Lee-Wick models and quantum gravity

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CERN, 5.5.2017

In general higher-derivative theories cannot be defined directly in Minkowski space, because they generate nonlocal, non Hermitian divergences, which cannot be subtracted away with any known procedure

U.G. Aglietti and D. Anselmi, Inconsistency of Minkowski higher-derivative theories, Eur. Phys. J. C 77 (2017) 84 and arXiv:1612.06510 [hep-th]

Consider the propagator
$$S(p,m)=\frac{1}{p^2-m^2+i\epsilon}\,\frac{M^4}{(p^2)^2+M^4}$$

and the bubble diagram
$$\Sigma(p)=\int_{k_s\leqslant\Lambda_{UV}}\frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}}\int_{-\infty}^{+\infty}\frac{\mathrm{d}k_0}{2\pi}S\left(k,m_1\right)S\left(k-p,m_2\right)$$

It is ok in four dimensions, but not in six, where (at m1 = m2 = 0)
$$\Sigma(p)=-\frac{M^4}{2(4\pi)^3}\left[\frac{M^2}{(p^2)^2}-\frac{i}{p^2}\right]\ln\left(\frac{\Lambda_{UV}^2}{M^2}\right)+\cdots$$

Decompose the propagator in partial fractions

$$S(p,m)=\frac{M^4}{M^4+m^4}\frac{1}{2\omega_\epsilon(p_s)}\left[\frac{1}{p_0-\omega_\epsilon(p_s)}-\frac{1}{p_0+\omega_\epsilon(p_s)}\right]$$
$$-\frac{M^2}{M^2-im^2}\frac{1}{4\Omega(p_s)}\left[\frac{1}{p_0-\Omega(p_s)}-\frac{1}{p_0+\Omega(p_s)}\right]+$$
$$-\frac{M^2}{M^2+im^2}\frac{1}{4\bar\Omega(p_s)}\left[\frac{1}{p_0-\bar\Omega(p_s)}-\frac{1}{p_0+\bar\Omega(p_s)}\right]$$

where $\omega_\epsilon(p_s)\equiv\sqrt{p_s^2+m^2-i\epsilon}$, $\Omega(p_s)\equiv\sqrt{p_s^2-iM^2}$, $p^\mu=(p_0,\mathbf{p})$ and $p_s=|\mathbf{p}|$

$$S(p,m)=\frac{1}{p^2-m^2+i\epsilon}\frac{M^4}{(p^2)^2+M^4}\qquad \Sigma(p)=\int_{k_s\leqslant\Lambda_{UV}}\frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}}\int_{-\infty}^{+\infty}\frac{\mathrm{d}k_0}{2\pi}S(k,m_1)\,S(k-p,m_2)$$

Apply the residue theorem to the bubble diagram. We have contributions at $k^2=\text{constant}$

$$S(k,m_1)\sim 1/k_s$$ for $k_s=|\mathbf{k}|$ large

$$S(k-p,m_2)\sim 1/((k-p)^2)^3\sim 1/(p\cdot k)^3\sim 1/k_s^3$$

At large space loop momentum, the integral behaves as

$$\int^{\Lambda_{UV}}\frac{k_s^{D-2}\mathrm{d}k_s}{k_s^4}$$

In four dimensions you get the problem with nontrivial numerators, which are certainly present in higher-derivative theories of quantum gravity

$$\mathcal{L}_{\rm HD} = -\frac{\sqrt{-g}}{2\kappa^2}\left[R - \frac{1}{M^4}(D_\rho R_{\mu\nu})(D^\rho R^{\mu\nu}) + \frac{1}{2M^4}(D_\rho R)(D^\rho R)\right]$$

Propagator
In harmonic gauge

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_0 = \frac{iM^4}{2(p^2+i\epsilon)}\frac{\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}}{(p^2)^2+M^4}$$

$$\begin{aligned}\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_1^{\rm nld} = \frac{\kappa^2 M^8}{240\pi^2(p^2)^2}\Big[&(68r+i)(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\nu\rho}\eta_{\mu\sigma}) + (373r-4i)\eta_{\mu\nu}\eta_{\rho\sigma}\\ &-\frac{1}{8p^2}(125ir^2+544r+8i)\left(p_\mu p_\rho \eta_{\nu\sigma}+p_\mu p_\sigma\eta_{\nu\rho}+p_\nu p_\rho\eta_{\mu\sigma}+p_\nu p_\sigma\eta_{\mu\rho}\right)\\ &+\frac{1}{4p^2}(255ir^2-1522r+36i)\left(p_\mu p_\nu\eta_{\rho\sigma}+p_\rho p_\sigma\eta_{\mu\nu}\right)\\ &-\frac{1}{2(p^2)^2}(185r^3+75ir^2-1048r+24i)p_\mu p_\nu p_\rho p_\sigma\Big]\ln\left(\frac{\Lambda_{UV}^2}{M^2}\right)\end{aligned}$$

$$r \equiv p^2/M^2$$

Thus, higher-derivative theories must be defined from Euclidean space

An alternative proposal for building higher-derivative theories is due to Lee and Wick

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix,
Nucl. Phys. B 9 (1969) 209
T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics,
Phys. Rev. D 2 (1970) 1033
R.E. Cutkosky, P.V. Landshoff, D.I. Olive, and J.C. Polkinghorne, A non-analytic S-matrix,
Nucl. Phys. B 12 (1969) 281,
B. Grinstein, D. O'Connell and M.B. Wise, Causality as an emergent macroscopic phenomenon: The Lee-Wick O(N) model, Phys. Rev. D 79 (2009) 105019 and arXiv:0805.2156 [hep-th].

