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Laboratori Nazionali di Legnaro (Istituto Nazionale di Fisica Nucleare)

Towards Kinetic Models of Electron Transport in Negative Ion Sources

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The extraction of negative ions from a plasma is necessarily accompanied by electrons, which are controlled with a transverse magnetic field. Theoretical model (mostly 1D in space) can clarify the transition from collisional regime to other regions. A full numerical analysis of the 3D geometry of the extraction is hindered by the computational load and by the rapidly growing electric field towards extraction, while theoretical analysis should include solution of equation in the phase space and adequately simplified models of collisions. The case of Coulomb collision is here addedto the previously discussed terms (constant mean free path or collision frequency), notwithstanding large divergence of its cross section in the low speed limit. Preliminary results for related transport integrals show that divergences are greatly suppressed by the averaging and integration process, and restricted to particular orbits.The electrons surfing sideways the sheath is substantially confirmed, with a large angle, due to previously discussed collision terms, with the same general recommendation for numerical ray tracing methods.

I. INTRODUCTION

Negative ion extraction modelling requires a kinetic description of e and H- passage from collision inside plasma to acceleration, both in the presence of a strong magnetic field. Fluid model are helpful, but restricted to plasma or beam at a time.

SO WE STUDY KINETIC MODELS:

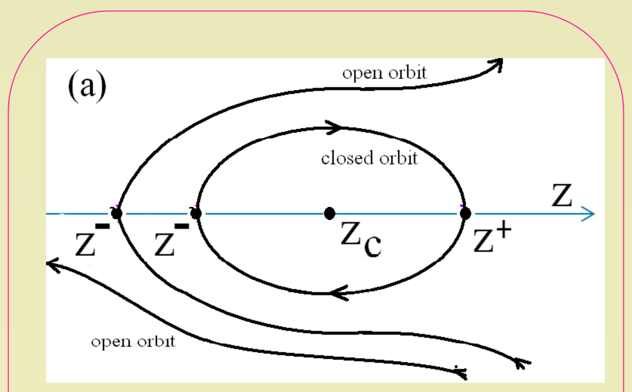


figure: Hamiltonian orbits; most orbit are closed for strong Bx or large plasma

SAMPLE OUTPUTS FROM BYPO18

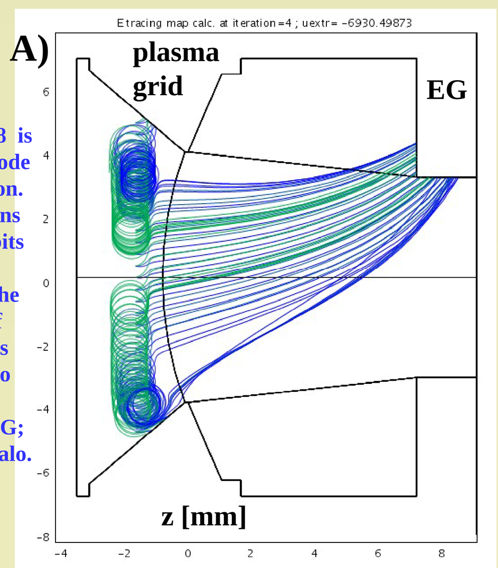
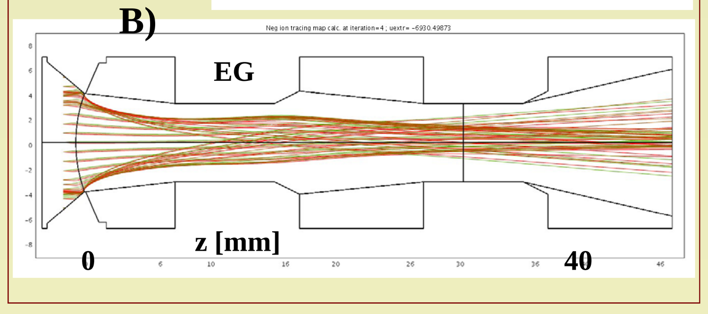


Figure 1: BYPO18 is a 2D simulation code for H-/e- extraction. Note that electrons make several orbits before being extracted from the plasma, even if magnetic field is barely enough to dump them on extraction grid EG; B) H- ions; note halo.



