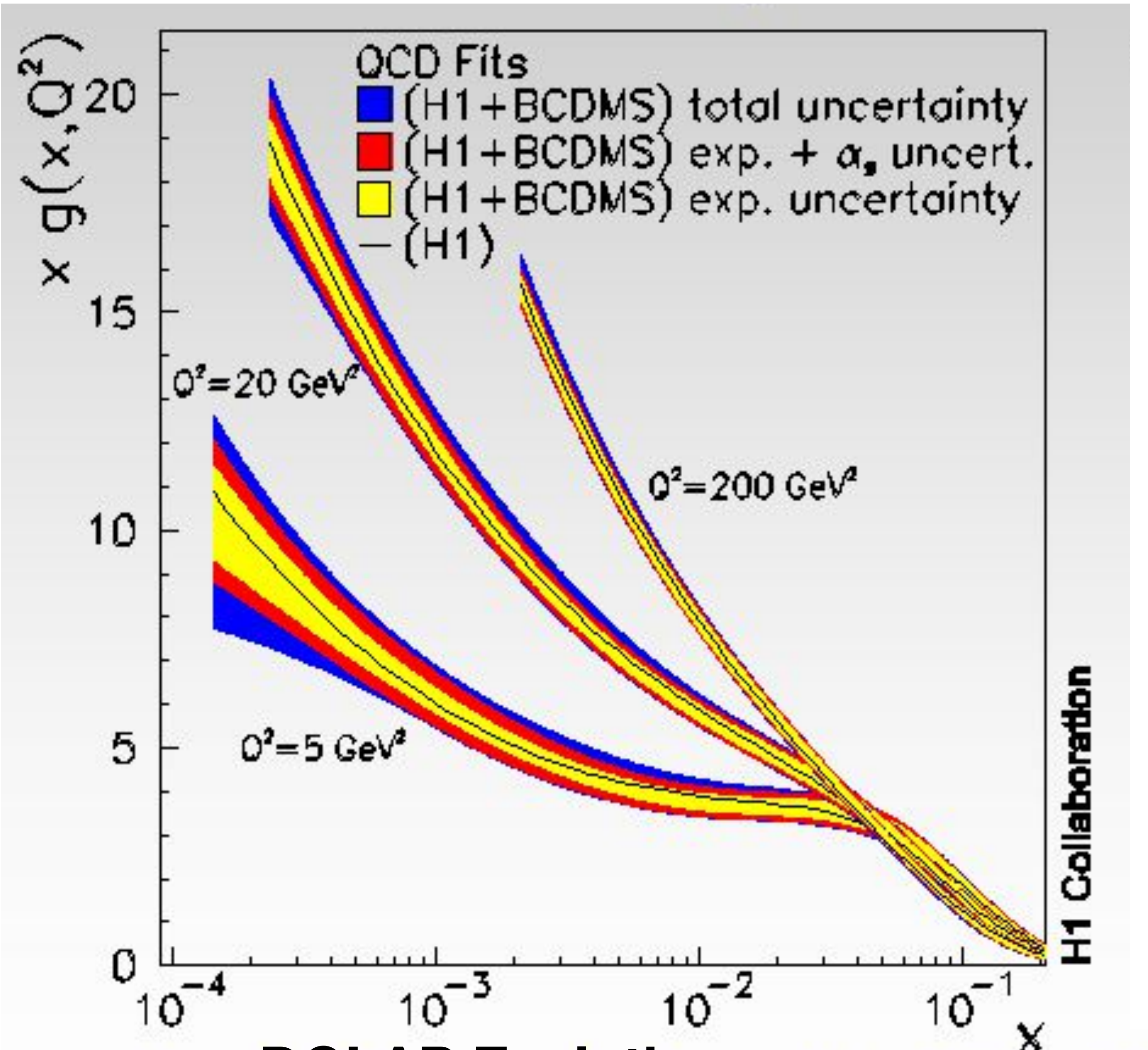
The Remarkable Achievements of Lev Lipatov



DGLAP Evolution

Universidad Técnica Federico Santa María,
Valparaiso, Chile
January 8-12, 2018

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

Lev Nikolaevich Lipatov

Head of the Theoretical Physics Division at
St. Petersburg's Nuclear Physics Institute
of the Russian Academy of Sciences

Academician of the Russian Academy of Sciences

Pomeranchuk Prize (2001)

Gribov and Lipatov (DGLAP)

Balitsky Fadin Kuraev Lipatov (BFKL)

Renormalon Analysis

Tunneling Theory

Critical Phenomena

…..

Pomeranchuk Prize

The **Pomeranchuk Prize** is an international award for theoretical physics
Institute for Theoretical and Experimental Physics (ITEP) from Moscow
Isaak Yakovlevich Pomeranchuk, who together with Landau established the
Institute.

Laureates

Source: Institute of Theoretical and Experimental Physics[1]

- 2016 Curtis J. Callan and Yuri A. Simonov[2]
- 2015 Stanley J. Brodsky and Victor Fadin[3]
- 2014 Leonid Keldysh and Alexander Zamolodchikov
- 2013 Mikhail Shifman and Andrey Slavnov
- 2012 Juan Martín Maldacena and Spartak Belyaev
- 2011 Heinrich Leutwyler and Semyon Gershtein
- 2010 André Martin and Valentine Zakharov
- 2009 Nicola Cabibbo and Boris Ioffe
- 2008 Leonard Susskind and Lev Okun
- 2007 Alexander Belavin and Yoichiro Nambu
- 2006 Vadim Kuzmin and Howard Georgi
- 2005 Iosif Khriplovich and Arkady Vainshtein
- 2004 Alexander F. Andreev and Alexander Polyakov
- 2003 Valery Rubakov and Freeman John Dyson
- 2002 Ludvig Faddeev and Bryce Seligman DeWitt
- 2001 Lev Lipatov and Tullio Regge
- 2000 Evgenii Feinberg and James Daniel Bjorken
- 1999 Karen Ter-Martirosian and Gabriele Veneziano
- 1998 Aleksander Ilyich Akhiezer and Sidney Drell

Lev’s Most Highly Cited Papers

BFKL pomeron in the next-to-leading approximation
Victor S. Fadin (Novosibirsk, IYF & Novosibirsk State U.), L.N. Lipatov (St. Petersburg, INP & St. Petersburg State U.). Feb 1998. 9 pp.
Published in **Phys.Lett. B429 (1998) 127-134**
DESY-98-033
DOI: 10.1016/S0370-2693(98)00473-0
e-Print: **hep-ph/9802290** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 983 records 500+

The Bare Pomeron in Quantum Chromodynamics
L.N. Lipatov (St. Petersburg, INP). Nov 1985. 32 pp.
Published in **Sov.Phys.JETP 63 (1986) 904-912, Zh.Eksp.Teor.Fiz. 90 (1986) 1536-1552**
LENINGRAD-85-1137
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 870 records 500+

The Pomeranchuk Singularity in Quantum Chromodynamics
I.I. Balitsky, L.N. Lipatov (St. Petersburg, INP). 1978. 15 pp.
Published in **Sov.J.Nucl.Phys. 28 (1978) 822-829, Yad.Fiz. 28 (1978) 1597-1611**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 3304 records 1000+

The Pomeranchuk Singularity in Nonabelian Gauge Theories
E.A. Kuraev, L.N. Lipatov, Victor S. Fadin (Novosibirsk, IYF). 1977. 6 pp.
Published in **Sov.Phys.JETP 45 (1977) 199-204, Zh.Eksp.Teor.Fiz. 72 (1977) 377-389**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 2997 records 1000+

Multi - Reggeon Processes in the Yang-Mills Theory
E. A. Kuraev, L. N. Lipatov, Victor S. Fadin (St. Petersburg, INP). 1976. 16 pp.
Published in **Sov.Phys.JETP 44 (1976) 443-450, Zh.Eksp.Teor.Fiz. 71 (1976) 840-855**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 1490 records 1000+

Reggeization of the Vector Meson and the Vacuum Singularity in Nonabe Gauge Theories
L.N. Lipatov (St. Petersburg, INP). 1976. 15 pp.
Published in **Sov.J.Nucl.Phys. 23 (1976) 338-345, Yad.Fiz. 23 (1976) 642-656**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 1188 records 1000+

On the Pomeranchuk Singularity in Asymptotically Free Theories
Victor S. Fadin, E.A. Kuraev, L.N. Lipatov (St. Petersburg, INP). 1975. 3 pp.
Published in **Phys.Lett. 60B (1975) 50-52**
DOI: 10.1016/0370-2693(75)90524-9
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 1085 records 1000+

The parton model and perturbation theory
L.N. Lipatov (St. Petersburg, INP). 1974. 18 pp.
Published in **Sov.J.Nucl.Phys. 20 (1975) 94-102, Yad.Fiz. 20 (1974) 181-198**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 1345 records 1000+

e+ e- pair annihilation and deep inelastic e p scattering in perturbation th
V.N. Gribov, L.N. Lipatov (St. Petersburg, INP). 1972. 20 pp.
Published in **Sov.J.Nucl.Phys. 15 (1972) 675-684, Yad.Fiz. 15 (1972) 1218-1237**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 1212 records 1000+

Deep inelastic e p scattering in perturbation theory
V.N. Gribov, L.N. Lipatov (St. Petersburg, INP). 1972. 27 pp.
Published in **Sov.J.Nucl.Phys. 15 (1972) 438-450, Yad.Fiz. 15 (1972) 781-807**
IPTI-381-71
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote

DGLAP Evolution Equations

Deep inelastic e p scattering in perturbation theory
V.N. Gribov, L.N. Lipatov (St. Petersburg, INP). 1972. 27 pp.
Published in
Sov.J.Nucl.Phys. 15 (1972) 438-450, Yad.Fiz. 15 (1972) 781-807
IPTI-381-7
Cited by 3726 records

V.N. Gribov, L.N. Lipatov. *Sov. J. Nucl. Phys.* 15:438 (1972)

G. Altarelli and G. Parisi. *Nucl. Phys.* B126:298 (1977)

Yu.L. Dokshitzer. *Sov. Phys. JETP* 46:641 (1977)

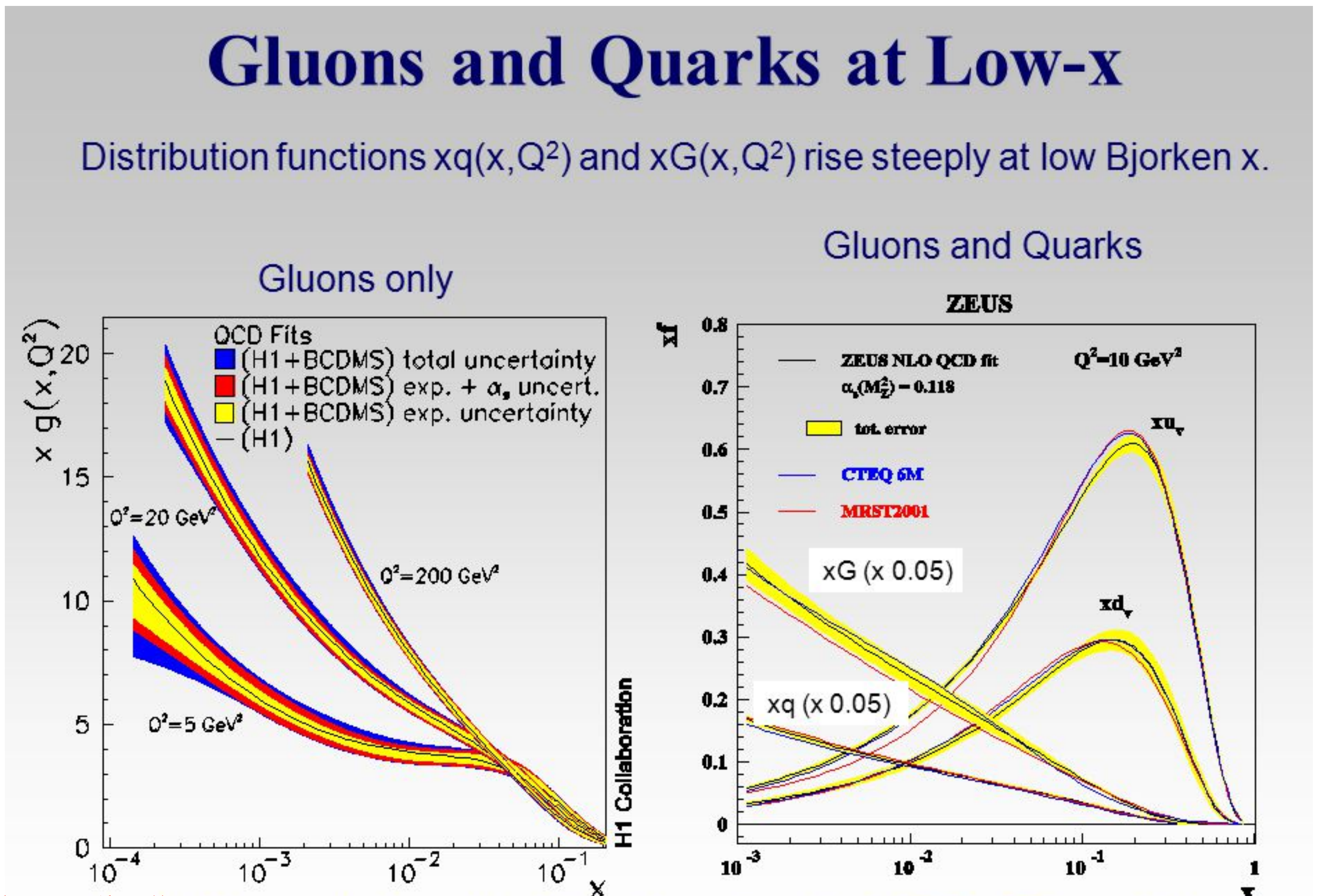
Motivated ERBL Evolution Equations

DGLAP Evolution Equations

V.N. Gribov, L.N. Lipatov. *Sov. J. Nucl.Phys.* 15:438 (1972)

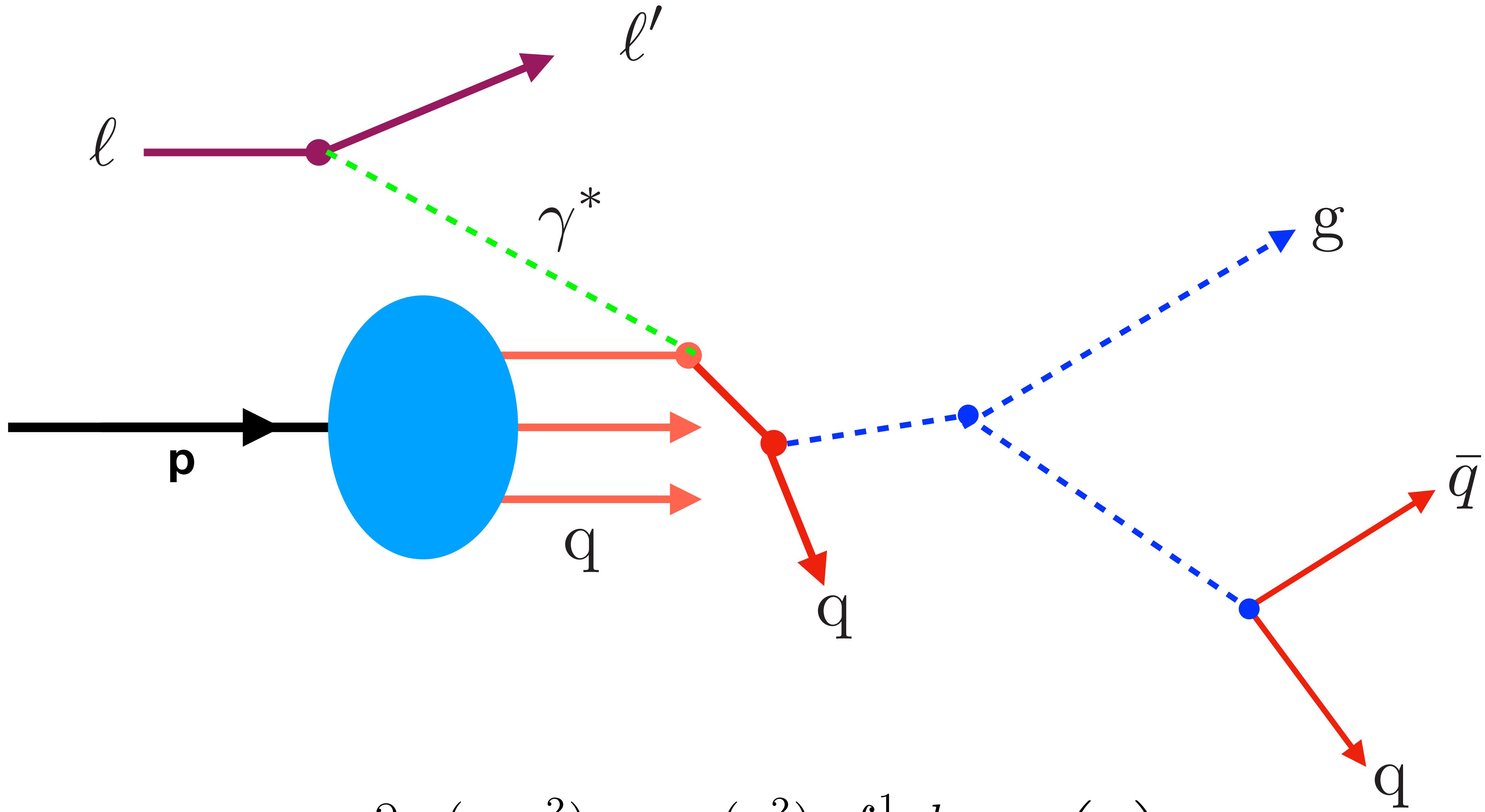
G. Altarelli and G. Parisi. *Nucl. Phys.* B126:298 (1977)