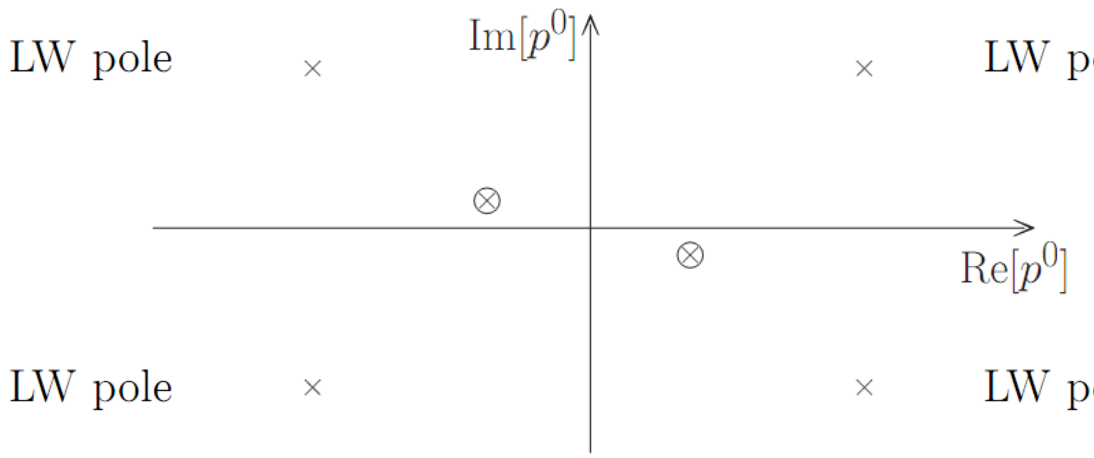
We are going to show that the two possibilities are actually the same thing

Even more, the Wick rotation from Euclidean space provides a new formulation of Lee-Wick quantum field theory, which cures the problems of the previous formulations and makes higher-derivative theories both renormalizable and unitary

D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory,
arXiv:1703.04584 [hep-th]
D. Anselmi and M. Piva, Perturbative unitarity of Lee-Wick models,
arXiv:1703.05563 [hep-th]

LW propagator

$$iD(p^2, m^2, \epsilon) = \frac{iM^4}{(p^2 - m^2 + i\epsilon)((p^2)^2 + M^4)}.$$



standard poles

$$p^0 = \pm\omega_m(\mathbf{p}) \mp i\epsilon,$$

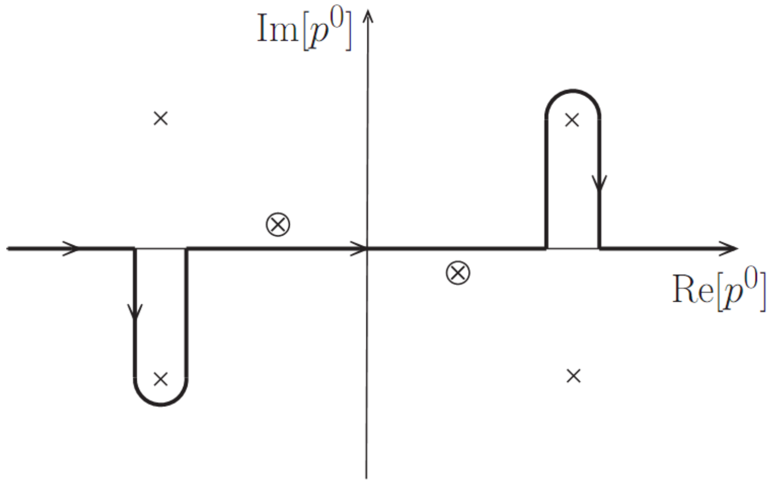
$$\omega_m(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}$$

LW poles

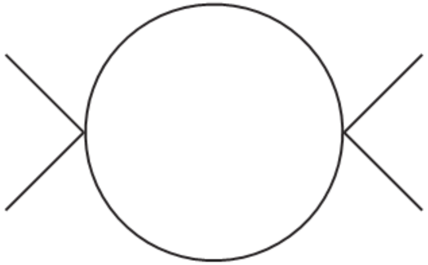
$$p^0 = \pm\omega_M(\mathbf{p}), \qquad p^0 = \pm\omega_M^*(\mathbf{p}),$$