Local oscillation expansion

electrical potential gradient $v_1 = v'(z)$

mag. potential gradient $a_1 = a'(z) > 0$

$1/a_1$ (Larmor radius)

L_e = Larmor radius modified by electric field gradient $v_2 = v''(z)$

$$L_e = a_1/(a_1^2 + v_2)$$

$v_d = v_1/a_1$ is the drift velocity

we define the coordinate transform

$$t = \frac{z - z^-}{2L_e} \quad , \quad q = \frac{z^+ - z}{2L_e} \quad , \quad q_0 = \frac{z_2 - z}{2L_e}$$

its inverse transform is

$$p_y = v_d + q - t$$

$$p_z = 2s_p (a_1 L_e)^{1/2} (q\,t)^{1/2}$$

$$\eta_z = 2s_p (a_1 L_e)^{1/2} (q - q_0)^{1/2} (t + q_0)^{1/2}$$

where $\eta_z = p_z(z_2)$ and $s_p = 1$ is the forward semiplane

The gaussian factor becomes:

$$E_1 \equiv e^{-(p_z^2+p_y^2)/2} = e^{-\frac{1}{2}(q^2+t^2+v_d^2) - c_1 q t - v_d(q-t)}$$

orbit shape param: $c_1 \equiv 2L_e a_1 - 1$

Calculation with infinite number of particles [infty]

$$f = f_\beta + \int^z \mathrm{d}z_2 e^{-L(z_2,z)} s(z_2) f_g(\eta_z) f_g(\eta_y)/\eta_z \qquad \text{(6 a)}$$

$$f(z,p_z;P_y) = f(z_1, p_z^{\gamma+}(z_1), P_y) e^{-L_o(z_1,z)} + \int_{z_1}^{z} \mathrm{d}z_2\, e^{-L_o(z_2,z)} \frac{\nu_m n_a(z_2)}{|p_z^\gamma(z_2)|} G_o^\gamma(z_2)$$

with logarithmic attenuation (smaller formulas for case 'o' constant frequency):

$$L(z_1, z) \equiv \int_{z_1}^{z} \mathrm{d}z_2\, \nu(\eta_z, \eta_y)/\eta_z \qquad \text{(6 b)}$$

$$L_o(z_3, z) = \left| \int_{z_3}^{z} \frac{\mathrm{d}z_2 \nu_m}{|p_z^\gamma(z_2)|} \right|$$

eq. (6 a) is equivalent to ray-tracing [infty] eq. (6 b) to Monte-Carlo [infty

Basic equation: coupled transport integral an Poisson eq. (we note, as matter of fact, quasi-neutrality)

$$s(z) = \int \mathrm{d}z_2\, N(z, z_2; [a], [v])\, s(z_2) + s_B(z) \qquad \lambda_D^2 v_{,zz} = n_{p0}\, e^{v(z)} - n_{H^-} - n_e \cong 0$$

Calculating N, requires to calculate attenuation L_c for Coulomb collisions, for example near pole P+; with many approximations, we get

$$L = L_c + L_h + L_o \quad , \quad L_c = \gamma_c \frac{\sqrt{1+x} - \sqrt{x}}{y^2} \qquad \gamma_c = \frac{8\hat{k}_c \sqrt{q_a}}{(1+c_1)|v_d|^{(5/2)-d}} \quad , \quad y \equiv t - v_d + c_1 q$$

$$q_a = |q_0| \quad x \equiv q/q_a \quad \hat{k}_c = k_c (L_e/a_1)^{1/2}$$

2015 term[7]: $L_h = |w/\lambda| = |c_d q_0|$ $\quad c_d = 2L_e/\lambda \quad \hat{\nu}_o = \nu_o (L_e/a_1)^{1/2}$

2016 term[8]: $L_o = \hat{\nu}_o |\alpha(z) - \alpha(z_1)|$ $\quad \alpha(z) = (a_1/L_e)^{1/2} \int_{z^-}^{z} dz_2/\eta_z$

A formula for direct forward contribution (near pole P+)

$$N_c^d \cong \frac{k_c e^{-\beta - c_d q_a}}{\pi \sqrt{1 + (q_a/v_d)}} \int_0^\infty \mathrm{d}x \int_0^\infty \frac{\mathrm{d}y\, e^{-y^2/2} e^{-\beta x - \gamma_o r - \gamma_c r/y^2}}{\sqrt{x(1+x)}(y^2 + 2\beta x)^{1-(d/2)}}$$