Yu.L. Dokshitzer. *Sov.Phys. JETP* 46:641 (1977)



Growth of quark and gluon distributions with Q^2 at low x

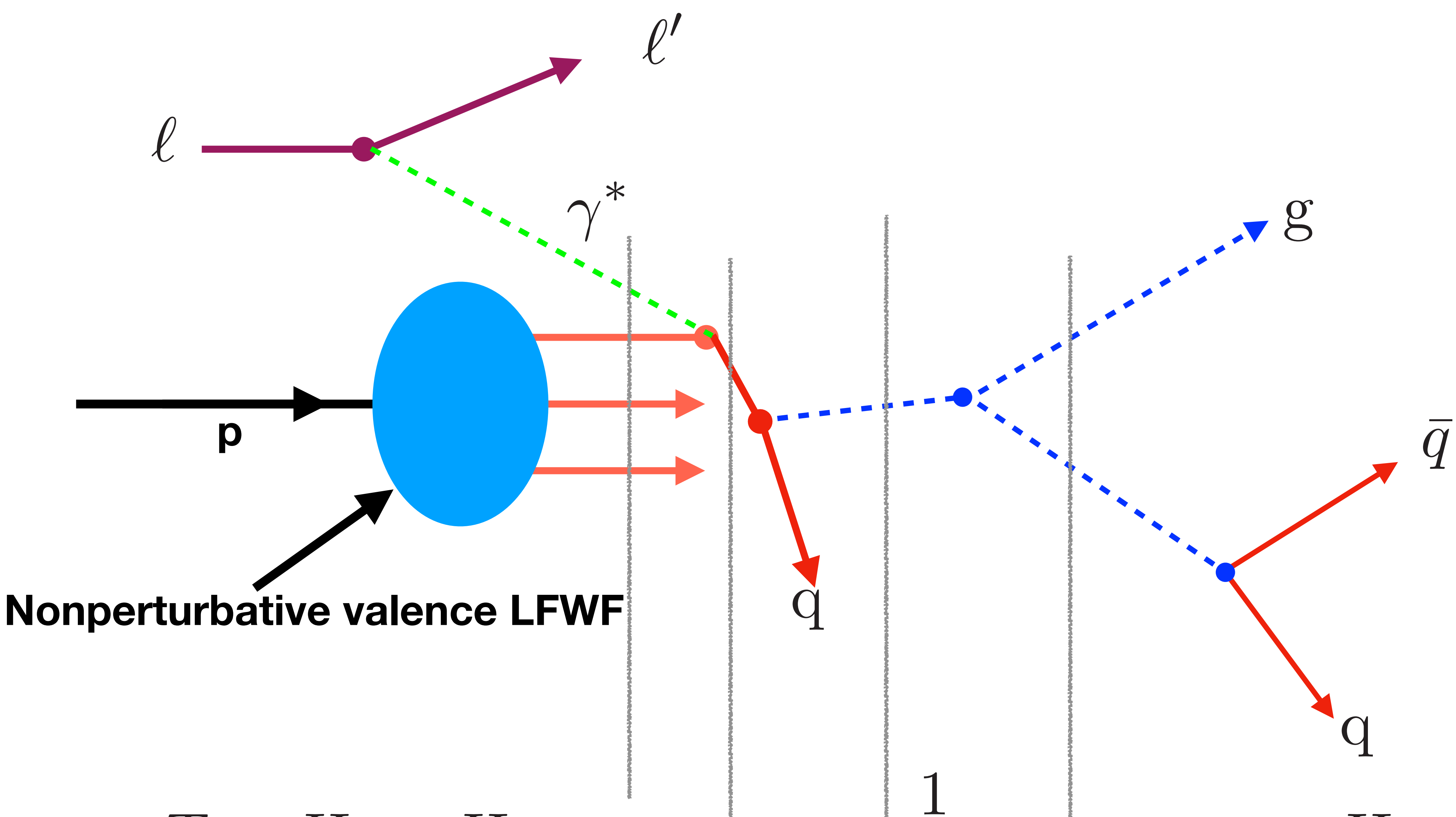
DGLAP Evolution in QCD



$$\mu^2 \frac{\partial q_v(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) q_v(y,\mu^2)$$

Use LF perturbation theory

$$\ell p \to \ell' + (qq) + q + g + q\bar{q}$$



$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{intermediate} - \mathcal{M}^2_{initial} + i\epsilon} H_I + \cdots$$

$$t = \log \frac{Q^2}{Q_0^2}$$

$$\frac{d}{dt} q_i(x,t) = \frac{\alpha_s(Q)}{2\pi} [q_i \times P_{qq}] + \frac{\alpha_s(Q)}{2\pi} [g \times P_{qg}]$$

$$\frac{d}{dt} g(x,t) = \frac{\alpha_s(Q)}{2\pi} [\textstyle\sum_i (q_i + \bar{q}_i) \times P_{qg}] + \frac{\alpha_s(Q)}{2\pi} [g \times P_{gg}]$$

$$P_{qq} = \tfrac{4}{3} [\tfrac{1+x^2}{(1-x)_+} + \tfrac{3}{2} \delta(1-x)]$$

$$P_{qq} = \tfrac{4}{3} [\tfrac{1+(1-x)^2}{x}] + \tfrac{3}{2} \delta(1-x)$$

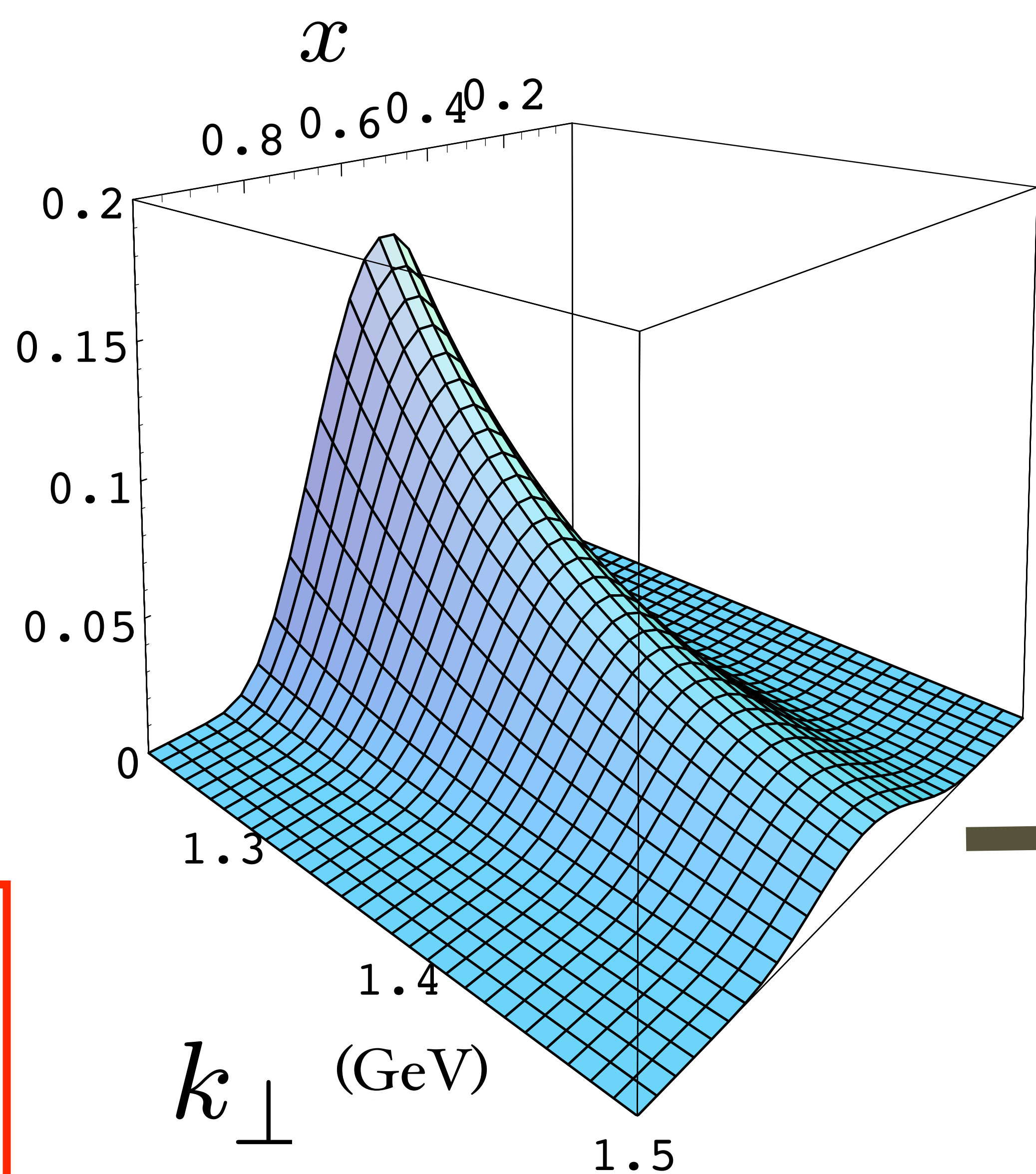
$$P_{gq} = \tfrac{1}{2} [x^2 + (1-x)^2]$$

$$P_{gg} = 6 [\tfrac{x}{(1-x)_+} + \tfrac{1-x}{x} + x(1-x)] + \tfrac{33-2n_f}{6} \delta(1-x)$$

Prediction from AdS/QCD: Meson LFWF

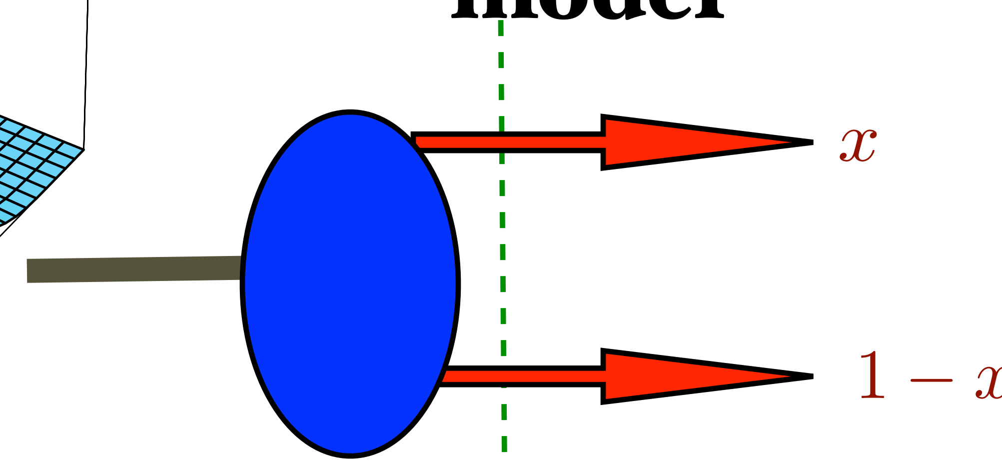
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

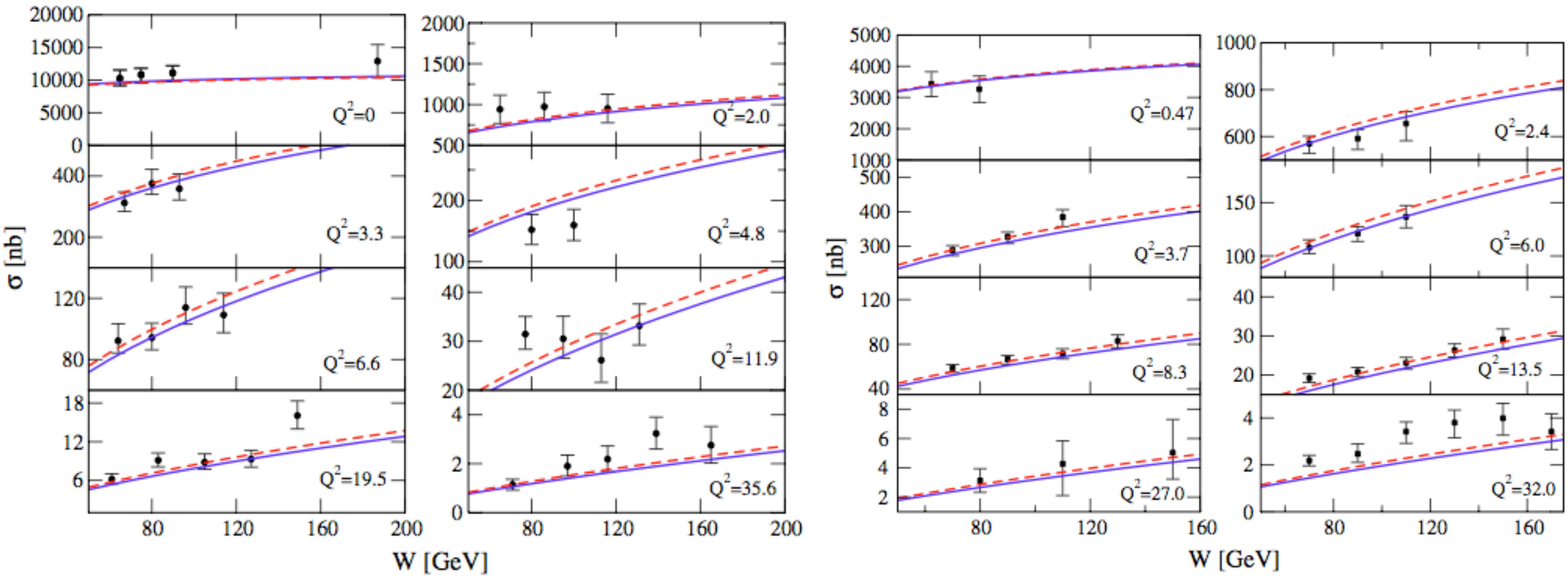
$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

