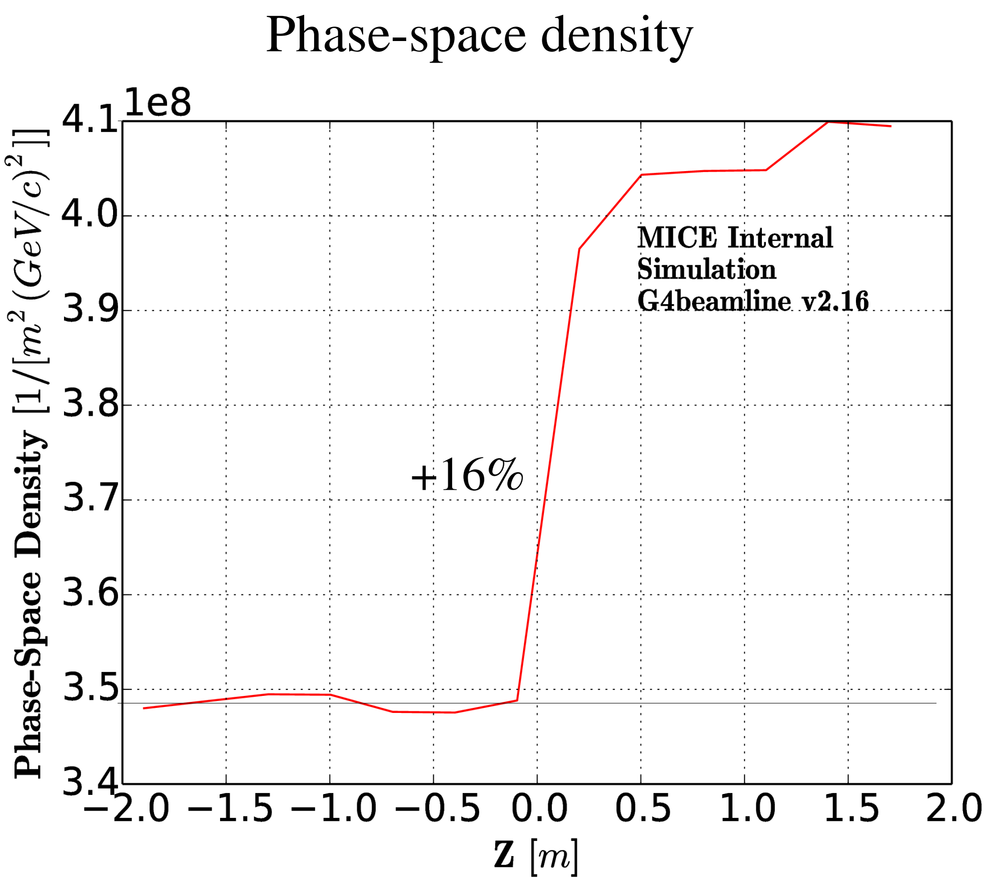
$$\mathrm{MISE}(\hat{f}(x)) = \mathrm{E}\int\left[\hat{f}(x) - f(x)\right]^2 dx$$



Simulation Study – PDF vs. Density

- 6-140 beam setting
- Evolution of 9th percentile contour (fixed muon count inside the contour).
- Phase-space PDF: slide 2's original KDE definition.
- Phase-space Density: scaled with sample size.





- 6-140 beam setting
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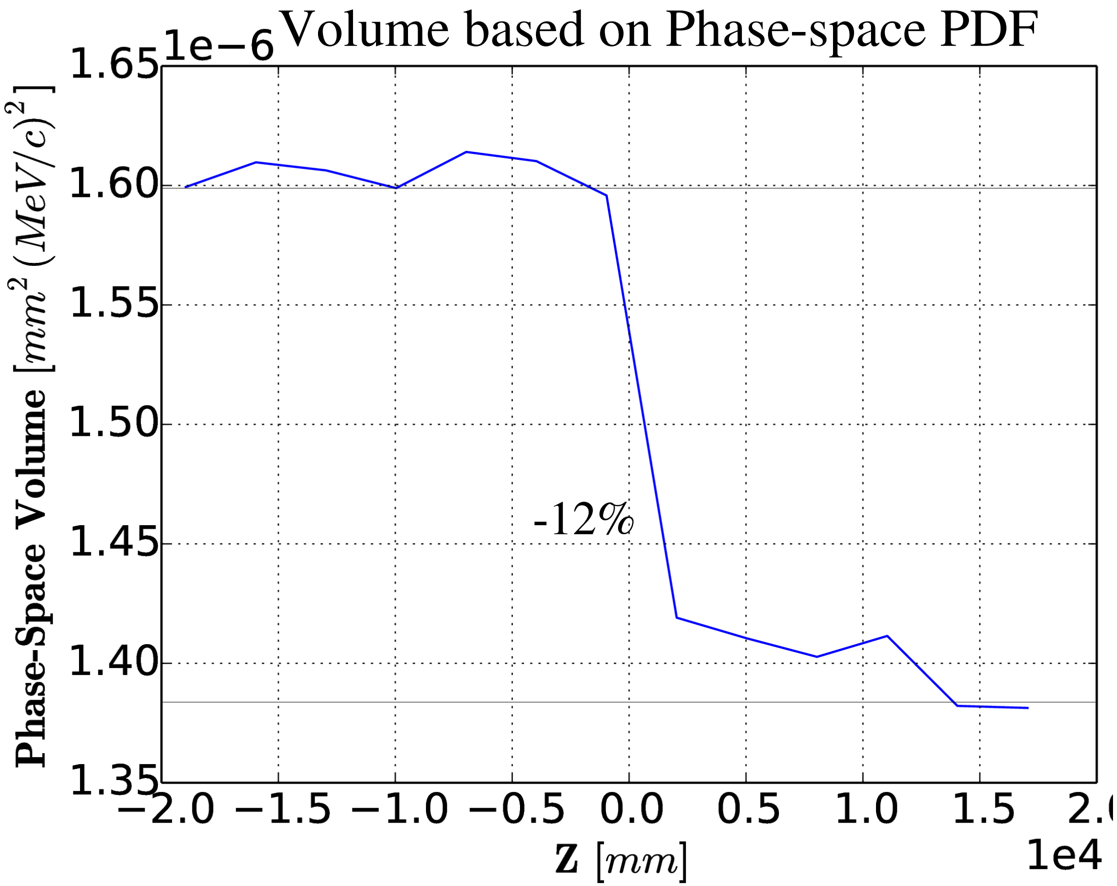
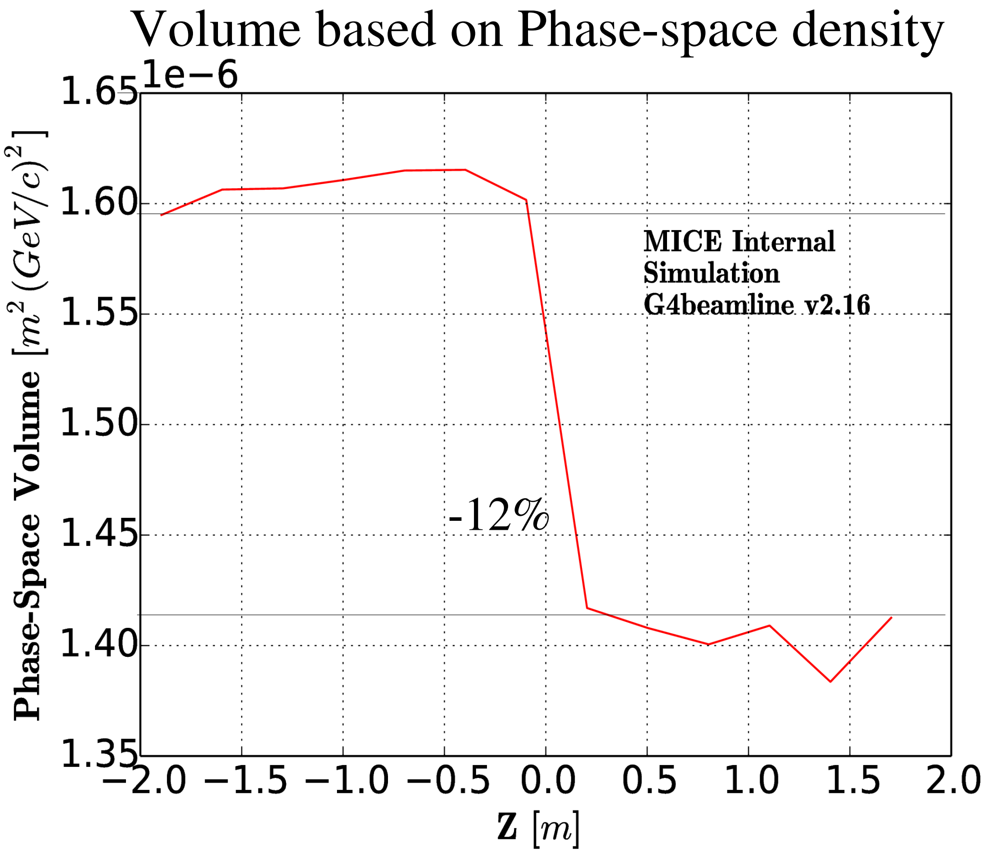
- Phase-space PDF: slide 2's original KDE definition.
- Phase-space Density: scaled with sample size.
- Volume measurement is robust against the choice of technique.





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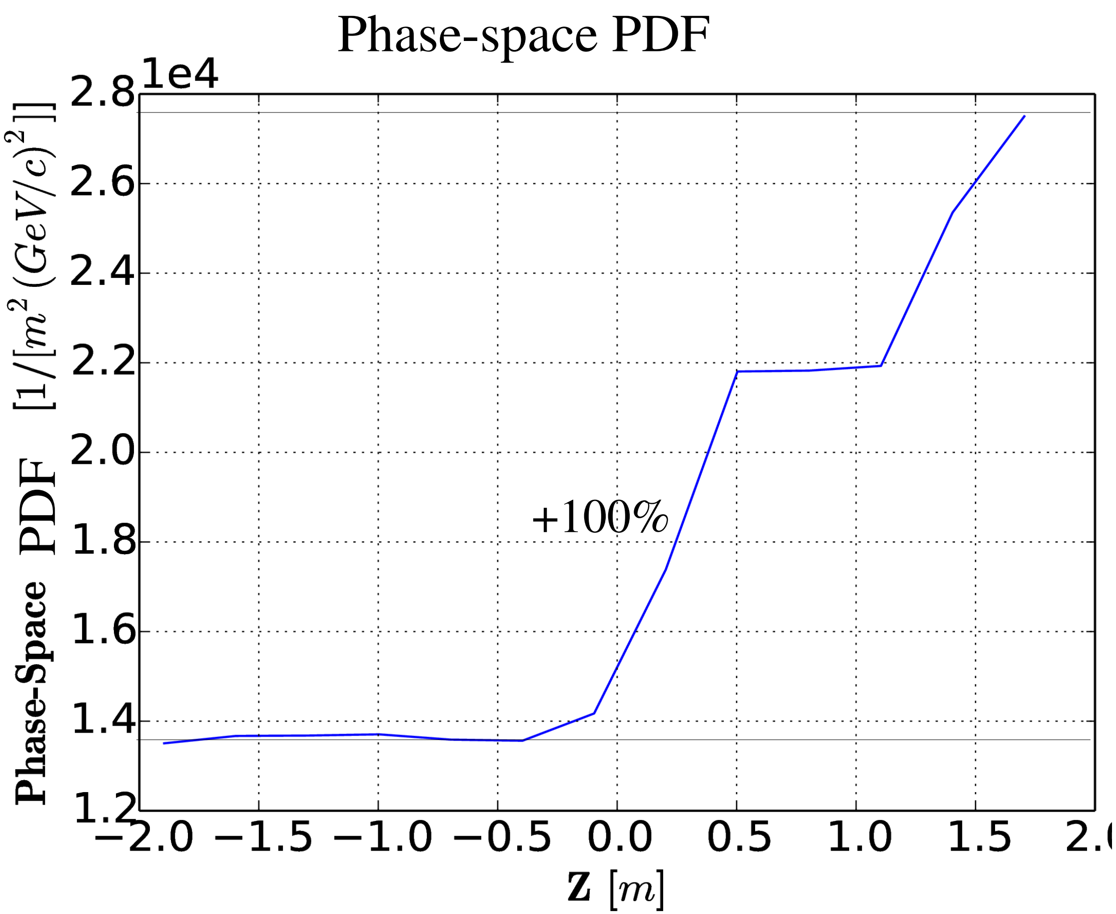