$$\omega_M(\mathbf{p}) = \sqrt{\mathbf{p}^2 + iM^2}$$

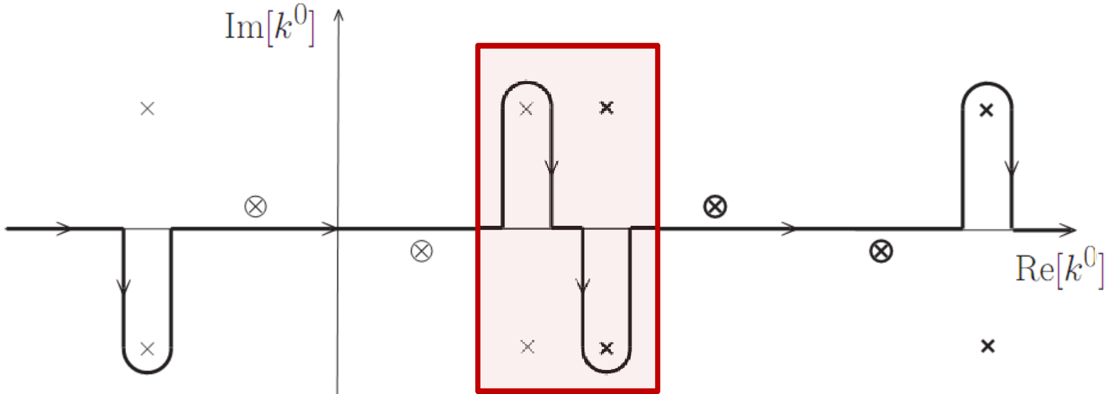
Wick rotation:



Bubble diagram:



$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} D(k^2, m_1^2, \epsilon_1) D((k-p)^2, m_2^2, \epsilon_2)$$



LW pinching

LW pinching $\quad$ At $\mathbf{p} = 0$

Poles involved in the LW pinching

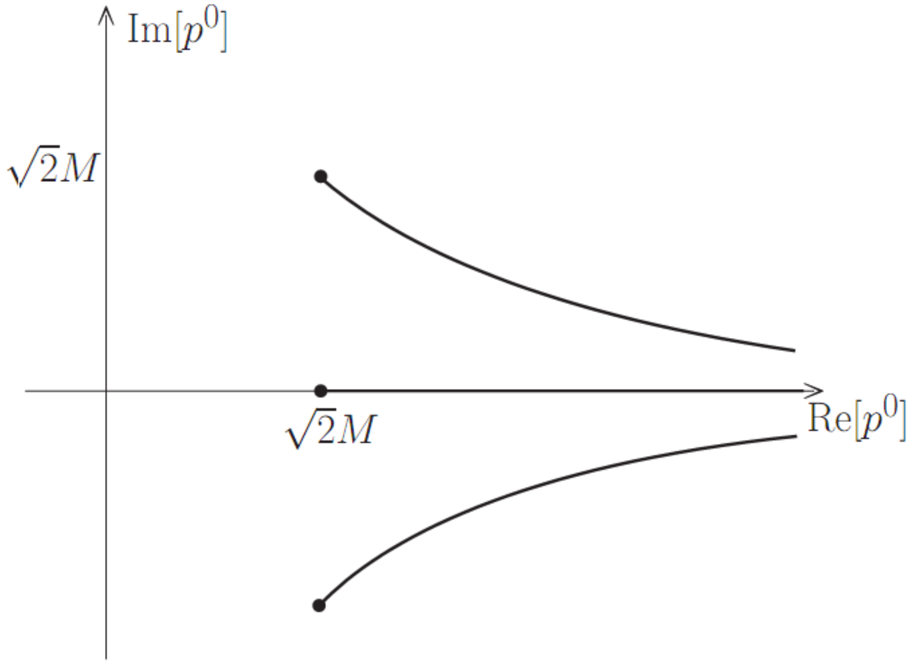
$$\frac{1}{k^0 - p^0 + \omega_M^*(\mathbf{k})} \frac{1}{k^0 - \omega_M(\mathbf{k})}$$

Conditions of LW pinching

$$p^0 = \sqrt{k_s^2 + iM^2} + \sqrt{k_s^2 - iM^2}$$

Solutions to the conditions of LW pinching

$$k_s^2 = \frac{(p^0)^4 - 4M^4}{4(p^0)^2}$$



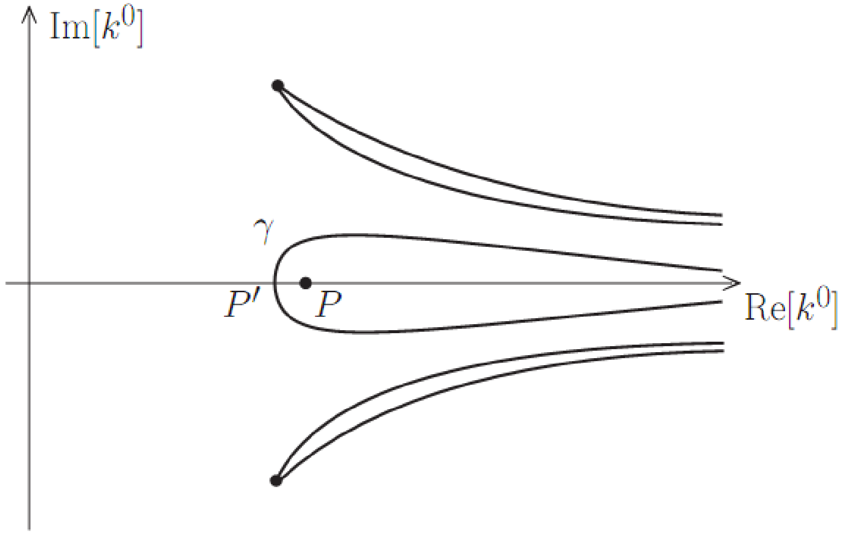
LW pinching   At $\mathbf{p} \neq 0$

$$p^0 = \sqrt{\mathbf{k}^2 + iM^2} + \sqrt{(\mathbf{k}-\mathbf{p})^2 - iM^2}$$

$$\sqrt{\frac{\mathbf{p}^2}{2} + \sqrt{\frac{(\mathbf{p}^2)^2}{4} + 4M^4}} \equiv E_{P'}$$

$$\sqrt{2M^2 + \mathbf{p}^2} \equiv E_P$$

Lorentz invariance is violated
The Wick rotation is not analytic



N. Nakanishi, Lorentz noninvariance of the complex-ghost relativistic field theory, Phys. Rev. D 3, 811 (1971).

Solutions to the conditions of LW pinching for p° real

$$\mathbf{k}^2 = \frac{(p^0)^4 - 4M^4}{4(p^0)^2}, \qquad \mathbf{p}^2 = 2\mathbf{p}\cdot\mathbf{k}.$$