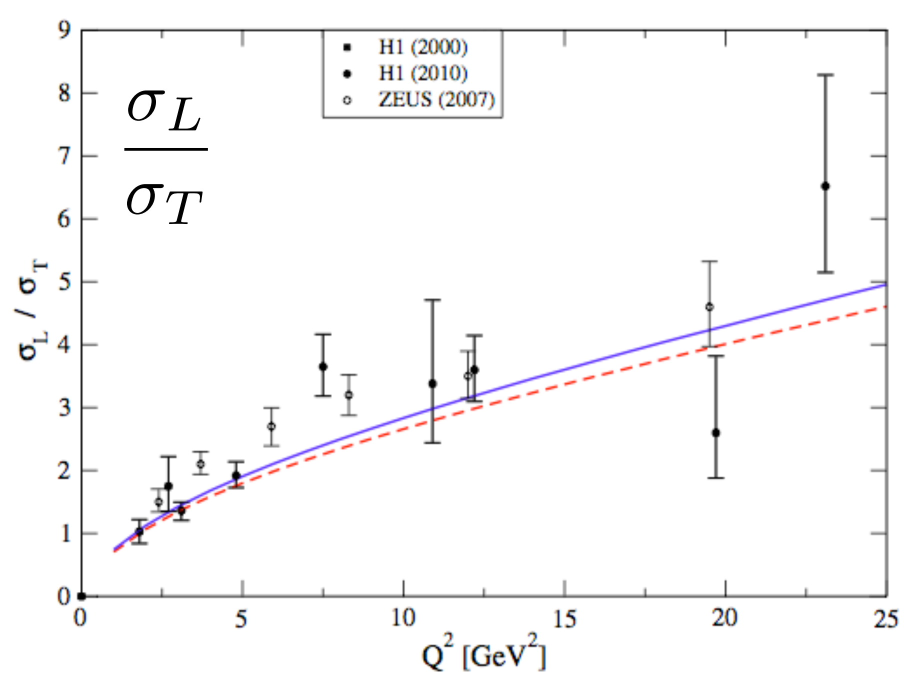
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



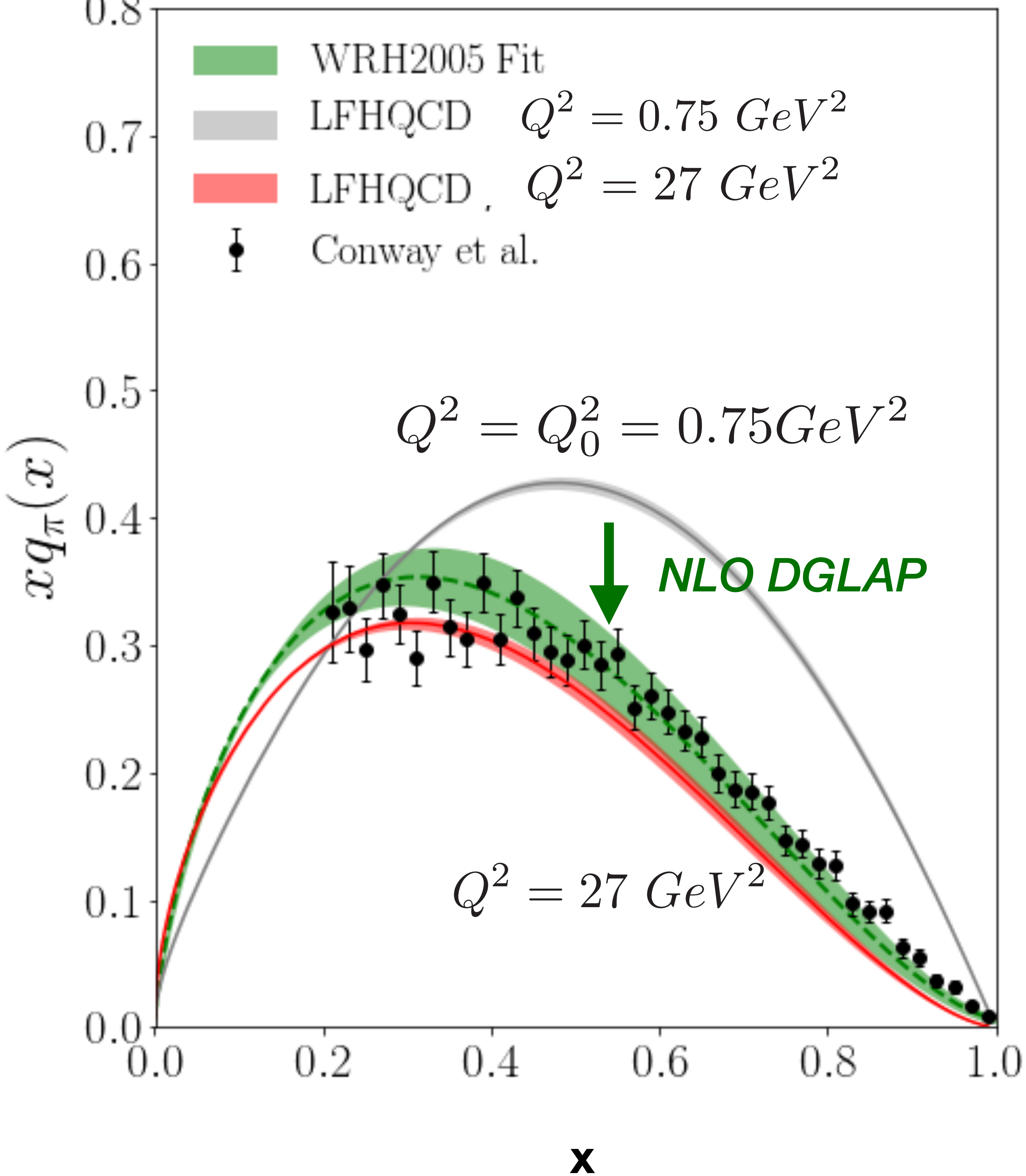
(a) H1 (b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$



T. Liu,
G. de Tèramond,
G. Dosch, A. Deur,
R.S. Sufian, sjb
(preliminary)

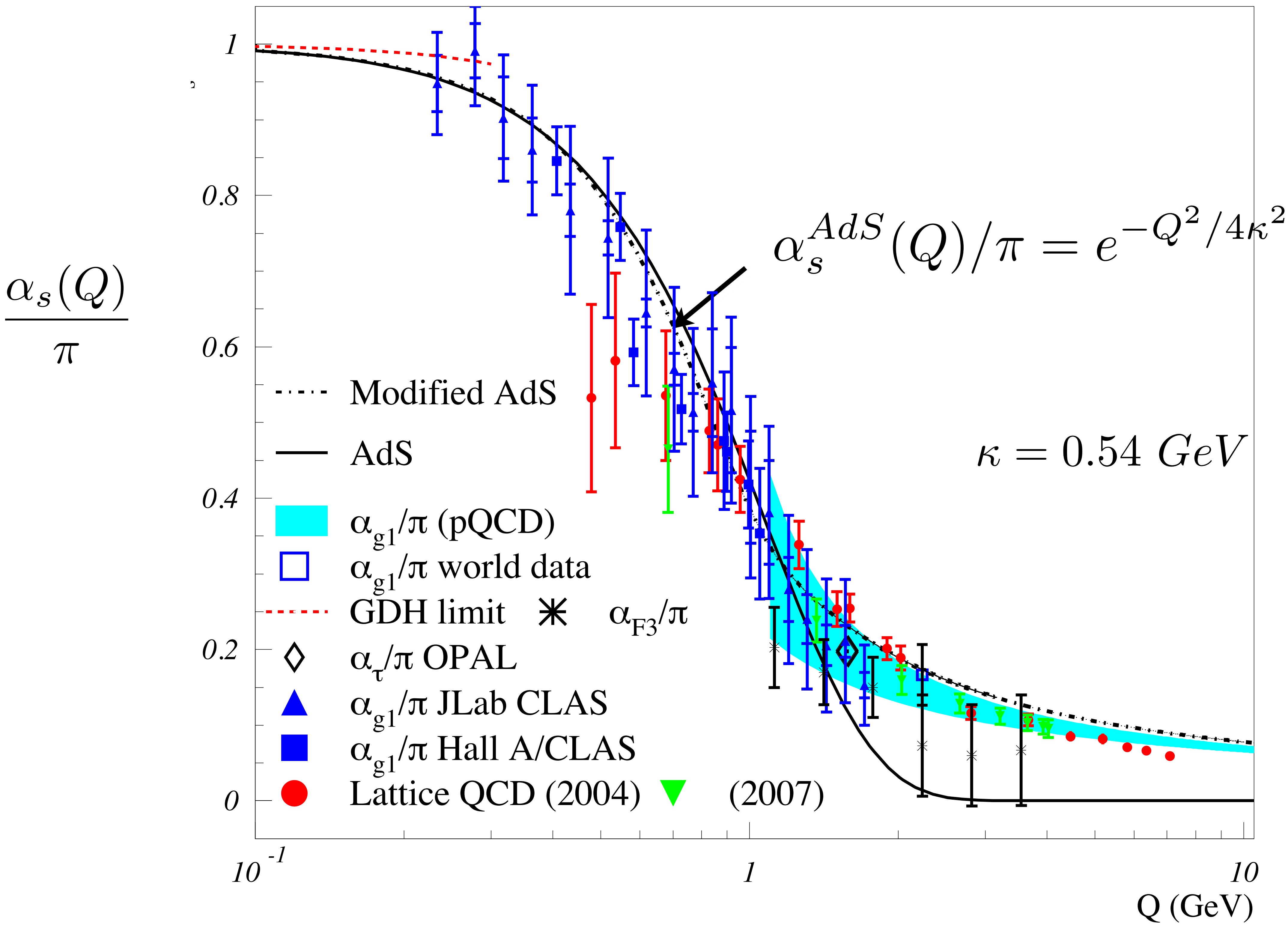
$$q_\pi(x, Q^2 < Q_0^2) = \int d^2\vec{k}_\perp |\psi_\pi(x, \vec{k}_\perp)|^2$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

"No parameters"

Start DGLAP evolution at transition scale Q^2_0

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

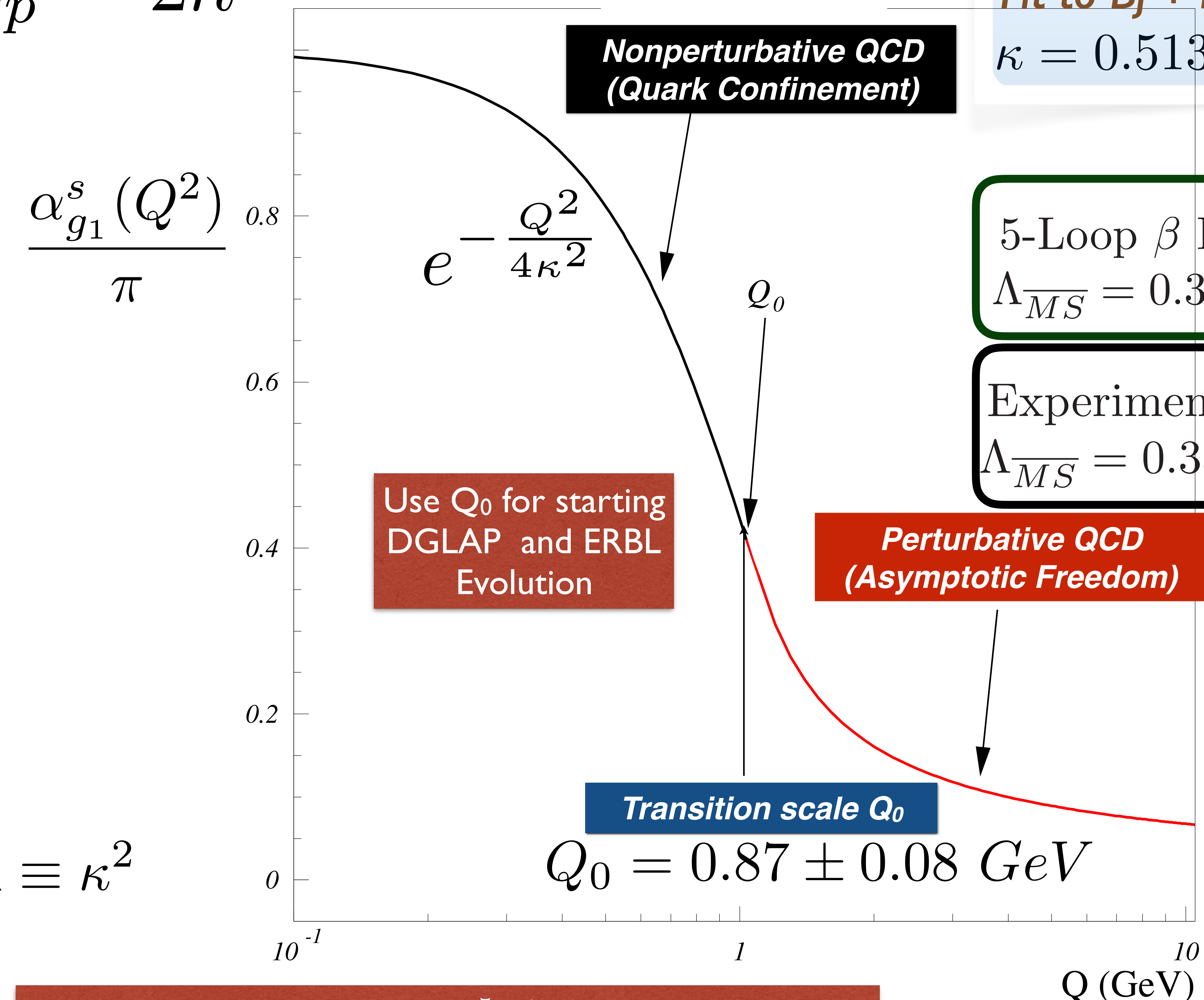
Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$



5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

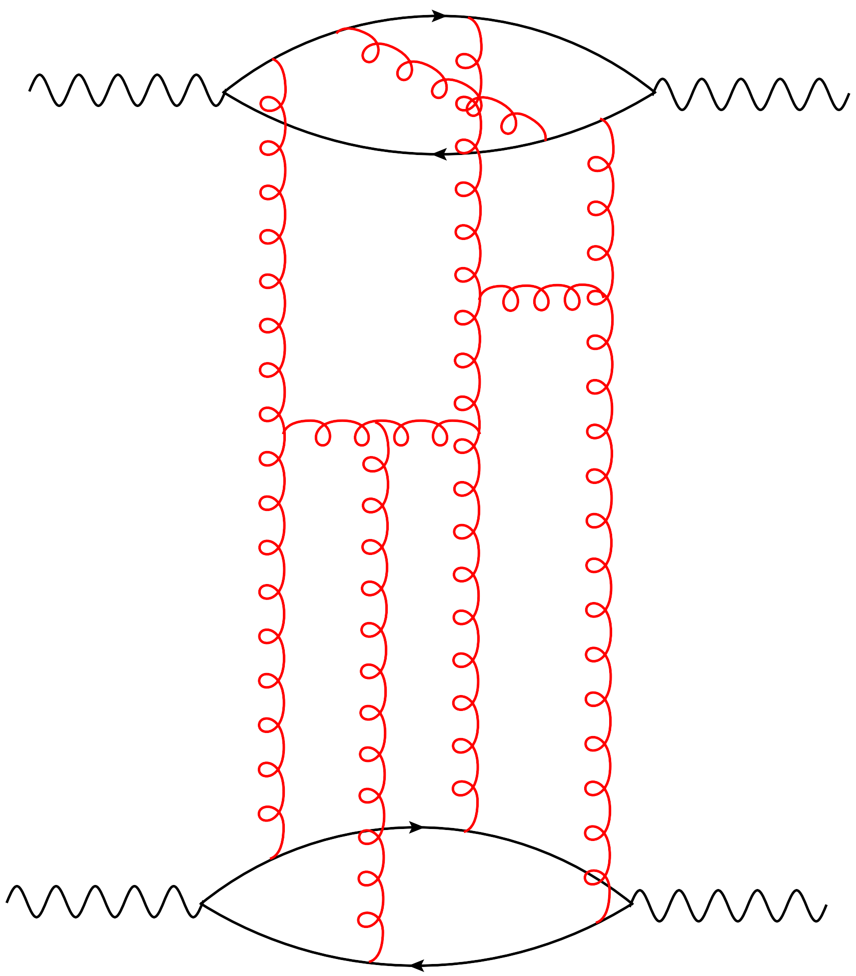
$\lambda \equiv \kappa^2$

$\overline{MS}$ scheme

Reverse Dimensional Transmutation!

BFKL Dynamics

V. S. Fadin, E. A. Kuraev, L. N. Lipatov, Phys. Lett. B60 (1975) 50;
Ya. Ya. Balitsky, L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.