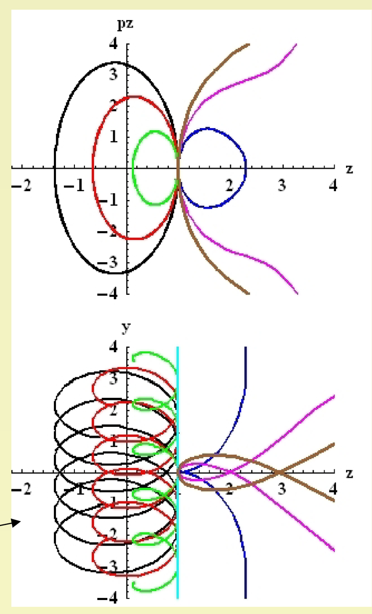
Plotted below $\quad r = \sqrt{1+x} - \sqrt{x}$

II.a 1D-2V MOTION IN CROSSED MAGNETIC AND ELECTRIC FIELDS

unit velocity $c_a = \sqrt{T_0/m_a}$

scaled magnetic pot. $a = q_a A_y / \sqrt{m_a T_0}$

scaled electric pot. $v = -e\phi/T_0 \equiv u$



velocity and distribution G after scatter at point z'

$$\eta_y = P_y - a(z') \ , \ \eta_z = p_z(z')$$

$$G = \frac{e^{-(\eta_z^2+\eta_y^2)/2}}{2\pi c_0}$$

Unperturbed orbit follow Hamiltonian h=H/T , P_y motion invariant

$$h(z, p_z; P_y) = \tfrac{1}{2} p_z^2 + \tfrac{1}{2}(P_y - a(z))^2 + v(z)$$

Equilibrium or Drift orbit: z(s) constant, where $s=c_a t$ is "scaled time"

equilibrium momentum $P_y^D(z) = a(z) + \{v_{,z}(z)/a_{,z}(z)\}$ (from $-h_{,z}=0$)

drift velocity $\mathrm{d}y/\mathrm{d}s = P_y^D - a(z) = \{v_{,z}(z)/a_{,z}(z)\} = v_D$

Motion reversal points z^+ or z^-, where dz/ds=0

momentum deviation from equilibrium (measured at $p = P_y - P_y^D(z)$

$$p + v_D = P_y - a(z) = \eta_y - \bar{a}$$

$$\bar{v} = v(z) - v(z') \qquad \bar{a} = a(z) - a(z')$$

Effective potential

$$V_{eff}(z, z', p) = \bar{v} + \tfrac{1}{2}\bar{a}^2 - \bar{a}\eta_y = \bar{v} - \tfrac{1}{2}\bar{a}^2 - \bar{a}(p + v_D(z))$$

Note scattered distribution G on an unperturbed orbit depends on z' only through potential

$$G = \frac{e^{-\bar{v} - \frac{1}{2}(p+v_D)^2}}{(2\sqrt{2\pi})} \Theta(2V_{eff})$$

Vlasov equation with

$$p_z f_{,z} - h_{,z} f_{,p_z} = C$$

scatter only collisional term

new terms with different collision frequency

$$C_{2V} = -f\nu(p_z, p_y) + f_g(p_z) f_g(p_y) s(z)$$

$$s(z) = \int \mathrm{d}p_z\, \mathrm{d}p_y\, f\nu(p_z, p_y) \qquad f_g(x) \equiv e^{-x^2/(2t_e)} (2\pi t_e)^{-1/2}$$

Coulomb collision after p_x averaging

$$\nu = \nu_c \cong k_c p_s^{d-2} \qquad p_s = (p_z^2 + p_y^2)^{1/2} \quad d \cong 2/\ln\Lambda$$

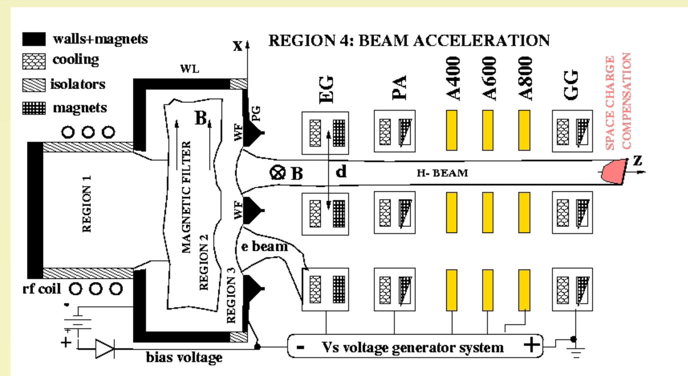
$$k_c = (N_p + N_{H^-}) \sigma_0 \Gamma(1 - \tfrac{1}{2}d)/\Gamma(\tfrac{3}{2} - \tfrac{1}{2}d)$$

1 eV cross section and closed approach distance

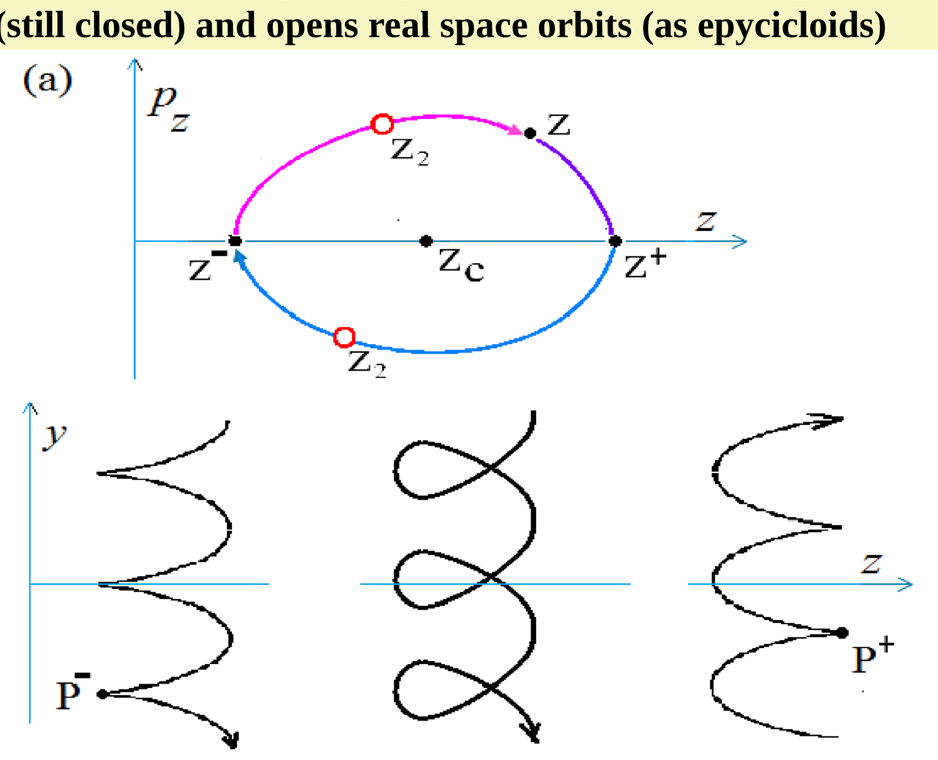
$$\sigma_0 = \pi b_0^2 \quad , \quad b_0 = e^2/(4\pi\epsilon_0 T_0)$$

SCHEME OF THE NEGATIVE ION ACCELERATORS

Figure 2:Scheme (not to scale) of a negative ion source and acceleration: additional grids A400, A600 and A800 exist only in MAMUG concept



In bulk magnetized plasma no electric field, electrons' orbits are circles, both in phase space zp_z and in real space zy. But the growing electric field toward extraction makes phase space orbits similar to ellipses (still closed) and opens real space orbits (as epycicloids)



Only two orbits (P^+ or P^-) for each given z and drift speed $v_d = -E_z/B_x/v_{th}$ shows $1/p_s$ poles (zero speed $p_z = p_y = 0$)

An example of equipotential lines in 2D

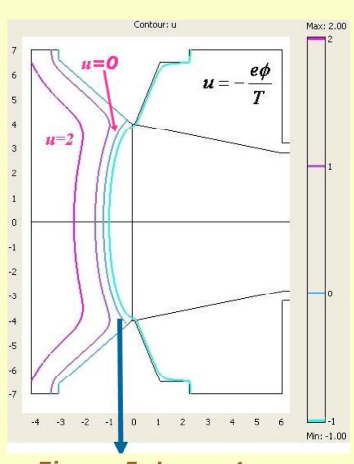


Figure 5: Large λ_D case from BYPO16, H^- and e^- extraction

Key issue is log. attenuation

$$L^o(z^-, z_2) = \frac{s_m}{2} - \frac{s_m}{\pi} \mathrm{asin} \frac{q - t - 2q_0}{q + t}$$

