FCC-ee Requirements on Beam Polarization and Energy Calibration

Alain Blondel, University of Geneva

Beam Polarization can provide two main ingredients to Physics Measurements

1. **Transverse beam polarization provides beam energy calibration** by resonant depolarization

   -- low level of polarization is required (~10% is enough)

   → at W pair threshold comes naturally

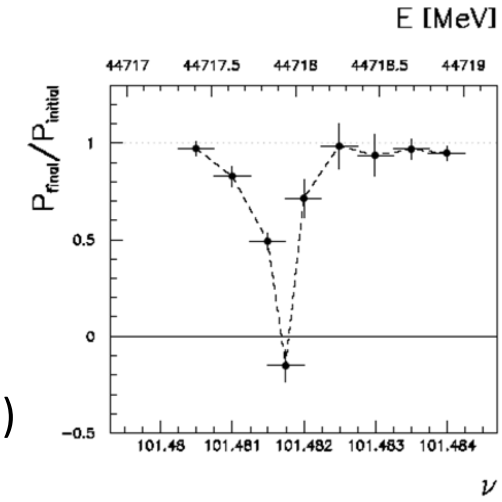
   → at Z use of wigglers at beginning of fills since polarization time is otherwise very long.

   → could be used also at ee $\rightarrow$ H(126) (depending on exact M_H!)

   → use 'single' non-colliding bunches

   not available at beam energies higher than ~90 GeV

   but for H and top can use ee $\rightarrow$ Z γ or ee $\rightarrow$ ZZ, WW to calibrate E_CM at ~5 MeV level.



-- must be done continuously during physics filly to avoid issues encountered at LEP

→ this is possible with single bunches and Compton polarimeter (commercial laser)

-- must be complemented by analysis of «average E_beam» to E_CM relationship

> Aim: Z mass and width to ~0.1 MeV (stat: 0.01 MeV)
> W mass to ~0.5 MeV (stat : 0.3MeV

«EPOL» working group on polarization and beam energy:
J. Wenninger, E. Gianfelice, D. Barber, W. Hillert, A. Bogomyagkov, I. Kopp, N. Munchoi, M. Koratzinos, K. Oide, A.B., et al. see already **arXiv:1506.00933**

Beam Polarization can provide two main ingredients to Physics Measurements

2. Longitudinal beam polarization provides chiral e+e- system

-- High level of polarization is required (>40%)

-- Must compare with natural e+e- polarization due to chiral couplings of electrons (15%) or with final state polarization analysis for CC weak decays (100% polarized) (tau and top)

-- Physics case for Z peak is very well studied and motivated:

A_LR, A_FB^Pol(f) etc… (CERN Y.R. 88-06)

figure of merit is L.P² --> must not lose more than a factor ~10 in lumi.

self calibrating polarization measurement *→

-- uses : enhance Higgs cross section (by 30%)

top quark couplings? final state analysis does as well (Janot **arXiv:1503.01325)**

enhance signal, subtract/monitor backgrounds, for ee→WW , ee →H

-- requires High polarization level and often both e- and e+ polarization

➔ **not interesting If loss of luminosity is too high**

-- Obtaining high level of polarization in high luminosity collisions is delicate in top-up mode

We have concluded that first priority is to achieve transverse polarization
in a way that allows continuous beam calibration by resoenant depolarization
- We believe that this is all possible
- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy

The multi-million question is 'do we want longitudinal polarization'?
we will discuss this in the following.

EXPERIMENTS ON BEAM-BEAM DEPOLARIZATION AT LEP

R. Assmann*, A. Blondel*, B. Dehning, A. Drees°, P. Grosse-Wiesmann, H. Grote, M. Placidi, R. Schmidt, F. Tecker†, J. Wenninger

PAC 1995

- With the beam colliding at one point, a polarization level of 40 % was achieved. The polarization level was about the same for one colliding and one non colliding bunch.
- It was observed that the polarization level depends critically on the synchrotron tune : when Q_s was changed by 0.005, the polarization strongly decreased.

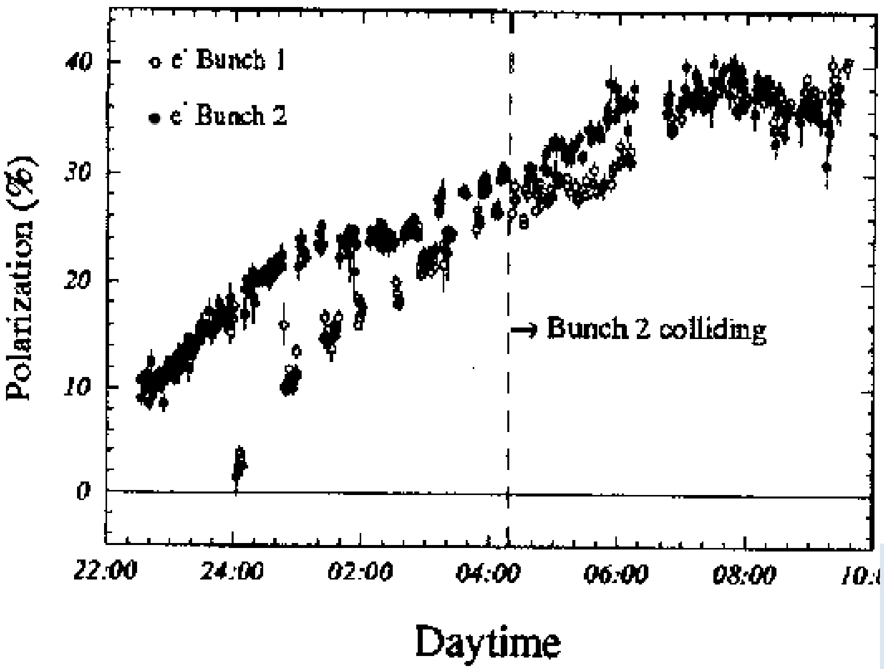


Figure. 3. Polarization level during third experiment

experiment performed at an energy of 44.71 GeV the polarization level was 40 % with a linear beam-beam tune shift of about 0.04/IP. This indicates, that the beam-beam depolarization does not scale with the linear beam-beam tune shift at one crossing point. Other parameters as spin tune and synchrotron tune are also of importance.