BFKL pomeron in the next-to-leading approximation
V. S. Fadin* and L. N. Lipatov**

Lipatov Lectures

An introduction to BFKL dynamics

Lev Lipatov
Petersburg Nuclear Physics Institute

1. Gluon reggeization
2. BFKL equation and its solution
3. Möbius invariance
4. Holomorphic separability
5. Integrability at large N_c
6. Odderon problem in QCD
7. Baxter-Sklyanin representation
8. Solution of the Baxter equation
9. Pomeron in the thermostat
10. Next-to-leading corrections in $N = 4$ SUSY

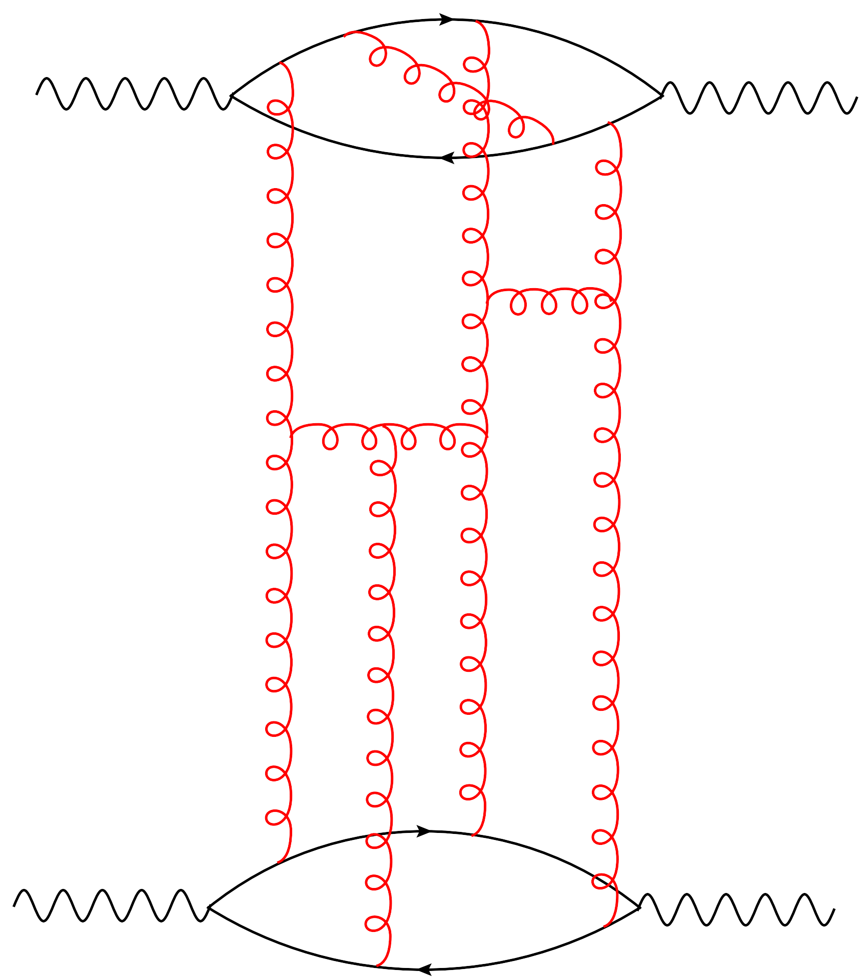
Lipatov Lectures

Results

1. Reggeization of gluons and quarks in QCD
2. Pomeron as a composite state of two reggeons
3. Odderon as a composite state of three reggeons
4. Möbius invariance of the BFKL equation
5. Holomorphic separability of BFKL Hamiltonian
6. Duality symmetry of BKP equations at $N_c \to \infty$
7. Integrability of the BFKL dynamics at $N_c \to \infty$
8. Effective action for reggeized gluon interactions
9. s- and t- chanel unitarity
10. Next-to-leading corrections to the BFKL equation
11. Remarkable properties of high energy dynamics in N=4 SUSY

BFKL Dynamics

V. S. Fadin, E. A. Kuraev, L. N. Lipatov, Phys. Lett. B60 (1975) 50;
Ya. Ya. Balitsky, L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.



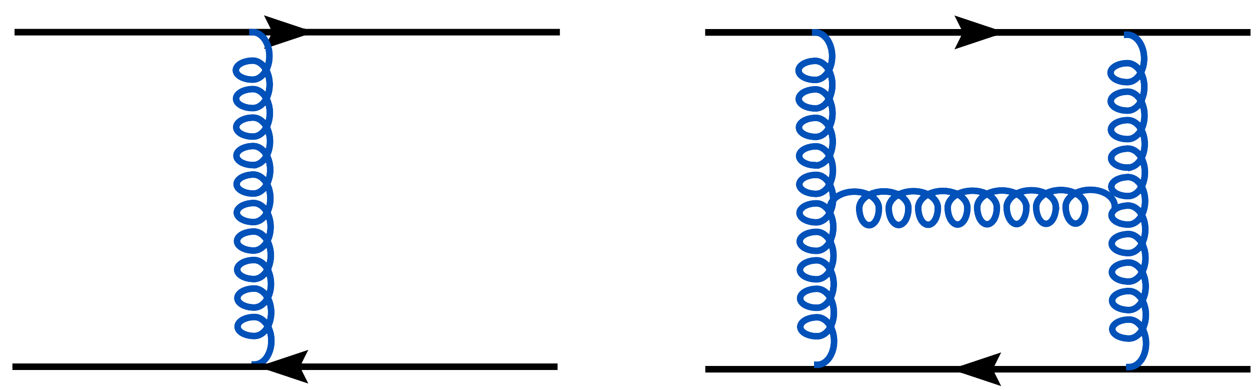
Connection to
'H'diagrams
Gluonic Flux Tube
Confining Potential

BFKL pomeron in the next-to-leading approximation
V. S. Fadin* and L. N. Lipatov**

Static Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$
$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + \boxed{8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3} \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



Summation of H graphs: confining potential?

Confinement eliminates IR divergences

Self-consistent mass scale κ

BFKL pomeron with massive gluons and running coupling

Eugene Levin[a,b], Lev Lipatov[c,d] and Marat Siddikov[a]
[a] *Departamento de Física, Universidad Técnica Federico Santa María, y Centro Científico - Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile*
[b] *Department of Particle Physics, School of Physics and Astronomy, Tel Aviv University, Tel Aviv, 69978, Israel*
[c] *Theoretical Physics Department, Petersburg Nuclear Physics Institute, Orlova Roscha, Gatchina, 188300, St. Petersburg, Russia and*
[d] *Physics Department, St.Petersburg State University, Ulyanovskaya 3, St.Petersburg 198504, Russia*

In this paper we proceed with the study of the Pomeron spectrum, by solving numerically the BFKL equation with massive gluons and running coupling. The spectrum of Regge singularities is discrete and the leading Pomeron has a considerable dependence on nonperturbative effects, for which we use Higgs mechanism as a model. We cross-checked this result with variational method and confirmed the infrared sensitivity of leading Pomeron. This fact is related to the infrared instability of the BFKL equation in QCD, with a running coupling. The subleading poles have a mild sensitivity to the soft physics, and are well described by known semiclassical methods. We also discuss the dependence on various prescriptions of the running coupling arguments.

Published in **Phys.Rev. D94 (2016) no.9, 096004**

The QCD pomeron with optimal renormalization

Stanley J. Brodsky (SLAC), Victor S. Fadin (Novosibirsk, IYF), Victor T. Kim (St. Petersburg, INP & Iowa State U., IITAP), Lev N. Lipatov (St. Petersburg, INP), Grigorii B. Pivovarov (Moscow, INR & Iowa State U., IITAP). Dec 1998. 11 pp.

Published in **JETP Lett. 70 (1999) 155-160**

SLAC-PUB-8037, IITAP-98-010

High-energy QCD asymptotics of photon-photon collisions

Stanley J. Brodsky (SLAC), Victor S. Fadin (Novosibirsk, IYF), Victor T. Kim (CERN & St. Petersburg, INP), Lev N. Lipatov (St. Petersburg, INP), Grigorii B. Pivovarov (Moscow, INR). Jul 2002. 7 pp.

Published in **JETP Lett. 76 (2002) 249-252, Pisma Zh.Eksp.Teor.Fiz. 76 (2002) 306-309**

SLAC-PUB-9318, CERN-TH-2002-143, PNPI-2484

The QCD Pomeron with Optimal Renormalization[1]

Stanley J. Brodsky[*], Victor S. Fadin[†], Victor T. Kim[‡&], Lev N. Lipatov[‡]
and
Grigorii B. Pivovarov[§&]

$$s^{\alpha_{\mathcal{P}}-1} = s^{\omega_{\mathcal{P}}}$$

Lowest order BFKL $$\omega_P = 12 \ln 2 \times \alpha_s/\pi$$

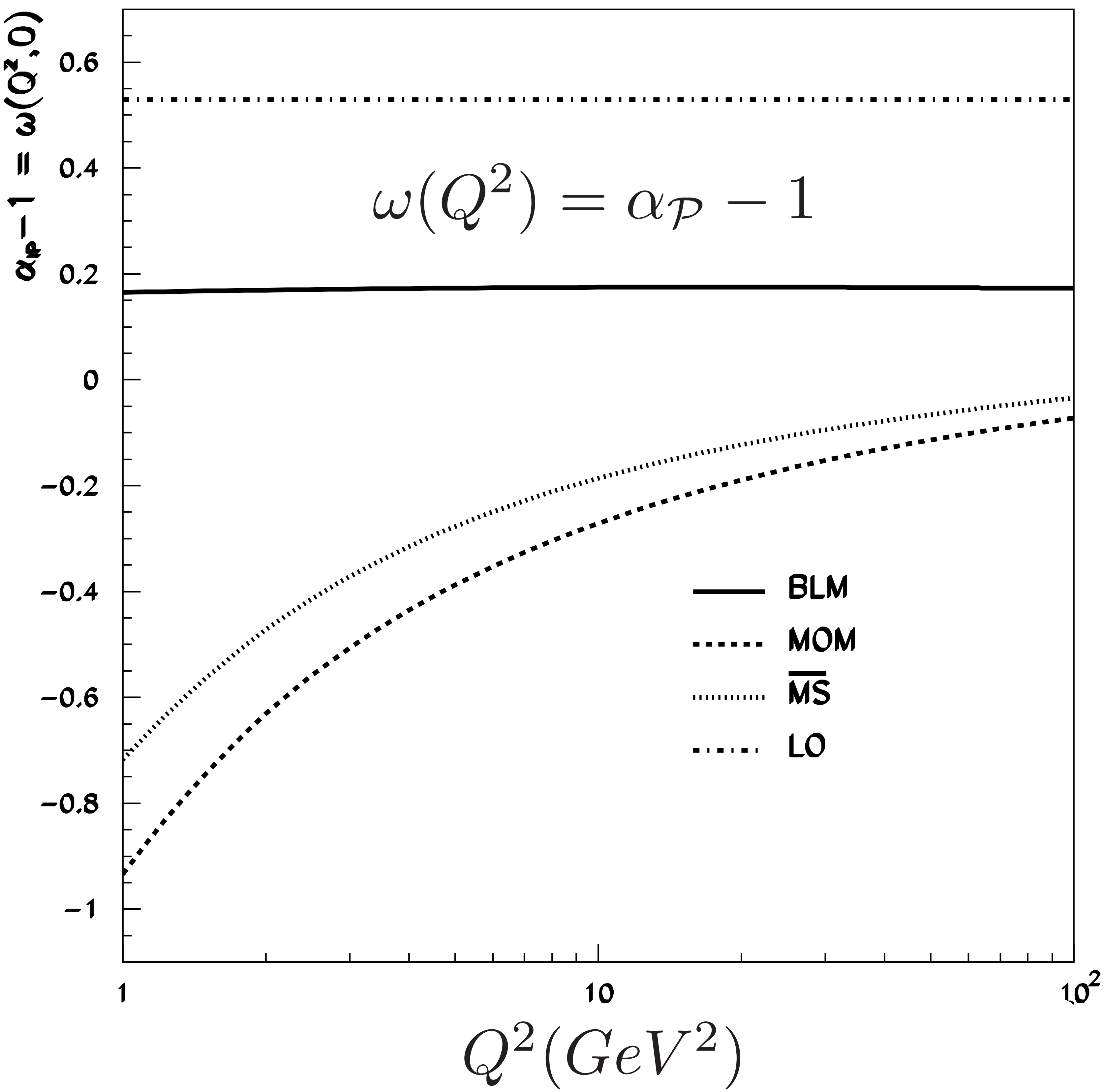
Adopting BLM scale setting, the NLO BFKL eigenvalue in the MOM-scheme is

$$\omega_{BLM}^{MOM}(Q^2,\nu) = N_C \chi_L(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi} \left[1 + r_{BLM}^{MOM}(\nu) \frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi} \right],$$

$$r_{BLM}^{MOM}(\nu) = r_{MOM}^{conf}(\nu)\,.$$

BLM/PMC: Scale chosen to eliminate β terms

BLM: ω(BFKL nlo) independent of Q^2



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SLAC-PUB-8037, IITAP-98-010

High-energy QCD asymptotics of photon-photon collisions

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Published in **JETP Lett. 76 (2002) 249-252, Pisma Zh.Eksp.Teor.Fiz. 76 (2002) 306-309**

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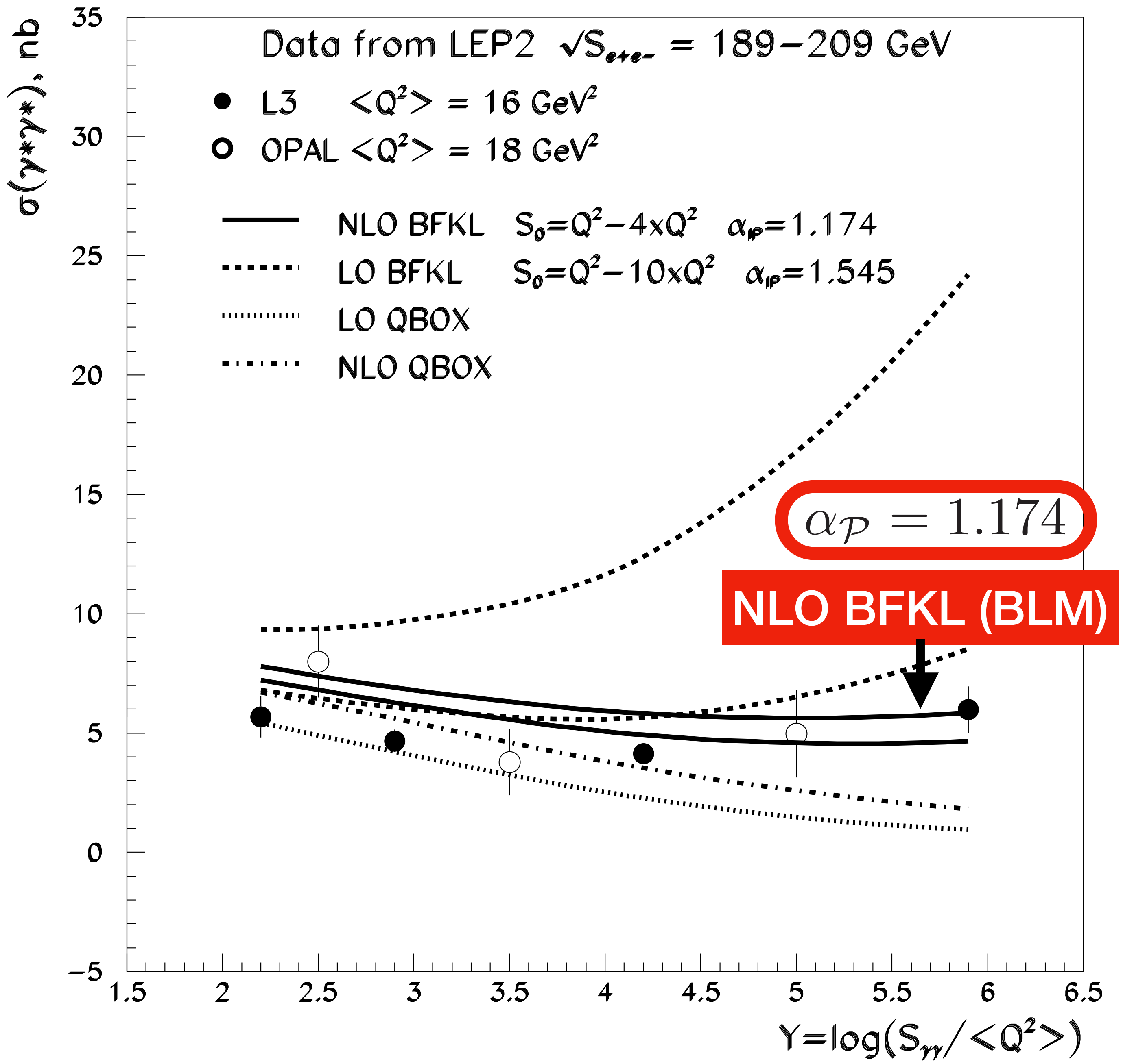