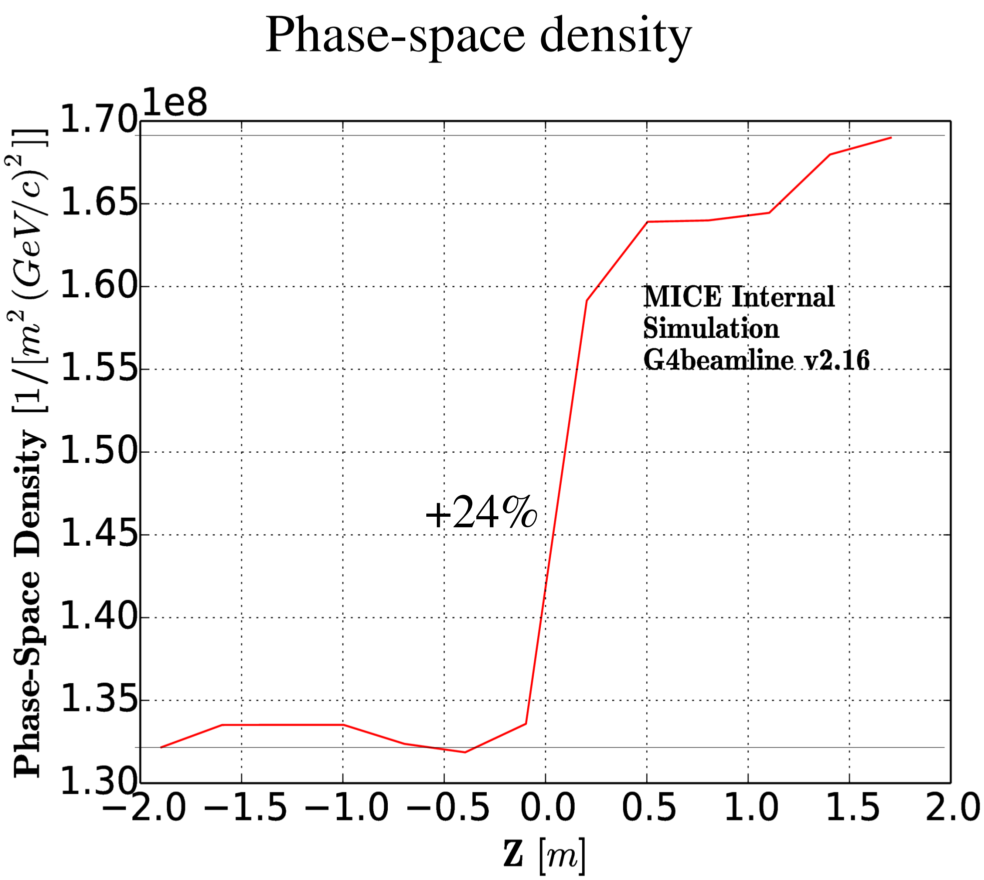

Simulation Study – Volume based on PDF vs. Density

- 10-140 beam setting
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- Phase-space Density: scaled with sample size.
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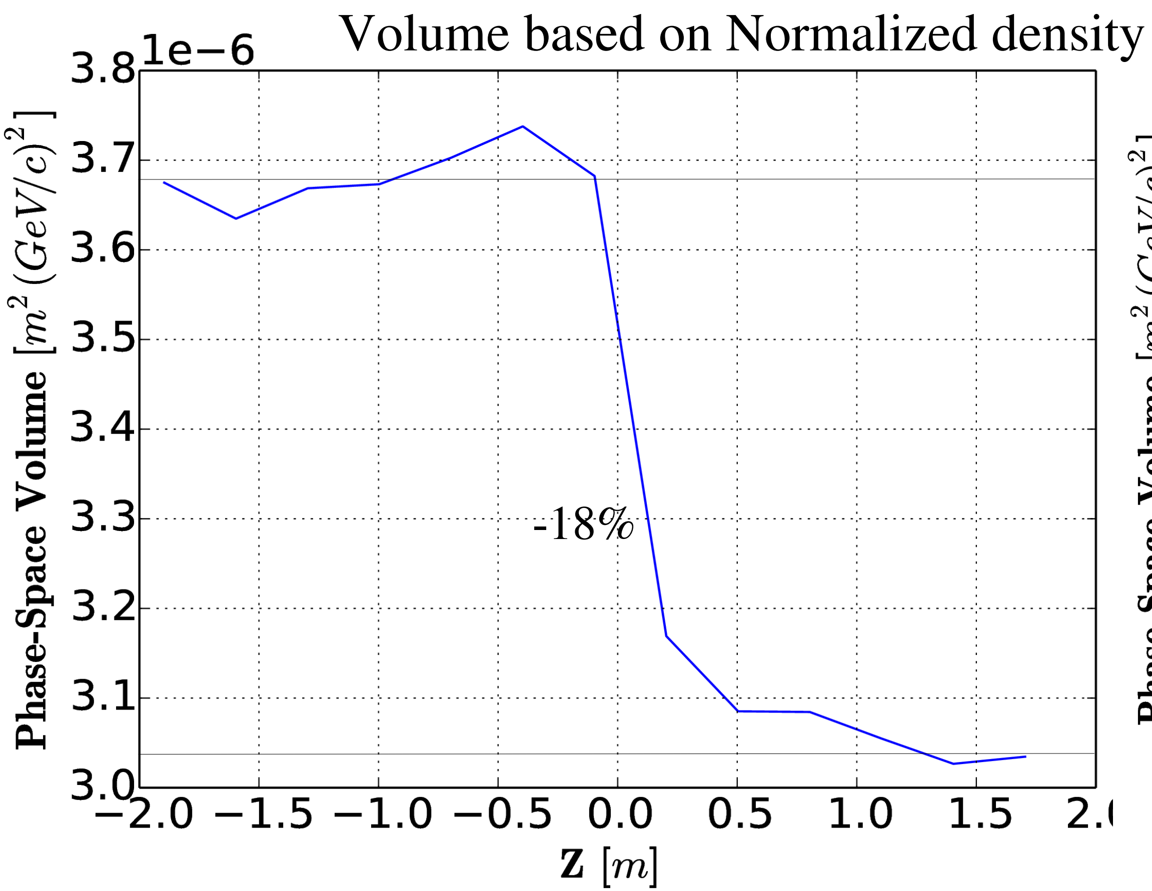
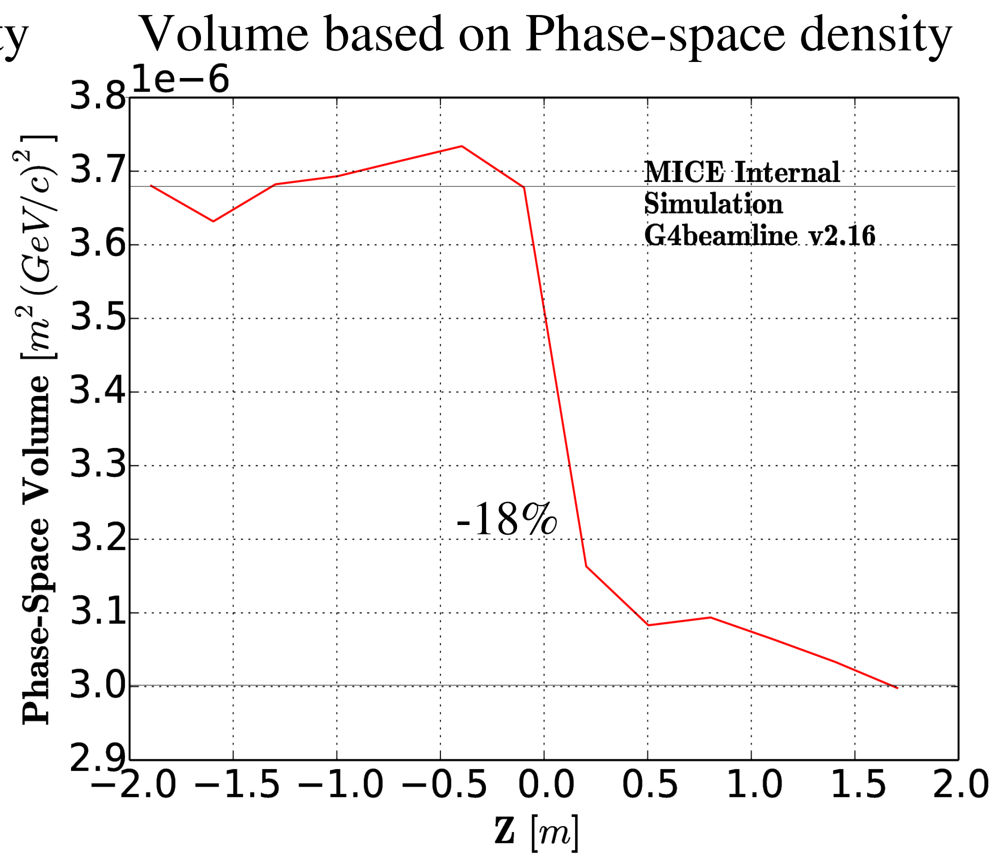
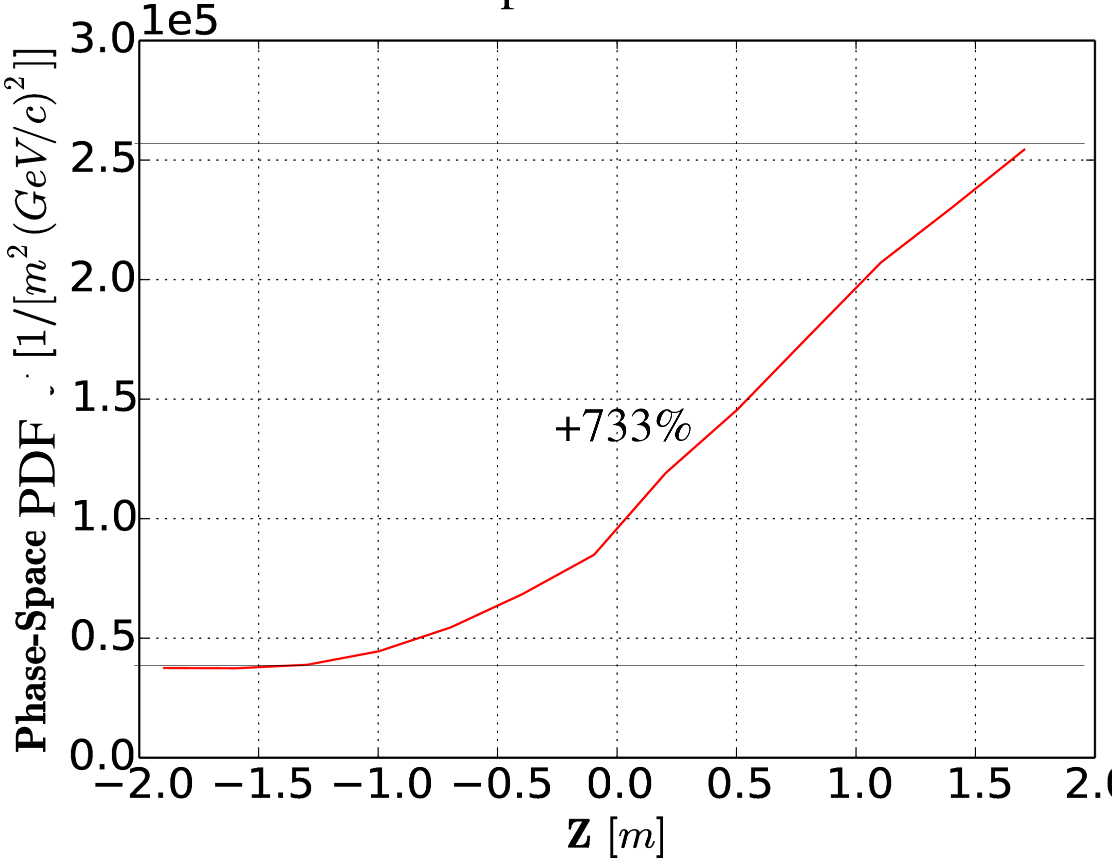