In the model [4], a simpler eq. applied

$$L(z_3, z) = \left| \int_{z_3}^{z} \frac{\mathrm{d}z_2}{\lambda |\eta_z|^0} \right| = \frac{|z - z_3|}{\lambda}$$

Then

$$L(z^-, z) = (z - z^-)/\lambda = c_d t \text{ where } c_d = 2L_e/\lambda$$

$$s_m \equiv L(z^-, z^+) = c_d(q + t)$$

ESCAPE INTEGRAL AND KERNELS (a simplification)

$$s_m = \nu_m \pi (L_e/a_1)^{1/2}$$

Average: v_z^e exit velocity, v_y^e side velocity, α^e exit angle are

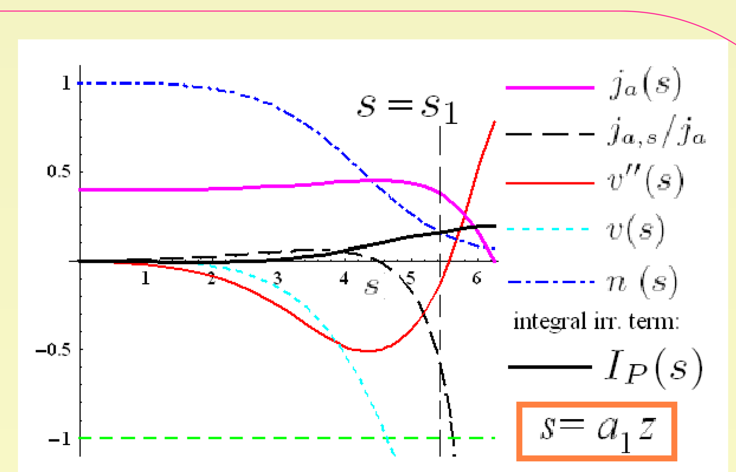
$$\alpha^e = \mathrm{atan}\frac{\langle p_y \rangle}{\langle p_z \rangle} \quad , \quad v_y^e = \frac{\langle p_y \rangle}{\langle 1 \rangle} \quad , \quad v_z^e = \frac{\langle p_z \rangle}{\langle 1 \rangle}$$

At extraction, only forward moving electron exit; since gas density decrease; formally we take the limit $s_m \to 0$

So kernels simplifies to

$$\mathcal{F}_e^o(q_0; a_f) = \frac{(a_1/L_e)^{1/2}}{4\pi^2} \int \mathrm{d}q\, \mathrm{d}t \frac{q+t}{\sqrt{q\,t}} \frac{a_f E_1 \theta_c}{\sqrt{(q - q_0)(t + q_0)}} \equiv \frac{(a_1/L_e)^{1/2}}{4\pi^2} \mathcal{I}_e^o(q_0; a_f)$$