$$\gamma^*\gamma^* \to X$$

High-Energy Asymptotics of Photon-Photon Collisions in QCD

S. J. Brodsky, V. S. Fadin, V. T. Kim, L.N. Lipatov, G. B. Pivovarov

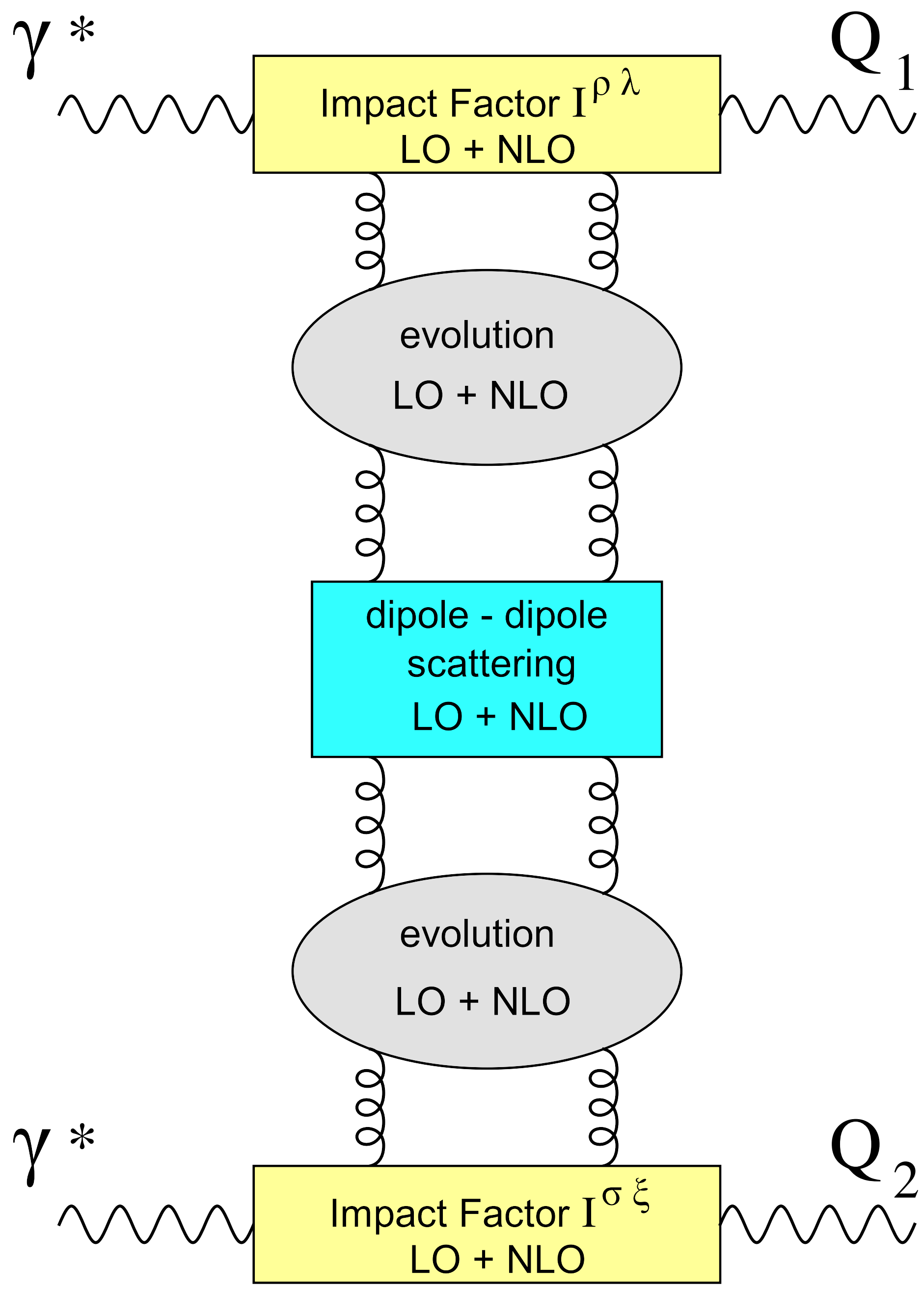
$$\sigma_{\gamma^*\gamma^*\to X}(s) \sim \left[\frac{s}{s_0}\right]^{\alpha_{\mathcal{P}}-1}$$

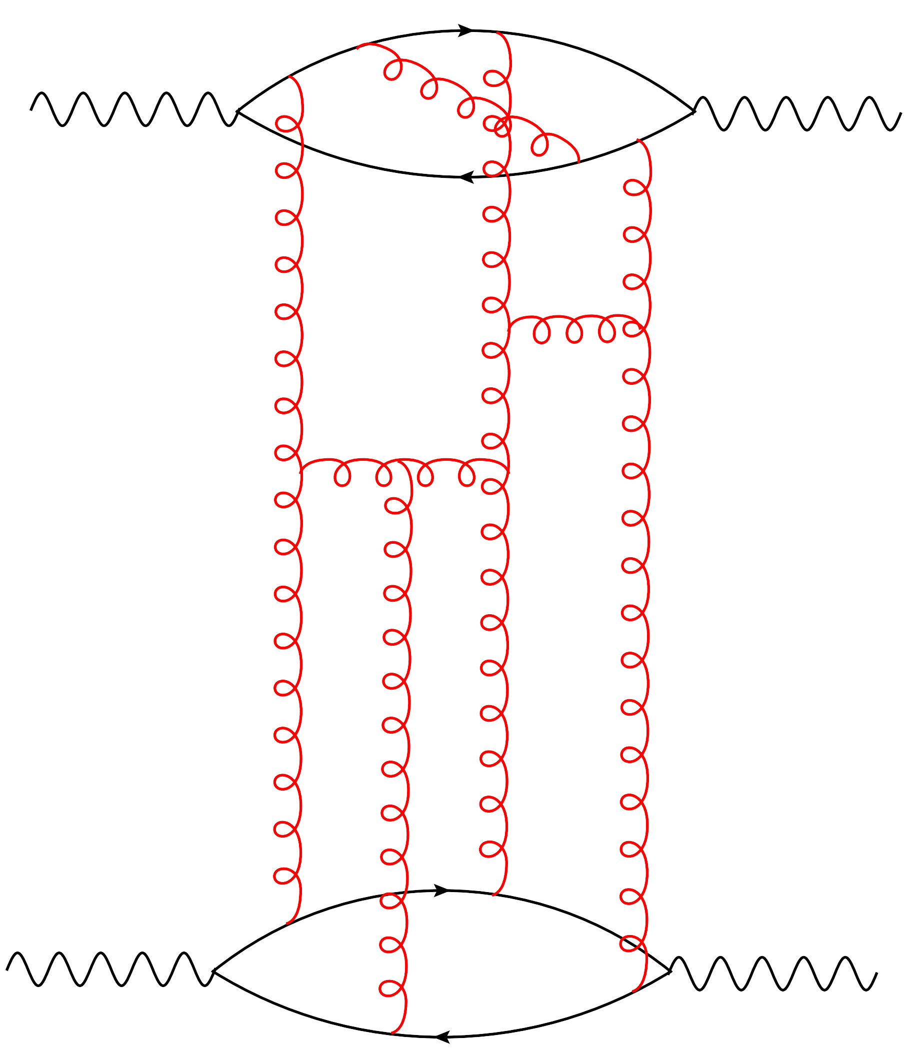
$$\alpha_{\mathcal{P}} - 1 = 12\log 2 \times \alpha_s/\pi \simeq 0.55 \qquad LO\ BFKL$$

$$\alpha_{\mathcal{P}} - 1 \simeq 0.13 - 0.18 \qquad NLO\ BFKL\ BLM/PMC$$

Giovanni A. Chirilli,* Yuri V. Kovchegov†

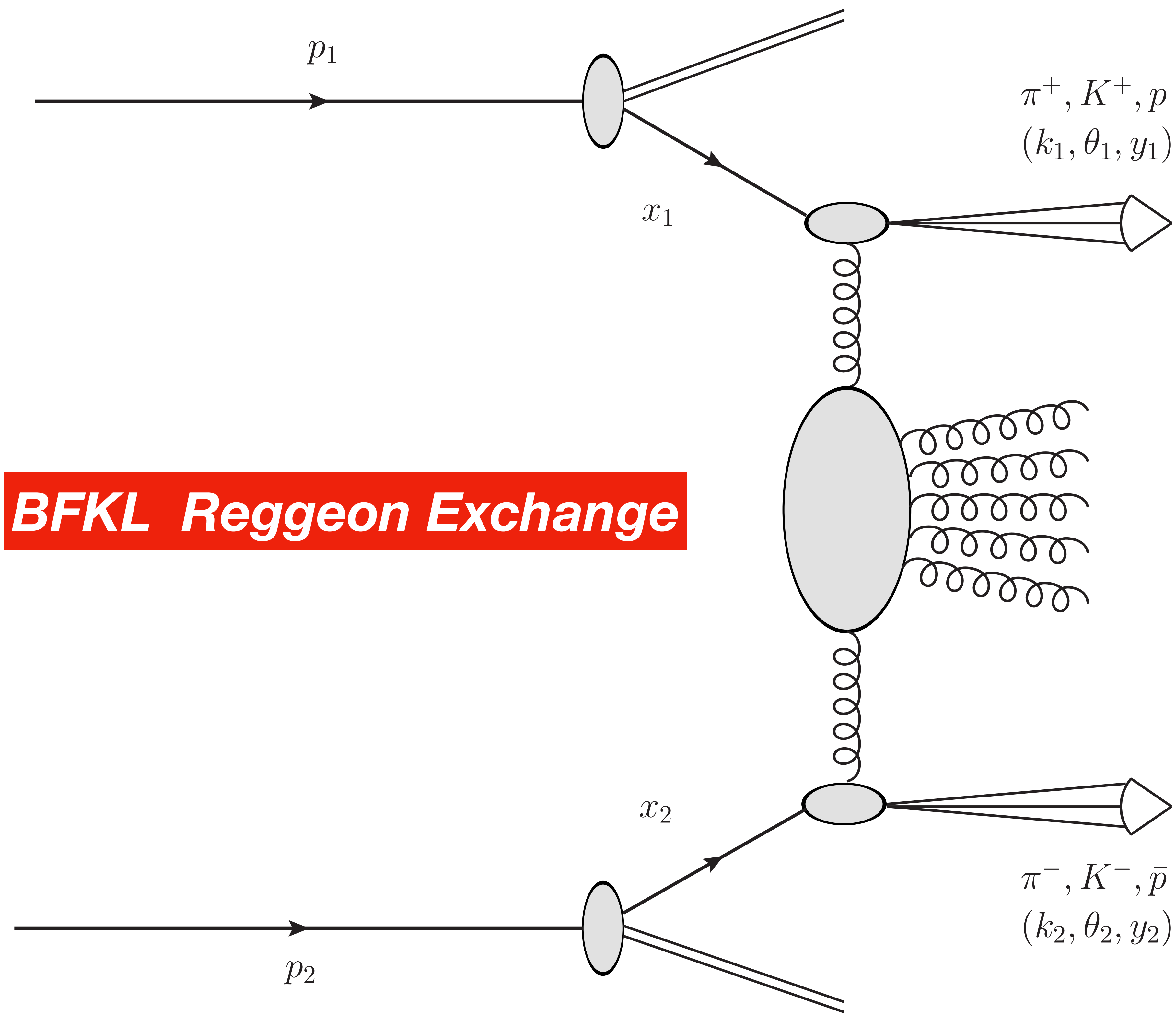
BFKL Analysis





Dihadron Production at LHC: BFKL Predictions for Cross Sections and Azimuthal Correlations

Francesco G. Celiberto, Dmitry Yu. Ivanov, Beatrice Murdaca, and Alessandro Papa



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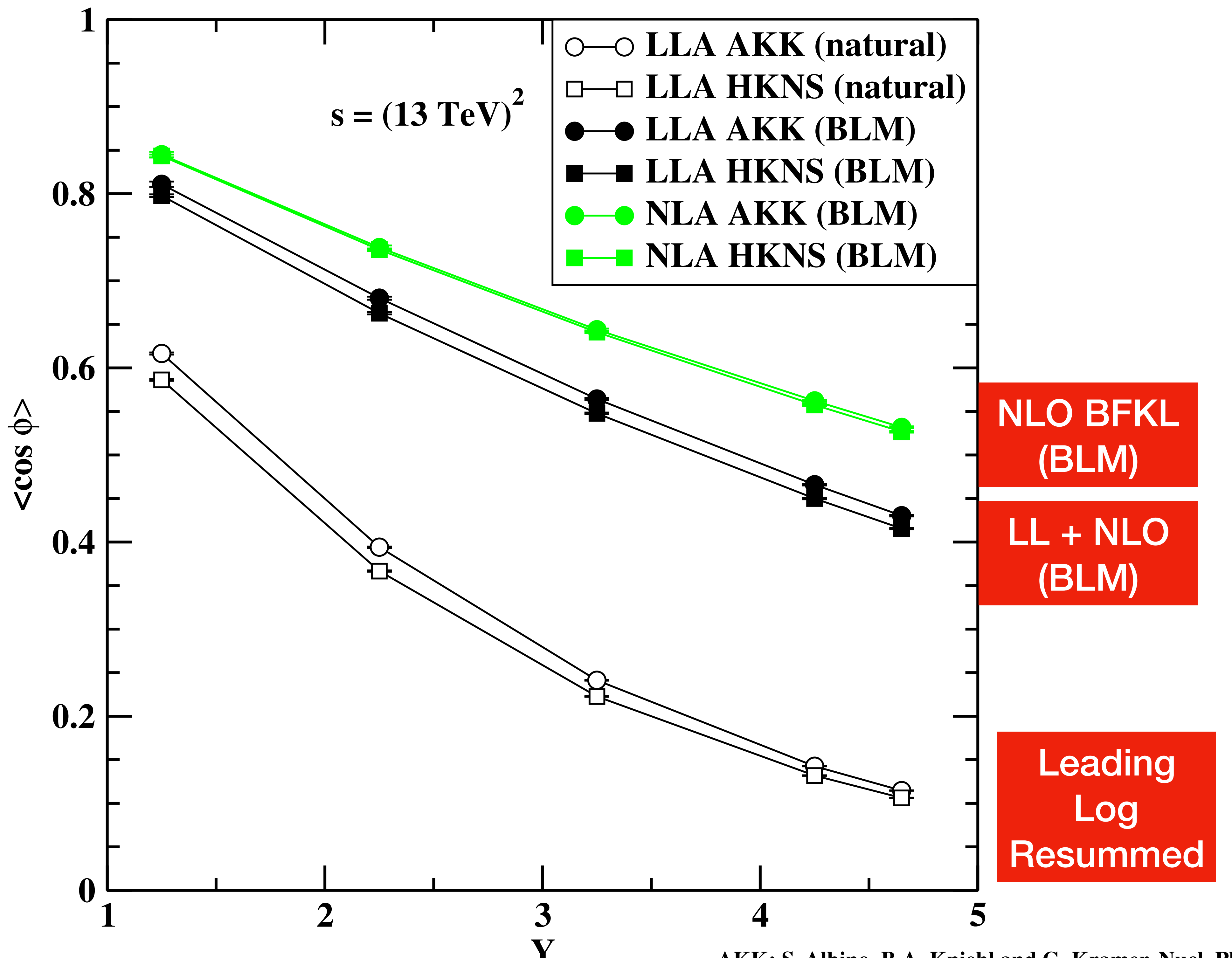