LEP:
This was only tried 3 times!
Best result: P = 40% , ξ^*_y= 0.04 , one IP

FCC-ee
Assuming 2 IP and ξ^*_y= 0.01 ➔
reduce luminosity, 10^{10} Z @ P~30%

Reduction due to continuous injection

The colliding bunches will lose intensity continuously due to collisions.
In FCC-ee with 4 IPs, L= 28 10^{34}/cm2/s beam lifetime is 213 minutes
In FCC-ee with 2 IPs, L= 220 10^{34}/cm2/s beam life time is 55minutes

Luminosity scales inversely to beam life time.
The injected e+ and e- are not polarized → asymptotic polarization is reduced.

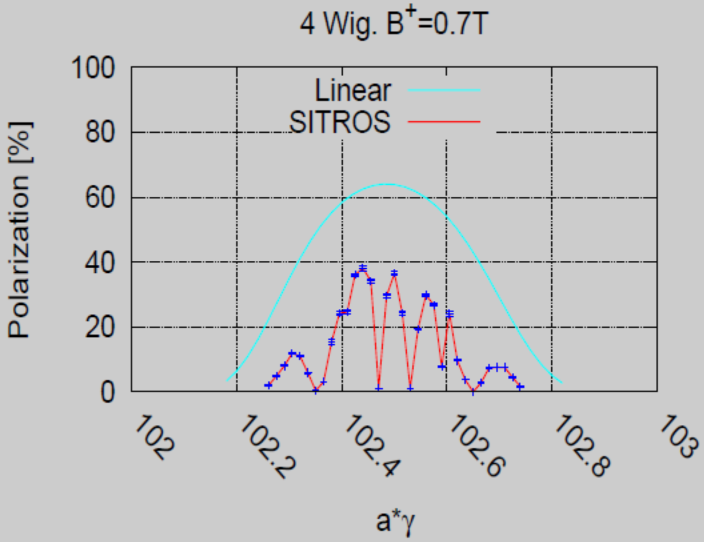
Assume here that machine has been well corrected and beams (no collisions, no injection) can be polarized to nearly maximum.
(***Eliana Gianfelice in Rome talk***)

- 45 GeV
  - limit ΔE=50 MeV (extrapolating from LEP)
  - 4 wigglers with B^{+}= 0.7 T
  - 10% polarization in 2.9 h for energy calibration

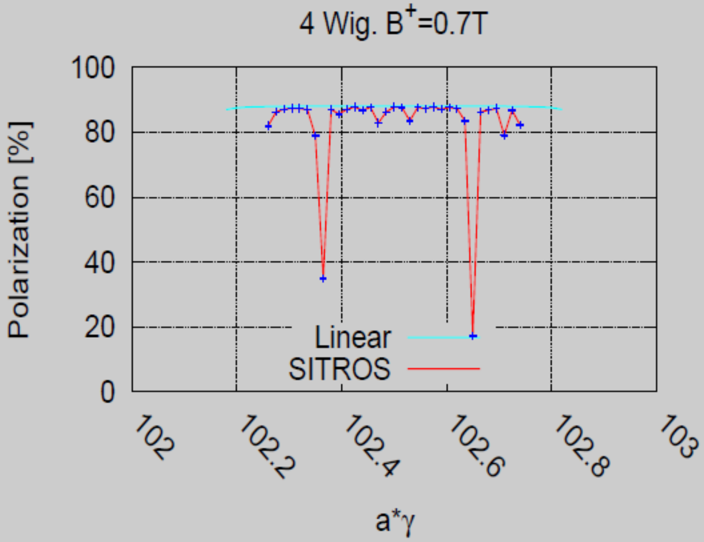
(polarization time is 26h)

- δ_y^Q= 200 μm
- BPMs errors
  - δ_y^M= 200 μm
  - 10% calibration errors

	y_{rms} (mm)	$\delta\hat{n}_{0,rms}$ (mrad)
	8.	26.4
SVD	0.8	3.9
+Harmonic bumps	0.9	2.0



SVD



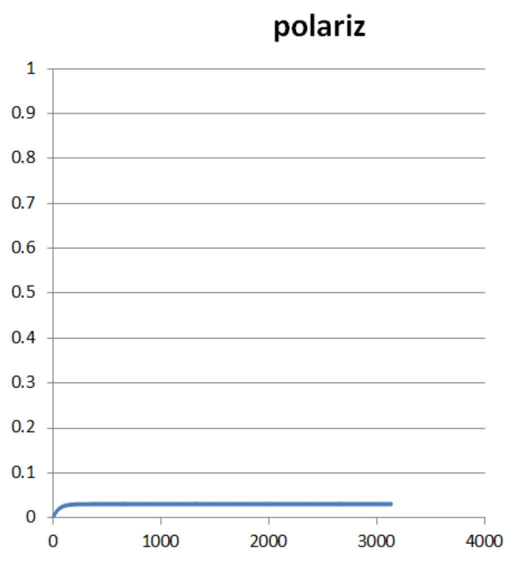
SVD + harmonic bumps

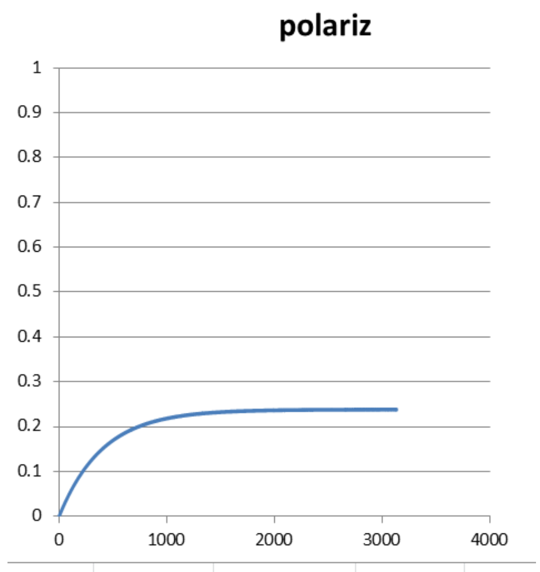
Mike Koratzinos and AB have simulated the simultaneous effect of
-- natural polarization
-- beam consumption by e+e- interactions
-- replenishment with unpolarized beams
assuming ***optimistically*** a maximal 90% asymptotic polarization