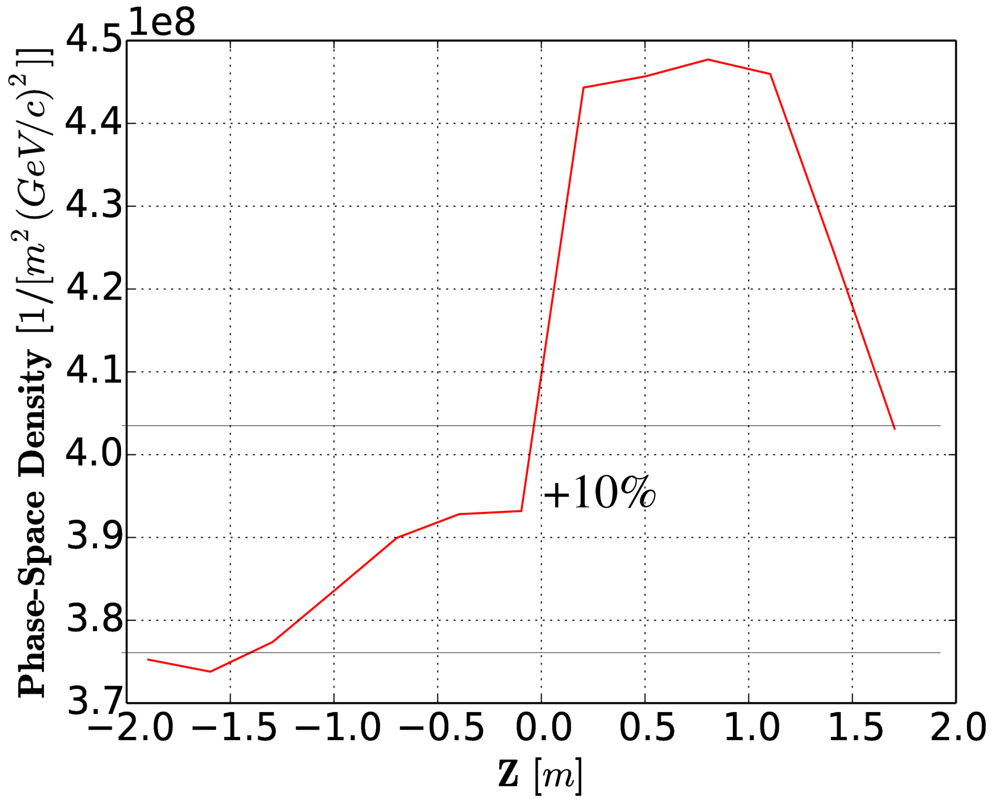
Toy MC Study – PDF vs. Density

- *6* mm beam passing through a LiH absorber only (no solenoids).
- Same transmission loss as the simulation study.
- Evolution of *9*th percentile contour (fixed muon count inside the contour).
- Phase-space PDF: slide 2's original KDE definition.
- Phase-space Density: scaled with sample size.

Phase-space PDF



Phase-space density



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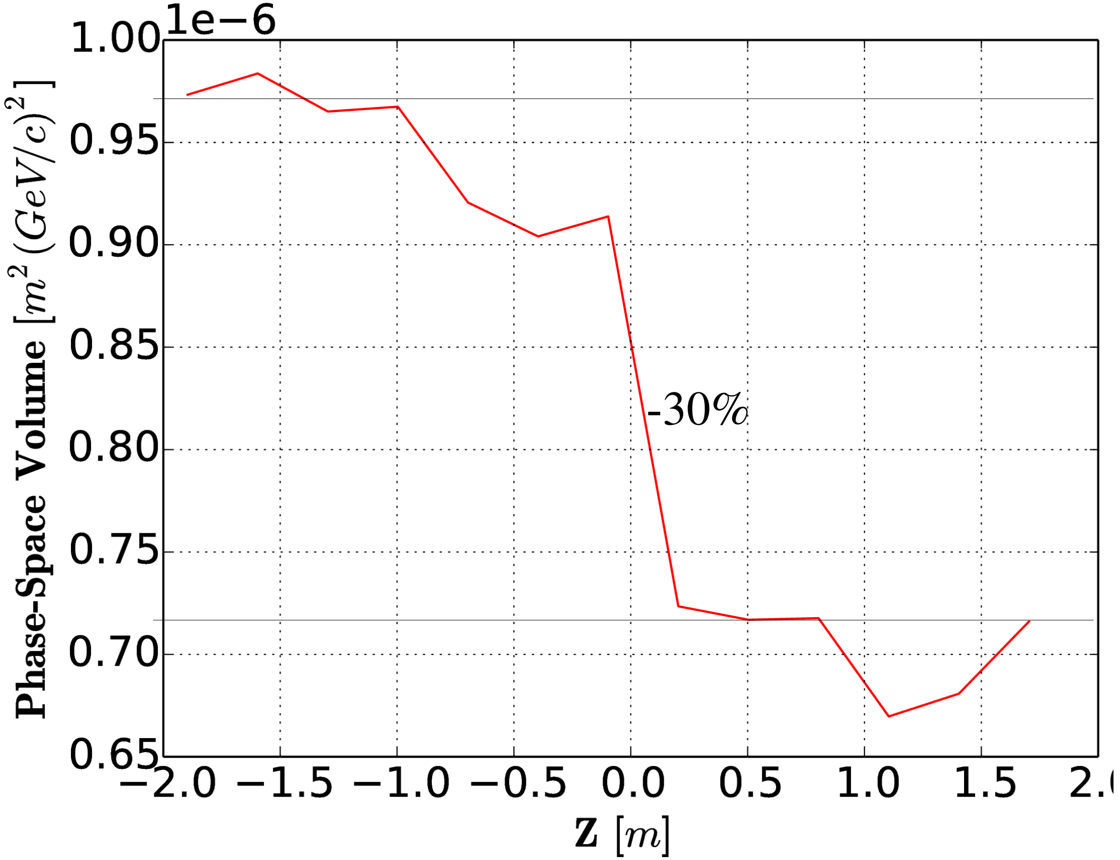
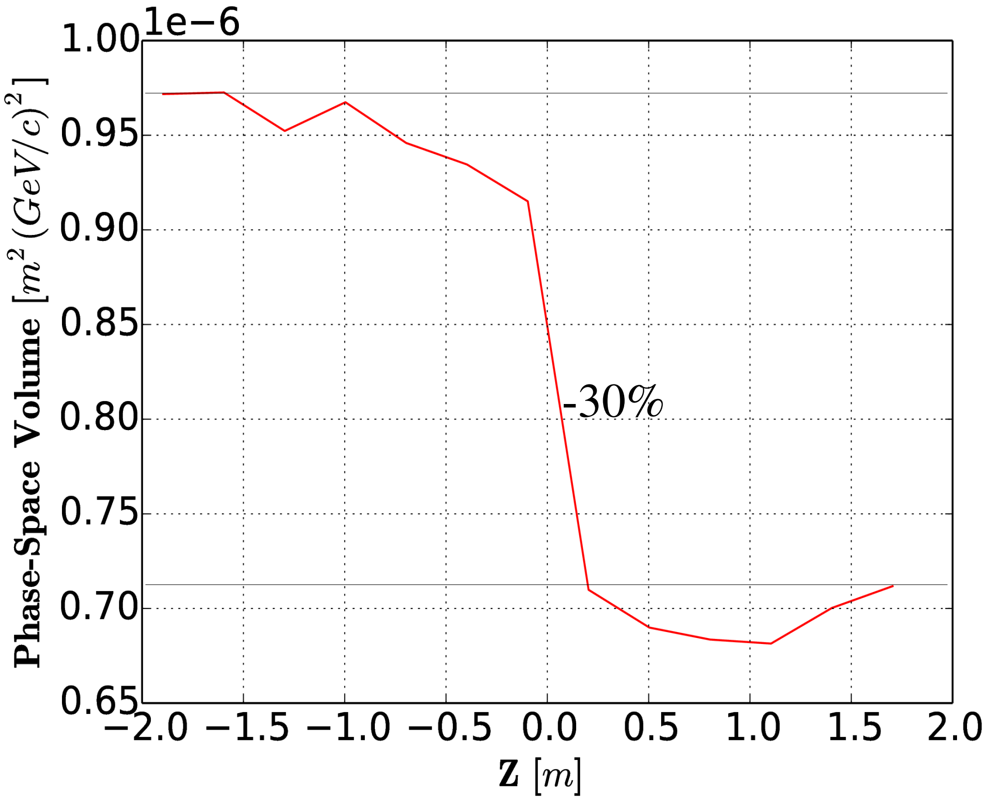
PDF-based Volume