NLA: Leading Log Resummed + Next-to Leading Order

LLA: Leading Log Resummed

- AKK: S. Albino, B.A. Kniehl and G. Kramer, Nucl. Phys. B 803, 42 (20
- HKNS: M. Hirai, S. Kumano, T.-H. Nagai and K. Sudoh, Phys. Rev. D

Dihadron Production at LHC: BFKL Predictions for Cross Sections and Azimuthal Correlations

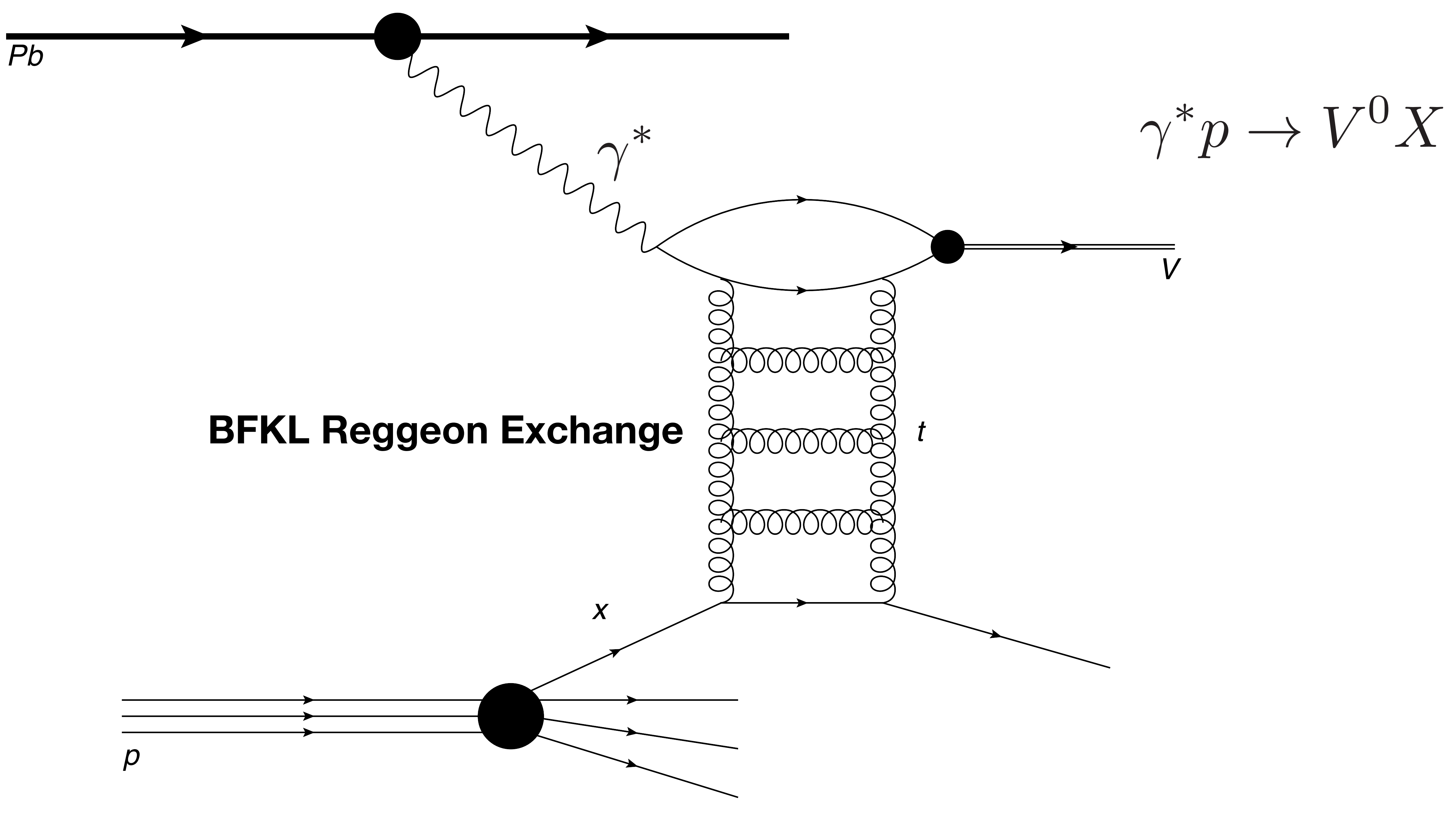
Francesco G. Celiberto Dmitry Yu. Ivanov, Beatrice Murdaca and Alessandro Papa

CONCLUSIONS AND OUTLOOK

In this paper we investigated the dihadron production process at the LHC at the center-of-mass energy o 13 TeV, giving the first theoretical predictions for cross sections and azimuthal angle correlations in the LLA and partial NLA BFKL approach. We implemented the exact version of the BLM optimization procedure in order to make completely vanish the β_0-dependence in our observables and minimize the size of the NLA corrections. We found that our NLA BLM predictions are close to the LLA BLM ones, while the LLA calculations at natural scales overestimate the total cross section C_0 and predict a stronge de-correlation for the azimuthal ratios R_{n0}. The good agreement between LLA and NLA at BLM scales is a direct consequence of the small size of the higher-order corrections, representing so a clear signal of the reliability of the BLM method. However, more accurate analyses are still needed: full NLA calculations including next-to-leading order hadron vertices, together with the study of larger rapidity intervals in the final state and considering the effect of a different choice for the factorization scale μ_F with respect to the renormalization scale μ_R, are underway. In view of all these considerations, we encourage experimental collaborations to include the study of the dihadron production in the program of future analyses at the LHC, making use of a new suitable channel to improve our knowledge about the dynamics of strong interactions in the Regge limit.

Probing BFKL dynamics in the Vector Meson Photoproduction at large – t in pPb collisions at the CERN LHC

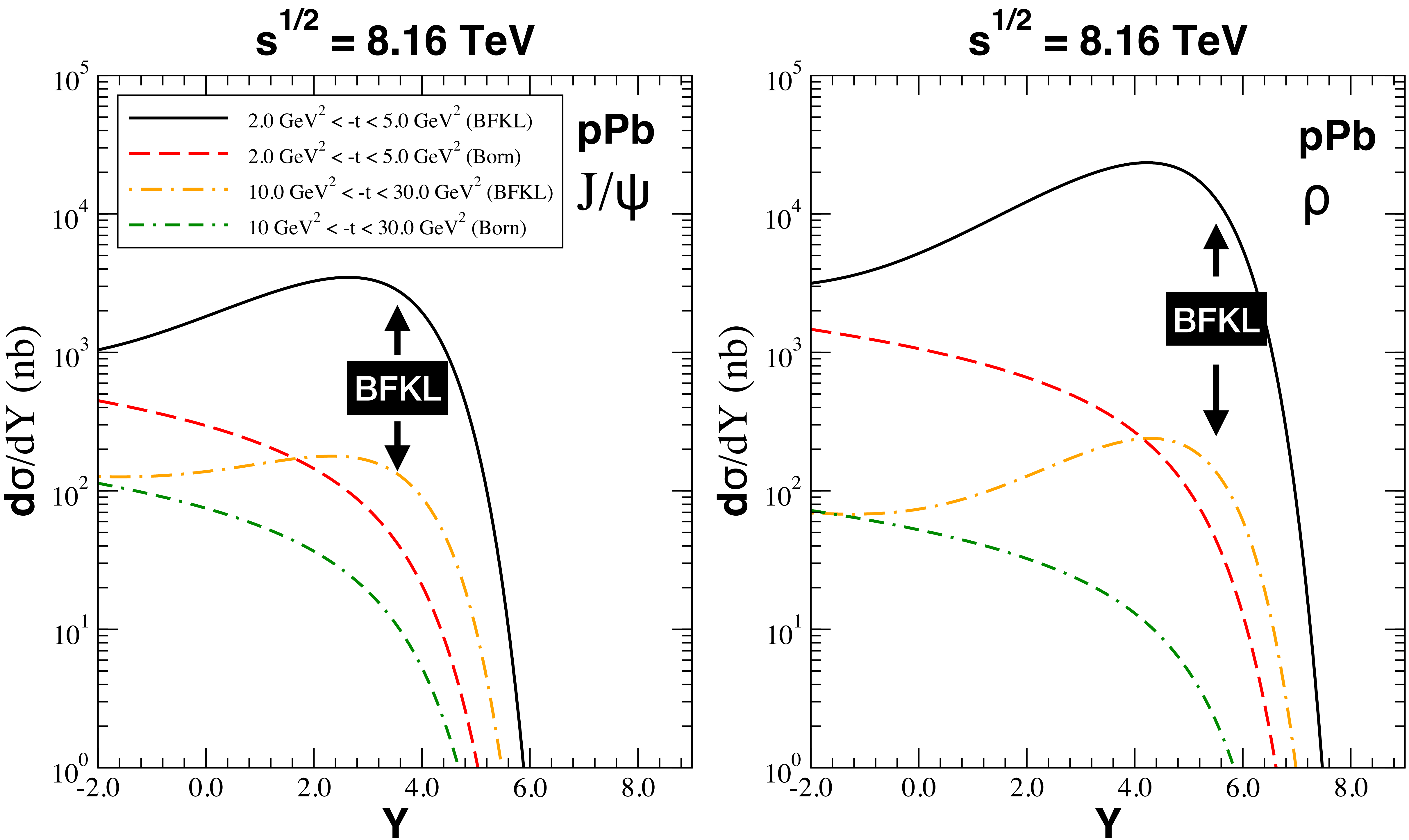
V. P. Goncalves and W. K. Sauter



Probing the BFKL dynamics in the Vector Meson Photoproduction at large – t in pPb collisions at the CERN LHC

V. P. Gon¸calves and W. K. Sauter

$$\gamma^* p \rightarrow V^0 X$$



Measure the BFKL Odderon

The Odderon intercept in perturbative QCD

P. Gauron, L.N. Lipatov, B. Nicolescu (Orsay, IPN & Paris U., VI-VII)

Jun 1993 - 7 pages

Z.Phys. C63 (1994) 253-256
DOI: 10.1007/BF01411017

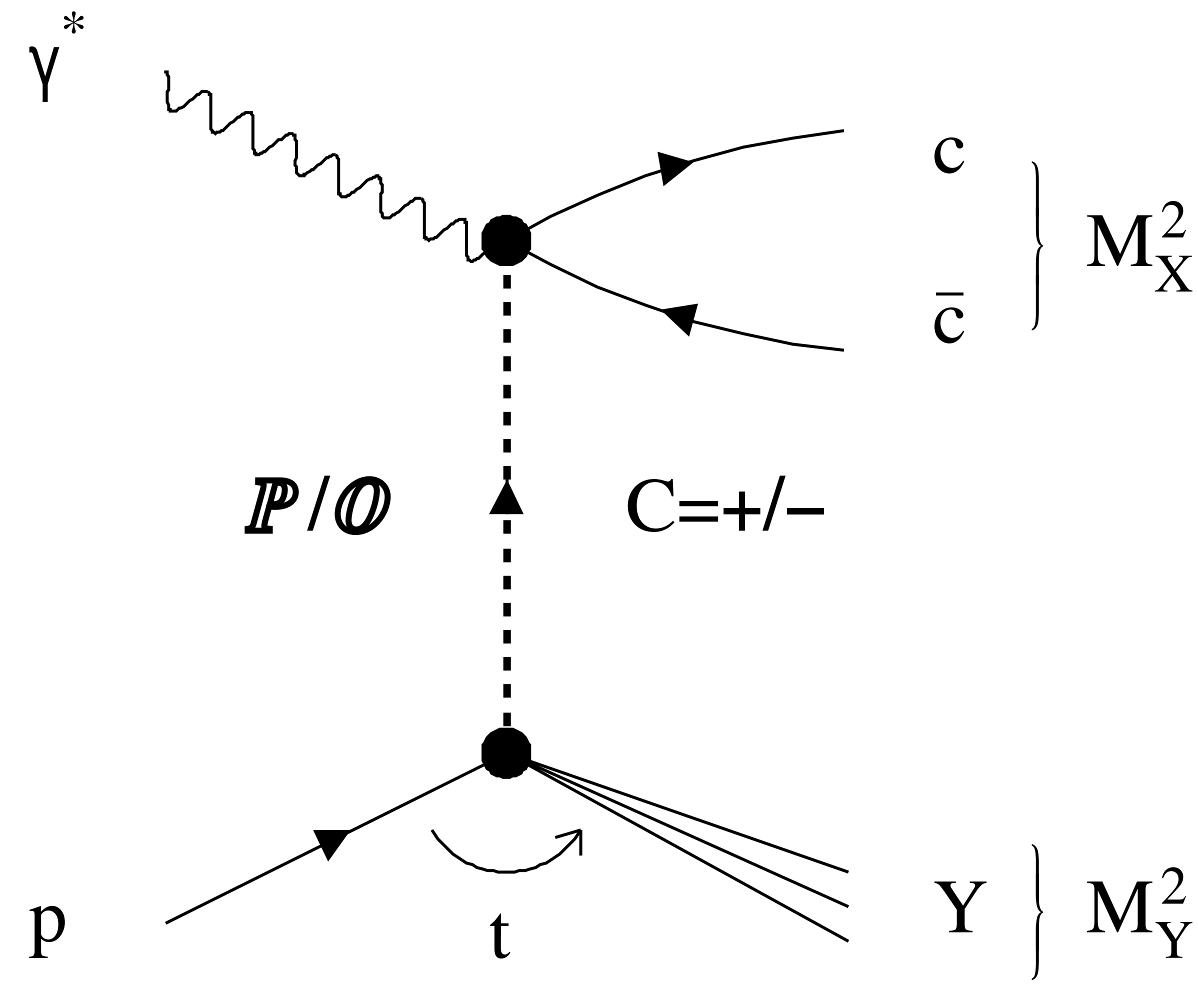
Odderon-Pomeron interference

Stanley J. Brodsky, Johan Rathsman (SLAC) , Carlos Merino (Santiago de Compostela U.)