Calculation around the LW pinching

denominator
$$D_0 = p^0 - \sqrt{\mathbf{k}^2 + iM^2} - \sqrt{(\mathbf{k}-\mathbf{p})^2 - iM^2}$$

change of variables around the pinching
$$k_s = \frac{\sigma_-}{2p^0} + \tau\frac{\sigma_+^2}{2\sigma_-(p^0)^2} + \eta\frac{p_s\sigma_+^2}{4\sigma_- M^2}, \qquad u = \frac{p_s}{2k_s} + \eta\frac{\sigma_+^2}{2\sigma_- M^2}$$
$$\sigma_\pm \equiv \sqrt{(p^0)^4 \pm 4M^4} \qquad u = \cos\theta$$

Key term of the integral:
$$\frac{\mathrm{d}^{D-1}\mathbf{k}}{D_0} = -\frac{2\pi^{(D-2)/2}}{\Gamma\left(\frac{D}{2}-1\right)}\frac{\sigma_+^4\left[\sigma_-^2 - (p^0)^2 p_s^2\right]^{(D-4)/2}}{(2p^0)^D M^2}\frac{\mathrm{d}\tau\mathrm{d}\eta}{\tau - ip_s\eta}$$

Limit

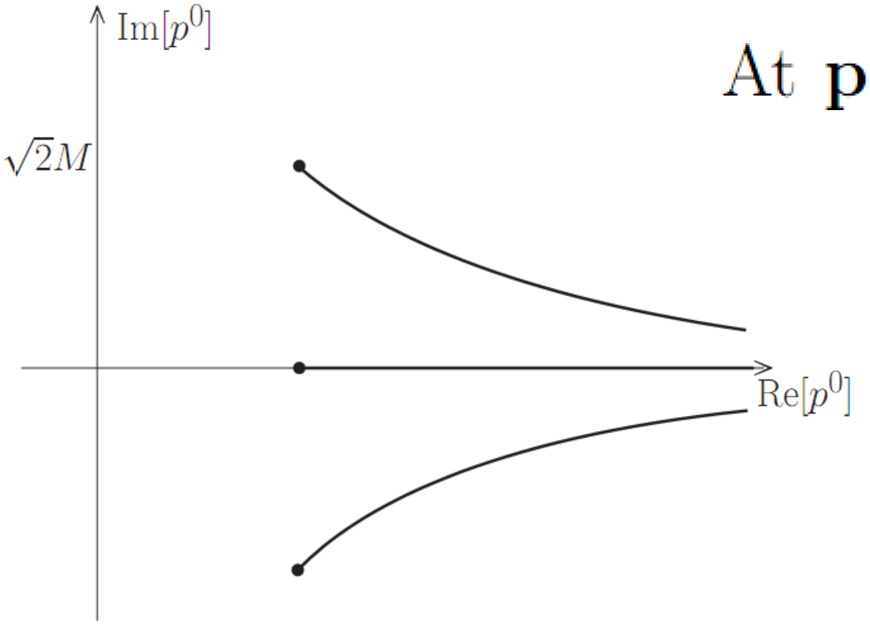
$$p_s \to 0 \qquad -\frac{2\pi^{(D-2)/2}}{\Gamma\left(\frac{D}{2}-1\right)}\frac{\sigma_+^4\sigma_-^{D-4}}{(2p^0)^D M^2}\mathrm{d}\tau\mathrm{d}\eta\left[\mathcal{P}\left(\frac{1}{\tau}\right) + \cancel{i\pi\,\mathrm{sgn}(\eta)\delta(\tau)}\right]$$

Other approaches have been proposed by introducing ad hoc prescriptions

$$\frac{2i}{(8\pi)^2}\frac{\sigma_-}{(p^0)^2}\frac{M^4}{(M^4+m_1^4)(M^4+m_2^4)}\left[(M^4+m_1^2m_2^2)\mathcal{P}\left(\frac{1}{\tau}\right)+aM^2(m_1^2-m_2^2)\delta(\tau)\right]\mathrm{d}\tau$$

CLOP prescription $\quad M' \neq M \quad a = \pi\,\mathrm{sgn}(M'-M)$

R.E. Cutkosky, P.V. Landshoff, D.I. Olive, and J.C. Polkinghorne, A non-analytic S-matrix, Nucl. Phys. B 12 (1969) 281.



At $\mathbf{p} = 0$

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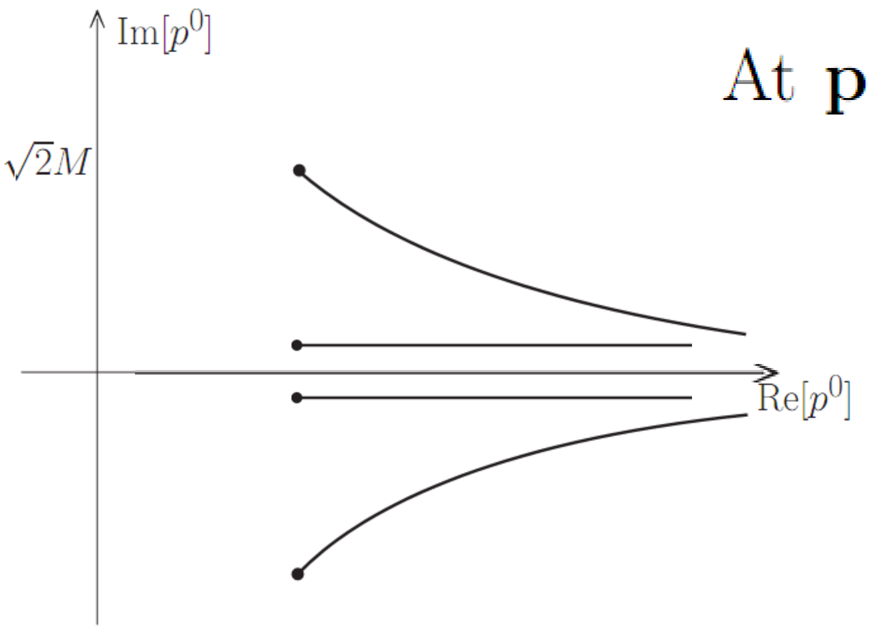

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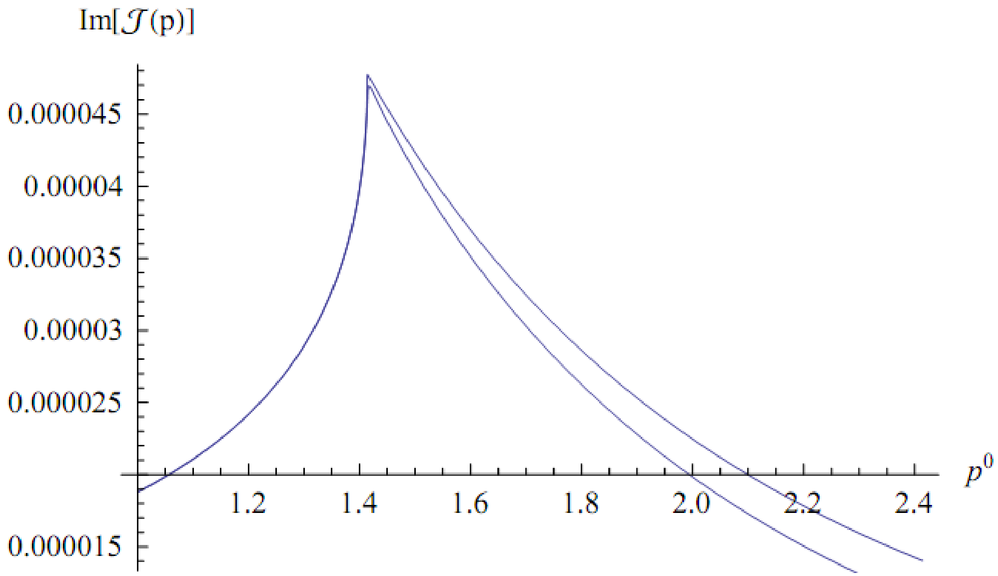
Comparison between our formulation and the CLOP prescription

$$m_1=3,\ m_2=5$$

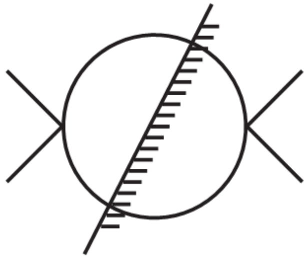
$$M=1$$



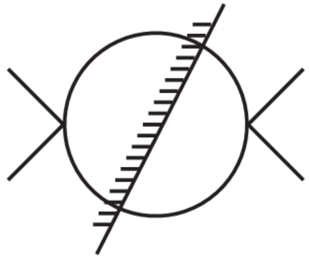
Unitarity: the real part vanishes both below and above the LW threshold

Cutting equations in the LW models

$$i\mathcal{M} - i\mathcal{M}^* \quad + $$



$$+$$



$$= \quad 0$$

$$i\mathcal{M} = \frac{\lambda^2}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D} D(k^2, m_1^2, \epsilon_1) D((k-p)^2, m_2^2, \epsilon_2)$$

$$= \frac{\lambda^2 M^8}{2}\int \frac{\mathrm{d}k^0 \mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^D} \prod_{j=1}^{2} \frac{1}{(e_j - \nu_j)(e_j + \nu_j)(e_j - \nu_j^*)(e_j + \nu_j^*)(e_j - \omega_j + i\epsilon_j)(e_j + \omega_j - i\epsilon_j)},$$

where $e_1 = k^0$, $e_2 = k^0 - p^0$, $\omega_1 = \sqrt{\mathbf{k}^2 + m_1^2}$ and $\omega_2 = \sqrt{(\mathbf{k}-\mathbf{p})^2 + m_2^2}$

$$\nu_1 = \sqrt{\mathbf{k}^2 + iM^2},\ \nu_2 = \sqrt{(\mathbf{k}-\mathbf{p})^2 + iM^2}$$

The residue theorem gives

$$i\mathcal{M} = -\frac{i\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} [\mathrm{Res}(z_1) + \mathrm{Res}(z_2) + \cancel{\mathrm{Res}(w_1)} + \mathrm{Res}(w_2) + \cancel{\mathrm{Res}(w_1^*)} + \mathrm{Res}(w_2^*)]$$

$$z_1 = \omega_1 - i\epsilon_1, \qquad z_2 = p^0 + \omega_2 - i\epsilon_2, \qquad w_1 = \nu_1, \qquad w_2 = p^0 + \nu_2,$$

The key identity

$$\mathrm{Res}(w_i^*) = (\mathrm{Res}(w_i))^*$$

ensures that

$$\mathrm{Disc}\mathcal{M} = 2i\mathrm{Im}\mathcal{M} = -\, i\lambda^2 \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \mathrm{Im}\left[\mathrm{Res}(z_1) + \mathrm{Res}(z_2)\right].$$

That is to say the LW poles do not propagate through the cuts. Only the physical degrees of freedom propagate

If we project the in and out states onto the physical subspace, the intermediate states are automatically projected onto the same subspace

This is perturbative unitarity, $SS^\dagger=1$

We checked it also in the triangle diagram. A paper with the general proof is in preparation

What does this mean for quantum gravity?

D. Anselmi, On the quantum field theory of the gravitational interactions, arXiv:1704.07728 [hep-th]

For previous alternative approaches (with the CLOP prescription or other methods), see

- *E. Tomboulis, 1/N expansion and renormalization in quantum gravity, Phys. Lett. B 70 (1977) 361*
- *E. Tomboulis, Renormalizability and asymptotic freedom in quantum gravity, Phys. Lett. B 97 (1980) 77*
- *Shapiro and L. Modesto, Superrenormalizable quantum gravity with complex ghosts, Phys. Lett. B755 (2016) 279-284 and arXiv:1512.07600 [hep-th]*
- *L. Modesto, Super-renormalizable or finite Lee--Wick quantum gravity, Nucl. Phys. B909 (2016) 584 and arXiv:1602.02421 [hep-th]*

There exist theories of quantum gravity that are (super)renormalizable and unitary, the simplest one being

$$
\begin{aligned}
-2\kappa^2\mu^\varepsilon \frac{\mathcal{L}_{\mathrm{QG}}}{\sqrt{-g}} = {} & 2\lambda_C M^2 + \zeta R - \frac{\gamma}{M^2} R_{\mu\nu}R^{\mu\nu} + \frac{1}{2M^2}(\gamma-\eta)R^2 \\
& -\frac{1}{M^4}(D_\rho R_{\mu\nu})(D^\rho R^{\mu\nu}) + \frac{1}{2M^4}(1-\xi)(D_\rho R)(D^\rho R) \\
& +\frac{1}{M^4}\left(\alpha_1 R_{\mu\nu}R^{\mu\rho}R_\rho^\nu + \alpha_2 R R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^3 + \alpha_4 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right. \\
& \left. + \alpha_5 R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + \alpha_6 R_{\mu\nu\rho\sigma}R^{\rho\sigma\alpha\beta}R^{\mu\nu}_{\alpha\beta}\right)
\end{aligned}
$$

The renormalization is the same as the one of the Euclidean theory:

$$p^0 = \sqrt{\mathbf{k}^2 + iM^2} + \sqrt{(\mathbf{k}-\mathbf{p})^2 - iM^2}$$

There are counterterms up to three loops

$$\frac{\mathcal{L}_{\mathrm{count}}}{\sqrt{-g}} = \frac{1}{(4\pi)^2\varepsilon}\left[2a_C M^4 + a_\zeta M^2 R - a_\gamma R_{\mu\nu}R^{\mu\nu} + \frac{1}{2}(a_\gamma - a_\eta)R^2\right]$$

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle^{\text{free}}_{\eta=\xi=0} = \frac{iM^4}{2}\frac{\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}}{P(1,\gamma,\zeta,2\lambda_C)}$$

$$P(a,b,c,d) \equiv a(p^2)^3 + bM^2(p^2)^2 + cM^4p^2 + dM^6$$

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle^{\text{free}} = \langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle^{\text{free}}_{\eta=\xi=0} - \frac{iM^4(\eta M^2+\xi p^2)}{2P(1,\gamma,\zeta,2\lambda_C)}\frac{(p^2\eta_{\mu\nu}+2p_\mu p_\nu)(p^2\eta_{\rho\sigma}+2p_\rho p_\sigma)}{P(1-3\xi,\gamma-3\eta,\zeta,2\lambda_C)}$$

$$iS(p) = \frac{i}{p^2-m^2+i\epsilon}\frac{M^4}{(p^2-\mu^2)^2+M^4}$$

With a vanishing cosmological constant, the unitarity conditions are

$$\gamma^2 < 4\zeta, \qquad (\gamma-3\eta)^2 < 4\zeta(1-3\xi), \qquad \gamma < 0, \qquad \gamma < 3\eta$$

The proof of unitarity cannot be carried out to the end in a strict sense when the cosmological constant is different from zero

It is not known how to define a consistent scattering theory in the presence of a cosmological constant (asymptotic states, S matrix, etc.)

This might be just our present lack of knowledge or it might be impossible from first principles

If it is impossible, it means that there is a unitarity anomaly in the universe and a nonvanishing cosmological constant is a measure of its magnitude

Let us inquire whether we can turn off the cosmological constant consistently in our model. That means that we have to solve a chain of RG conditions

$$f = 0, \qquad \beta_i f_i = 0, \qquad \beta_j(\beta_{ij} f_i + \beta_i f_{ij}) = 0,$$

$$\beta_k(\beta_{jk}\beta_{ij} f_i + \beta_j \beta_{ijk} f_i + 3\beta_j \beta_{ij} f_{ik} + \beta_j \beta_i f_{ijk}) = 0,$$

$$\beta_\zeta = -2a_\zeta^{(1)}\bar{\alpha} - 4z_1\bar{\alpha}^2, \qquad \beta_\gamma = -2a_\gamma\bar{\alpha}, \qquad \beta_\eta = -2a_\eta\bar{\alpha},$$
$$\beta_C = -2a_C^{(1)}\bar{\alpha} - 2(2u_1\gamma + 2v_1\eta + 3w_1\bar{\alpha})\bar{\alpha}^2, \qquad \beta_{\mathrm{QG}} = 0,$$

$$\bar{\alpha} = \frac{\kappa^2 M^2}{(4\pi)^2}$$

$$\xi = \alpha_6 = 0$$

$$a_C^{(1)} = \frac{3}{4}(4\zeta - 2\gamma^2 + 2\eta\gamma - 3\eta^2), \qquad a_\zeta^{(1)} = \frac{1}{4}\gamma\tau + \frac{1}{2}\eta\sigma,$$
$$\begin{aligned}240a_\gamma = {} & 756 - 1080\alpha_1 + 360\alpha_1^2 + 480\alpha_2 - 480\alpha_1\alpha_2 - 640\alpha_2^2 + 960\alpha_4 - 960\alpha_1\alpha_4 \\ & -2560\alpha_2\alpha_4 - 5440\alpha_4^2 + 1940\alpha_5 - 1080\alpha_1\alpha_5 - 1280\alpha_2\alpha_5 - 3040\alpha_4\alpha_5 - 225\alpha_5^2, \\ 240a_\eta = {} & -508 - 1440\alpha_1 + 1395\alpha_1^2 - 2400\alpha_2 + 5160\alpha_1\alpha_2 + 6880\alpha_2^2 - 8640\alpha_3 + 12960\alpha_1\alpha_3 \\ & +43200\alpha_2\alpha_3 + 77760\alpha_3^2 - 1920\alpha_4 + 9600\alpha_1\alpha_4 + 17920\alpha_2\alpha_4 + 34560\alpha_3\alpha_4 \\ & +20800\alpha_4^2 + 180\alpha_5 + 1170\alpha_1\alpha_5 + 3560\alpha_2\alpha_5 + 11520\alpha_3\alpha_5 + 5120\alpha_4\alpha_5 + 520\alpha_5^2,\end{aligned}$$

$$\tau = 8 + 9\alpha_1 - 72\alpha_3 + 64\alpha_4 + 3\alpha_5, \qquad \sigma = -12 + 9\alpha_1 + 30\alpha_2 + 108\alpha_3 + 24\alpha_4 + 8\alpha_5$$

$$\lambda_C = \eta = 0 \qquad \beta_C = \beta_\eta = 0$$

$$\alpha_4 = \alpha_5 = \alpha_6 = 0$$

$$(\alpha_1, \alpha_2) = (4.51163.., -3.91524...), \qquad (2.89114..., -1.93684...),$$
$$(0.800169..., -0.368609...), \qquad (0.197062..., -0.679314...)$$

$$\alpha_3 = (8 + 9\alpha_1)/72$$

The theory turns out to be finite in this case

Relaxing the assumption $\eta = 0$ there are solutions that are not finite:

$$\zeta = 3.1545\eta^2, \qquad \gamma = 2.7492\eta, \qquad \alpha_2 = 0.5589, \qquad \lambda_C = \xi = \alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0.$$

$$\beta_\eta = -2.4988\bar{\alpha}$$

Similar results hold when the theory is coupled to matter (up to matter self-interactions)

The problem of uniqueness

The model we have studied is just the simplest one in a class that contains infinitely many similar models

$$-2\kappa^2\mu^\varepsilon \frac{\mathcal{L}'_{\mathrm{QG}}}{\sqrt{-g}} = 2\lambda_C M^2 + \zeta R + \frac{1}{M^2} R_{\mu\nu} P_n(\Box_c/M^2) R^{\mu\nu} - \frac{1}{2M^2} R Q_n(\Box_c/M^2) R + V(R)$$

It is relatively easy to build consistent models with a vanishing cosmological constant for $n > 2$

$$-2\kappa^2\mu^\varepsilon \frac{\mathcal{L}'_{\mathrm{QG}}}{\sqrt{-g}} = \zeta R - \frac{\gamma}{M^2} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2M^2}(\gamma - \eta) R^2 + \frac{1}{M^{2n+2}} R_{\mu\nu} \Box_c^n R^{\mu\nu} - \frac{1}{2M^{2n+2}}(1-\xi) R \Box_c^n R$$

Only γ and η run in such models

To have uniqueness, we should have a strictly renormalizable theory, instead of superrenormalizable ones

Fake degrees of freedom

Consider the theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{\lambda}{4!}\varphi^4$$

and the modified Euclidean propagator

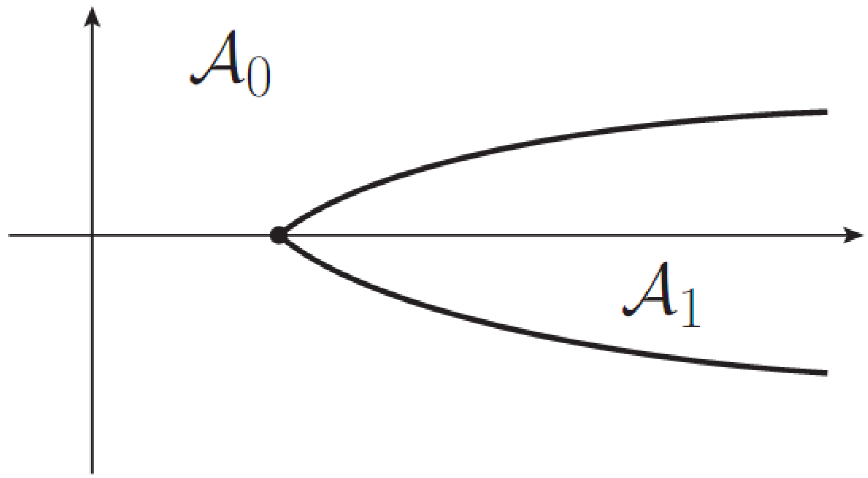
$$\frac{p_E^2}{(p_E^2)^2 + \mathcal{E}^4}$$

Consider the new prescription

$$\lim_{\mathcal{E}\to 0} \frac{p^2}{[(p^2)^2 + \mathcal{E}^4]_{\mathrm{LW}}}$$

Bubble diagram with this prescription:

$$-\frac{i}{4(4\pi)^2}\ln\frac{(p^2)^2}{\mu^4}$$



We have three disjoint regions of analyticity

Bubble diagram with the Feynman prescription:

$$-\frac{i}{2(4\pi)^2}\ln\frac{-p^2-i\epsilon}{\mu^2}$$

basically, two regions are squeezed onto the real axis

The scalar degree of freedom has been turned into a fake one by our prescription

We can use this trick to turn ghosts into fake degrees of freedom

The higher-derivative gravity theory

$$-2\kappa^2\frac{\mathcal{L}_{\mathrm{QG}}}{\sqrt{-g}}=2\Lambda_C+\zeta R-\frac{\gamma}{M^2}R_{\mu\nu}R^{\mu\nu}+\frac{1}{2M^2}(\gamma-\eta)R^2$$

does not belong to the LW class of theories shown before, because

at $\eta=0$ the propagator

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_{\eta=0}^{\mathrm{free}}=\frac{iM^2}{2p^2(\zeta M^2+\gamma p^2)}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma})$$

has a ghost on the real axis

However, we can kill the ghost with the new prescription:

$$\left\{\frac{1}{p^2+i\epsilon}-\frac{\gamma(\zeta M^2+\gamma p^2)}{[(\zeta M^2+\gamma p^2)^2+\mathcal{E}^4]_{\mathrm{LW}}}\right\}\frac{i}{2\zeta}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma})$$

at $\eta \neq 0$ we find

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_{\eta}^{\text{free}} = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_{\eta=0}^{\text{free}} - \frac{i\eta M^2}{2(p^2)^2} \frac{(p^2\eta_{\mu\nu} + 2p_\mu p_\nu)(p^2\eta_{\rho\sigma} + 2p_\rho p_\sigma)}{(\zeta M^2 + \gamma p^2)[\zeta M^2 + (\gamma - 3\eta)p^2]}$$

The extra term can be rendered unitary in a similar way:

$$-\frac{i\eta}{2} \frac{(p^2\eta_{\mu\nu} + 2p_\mu p_\nu)(p^2\eta_{\rho\sigma} + 2p_\rho p_\sigma)}{[(p^2)^2 + \mathcal{E}^4]_{\text{LW}}} \frac{M^2(\zeta M^2 + \gamma p^2)(\zeta M^2 + (\gamma - 3\eta)p^2)}{[(\zeta M^2 + \gamma p^2)^2 + \mathcal{E}^4]_{\text{LW}}[(\zeta M^2 + (\gamma - 3\eta)p^2)^2 + \mathcal{E}^4]_{\text{LW}}}.$$

The cosmological constant cannot be turned off at all energies in this model (it might be in supersymmetric extensions of it)

This may be the signal of a unitarity anomaly in the universe
and the reason why the cosmological constant is so small

Conclusions

We have a better understanding of higher-derivative quantum field theories

They do not like Minkowski spacetime, they prefer “Wick spacetime”,
which is the Wick rotated Euclidean space

However, the Wick rotation is not analytic

Nevertheless, it is consistent and shows that the Lee-Wick models do not need external ad hoc prescriptions to be formulated.
The physical predictions are different from those of the previous proposals

There exist unitary and superrenormalizable theories of quantum gravity, however they are infinitely many

A new quantization prescription, which turns physical and/or ghost degrees of freedom into fake ones, is able to make sense of the unique theory of higher-derivative gravity with a dimensionless gauge coupling, which is renormalizable and unitary