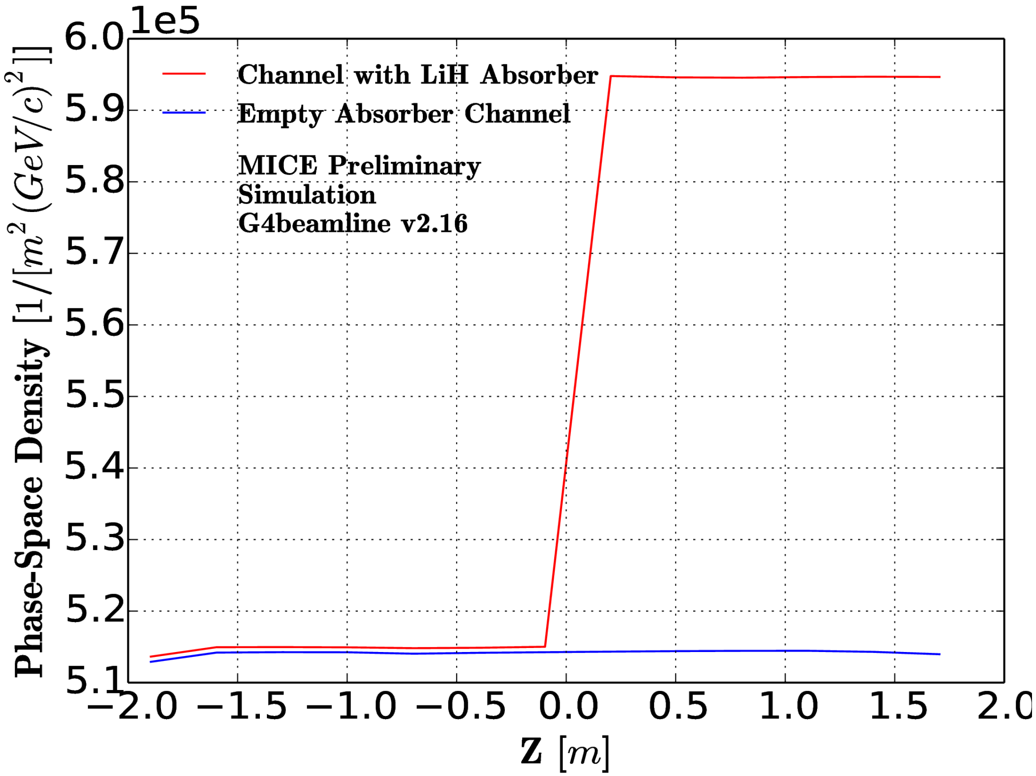


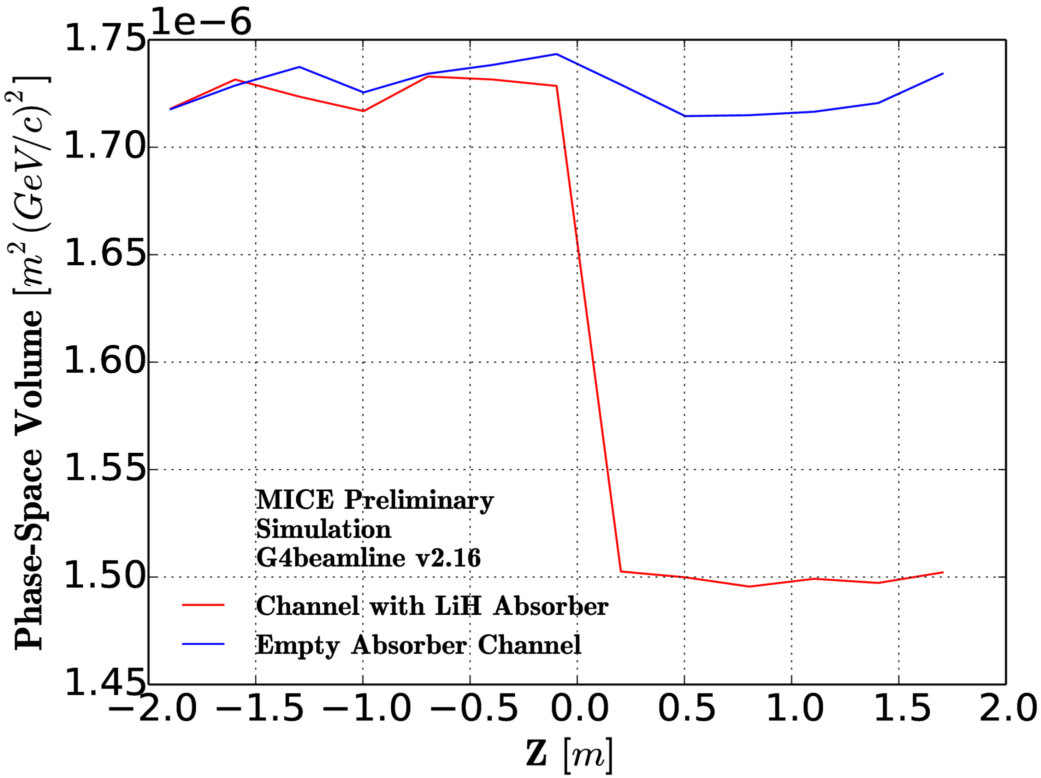
Density-based Volume



Alternative Simulated KDE Study

- The "routine" KDE routine:
  - Let all muons contribute to density.
  - Take a magnifying glass and only track densities of the 9th percentile contour.
- KDE routine on this slide:
  - Record coordinates of core muons (*9th* percentile).
  - Re-run KDE on this subsample.
  - Same 6-140 beam setting.

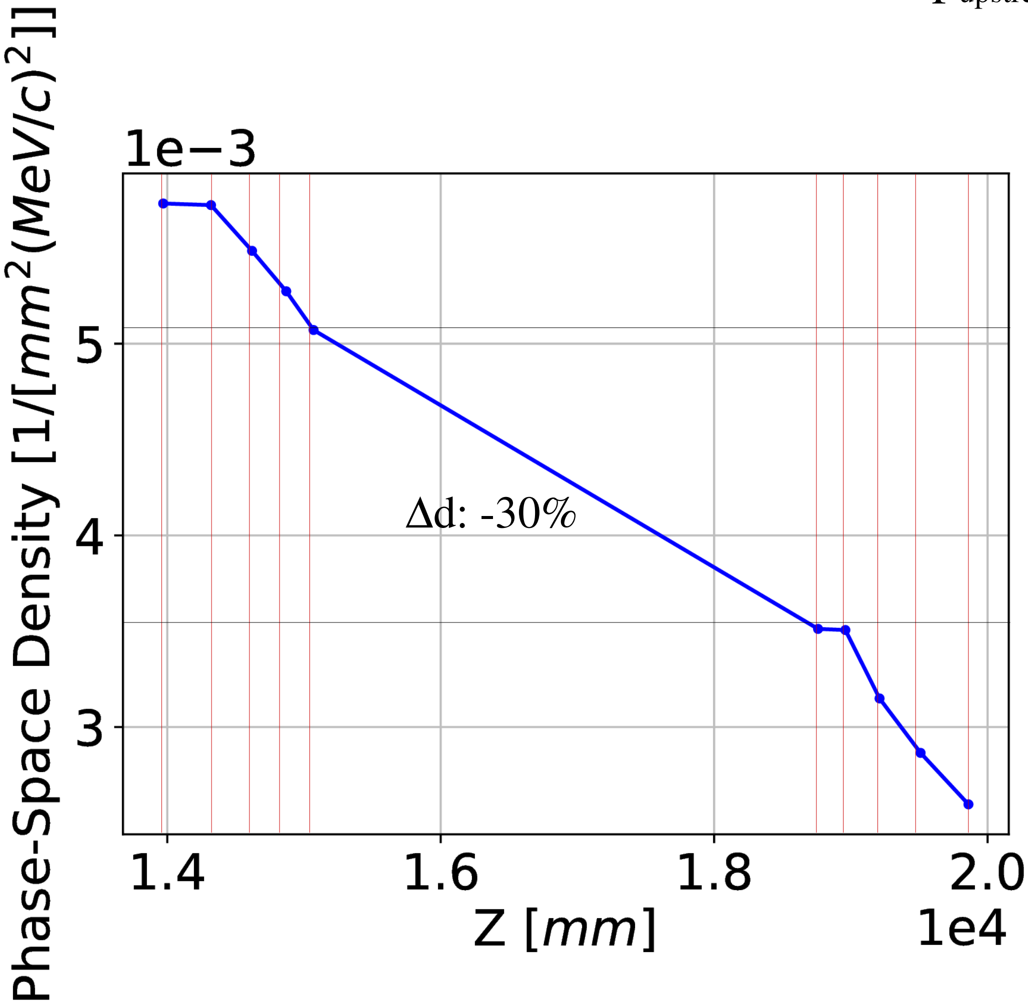


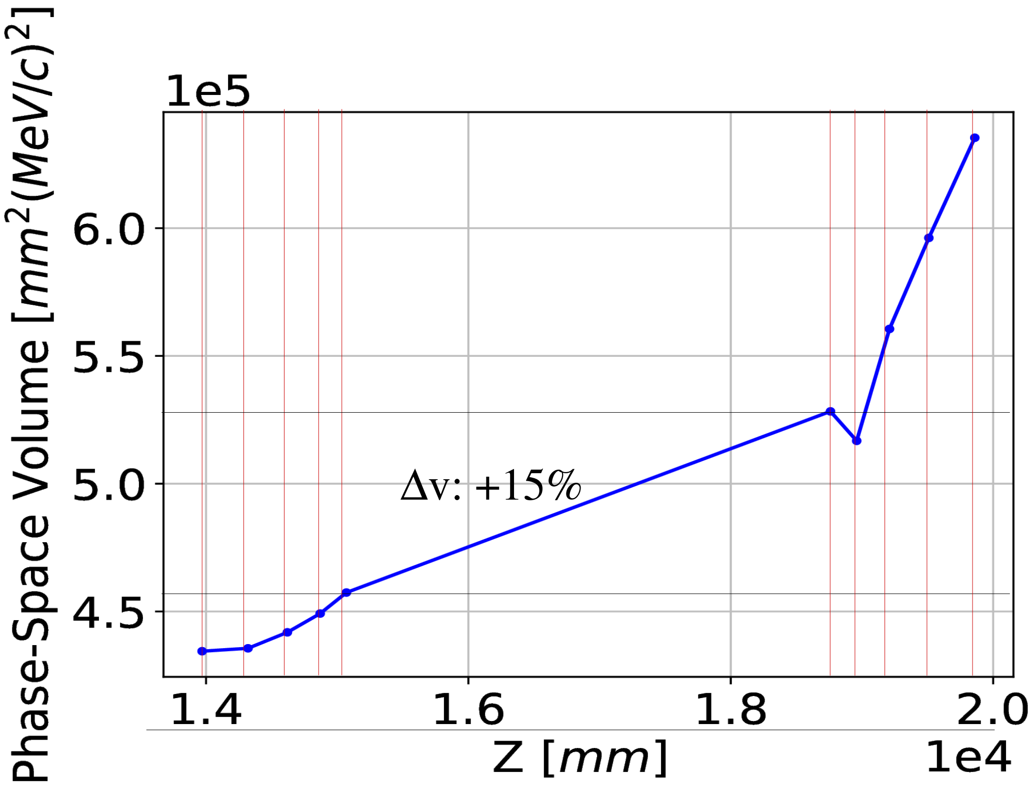


T. A. Mohayai, et al., "Novel Implementation Of Non-Parametric Density Estimation in MICE", IPAC'17, IPAC-2017-WEPAB135 (2017).

Modified KDE Analysis with Data

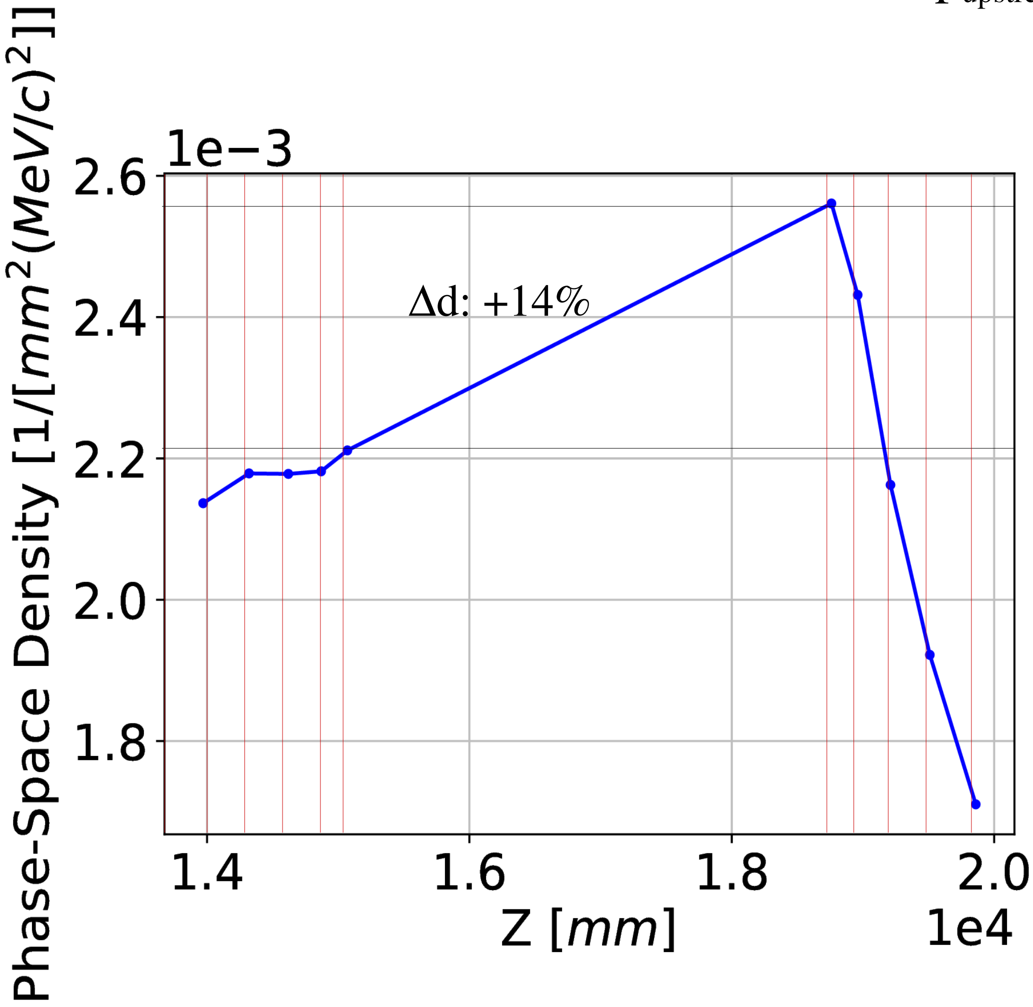
- Run 8681:
  - 3-140 beam setting with LiH absorber and no currents in M1D & M2D coils.
  - Vertical lines: tracker stations. Horizontal lines: locations of the tracker reference planes.
  - Cuts: *28* < TOF01 < *30.5* ns, *135* < $p_{upstream}$ < *145* MeV/c, p_{value} > *0.02*.

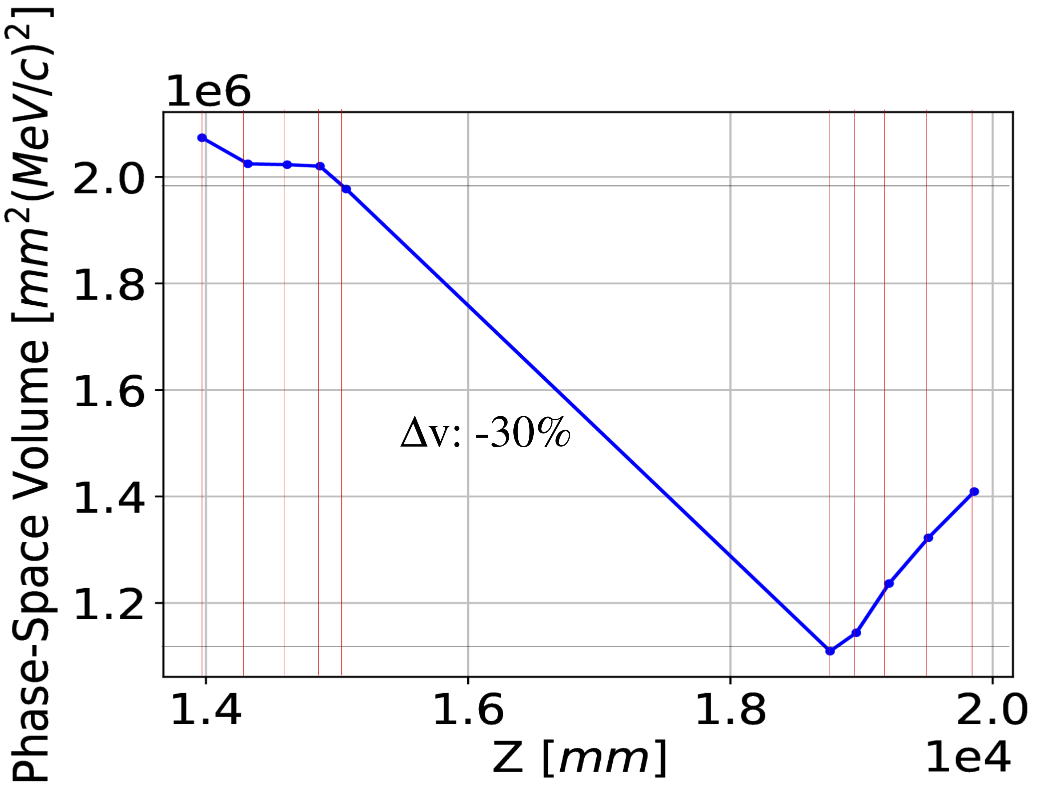




Modified KDE Analysis with Data

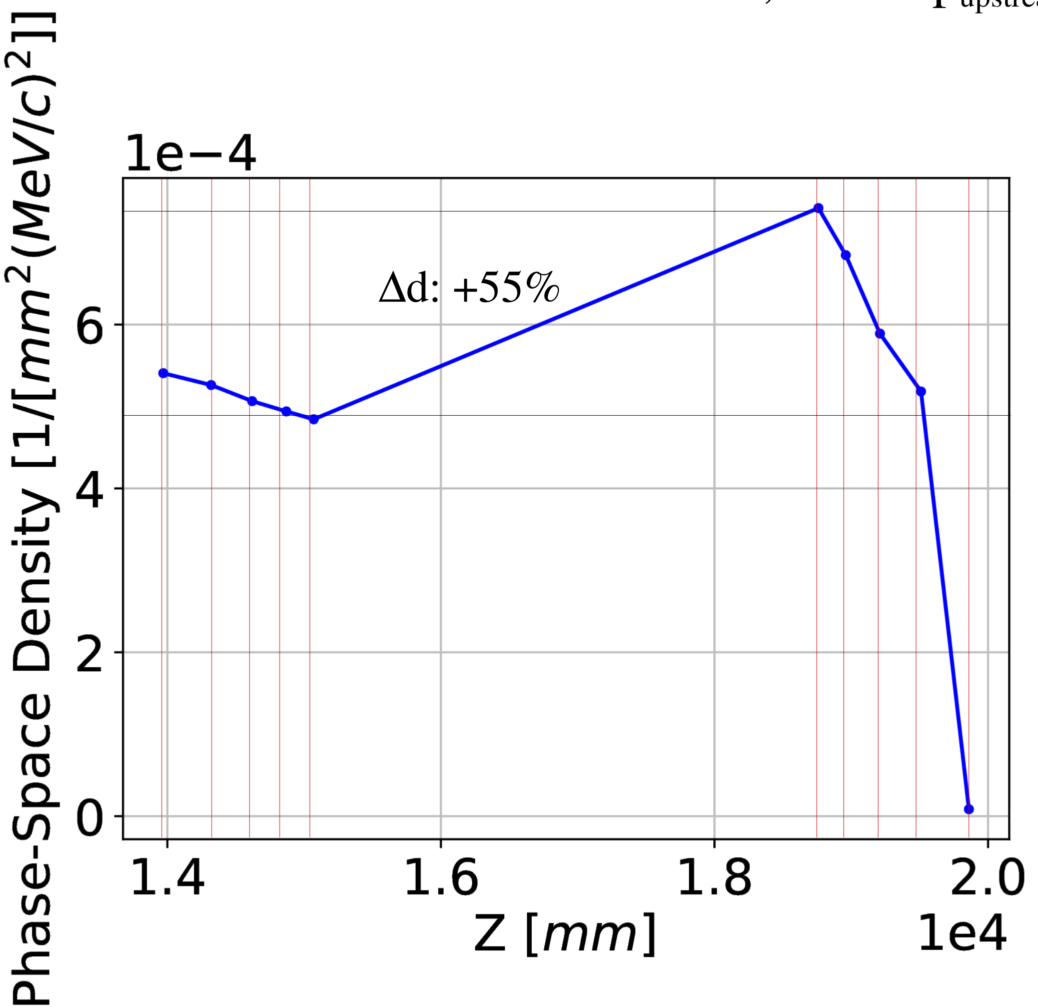
- Run 8699:
  - 6-140 beam setting with LiH absorber and no currents in M1D & M2D coils.
  - Vertical lines: tracker stations. Horizontal lines: locations of the tracker reference planes.
  - Cuts: *28* < TOF01 < *30.5* ns, *125* < $p_{upstream}$ < *162* MeV/c, p_{value} > *0.02*.

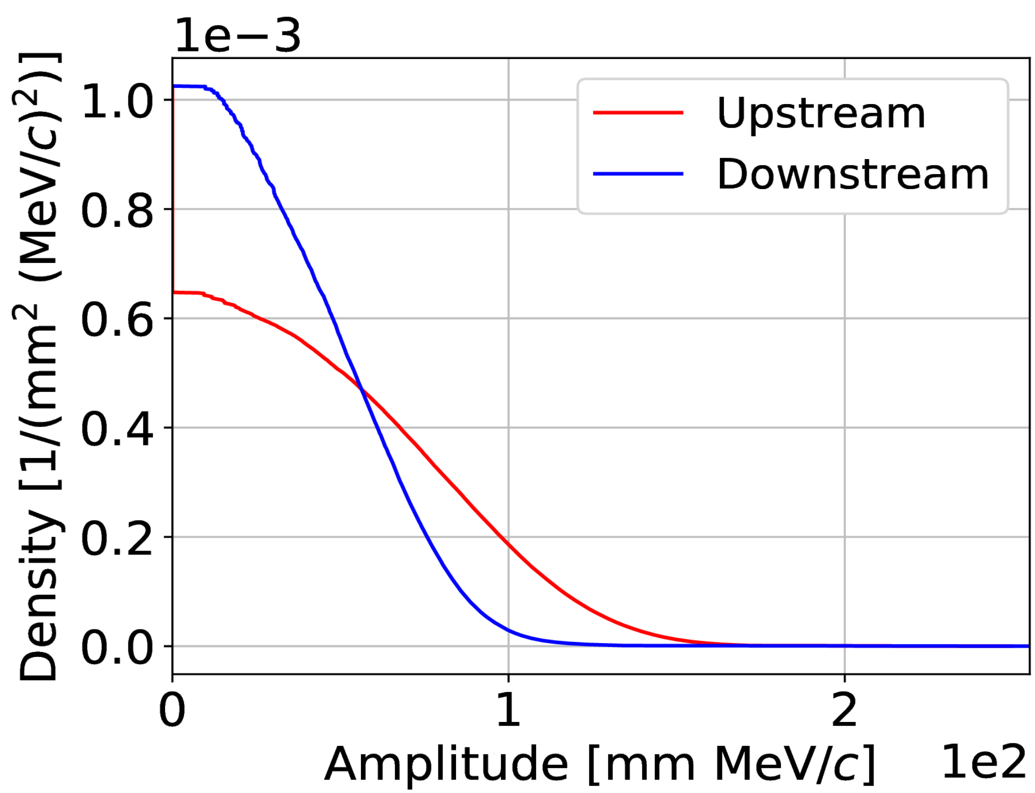




Modified KDE Analysis with Data

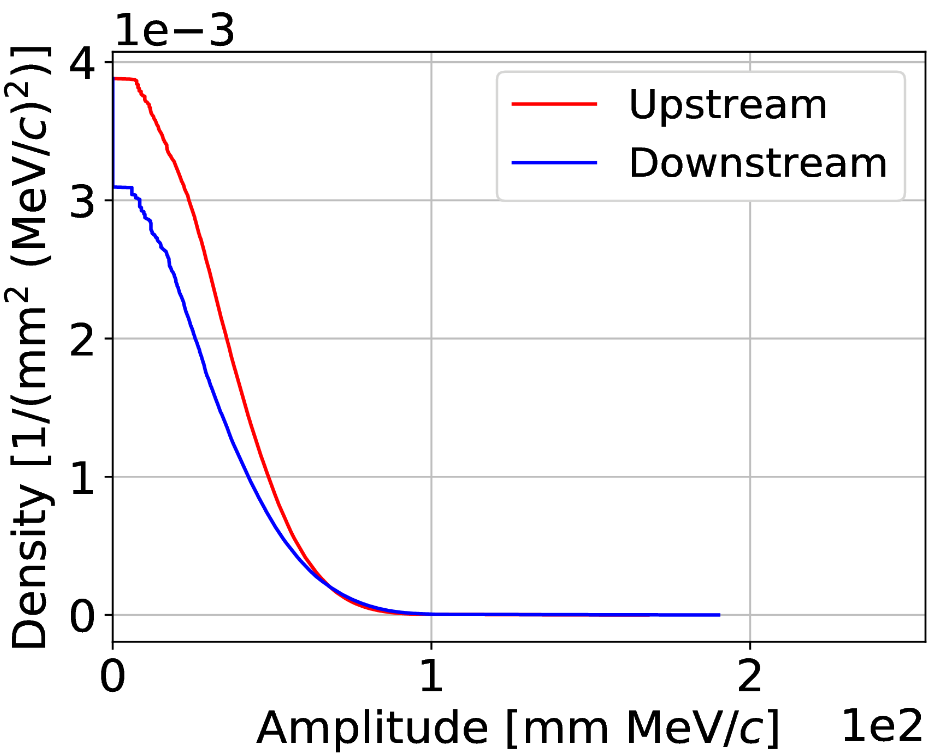
- Run 8685:
  - 10-140 beam setting with LiH absorber and no currents in M1D & M2D coils.
  - Vertical lines: tracker stations. Horizontal lines: locations of the tracker reference planes.
  - Cuts: *28* < TOF01 < *30.5* ns, *125* < $p_{upstream}$ < *162* MeV/c, p_{value} > *0.02*.

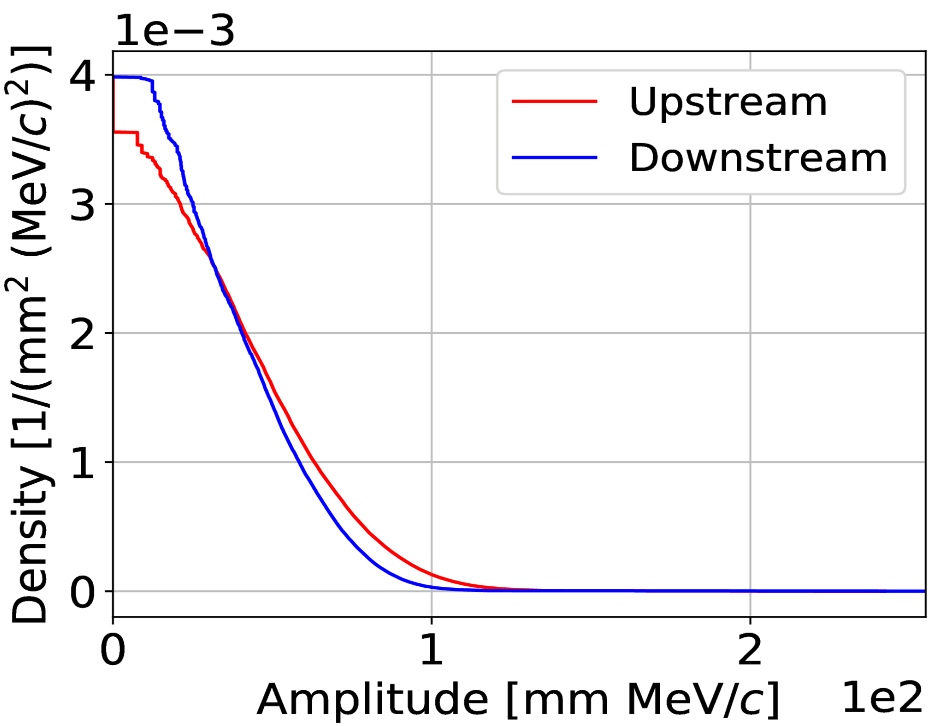




Density Versus Amplitude

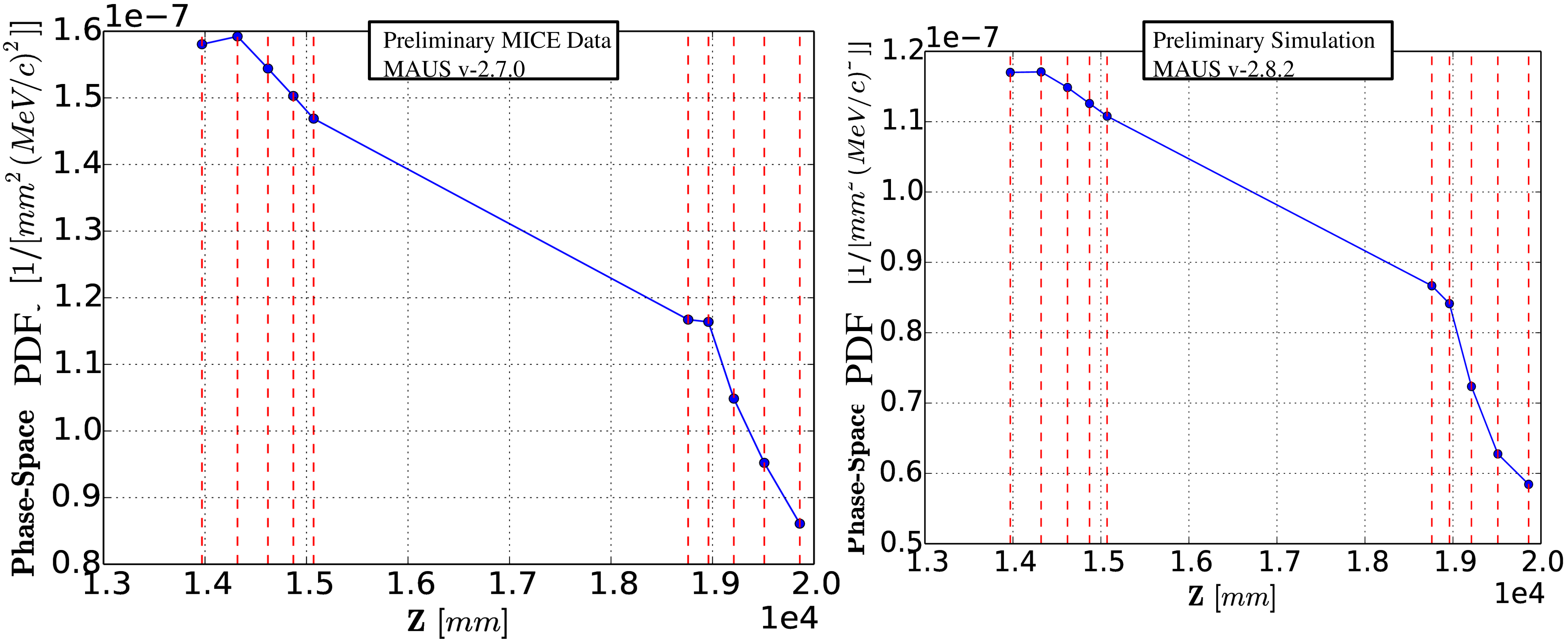
- Amplitude: *4th* root of volume or the mean radius of the hyper-ellipsoid.
- Increase in core density while decrease in density in the periphery.





KDE Application to MICE Data & Simulation

- Run 8681, Data versus Simulation:
  - 3-140, emittance-momentum setting, LiH absorber, downstream Match coils off.
  - Dashed red lines: tracker stations.
  - 28 < TOF01 < 30.5 ns, 125 < $p_{upstream}$ < 162 MeV/c, fixed contour evolution.



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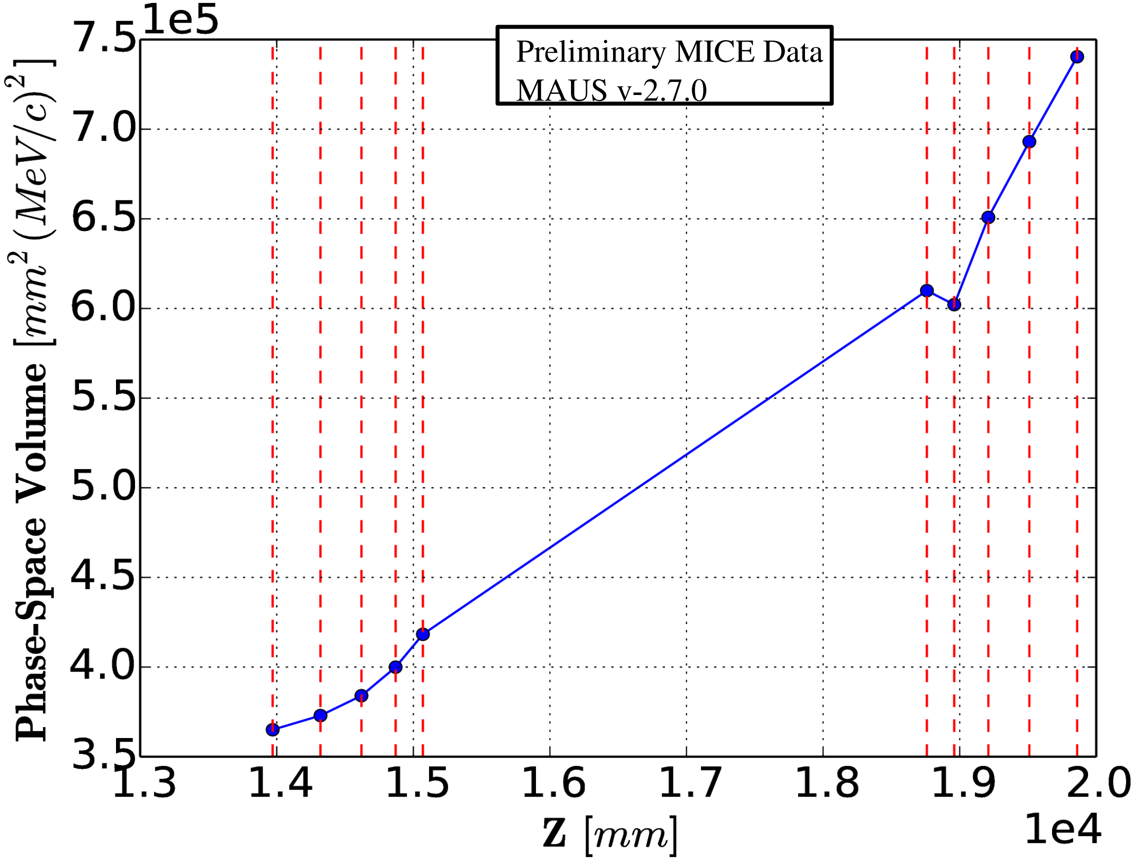
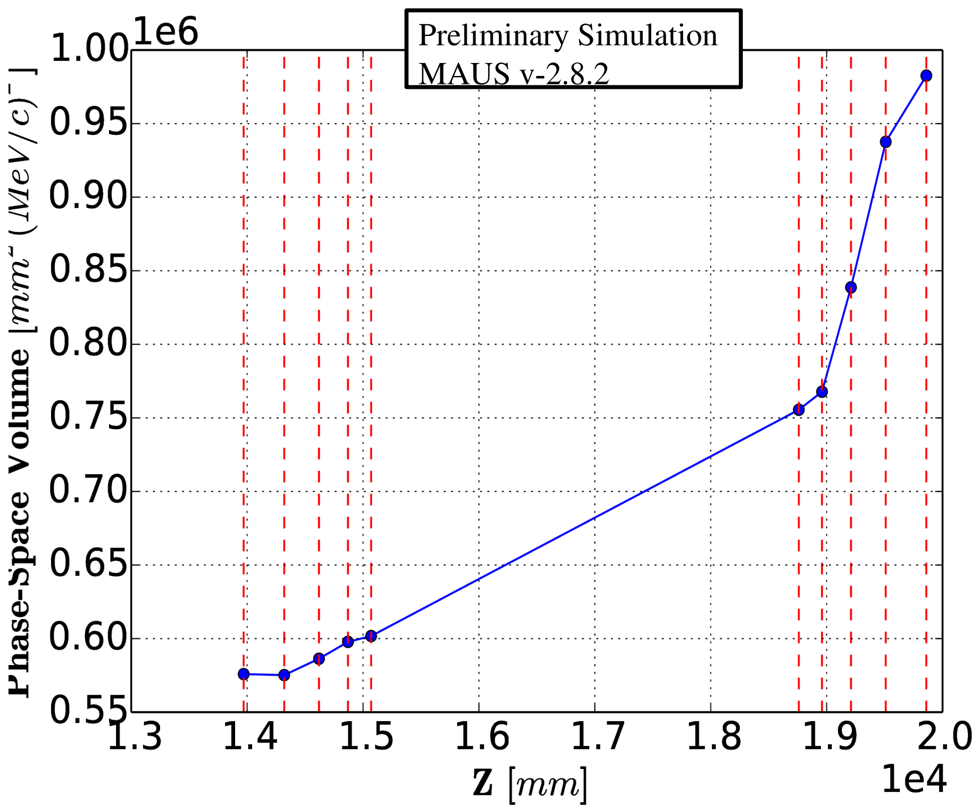




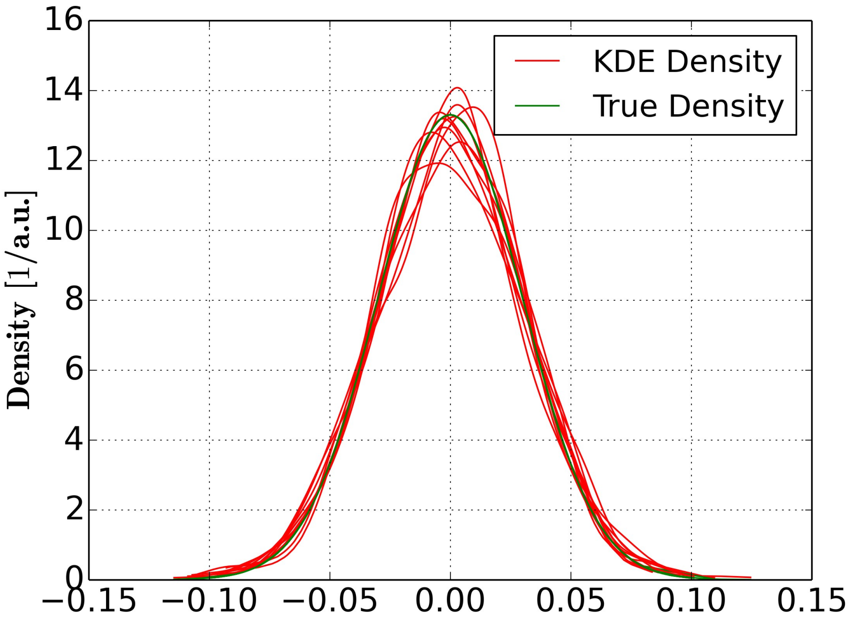
Conclusion and Future Prospects

- Phase-space PDF (normalized phase-space density) is more sensitive to beam loss than phase-space density (scaled phase-space density).
- The errors (MISE, systematics, etc) get too large in 4D. Further investigation into the error analysis in progress.
- Further extension of the toy MC studies under way.
- Kernel-based NNDE currently being applied to data. To be extended to a hybrid of the KDE and the NNDE methods.
- Stay tuned!

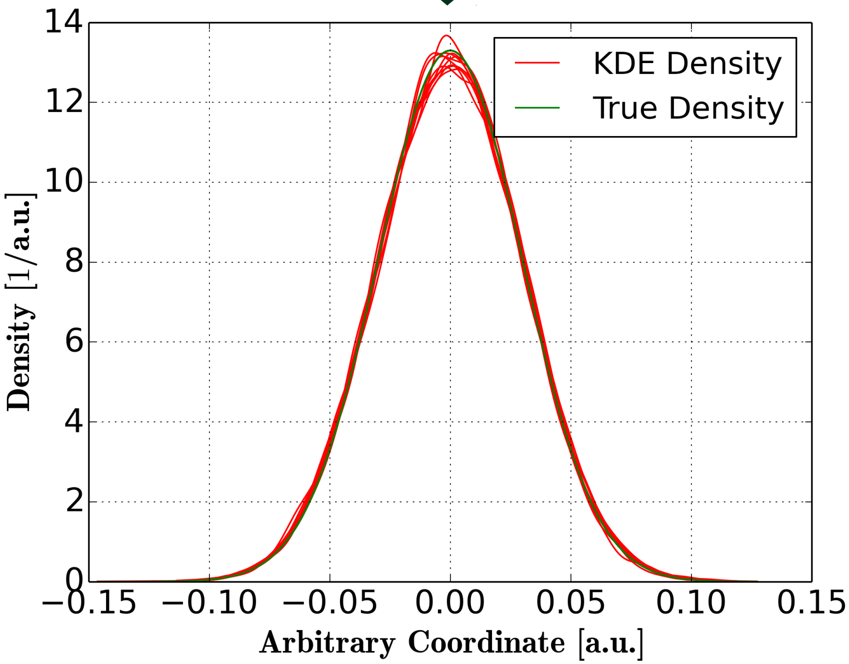
Additional Slides

KDE Error Analysis in 1D – sample size study, True vs. KDE

- Generated 10 Gaussian distributions (σ = 0.03 a.u), each with 1k, 10k, and 100k data points.
- Compared their KDE and true densities.
- The bandwidth is optimal (minimized MISE).

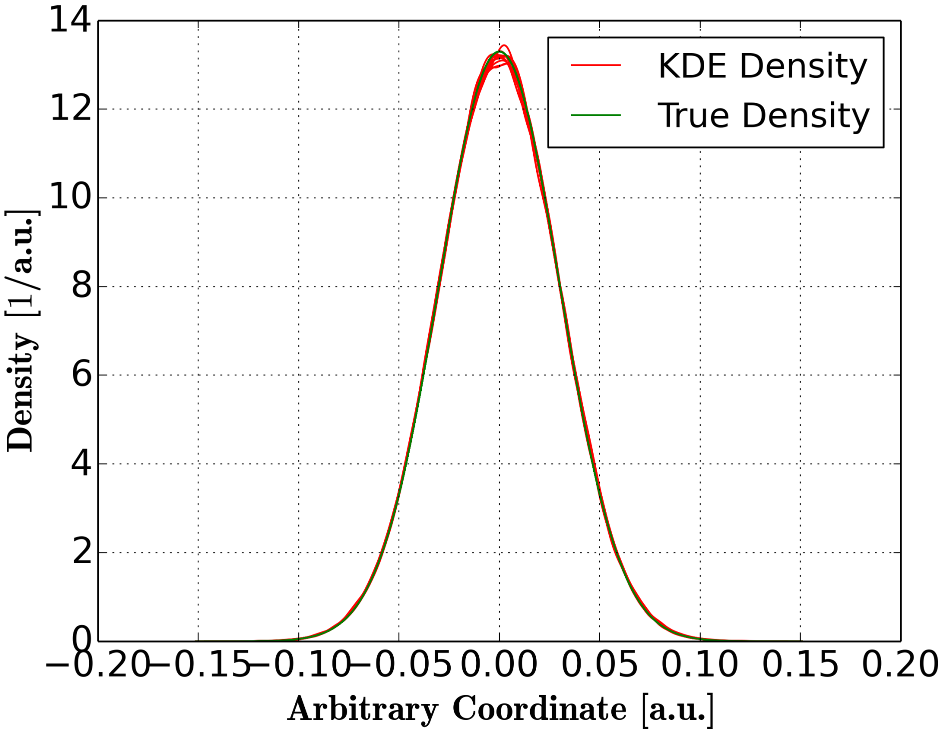


1k



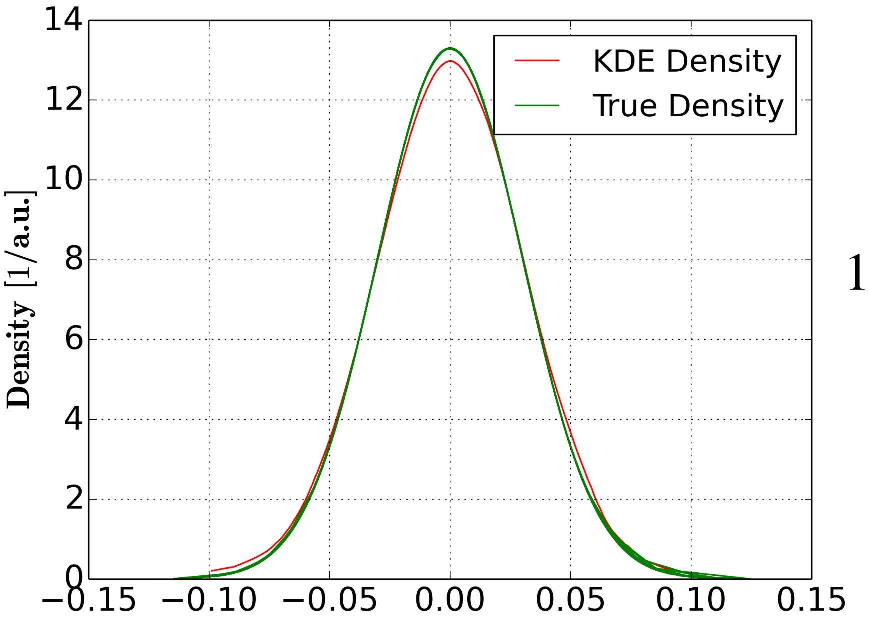
10k

100k

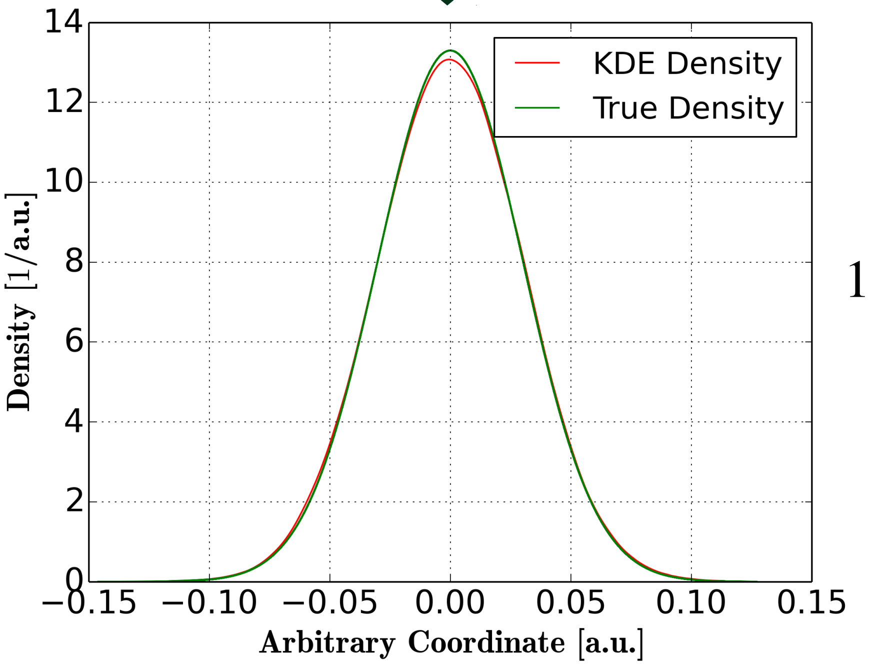


KDE Error Analysis in 1D cont. – sample size study, True vs. KDE

- Generated 10 Gaussian distributions (σ = 0.03 a.u), each with 1k, 10k, and 100k data points.
- Compared the **averages** of their KDE and true densities.
- The bandwidth is optimal (minimized MISE).

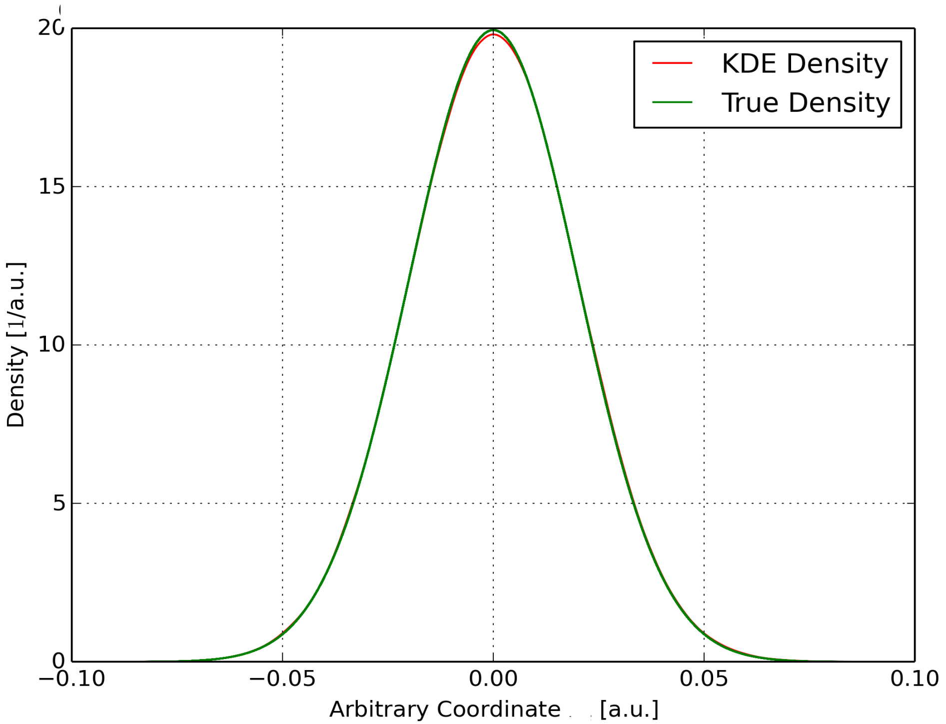


1k



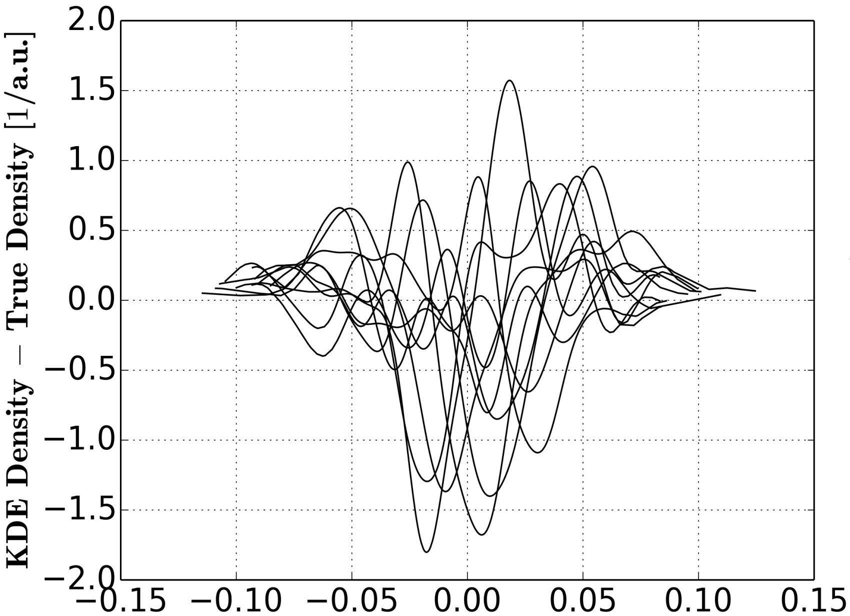
10k

100k



KDE Error Analysis in 1D cont. – sample size study, True vs. KDE

- Generated 10 toy Gaussian distributions (σ = 0.03 a.u), each with 1k, 10k, and 100k data points.
- Compared their **KDE errors** (differences between KDE density and true densities).
- The bandwidth is optimal (minimized MISE).



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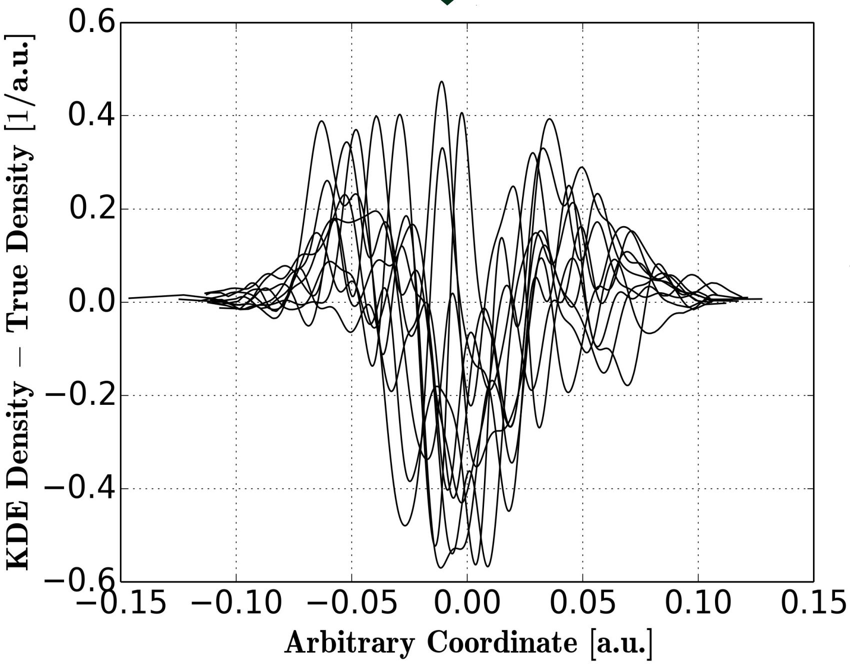
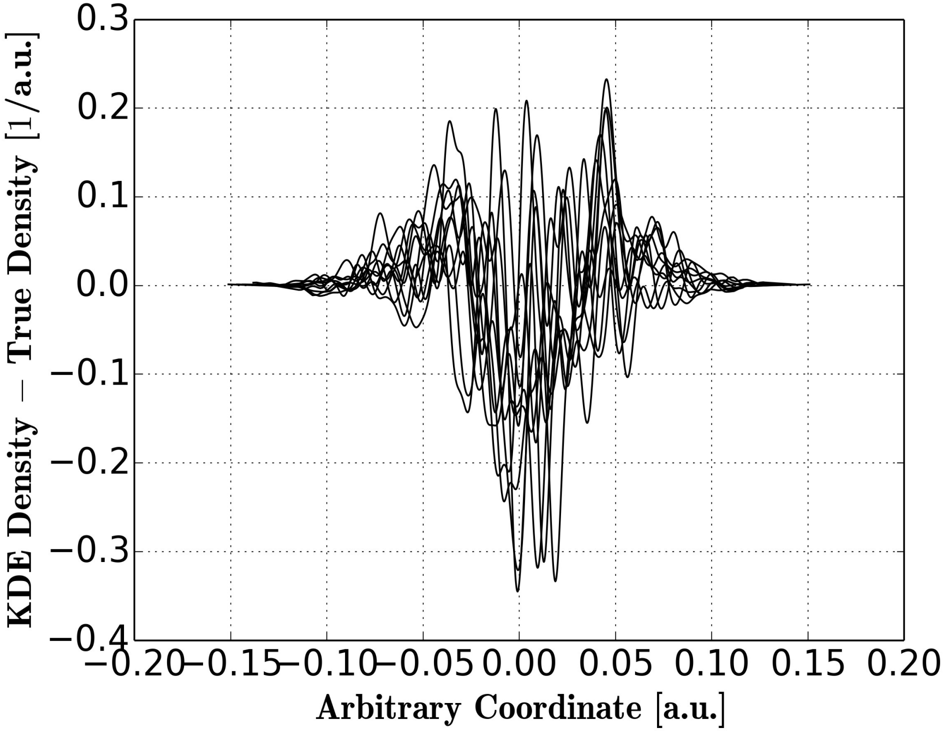


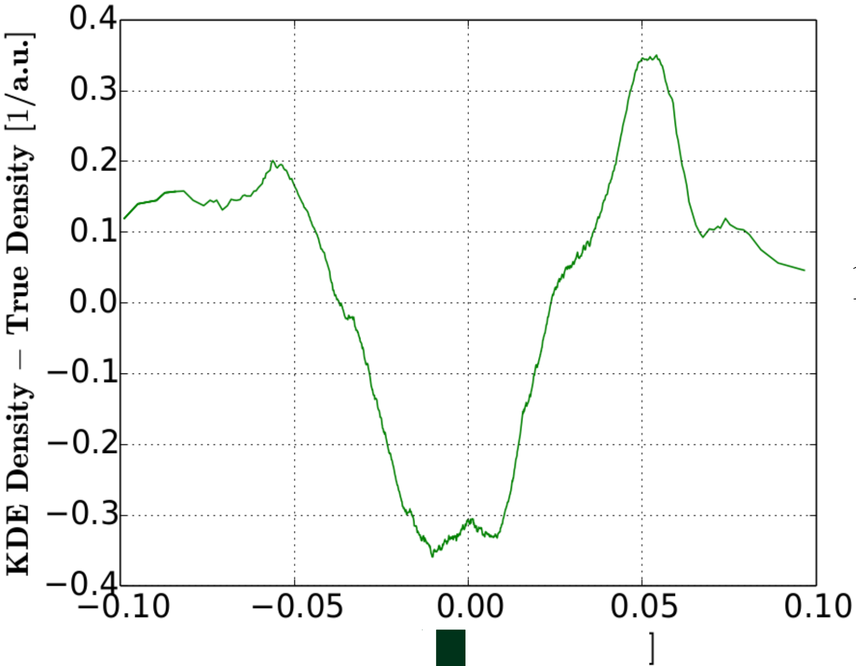
10k



100k

KDE Error Analysis in 1D cont. – sample size study, True vs. KDE

- Generated 10 toy Gaussian distributions (σ = 0.03 a.u), each with 1k, 10k, and 100k data points.
- Compared **their KDE error averages.**
- The bandwidth is optimal (minimized MISE).



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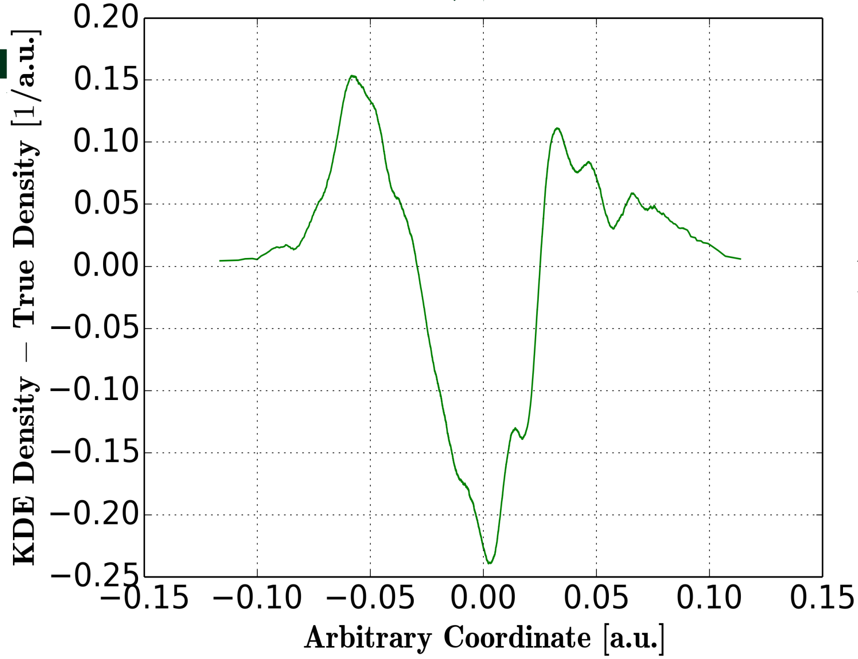

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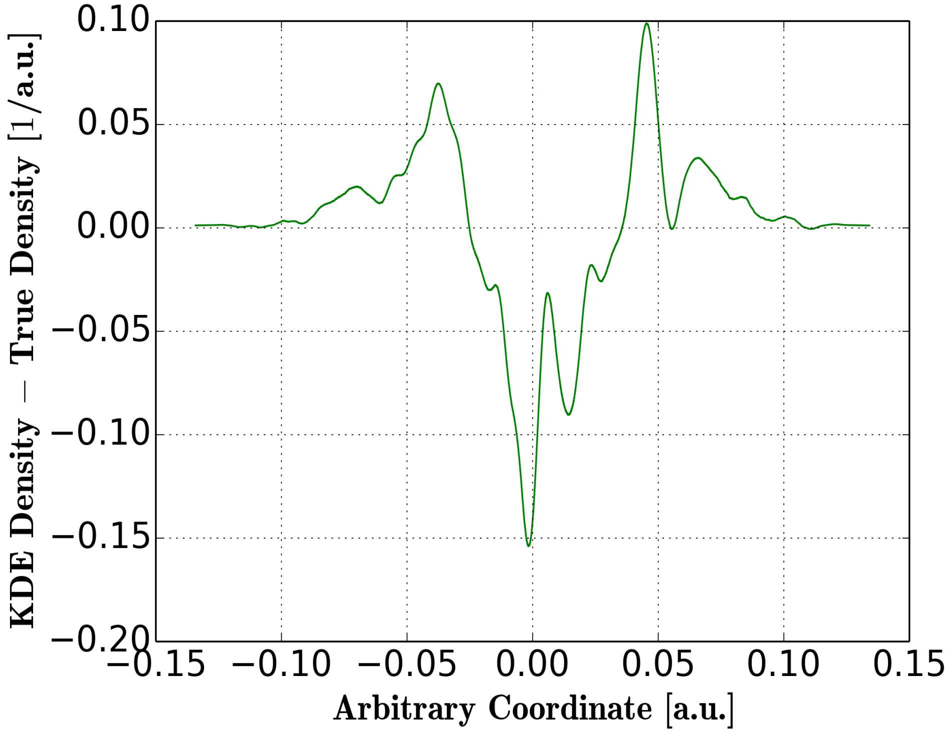

100k

KDE Error Analysis in 1D cont. – summary

- To summarize, isolated each curve's peak density (density curve enclosing 9% of the sample size):
  - KDE density stabilizes and approaches the true density curve as sample size grows.
  - Slight increase in mean density with growing sample size (caused by the optimal bandwidth's dependence on sample size).