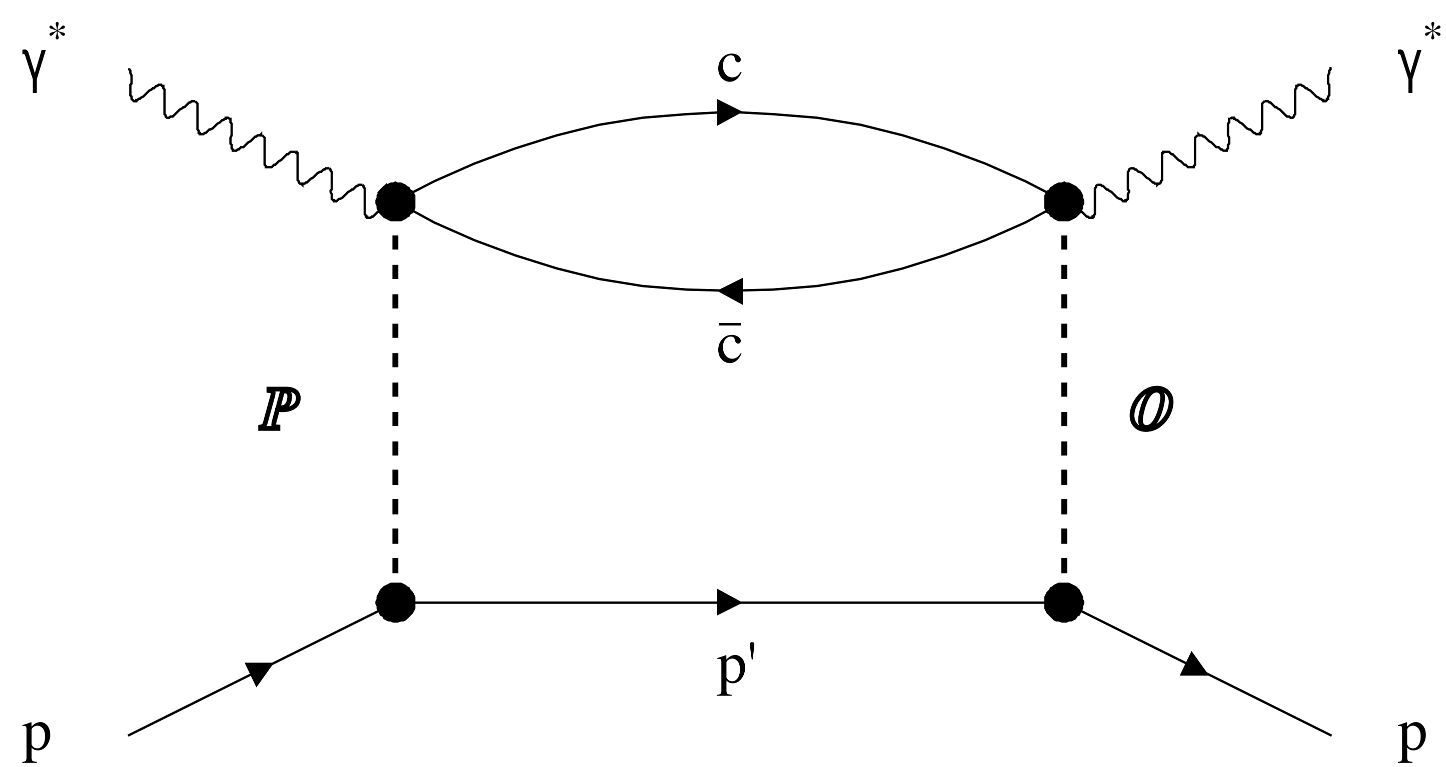
Apr 1999 - 10 pages

Phys.Lett. B461 (1999) 114-122
DOI: 10.1016/S0370-2693(99)00807-2
SLAC-PUB-8095
e-Print: hep-ph/9904280 | PDF

$$\mathcal{A}(t, M_X^2, z_c) \simeq 2 \frac{\kappa_{pp'}^{\mathcal{O}} \kappa_{\mathcal{O}}^{\gamma c\bar{c}}}{\kappa_{pp'}^{\mathcal{P}} \kappa_{\mathcal{P}}^{\gamma c\bar{c}}} \sin\left[\frac{\pi\left(\alpha_{\mathcal{O}} - \alpha_{\mathcal{P}}\right)}{2}\right] \left(\frac{s_{\gamma p}}{M_X^2}\right)^{\alpha_{\mathcal{O}} - \alpha_{\mathcal{P}}} \frac{\sin\frac{\pi\alpha_{\mathcal{P}}}{2}}{\cos\frac{\pi\alpha_{\mathcal{O}}}{2}} \frac{2z_c - 1}{z_c^2 + (1 - z_c)^2} \,.$$



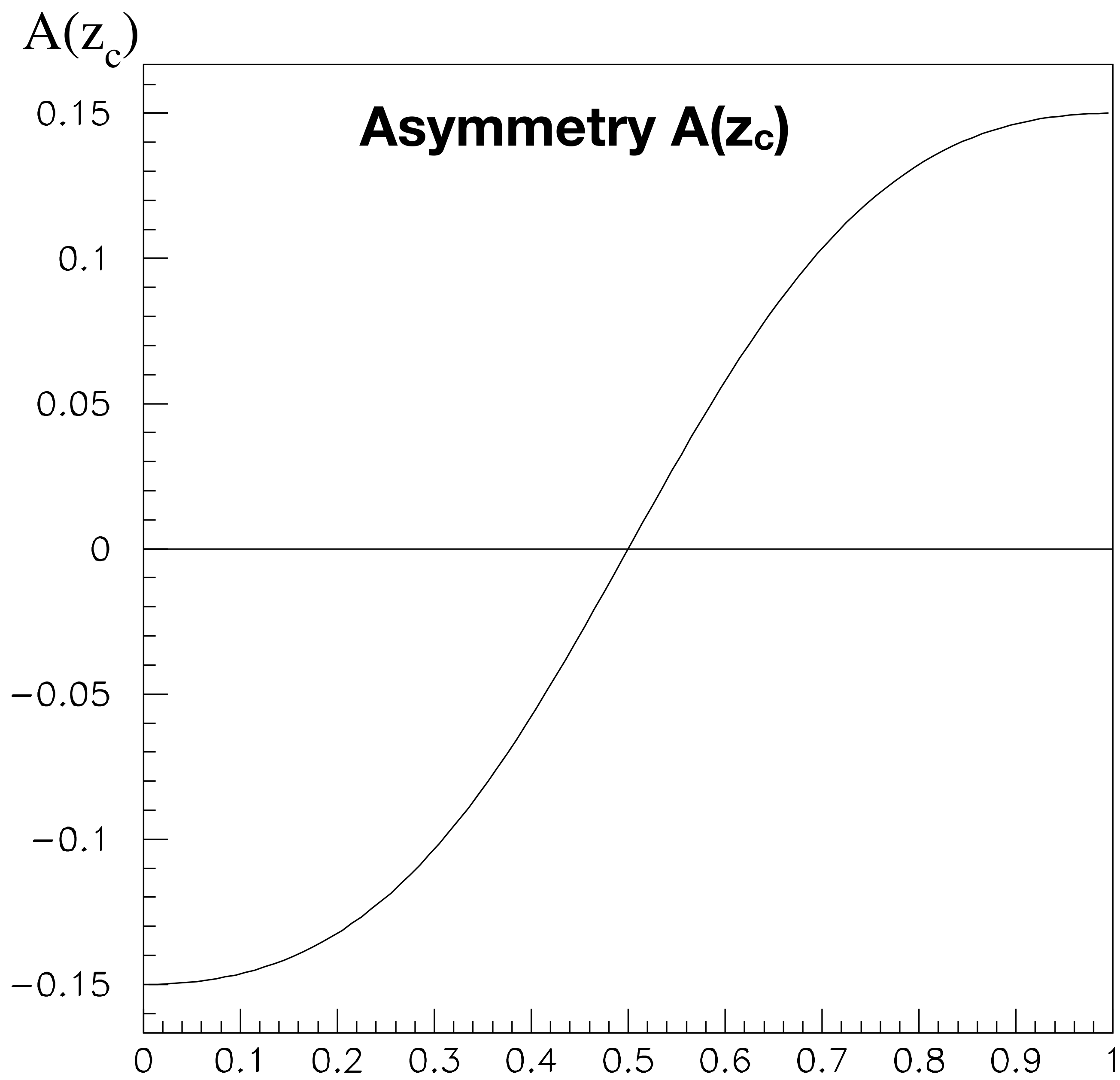
γ*
c
c̄
M_X^2
ℙ/𝕆
C=+/−
p
t
Y
M_Y^2



γ*
c
γ*
c̄
ℙ
𝕆
p'
p
p

Rathsman, Merino, SJB

$$\gamma p \to c\ \bar{c}\ p'$$



The asymmetry in fractional energy z_c of charm versus anticharm jets predicted using the Donnachie-Landshoff Pomeron for $\alpha_{\mathcal{P}} = 1.2$, $\alpha_{\mathcal{O}} = 0.95$ and $s_{\gamma p}/M_X^2 = 100$.

Also: Odderon Exchange in pion photoproduction

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

- **Dressed Photon Propagator sums all β (vacuum polarization) contributions, proper and improper**

$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_o)}{1-\Pi(t_0)}$$

- **Initial Scale Choice t_o is Arbitrary!**

- **Any renormalization scheme can be used** $\alpha(t) \to \alpha_{\overline{MS}}(e^{-\frac{5}{3}}t)$

BLM Scale Setting

$\beta_0 = 11 - \frac{2}{3} n_f$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2}\beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right],$$

Conformal coefficient - independent of β

where

$$Q^* = Q \exp(3A_{VP}),$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion:

Gardi, Grunberg, Rathsman, sjb

BLM/PMC: Set Scales

$$a(Q) \equiv \frac{\alpha_s(Q)}{\pi}$$

such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\begin{aligned}\rho(Q^2) = r_{0,0} &+ r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1} \\ &+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3} \\ &+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1} \\ &+ \cdots\end{aligned}$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

How do we identify the β terms?

BLM: Use n_f dependence of β_0 and β_1

Principle of Maximum Conformality (PMC)

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky†

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu‡

Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

Principle of Maximum Conformality (PMC)

- **Subtract extra constant δ in dimensional regularization. Defines new scheme R_δ**

$$\log 4\pi - \gamma_E - \delta \qquad \overline{MS} : \delta = 0$$

(δ: Arbitrary constant!)

- **$\boxed{\text{Coefficients of δ identify β terms !}}$**

- **Shift β terms to argument of running coupling $\alpha_s(Q_n^2)$ at each order n (analogous to all-orders vacuum polarization summation in QED)**

- **Resulting PQCD series matches β= 0 conformal series**

- **scheme-independent predictions at each computed order**

- **almost independent of initial scale μ_0**

M. Mojaza, L. di Giustino, Xing-Gang Wu, sjb

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

Choose μ_R^{init}*; arbitrary initial renormalization scale*

Identify βᵢ via δ-dependence

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *− terms*

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at N_C=0

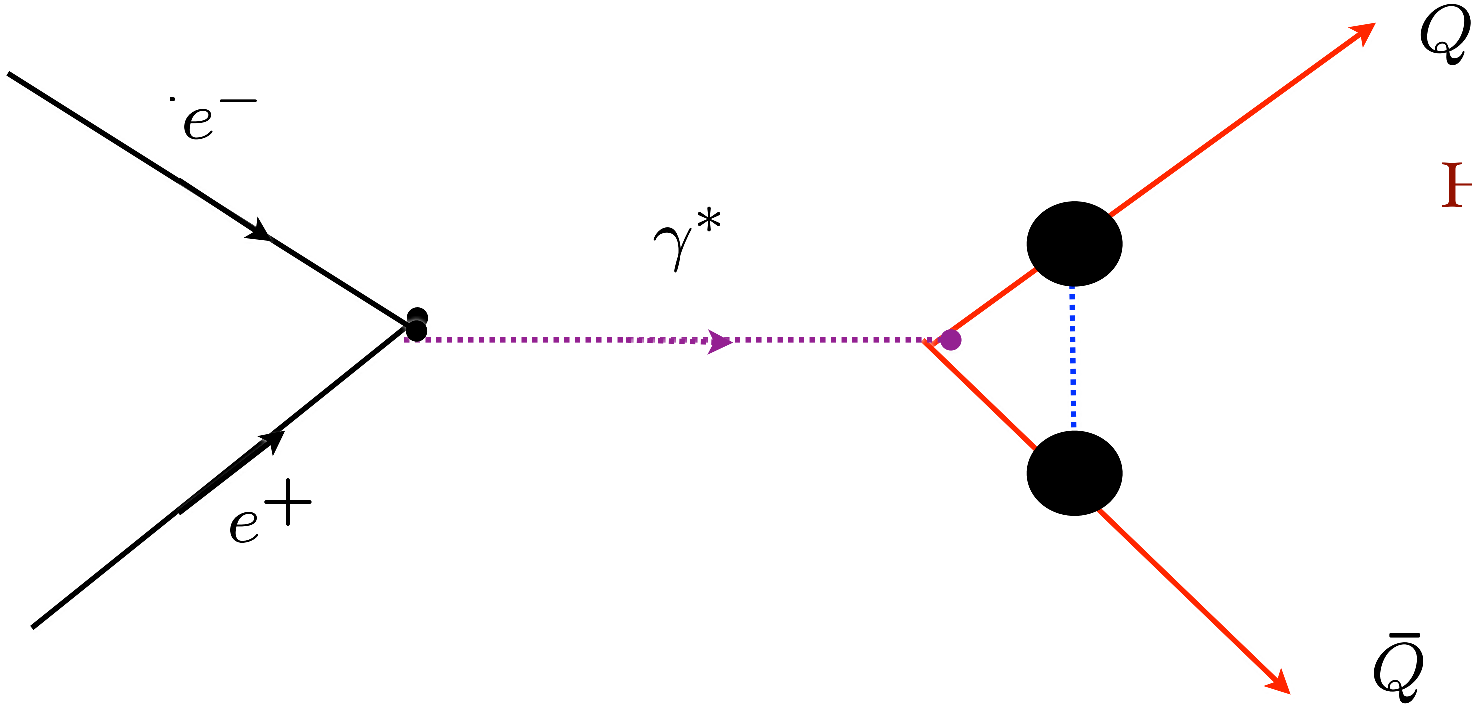
Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

Principle of Maximum Conformality

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB



Hoang, Kuhn, Teubner, sjb

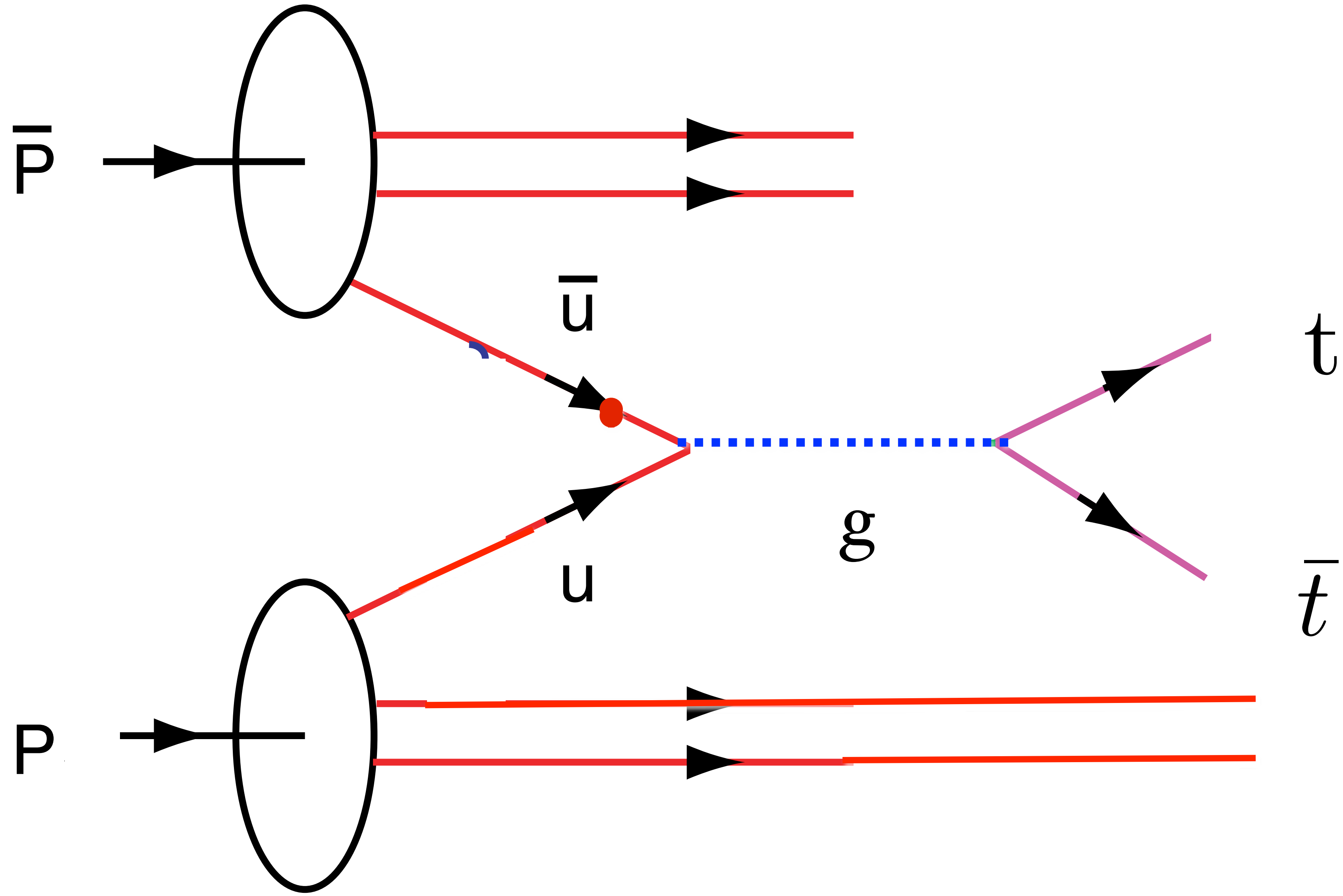
$$F_1 + F_2 = \left[1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}\right] \times \left[1 + \frac{\pi\alpha_s(sv^2)}{4v}\right]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM/PMC Scales