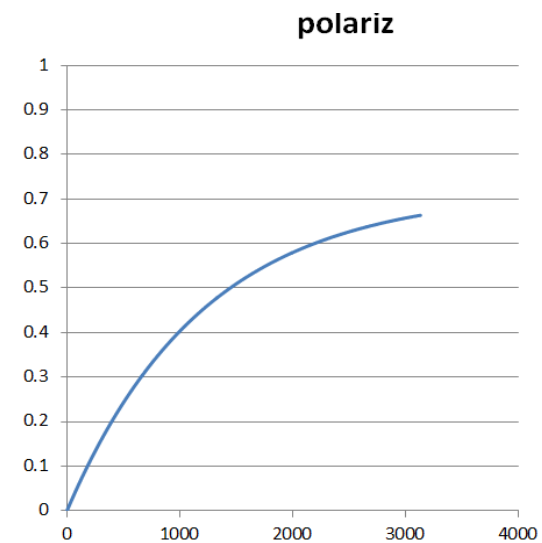
Running at full luminosity
P_max=0.03! P_eff=0.03

Running at 10% Lumi
P_max=0.24, P_eff=0.21

Running at 1% Lumi
P_max=0.66, P_eff=0.5







ΔA_{LR} scales as $1/\sqrt{(P^2L)}$

Lumi loss factor	L.10^34	**Figure of merit: sum(P^2L)**	effpol	Pmax
1	220	0.195	0.03	0.03
2	110	0.367	0.059	0.06
4	55	0.627	0.1078	0.11
6	37	0.805	0.149	0.16
8	27.5	0.924	0.184	0.2
10	22	1.003	0.214	0.24
12	18	1.053	0.24	0.27
15	14.7	1.09	0.27	0.32
18	12.2	**1.101**	0.3	0.35
22	10	1.088	0.33	0.4
26	8.5	1.059	0.354	0.43
30	7.3	1.023	0.37	0.46
40	5.5	0.92	0.41	0.52

Since one can see that there is an optimum
around a reduction of luminosity by a factor 18.

This is still a luminosity of $>10^{35}$ per IP… and the effective polarization is 30%.
This is equivalent to a 100% polarization expt with luminosity reduced by 180.

Longitudinal polarization at FCC-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of $\sin^2\theta^{\ell ept}_W = e^2/g^2\ (m_z)$

(-- not to be confused with -- $\sin^2\theta_W = 1- m_w^2/m_z^2$

Useful references from the past:

«polarization at LEP» CERN Yellow Report 88-02

Precision Electroweak Measurements on the Z Resonance

Phys.Rept.427:257-454,2006

http://arxiv.org/abs/hep-ex/0509008v3

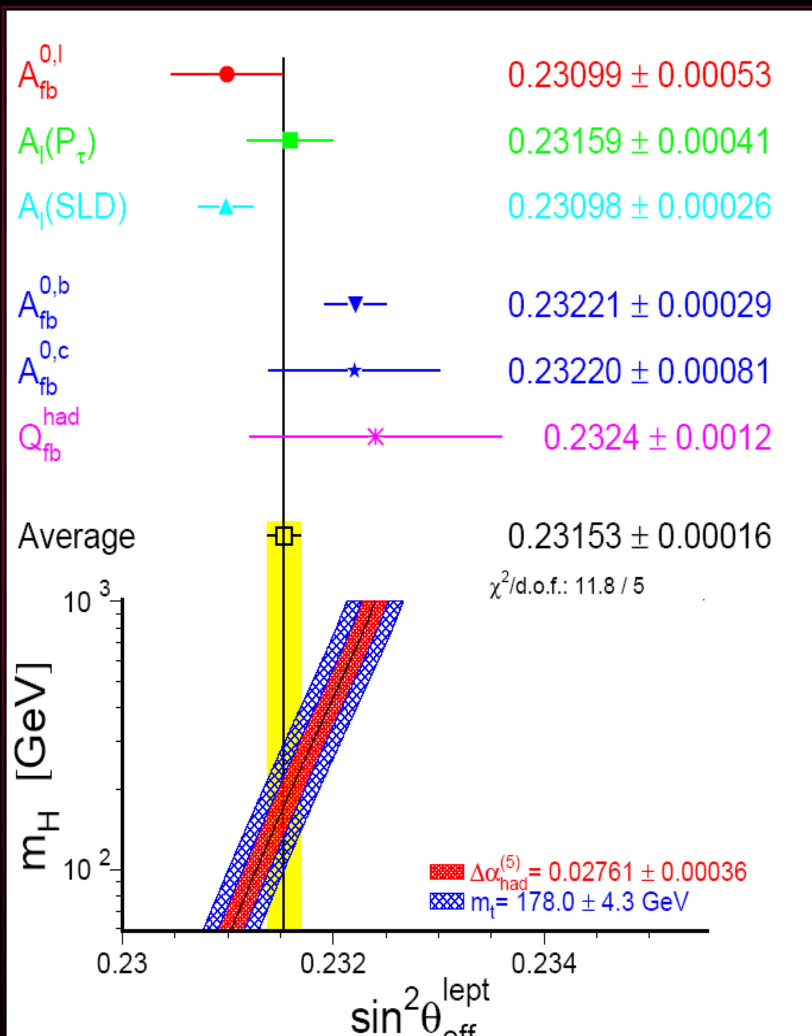
GigaZ @ ILC by K. Moenig

Measuring $\sin^2\theta_W^{eff}\ (m_Z)$

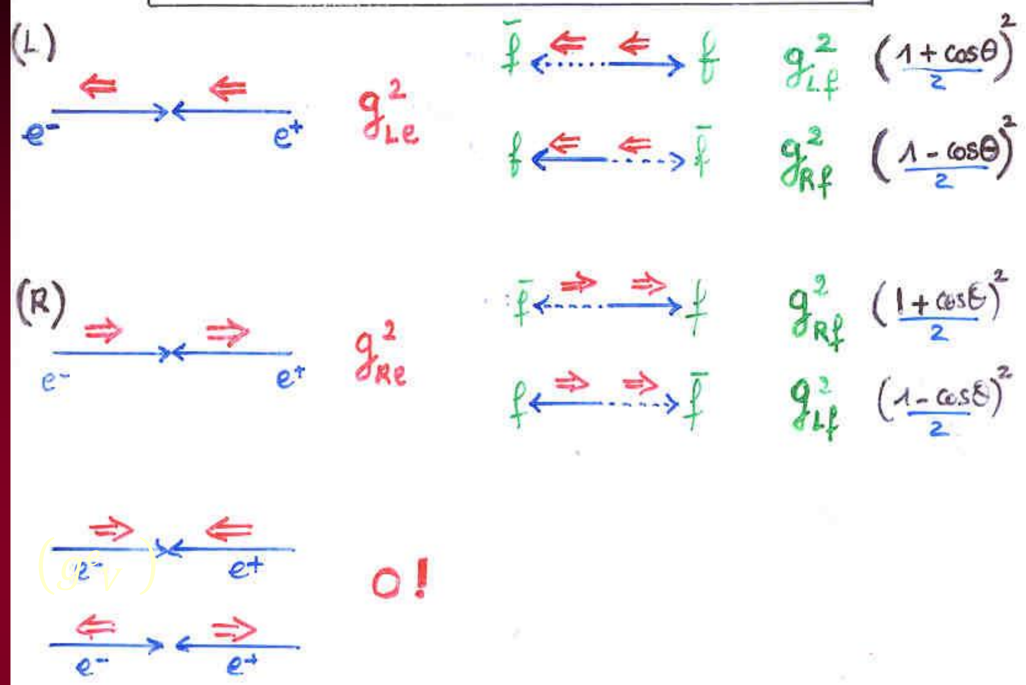
$$\sin^2\theta_W^{eff} \equiv \frac{1}{4}\left(1 - g_V/g_A\right)$$

$$g_V = g_L + g_R$$

$$g_A = g_L - g_R$$



$\boxed{\text{Helicity effects in } e^+e^- \to f\bar{f}}$



Pol.d BEAM $\Rightarrow$

$$A_{LR} = \frac{\sigma_L^{tot} - \sigma_R^{tot}}{\sigma_L^{tot} + \sigma_R^{tot}} = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} \equiv \mathcal{A}_e = \frac{2\, g_{Ve}\, g_{Ae}}{g_{Ve}^2 + g_{Ae}^2}$$

$$A_{FB}^{Pol\,f} = \frac{\sigma_L^{Ff} - \sigma_L^{Bf} - (\sigma_R^{Ff} - \sigma_R^{Bf})}{\sigma_L^{Ff} + \sigma_L^{Bf} + \sigma_R^{Ff} + \sigma_R^{Bf}} = \frac{3}{4}\mathcal{A}_f$$

no Pol available:

$$\boxed{A_{FB} = \frac{\sigma_U^{Ff} - \sigma_U^{Bf}}{\sigma_U^{Ff} + \sigma_U^{Bf}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f}$$

Pol.n analyses τ

$$\langle P_f \rangle = \frac{\sigma_U^R - \sigma_U^L}{\sigma_U^R + \sigma_U^L} = -\mathcal{A}_f$$

$$A_{FB}^{Pol} = \frac{\sigma_U^{RF} - \sigma_U^{LF} - (\sigma_U^{RB} - \sigma_U^{LB})}{\sigma_U^{RF} + \sigma_U^{LF} + \sigma_U^{RB} + \sigma_U^{LB}} = -\frac{3}{4}\mathcal{A}_e$$

$\Delta\rho$: $\equiv \varepsilon_1$

$$\Gamma_\ell = (1+\Delta\rho)\frac{G_F\, M_Z^3}{24\pi\sqrt{2}}\left(1+\left(\frac{g_{V\ell}}{g_{A\ell}}\right)^2\right)\left(1+\frac{3}{4}\frac{\alpha}{\pi}\right)$$

ε_3

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\,\frac{1}{1+\Delta\rho}\,\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

δ_{vb}

$$\Gamma_b = (1+\delta_{vb})\,\Gamma_d\left(1 - \underset{\alpha\, m_b^2/M_Z^2}{\text{mass corrections}}\right)$$

ε_2

$$M_W^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\sin^2\theta_W^{eff}}\cdot\frac{1}{(1-\varepsilon_3+\varepsilon_2)}$$

$\sin^2\theta_W^{eff}$ is defined from

$$\sin^2\theta_W^{eff} = \frac{1}{4}\left(1-\underbrace{\frac{g_{V\ell}}{g_{A\ell}}}\right) = \sin^2\theta_W^{eff}{}_{lept}$$

obtained from asymmetries at the Z.

also

Δr

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\,G_F}\cdot\frac{1}{\left(1-\frac{M_W^2}{M_Z^2}\right)}\,\frac{1}{(1-\Delta r)}$$

$$\Delta r = \Delta\alpha - \frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3 + \frac{c^2-s^2}{s^2}\varepsilon_2$$

EWRCs

relations to the well measured

G_F m_Z α_{QED}

at first order:

$\Delta\rho = \alpha/\pi\ (m_{top}/m_Z)^2$

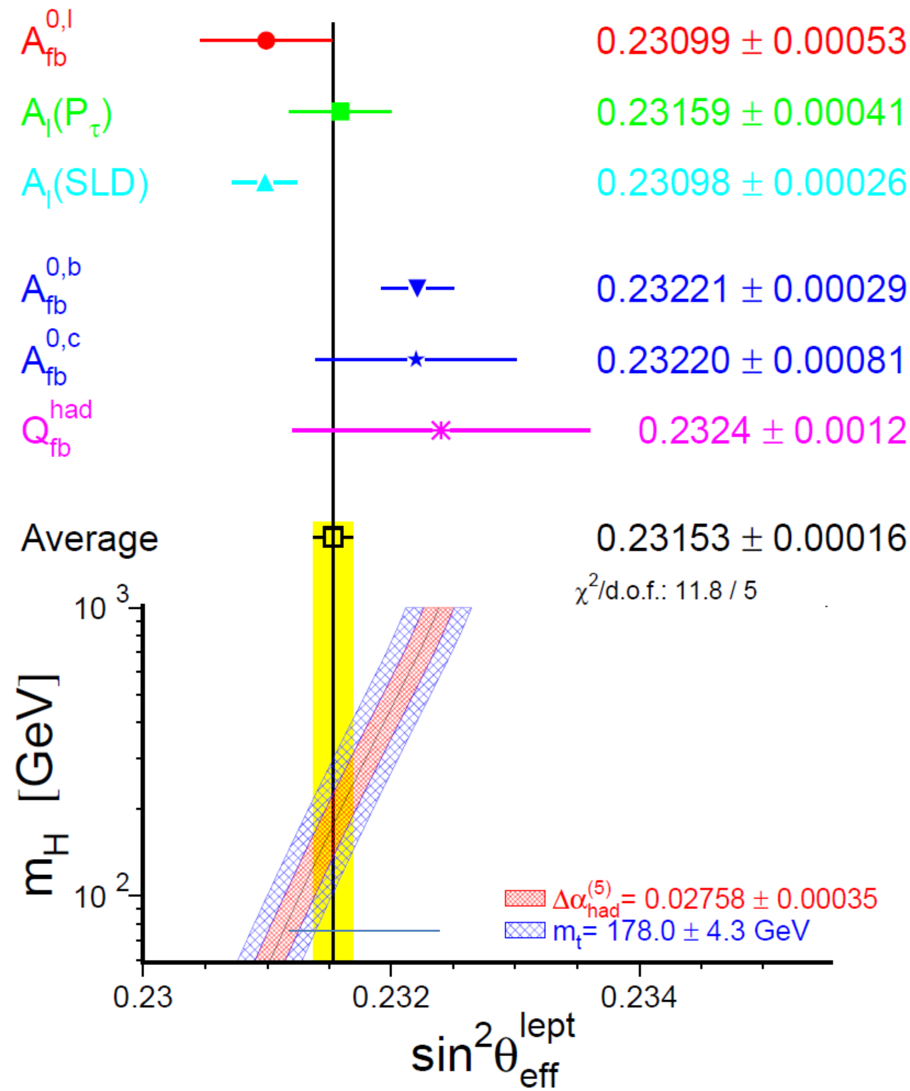
$-\ \alpha/4\pi\ \log\,(m_h/m_Z)^2$

$\varepsilon_3 = \cos^2\theta_W\ \alpha/9\pi\ \log\,(m_h/m_Z)^2$

$\delta_{vb} = 20/13\ \alpha/\pi\ (m_{top}/m_Z)^2$

complete formulae at 2d order
including strong corrections
are available in fitting codes

e.g. ZFITTER , GFITTER


$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23221 ± 0.00029
$A_{fb}^{0,c}$ 0.23220 ± 0.00081
Q_{fb}^{had} 0.2324 ± 0.0012
Average 0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
m_H [GeV]
10^3
10^2
$\Delta\alpha_{had}^{(5)}$= 0.02758 ± 0.00035
m_t= 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$

TLEP

A Sample of Essential Quantities:

X	Physics	Present precision		TLEP stat Syst Precision	TLEP key	Challenge
M_Z MeV/c2	Input	91187.5 ±2.1	Z Line shape scan	0.005 MeV <±0.1 MeV	E_cal	QED corrections
Γ_Z MeV/c2	$\Delta\rho$ (T) (no $\Delta\alpha$!)	2495.2 ±2.3	Z Line shape scan	0.008 MeV <±0.1 MeV	E_cal	QED corrections
R_ℓ	α_s , δ_b	20.767 ± 0.025	Z Peak	0.0001 ± 0.002 - 0.0002	Statistics	QED corrections
N_ν	Unitarity of PMNS, sterile ν's	2.984 ±0.008	Z Peak Z+γ(161 GeV)	0.00008 ±0.004 0.001	->lumi meast Statistics	QED corrections to Bhabha scat.
R_b	δ_b	0.21629 ±0.00066	Z Peak	0.000003 ±0.000020 - 60	Statistics, small IP	Hemisphere correlations
A_{LR}	$\Delta\rho$, ε_3 ,$\Delta\alpha$ (T, S)	0.1514 ±0.0022	Z peak, polarized	±0.000015	4 bunch scheme	Design experiment
M_W MeV/c2	$\Delta\rho$, ε_3 , ε_2, $\Delta\alpha$ (T, S, U)	80385 ± 15	Threshold (161 GeV)	0.3 MeV <0.5 MeV	E_cal & Statistics	QED corections
m_{top} MeV/c2	Input	173200 ± 900	Threshold scan	10 MeV	E_cal & Statistics	Theory limit at 100 MeV?

Extracting physics from $\sin^2\theta^{lept}_W$

1. Direct comparison with m_Z

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\,\frac{1}{1+\Delta\rho}\,\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

Uncertainties in m_{top}, $\Delta\alpha(m_z)$, m_H, etc….

$\Delta\sin^2\theta^{lept}_W \sim \Delta\alpha(m_z)/3 = 10^{-5}$ if we can reduce $\Delta\alpha(m_z)$ (see P. Janot)

2. Comparison with m_w/m_Z

Compare above formula with similar one:

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\cdot\frac{1}{1-\left(-\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho+2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3+\frac{c^2-s^2}{s^2}\varepsilon_2\right)}$$

Where it can be seen that **$\Delta\alpha(m_z)$** cancels in the relation.

The limiting error is the error on m_W.

For **Δm_W= 0.5 MeV this corresponds to $\Delta\sin^2\theta^{lept}_W = 10^{-5}$**

Assume for now ONE experiment at ECM=91.2

Luminosity «baseline» with beta*=1mm : 2.1 $10^{36}/cm^2/s$ = 2 pb^{-1}/s,
Sigma_had = 31 $10^{-33} cm^2$ ➔ 6.5 10^{11} qq events/10^7 year/exp.

Consider 3 years of 10^7 s
2 10^{12} Z→ $\bar{q}q$ events (typical exp at LEP was 4.10^6)
4 10^{11} Z→ $\bar{b}b$
10^{11} Z→ $\mu\mu$, $\tau\tau$ each

Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare

ILC measurement of A_{LR} with 10^9 Z and P_{e-} =80%, P_{e+} =30%

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{Pol}(\tau)$ with 2.10^{12} Z

Comparing A_{LR} (P) and A_{FB} ($\mu\mu$)

Both measure the weak mixing angle as **defined** by the relation $A_\ell = \frac{(g^e{}_L)^2 - (g^e{}_R)^2}{(g^e{}_L)^2 + (g^e{}_R)^2}$

with $(g^e{}_L) = ½ - \sin^2\theta^{lept}_W$ and $(g^e{}_R) = -\sin^2\theta^{lept}_W$ $A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$

$A_{LR} = A_e$

$A_{FB}^{\mu\mu} = ¾\, A_e A_\mu = ¾\, A_\ell^2$

-- $\mathbf{A_{FB}}^{\mu\mu}$ is measured using muon pairs (5% of visible Z decays) and unpolarized beams

-- $\mathbf{A_{LR}}$ is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization

both with very small experimental systematics

-- **parametric sensitivity** $\frac{dA_{FB}^{\mu\mu}}{d\sin^2\theta^{lept}_W} = 1.73$ **vs** $\frac{dA_{LR}}{d\sin^2\theta^{lept}_W} = 7.9$

-- **sensitivity to center-of-mass energy** (w.r.t. m_Z) is larger for $A_{FB}^{\mu\mu}$

$$\frac{\partial A_{FB}^{\mu\mu}}{\partial\sqrt{s}} = 0.09/\text{GeV} \quad \textbf{vs} \quad \frac{\partial A_{LR}}{\partial\sqrt{s}} = 0.019/\text{GeV}$$

"an 80 MeV uncertainty in Ecm corresponds to a 1% error on A_{LR}" (relative error)

But of course $A_{FB}^{\mu\mu}$ benefits from much larger statistics and Ecm precision of circular collider

Measurement of A_{LR}

electron bunches	$1\Leftarrow$	2	3	$4\Leftarrow$
positron bunches	1	$2\Rightarrow$	3	$4\Rightarrow$
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

$$\sigma_1 = \sigma_u (1 - P^-_e A_{LR})$$

$$\sigma_2 = \sigma_u (1 + P^+_e A_{LR})$$

$$\sigma_3 = \sigma_u$$

$$\sigma_4 = \sigma_u [1 - P^+_e P^-_e + (P^+_e - P^-_e) A_{LR}]$$

Verifies polarimeter with experimentally measured cross-section ratios

statistics

ΔA_{LR} = 0.0025 with about 10^6 Z^0 events,

ΔA_{LR} = 0.000045 with 5.10^{10} Z and 30% polarization in collisions.

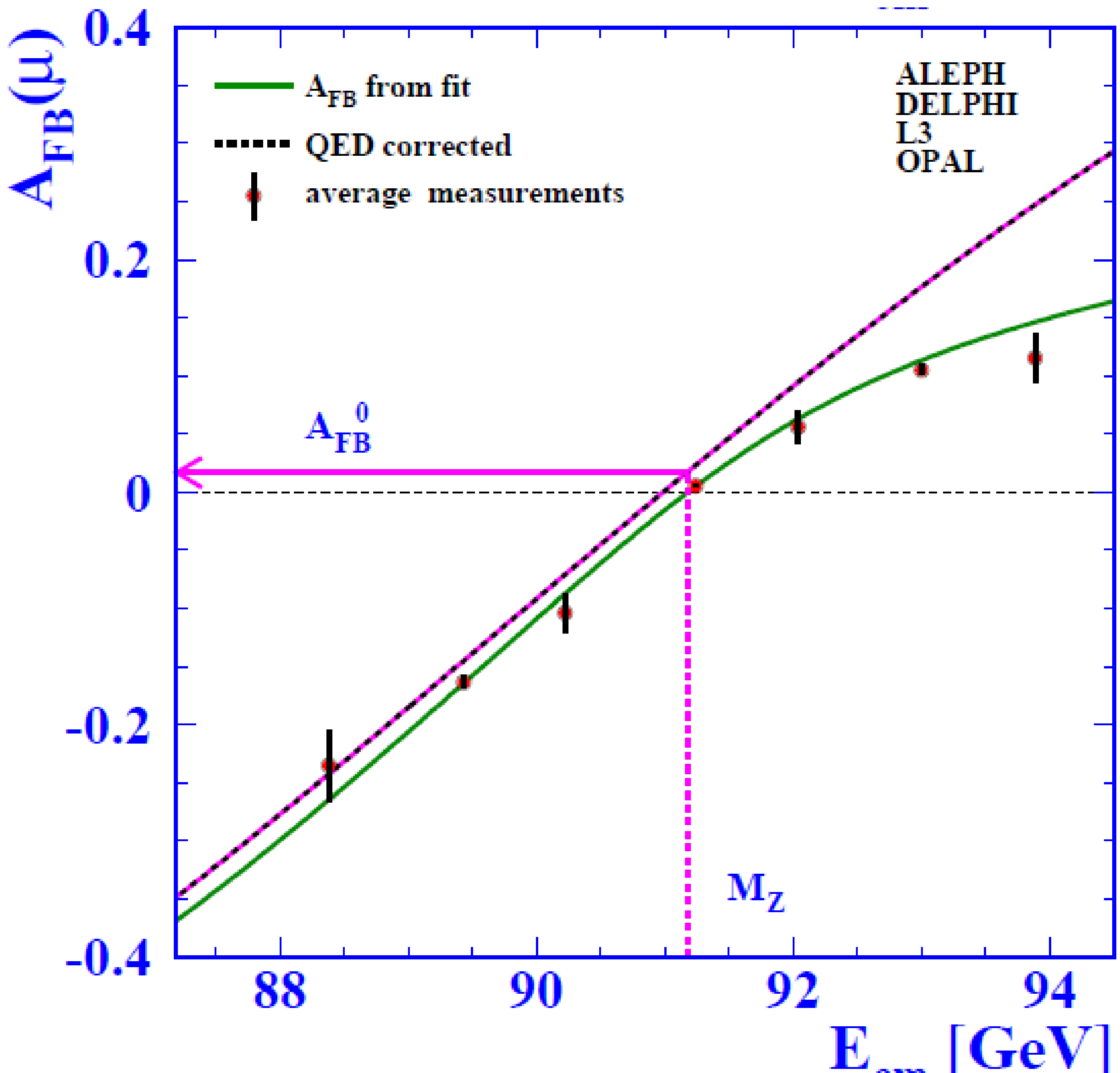
$\Delta \sin^2\theta_W^{eff}$ (stat) = O(2.10^{-6})

Will consider two sources of errors

-- statistics
-- uncertainty on center-of-mass energy (relative to the Z mass)

main inputs taken from
arXiv:hep-ex/0509008v3 precision measurements on the Z resonance
Phys. Rep. 427:257-454,2006

there are other uncertainties but they are very small for A_{FB}
This is a lower limit estimate for A_{LR} ; the systematics related to knowledge of the beam polarization (80% for e-, 30% for e+) should also be taken into account



$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^{0}
M_Z
88
90
92
94
E_{cm} [GeV]

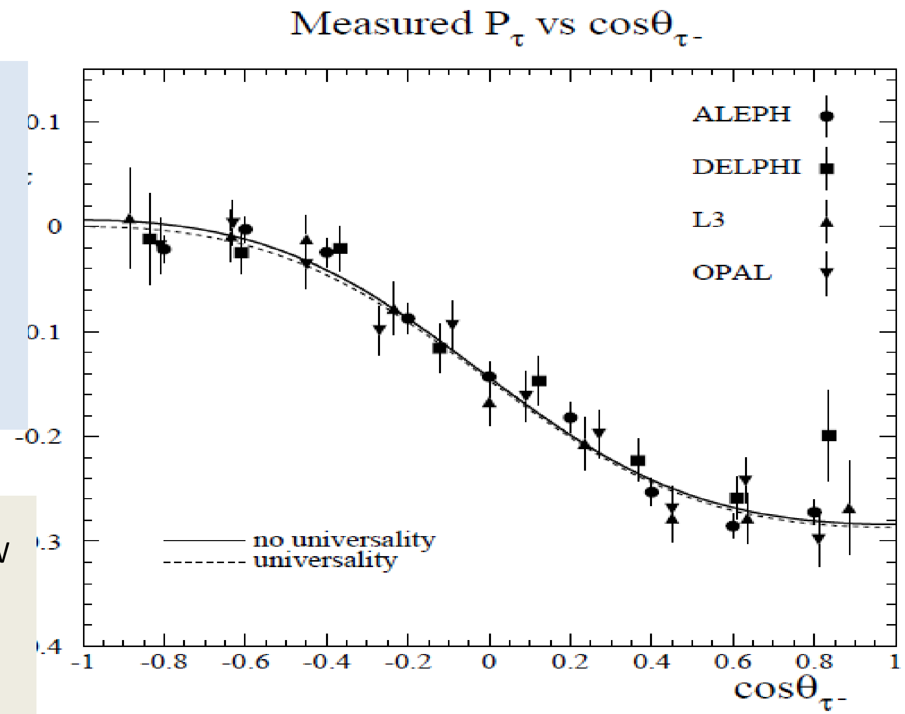
	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC	A_{LR} @ FCC-ee
visible Z decays	10^{12}	visible Z decays	10^{9}	5.10^{10}
muon pairs	10^{11}	beam polarization	90%	30%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$	$4.5\ 10^{-5}$
ΔE_{cm} (MeV)	0.1		2.2	?
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$	
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$	
$\Delta\sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$	$6\ 10^{-6}$ +?

All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ (310^{-5}) or the comparison with m_W (500keV)

But this precision on $\Delta\sin^2\theta^{\ell ept}_W$ can only be exploited at FCC-ee!

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.
At FCC-ee

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)



Already syst. level of 6 10^{-5} on $\sin^2\theta^{eff}_W$
much improvement possible
by using dedicated selection
e.g. tau→ π ν to avoid had. model

Figure 4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP experiments. Only the statistical errors are shown. The values are not corrected for radiation, interference or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.

Concluding remarks

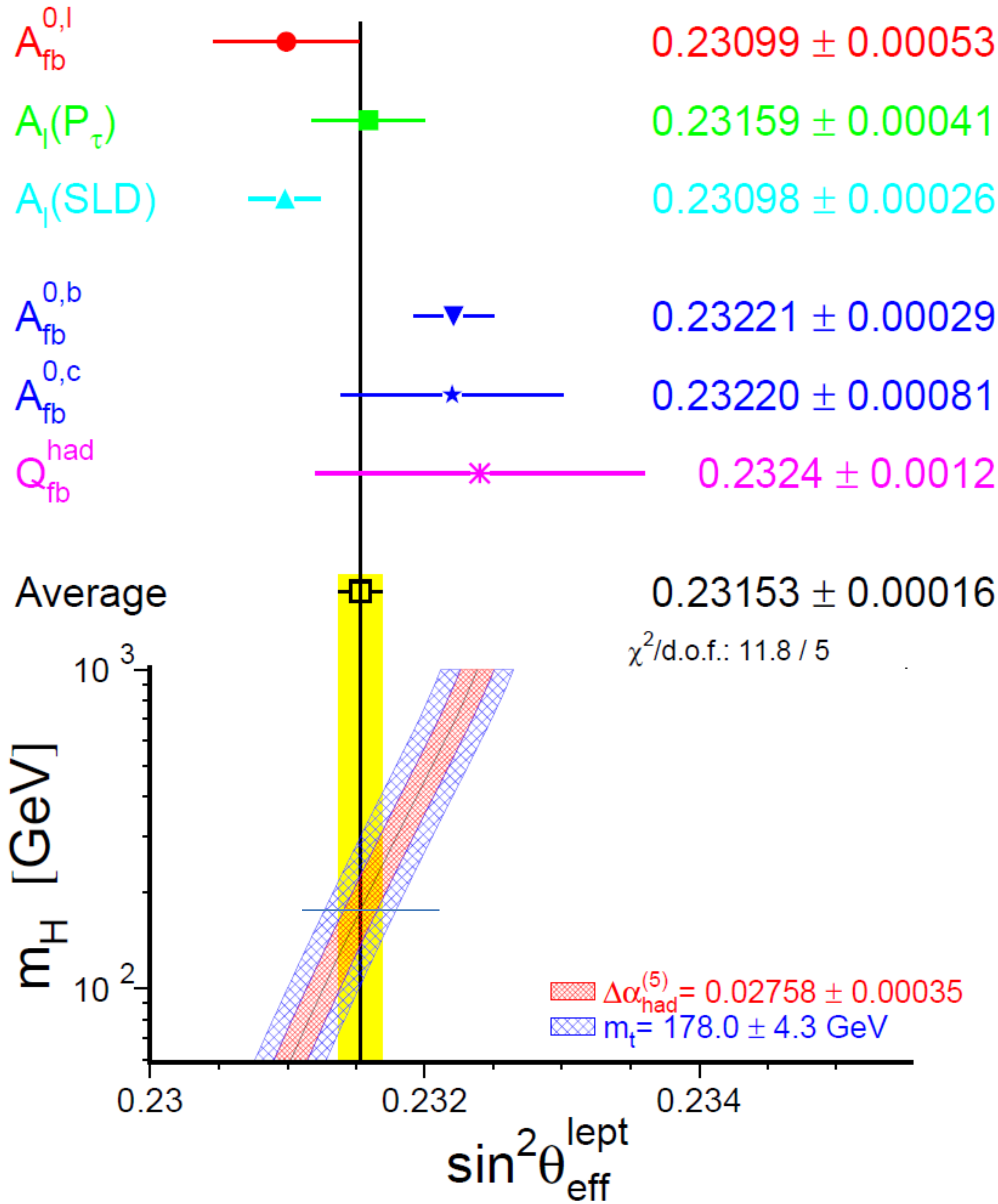
1. There are very strong arguments for precision energy calibration with transverse polarization at the Z peak.

2. Given the likely loss in luminosity, and the intrinsic uncertainties in the extraction of the weak couplings, the case for longitudinal polarization is limited.

➔ **We have concluded that first priority is to achieve transverse polarization** in a way that allows continuous beam calibration by resoenant depolarization

- We believe that this is all possible with a very high precision, both at the Z and the W. calibration at higher energies can be made from the data themselves at sufficient level.

- the question of the residual systematic error requires further studies of the relationship between beam energy and center-of-mass energy with the aim of achieving a precision of O(100 keV) on E_CM



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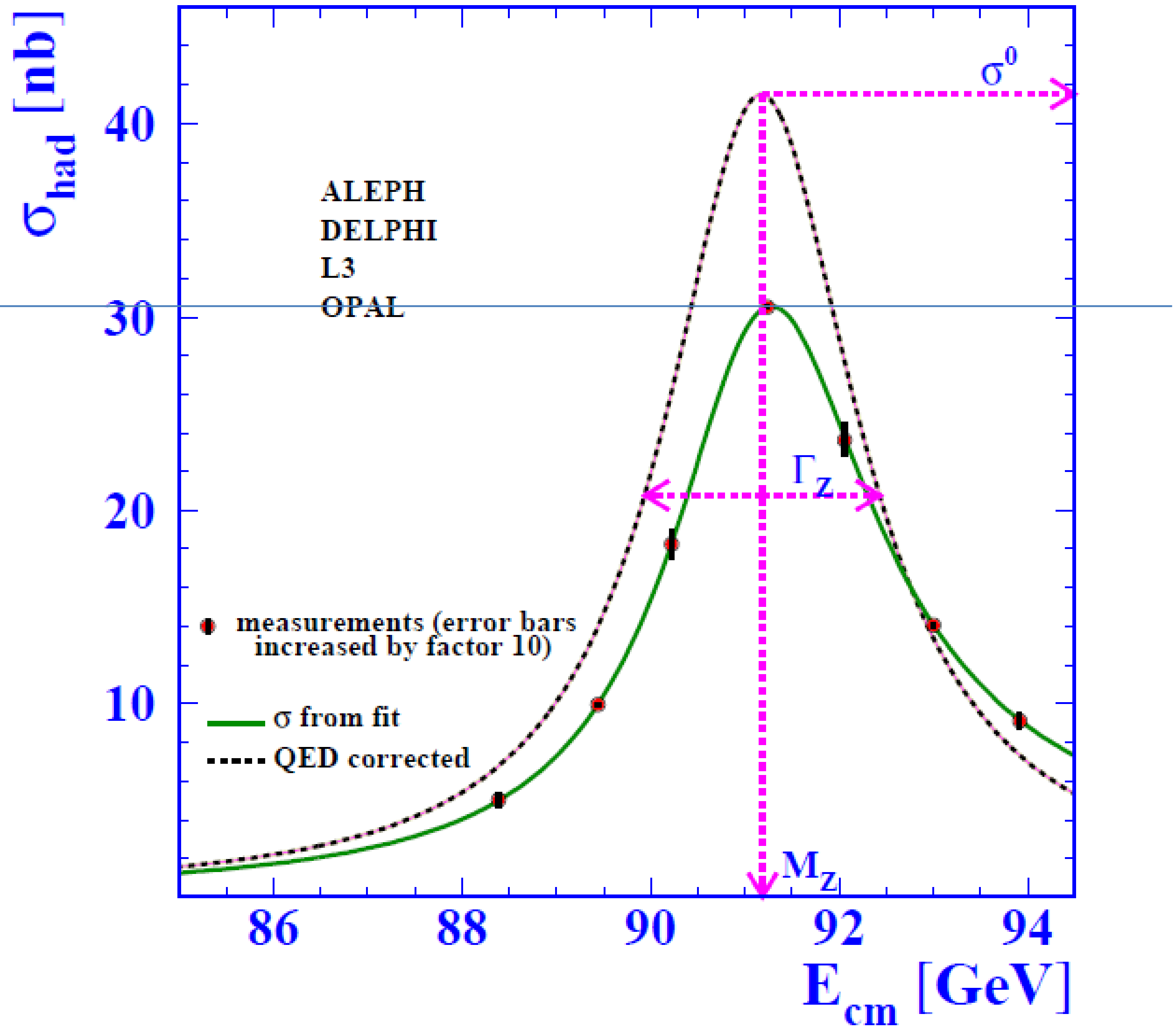
$A_{fb}^{0,b}$
0.23221 ± 0.00029
$A_{fb}^{0,c}$
0.23220 ± 0.00081
Q_{fb}^{had}
0.2324 ± 0.0012
Average
0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$



σ_had [nb]
40
30
20
10
86
88
90
92
94
E_cm [GeV]
σ^0
Γ_Z
M_Z
ALEPH
DELPHI
L3
OPAL
measurements (error bars increased by factor 10)
σ from fit
QED corrected

Going through the observables

the weak mixing angle as **defined** by the relation

$$A_\ell = \frac{2 g^e{}_V\, g^e{}_A}{(g^e{}_V)^2 + (g^e{}_A)^2} = \frac{(g^e{}_L)^2 - (g^e{}_R)^2}{(g^e{}_L)^2 + (g^e{}_R)^2}$$

with $(g^e{}_L) = ½ - \sin^2\theta^{lept}{}_W$ and $(g^e{}_R) = -\sin^2\theta^{lept}{}_W$

$A_\ell \approx 8(1/4 - \sin^2\theta^{lept}{}_W)$ very sensitive to $\sin^2\theta^{lept}{}_W$!

$A_{LR} = A_e$ measured from $(\sigma_{vis,L} - \sigma_{vis,R}) / (\sigma_{vis,L} - \sigma_{vis,R})$

(σ total visible cross-section had + $\mu\mu$ + $\tau\tau$ (35 nb) for 100% Left Polarization

$A_{FB}{}^{\mu\mu} = ¾\, A_e A_\mu = ¾\, A_\ell^2$

$$\begin{aligned} \mathcal{G}_{Vf} &= \sqrt{\mathcal{R}_f}\,(T_3^f - 2Q_f\mathcal{K}_f \sin^2\theta_W) \\ \mathcal{G}_{Af} &= \sqrt{\mathcal{R}_f}\,T_3^f . \end{aligned}$$

$$\begin{aligned} A_{FB}^{0,f} &= \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \\ A_{LR}^0 &= \mathcal{A}_e \\ A_{LRFB}^0 &= \frac{3}{4}\mathcal{A}_f \\ \langle\mathcal{P}_\tau^0\rangle &= -\mathcal{A}_\tau \\ A_{FB}^{pol,0} &= -\frac{3}{4}\mathcal{A}_e . \end{aligned}$$

$$\begin{aligned} A_{FB} &= \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ A_{LR} &= \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} \\ A_{LRFB} &= \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R}\frac{1}{\langle|\mathcal{P}_e|\rangle} . \end{aligned}$$