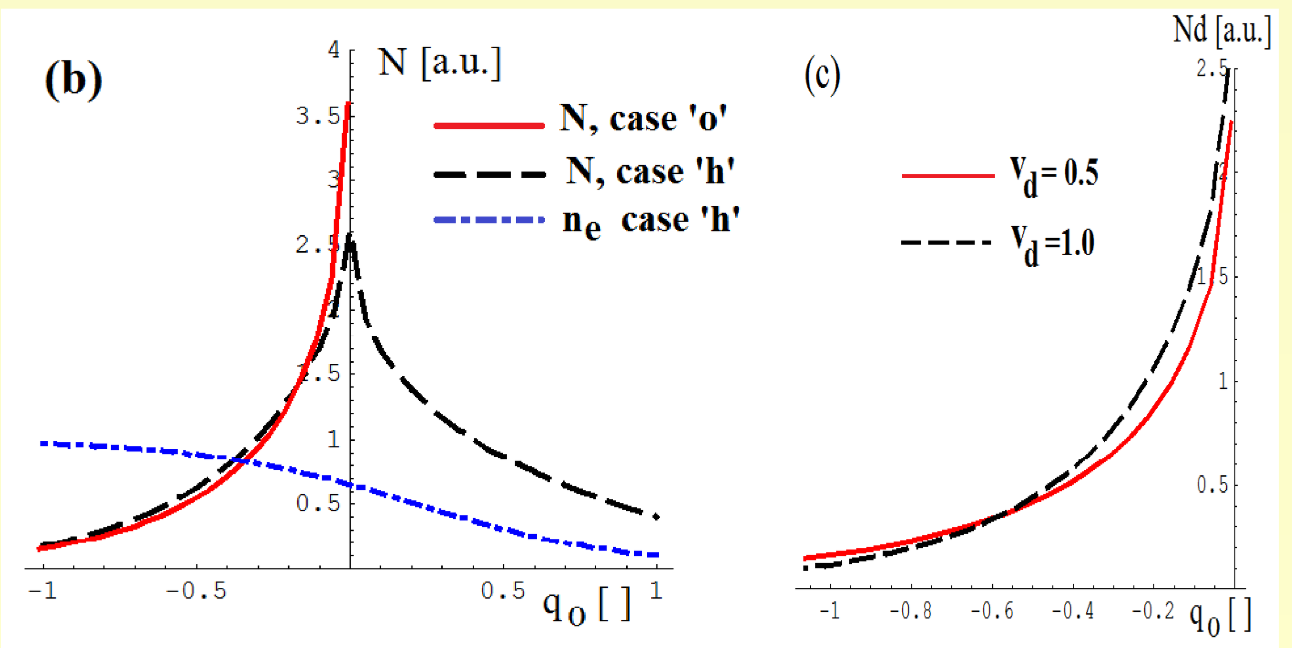
OLD [7,8] AND NEW RESULTS



The density n(z), from eq. (1) to be self-consistently solved , is in principle needed to compute escape integrals; but a n(z) approximation gives satisfactory result. I use the prevoiusly computed n [Ref 4] plotted above (note the expected smoothed trapezoidal shape).

$v_d = v'(z)/a'(z)$ goes from -0.2 to -1.2 which is the (minimum) reached at presheath end

Some graphs of KERNELS: Note 'o' is the constant collsion model; 'h' is the 2015 ad-hoc model , Ndc an approxiamtion of Coulomb case



Kernel terms vs space coordinate $q_0 = (z_2 - z)/2\, L_e$ (dimensionless)

Collisions make electron velocity to follow drift velocity order (1.2 vd) near extraction, so taking them away from zero speed pole. Consequently we have a large tilt angle in average electron emission velocity

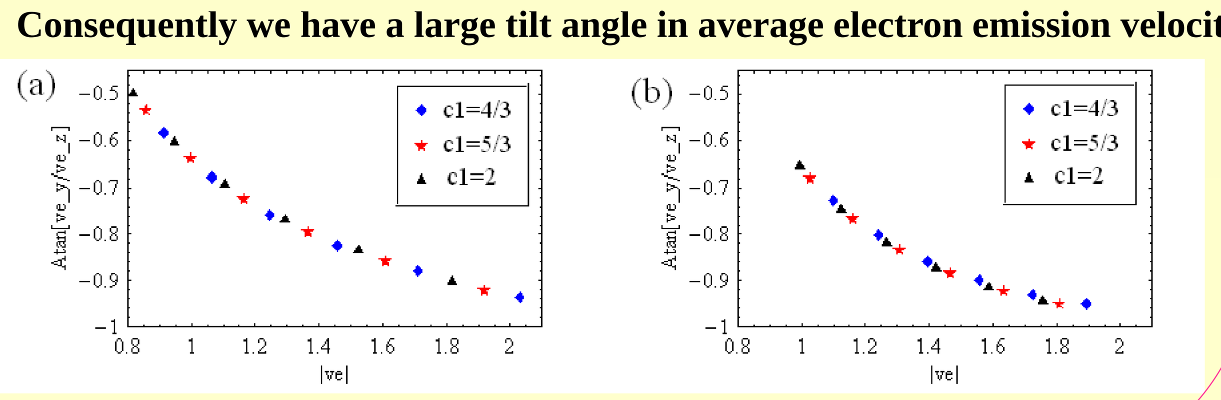


Figure: emission angle [rad] vs $-v_d$

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