QCD coupling at small scales at low relative velocity v

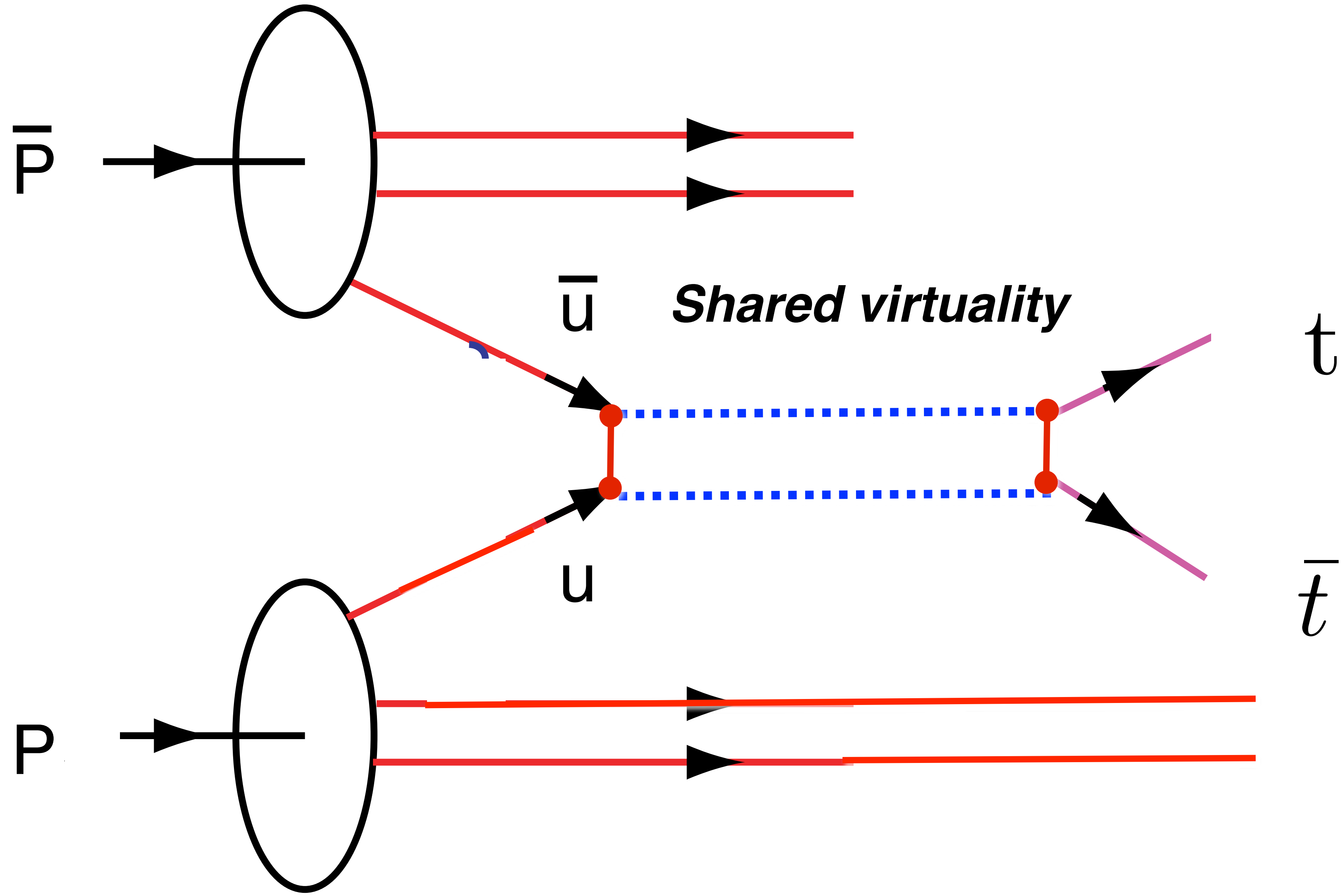
Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron



Born term.

Xing-Gang Wu, sjb

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron



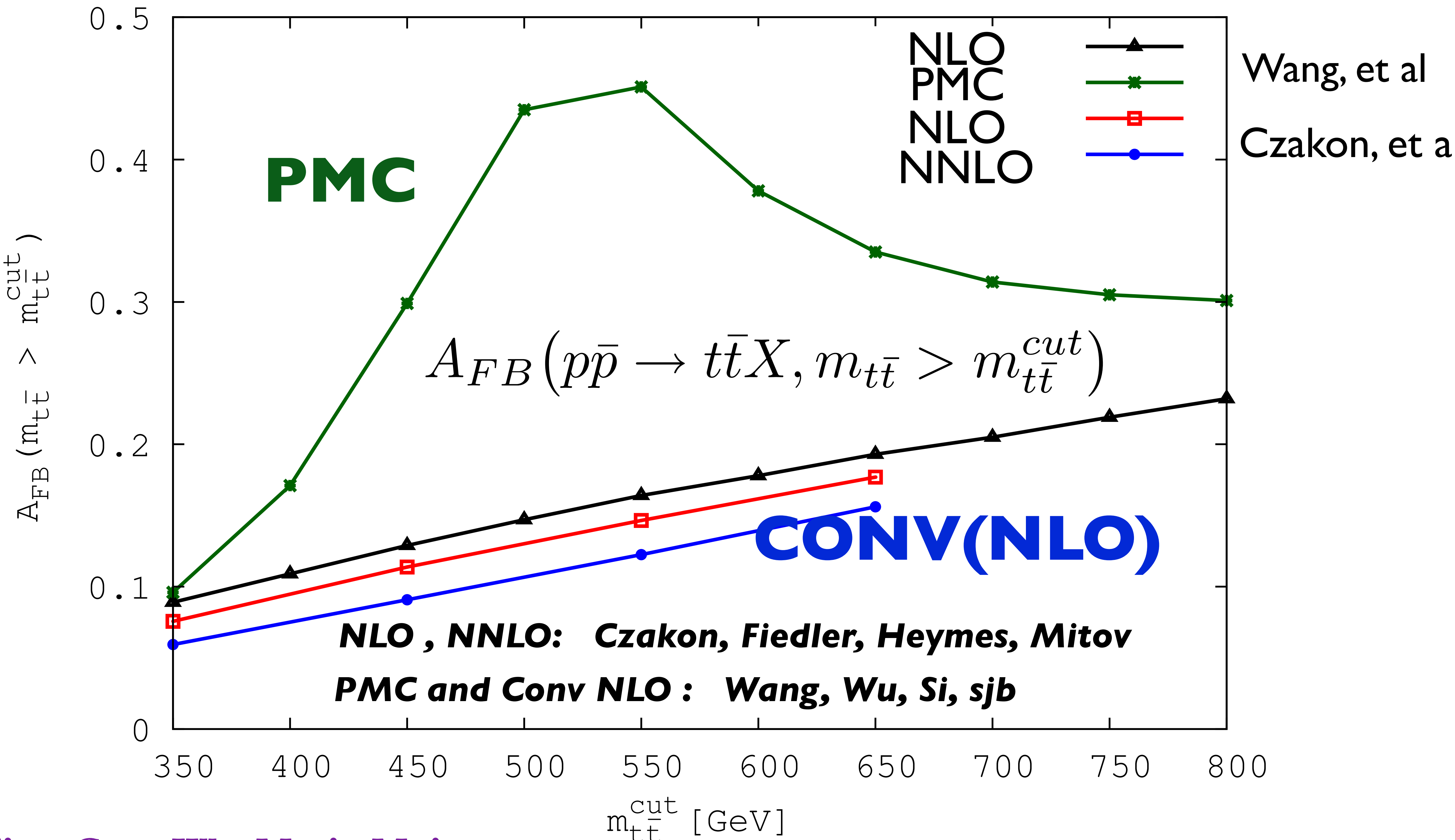
Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED!!

Xing-Gang Wu, sjb

NNLO QCD predictions for fully-differential top-quark pair production at the Tevatron arXiv:1601.05375

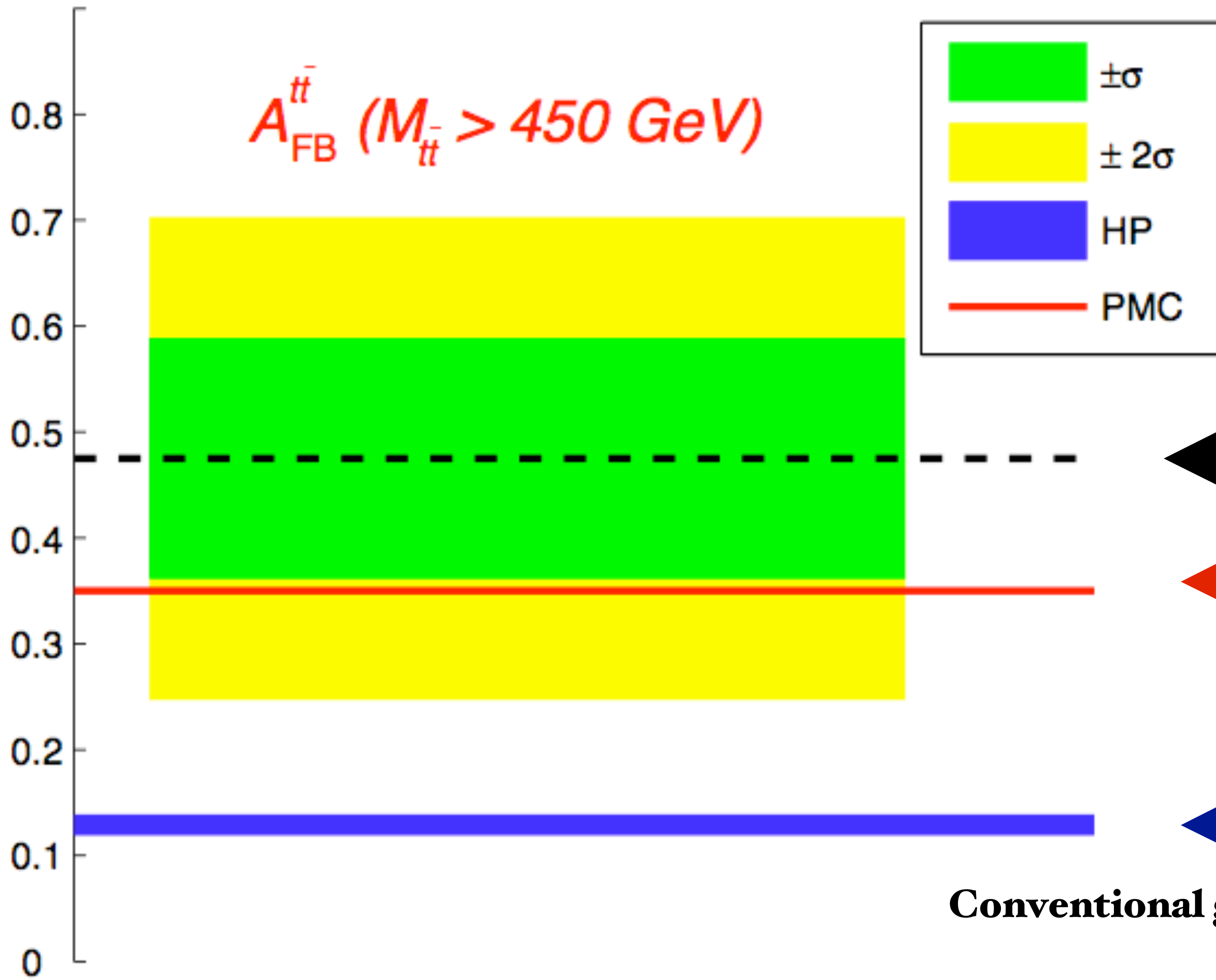
Michał Czakon,[a] Paul Fiedler,[a] David Heymes[b] and Alexander Mitov[b]



Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

Predictions for the cumulative front-back asymmetry.

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

PMC Prediction

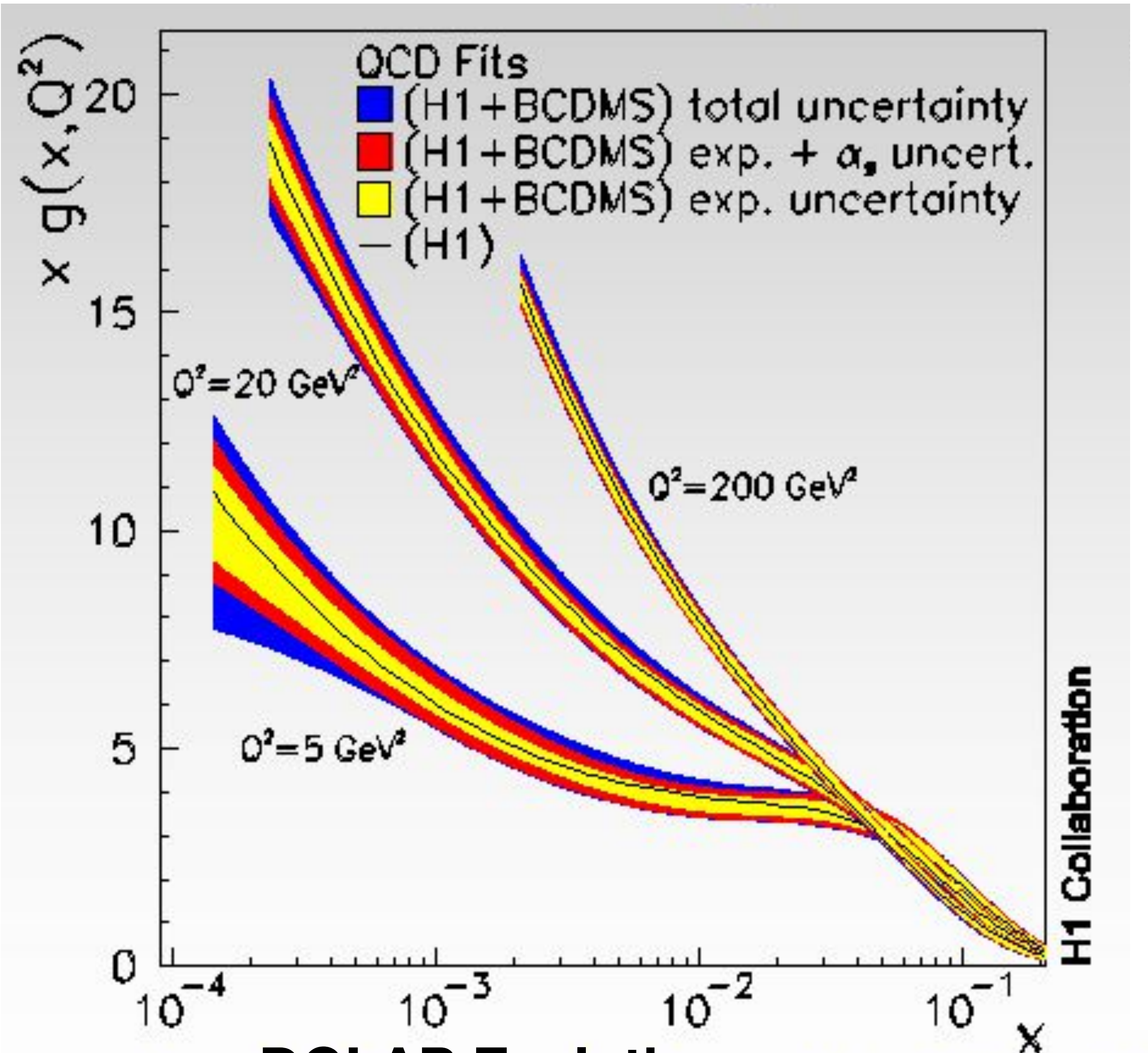
Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Features of BLM/PMC

- *Predictions are scheme-independent at every order*
- *Matches conformal series*
- *Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)*
- *No n! Renormalon growth (Lipatov)*
- *New scale appears at each order; n_F determined at each order - matches virtuality of quark loops*
- *Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)*
- *Rigorous: Satisfies all Renormalization Group Principles*
- *Realistic Estimate of Higher-Order Terms*
- *Same as Gell-Mann Low for QED* $N_C \to 0$
- *Eliminates unnecessary theory error*
- *Maximal sensitivity to new physics*
- *Example: BFKL intercept* (Fadin, Kim, Lipatov, Pivovarov, sjb)

The Remarkable Achievements of Lev Lipatov



DGLAP Evolution

Universidad Técnica Federico Santa María, Valparaiso, Chile
January 8-12, 2018

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY