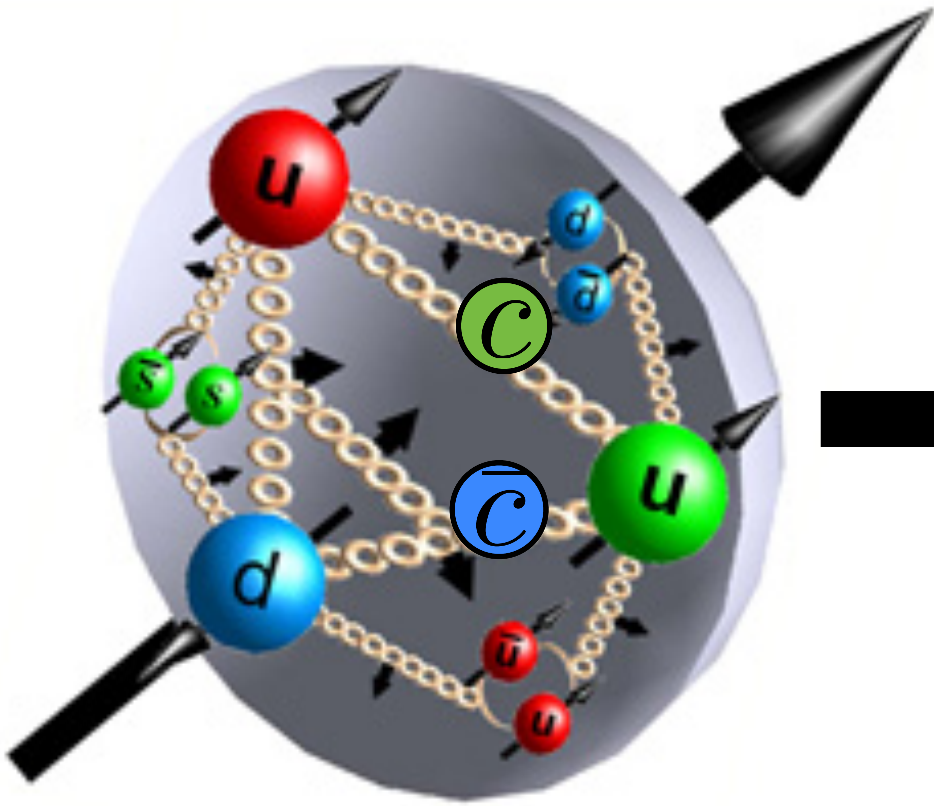
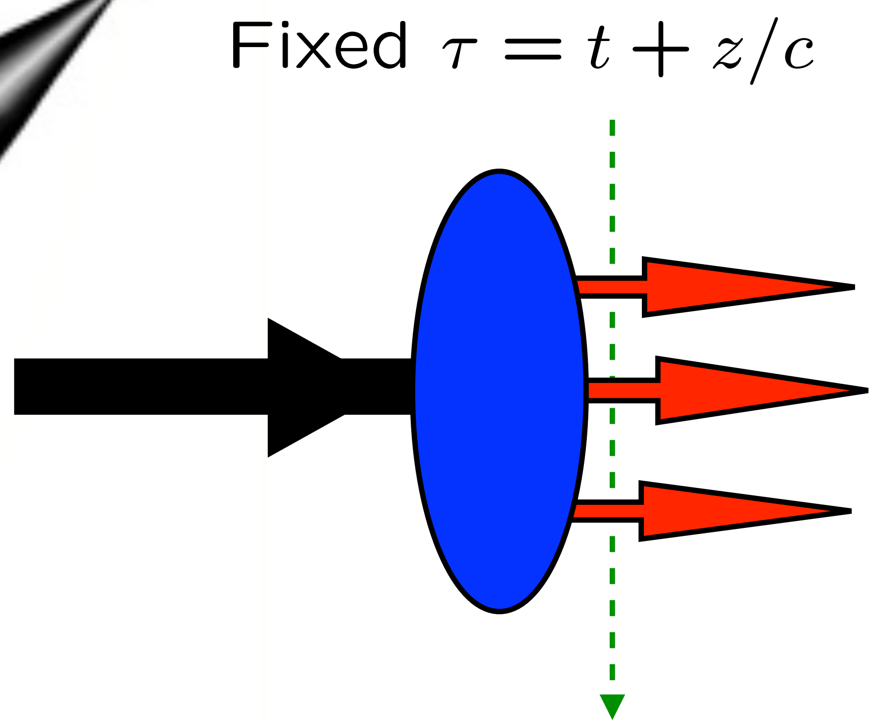
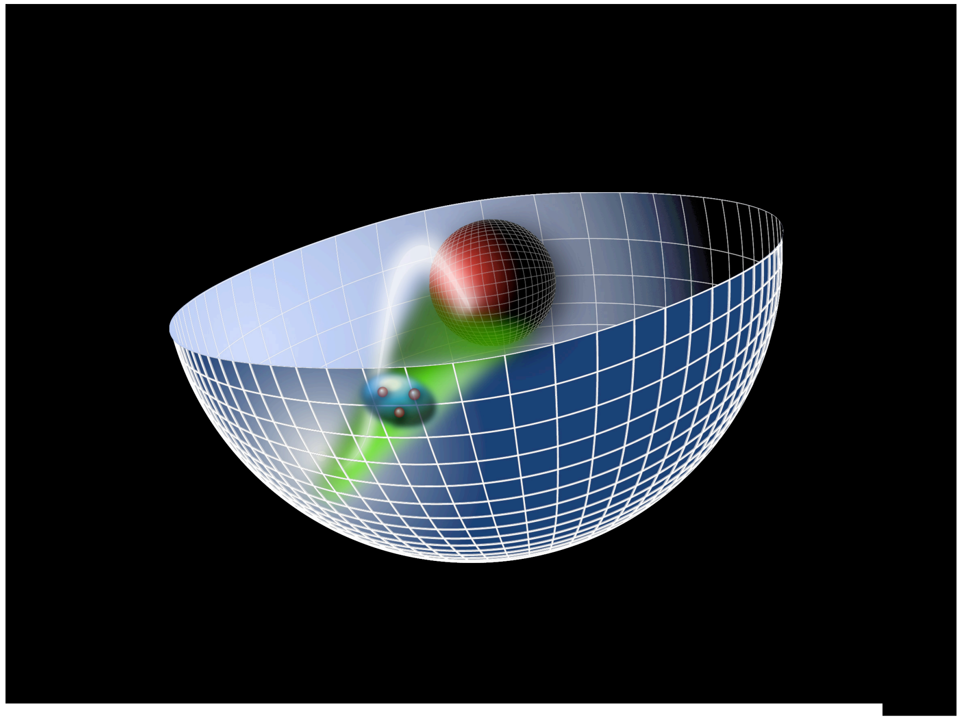
Supersymmetric Features of Hadron Physics and Properties of Quantum Chromodynamics from Light-Front Holography and Superconformal Algebra







Universidad Técnica Federico Santa María, Valparaiso, Chile
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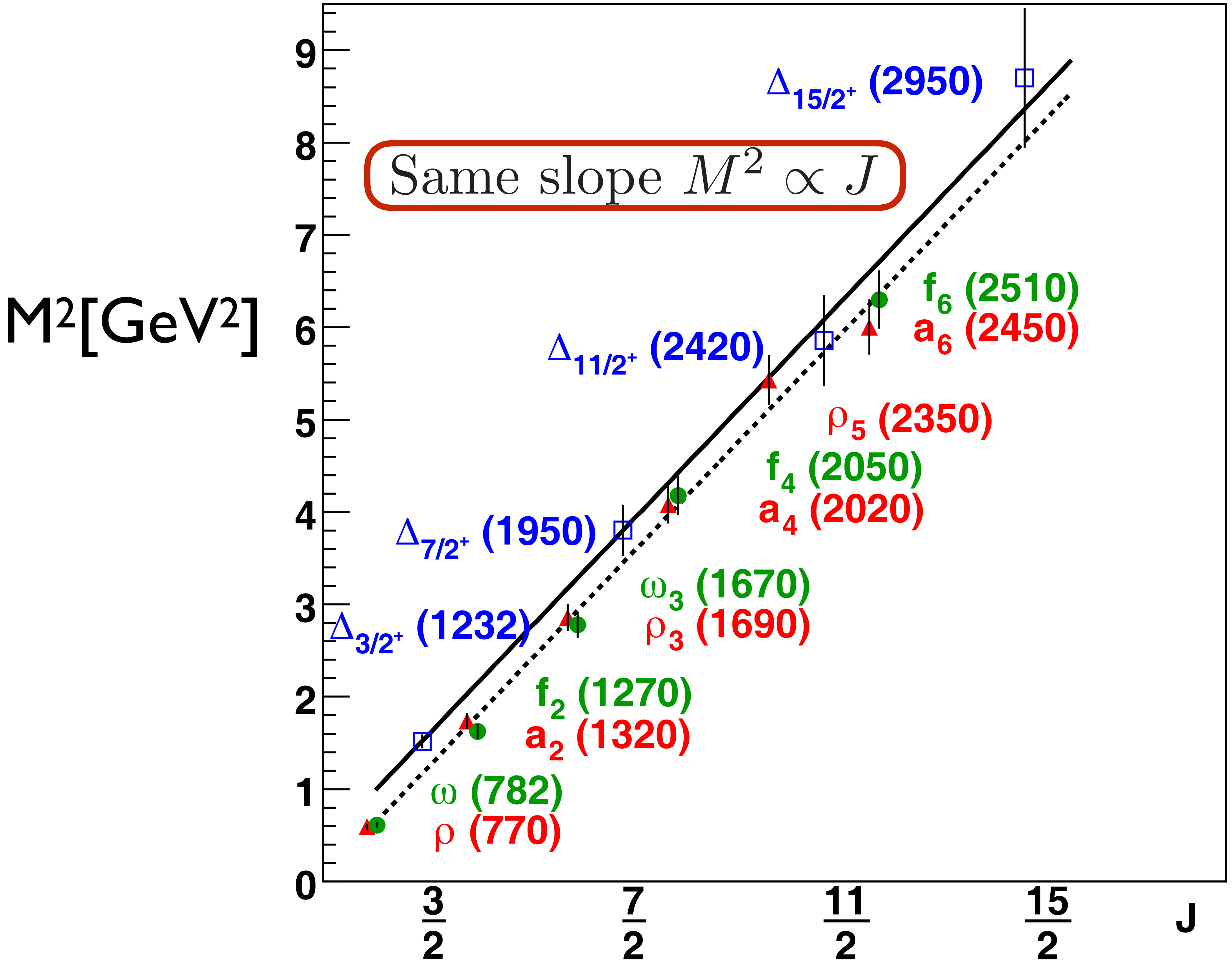
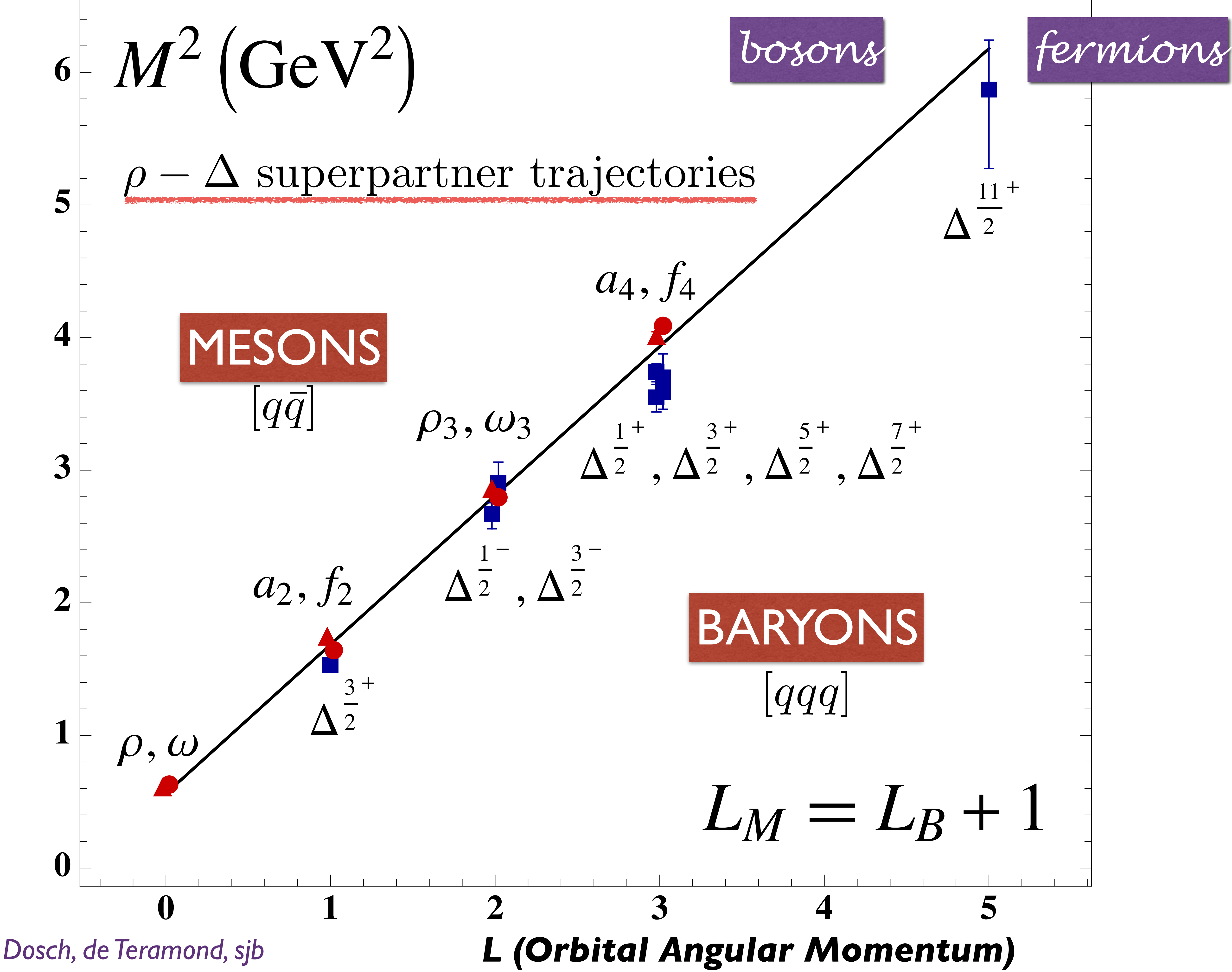
with Guy de Tèramond, Hans Günter Dosch, C. Lorcè, M. Nielsen, K. Chiu, R. S. Sufian, A. Deur

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY
LELAND STANFORD JUNIOR UNIVERSITY 1891



The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.



$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta\frac{3}{2}^+$
$\Delta\frac{1}{2}^-, \Delta\frac{3}{2}^-$
$\Delta\frac{1}{2}^+, \Delta\frac{3}{2}^+, \Delta\frac{5}{2}^+, \Delta\frac{7}{2}^+$
$\Delta\frac{11}{2}^+$
$L_M = L_B + 1$
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

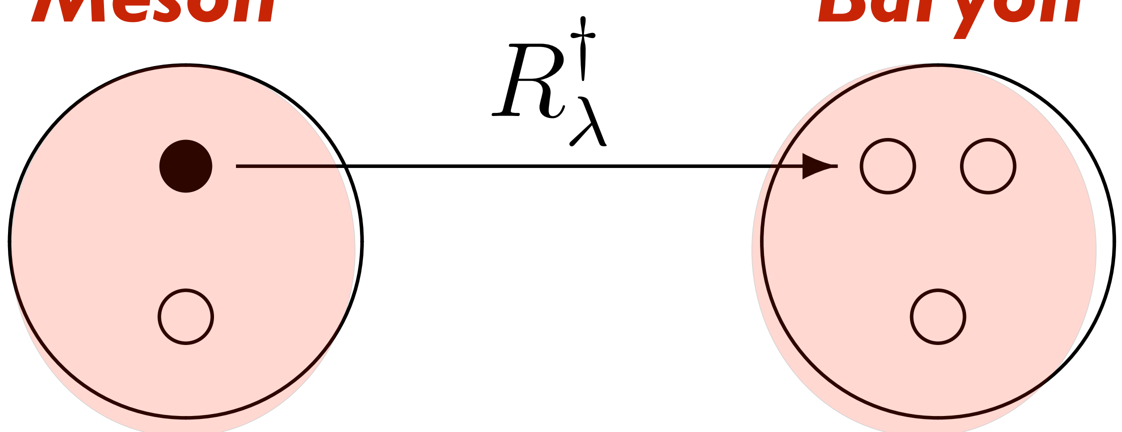
Guy de Tèramond, Hans Günter Dosch, sjb

Superconformal Algebra

2X2 Hadronic Multiplets: 4-Plet

Bosons, Fermions with Equal Mass!

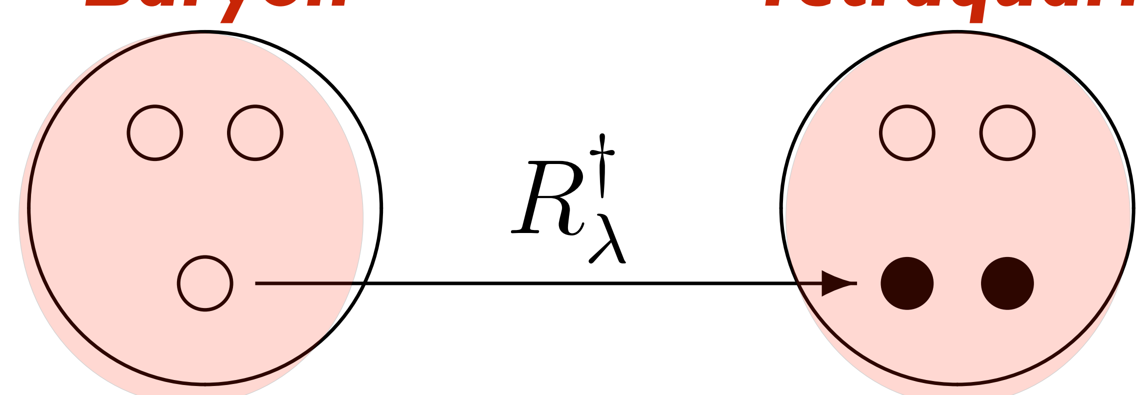
Meson **Baryon**



$\phi_M, \; L_B + 1$ $\psi_{B+}, \; L_B$

$$R^\dagger_\lambda \;\bar q \to [qq]$$
$$\bar 3_C \to \bar 3_C$$

Baryon **Tetraquark**



$\psi_{B-}, \; L_B + 1$ $\phi_T, \; L_B$

$$R^\dagger_\lambda \;q \to [\bar q \bar q]$$
$$3_C \to 3_C$$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$\sigma(500)$
$\bar{q}q$	2^{-+}	$\pi_2(1670),\ \eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	—
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
							1^{+-}	$b_1(1235)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar{u}\bar{d}]$	2^{--}	—
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar{u}\bar{d}]$	3^{++}	—
$\bar{q}s$	0^-	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	1^+	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^+	$K_0^*(1430)$
$\bar{q}s$	2^-	$K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][\bar{s}\bar{q}]$	1^-	—
$\bar{s}q$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	1^-	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	2^+	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar{u}\bar{d}]$	1^+	$K_1(1400)$
$\bar{s}q$	3^-	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar{u}\bar{d}]$	2^-	—
$\bar{s}q$	4^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar{u}\bar{d}]$	3^+	—
$\bar{s}s$	0^{-+}	$\eta(550),\ \eta'(958)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	—
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
								$a_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar{s}\bar{q}]$	2^{--}	—
$\bar{s}s$	2^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar{s}\bar{q}]$	1^+	$K_1(1650)$

New Organization of the Hadron Spectrum

M. Nielsen

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

$$P^\pm = P^0 \pm P^z$$

Eigenstate -- independent of τ

$$\text{Eigenvalue } P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

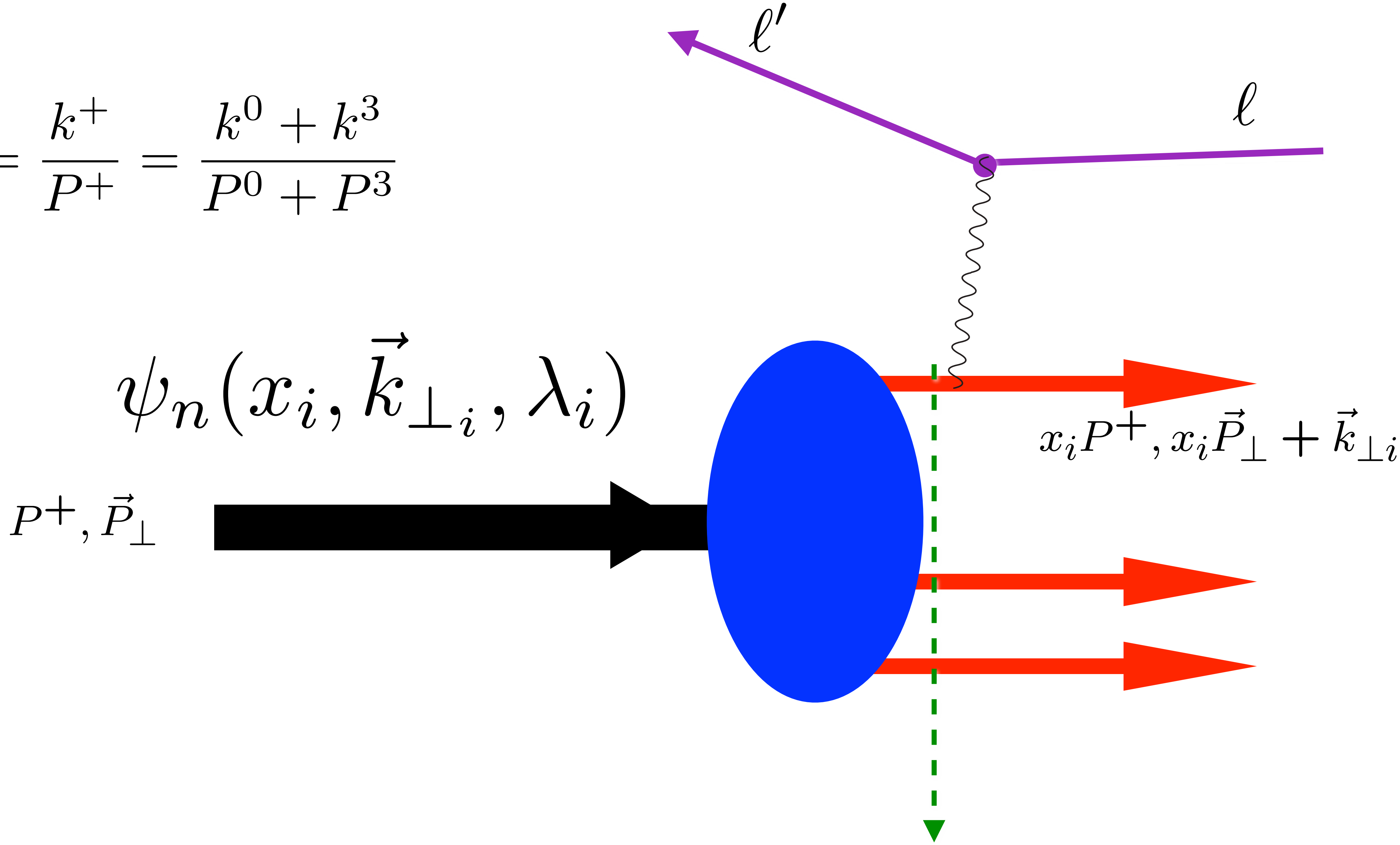
$$H_{LF} = P^\mu P_\mu = P^+ P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$



Measurements of hadron LF wavefunction are at fixed LF time

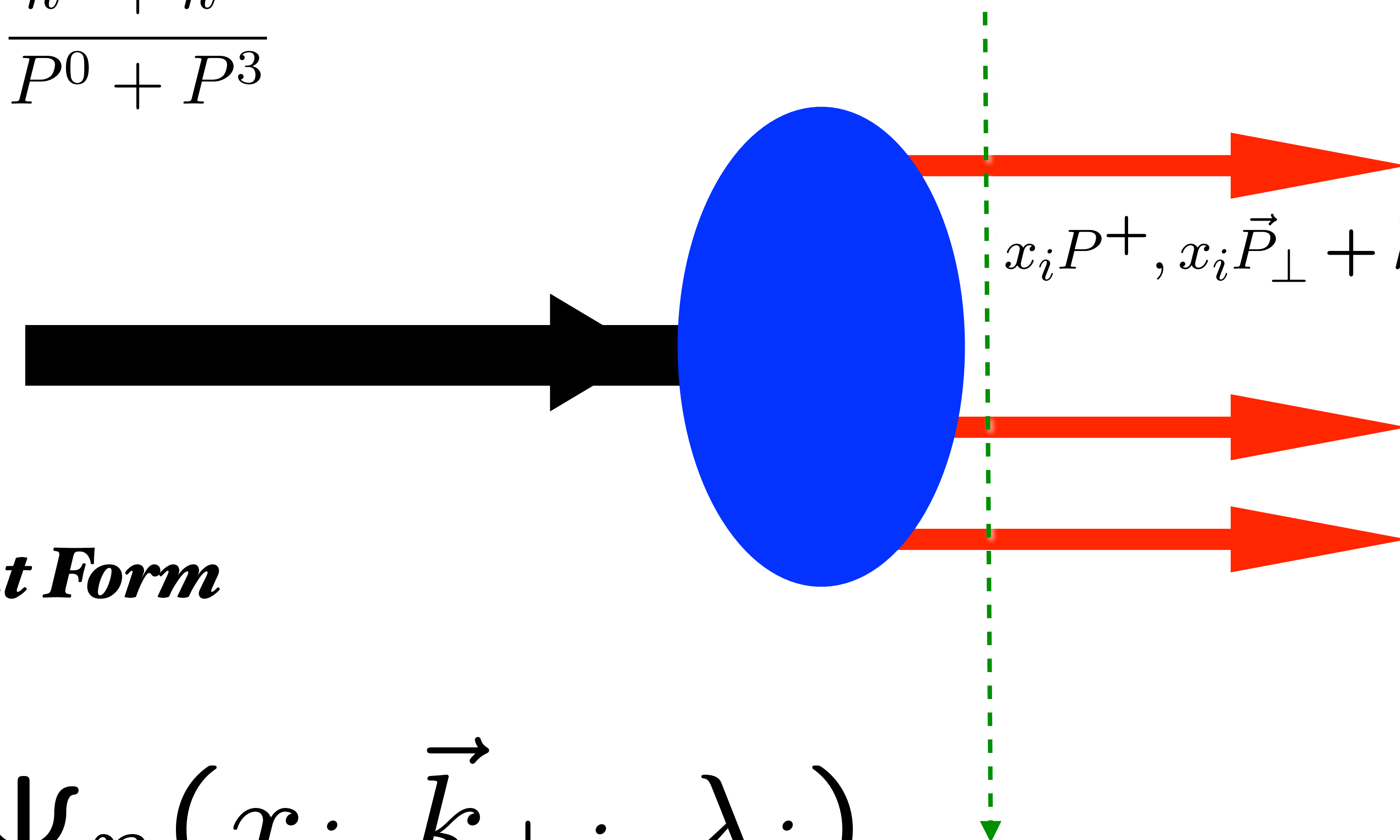
Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

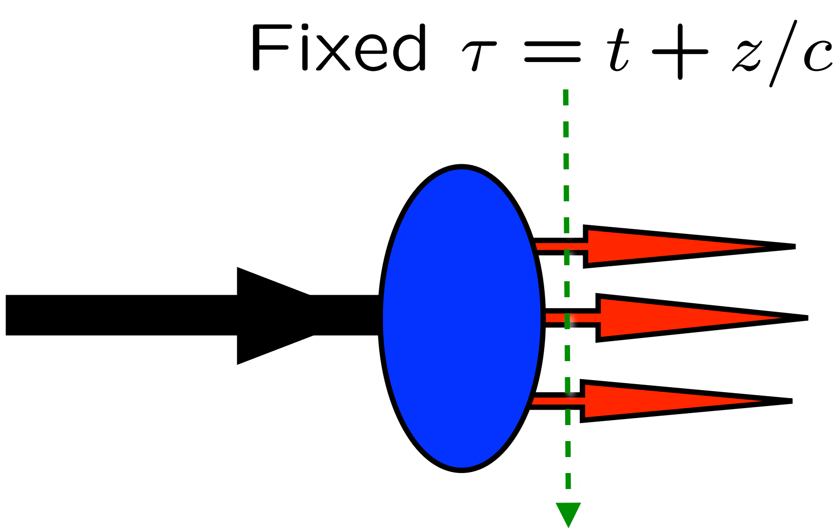
Invariant under boosts! Independent of P^μ

Causal, Frame-independent, Simple Vacuum, Current Matrix Elements are overlap of LFWFS

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{H^{QCD}_{LF} |\psi> = M^2 |\psi>}$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$\boxed{H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>}$$

$$|p,S_z> = \sum_{n=3}\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

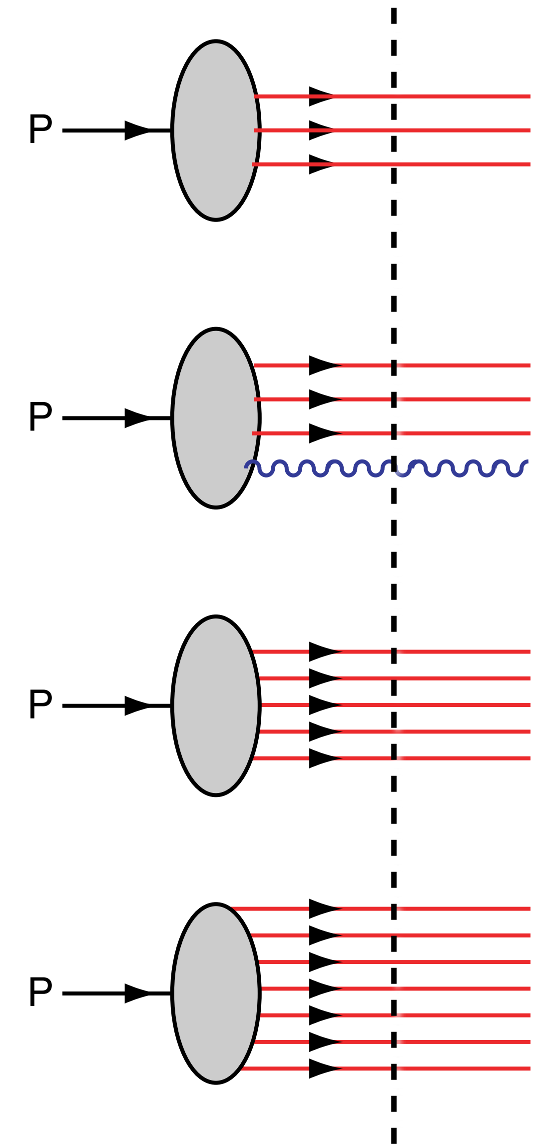
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

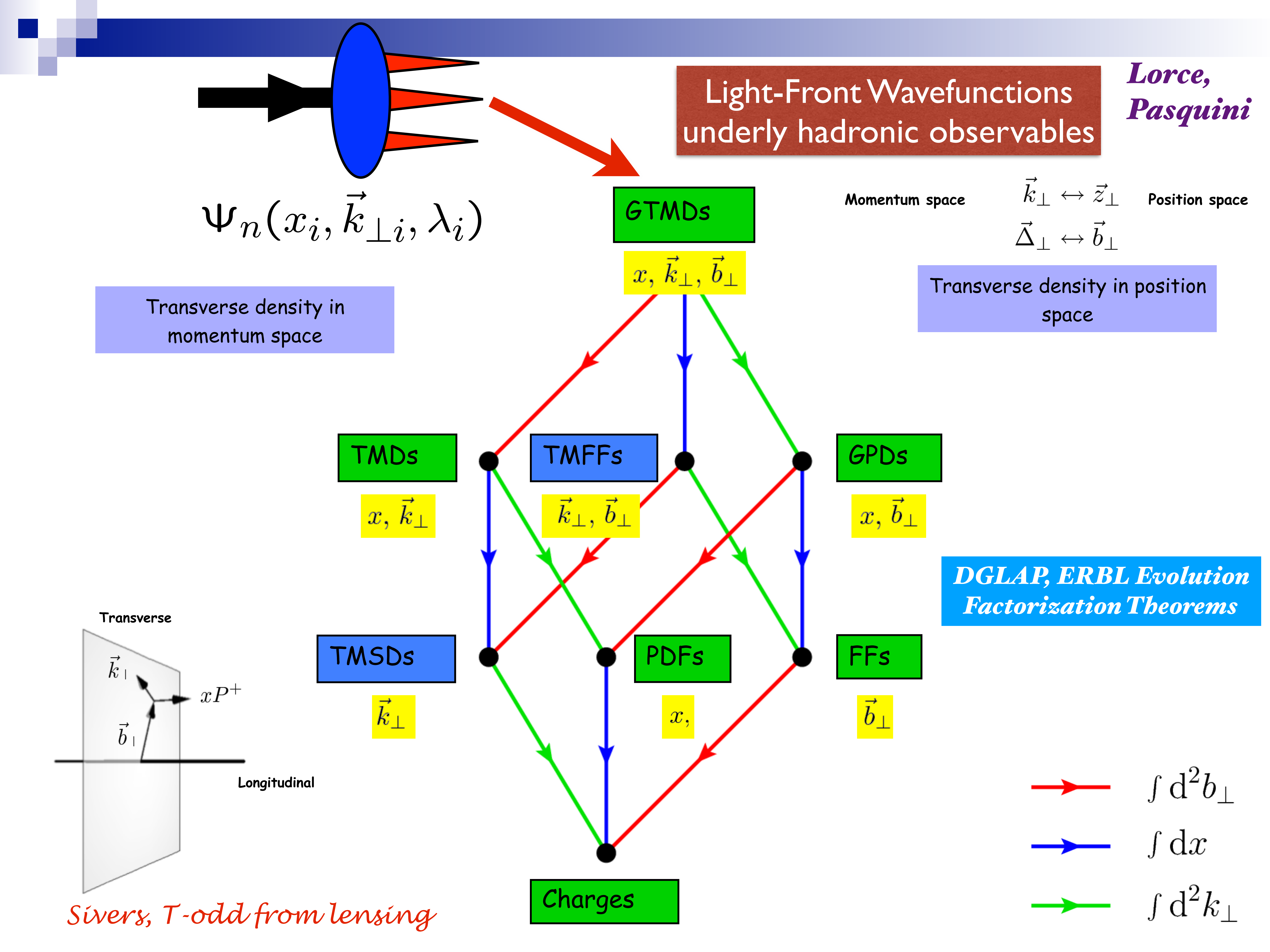


Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Hidden Color


Light-Front Wavefunctions underly hadronic observables
Lorce, Pasquini
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
Momentum space
$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$
Position space
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
GTMDs
$x,\, \vec{k}_\perp,\, \vec{b}_\perp$
Transverse density in momentum space
Transverse density in position space
TMDs
$x,\, \vec{k}_\perp$
TMFFs
$\vec{k}_\perp,\, \vec{b}_\perp$
GPDs
$x,\, \vec{b}_\perp$
DGLAP, ERBL Evolution
Factorization Theorems
TMSDs
$\vec{k}_\perp$
PDFs
$x,$
FFs
$\vec{b}_\perp$
Charges
$\int \mathrm{d}^2 b_\perp$
$\int \mathrm{d}x$
$\int \mathrm{d}^2 k_\perp$
Transverse
$\vec{k}_\perp$
xP^+
$\vec{b}_\perp$
Longitudinal
Sivers, T-odd from lensing

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

Physics Independent of Observer's Motion

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**

Penrose, Terrell, Weisskopf

- **Same structure function measured at an e p collider and the proton rest frame**
- **No dependence of hadron structure on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no vacuum condensates!**
- **Profound implications for Cosmological Constant**

Roberts, Shrock, Tandy, sjb

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

- *"History": Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!*
- *Wick Theorem applies, but few amplitudes since all $k^+ > 0$.*
- *J_z Conservation at every vertex* $\boxed{\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n}$ *at order g^n*

  K. Chiu, sjb
- *Unitarity is explicit*
- *Loop Integrals are 3-dimensional* $\int_0^1 dx \int d^2 k_\perp$
- *hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

$$\boxed{H_{QED}}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\boxed{[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)}$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

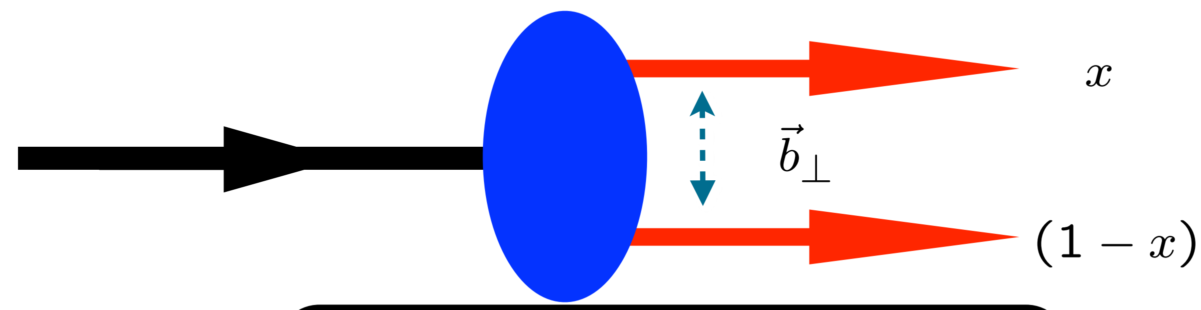
Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

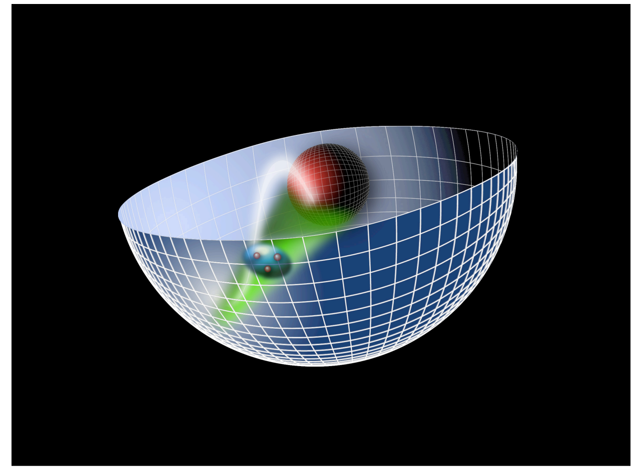
Sums an infinite # diagrams

Semiclassical first approximation to QCD

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\boxed{\zeta^2 = x(1-x)\mathrm{b}_\perp^2.}$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

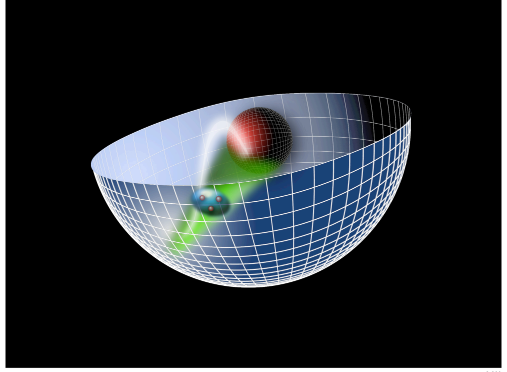
Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

AdS$_5$



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

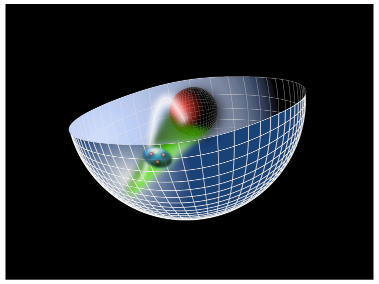
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu}x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

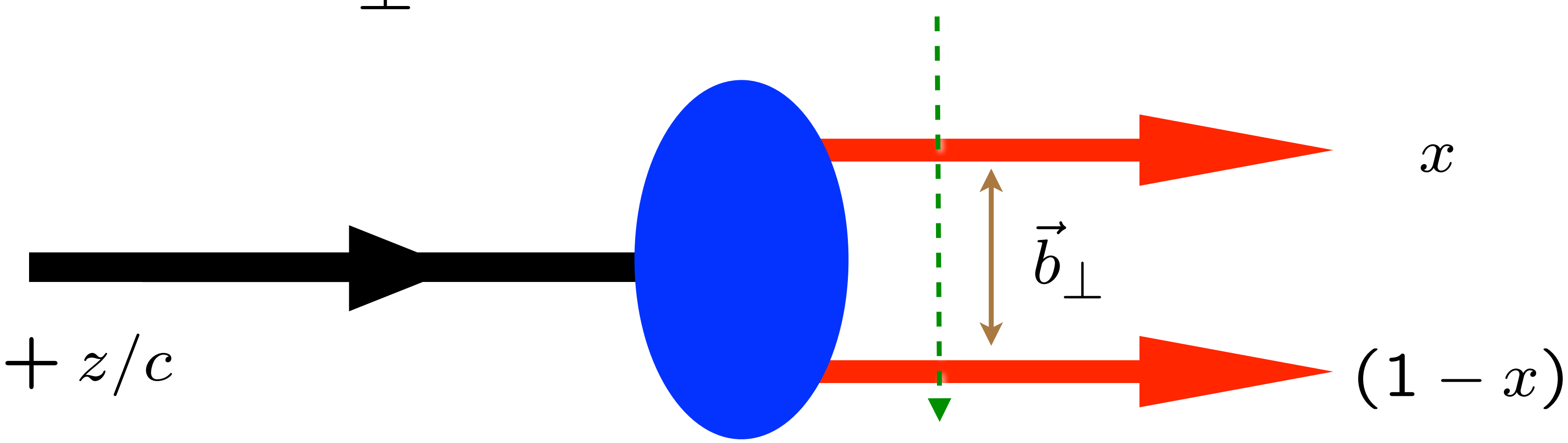
de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS$_5$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Meson Spectrum in Soft Wall Model

Massless pion!

$$\boxed{m_\pi = 0 \text{ if } m_q = 0}$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

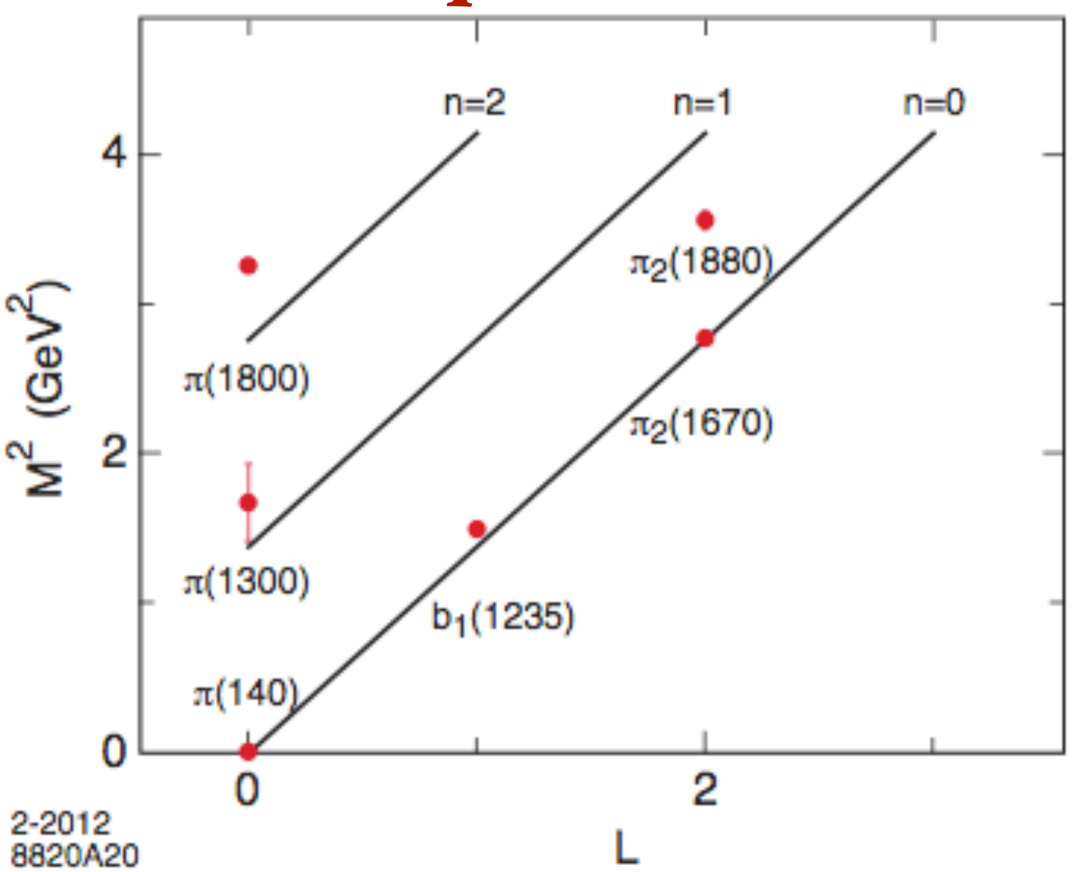
G. de Teramond, H. G. Dosch, sjb

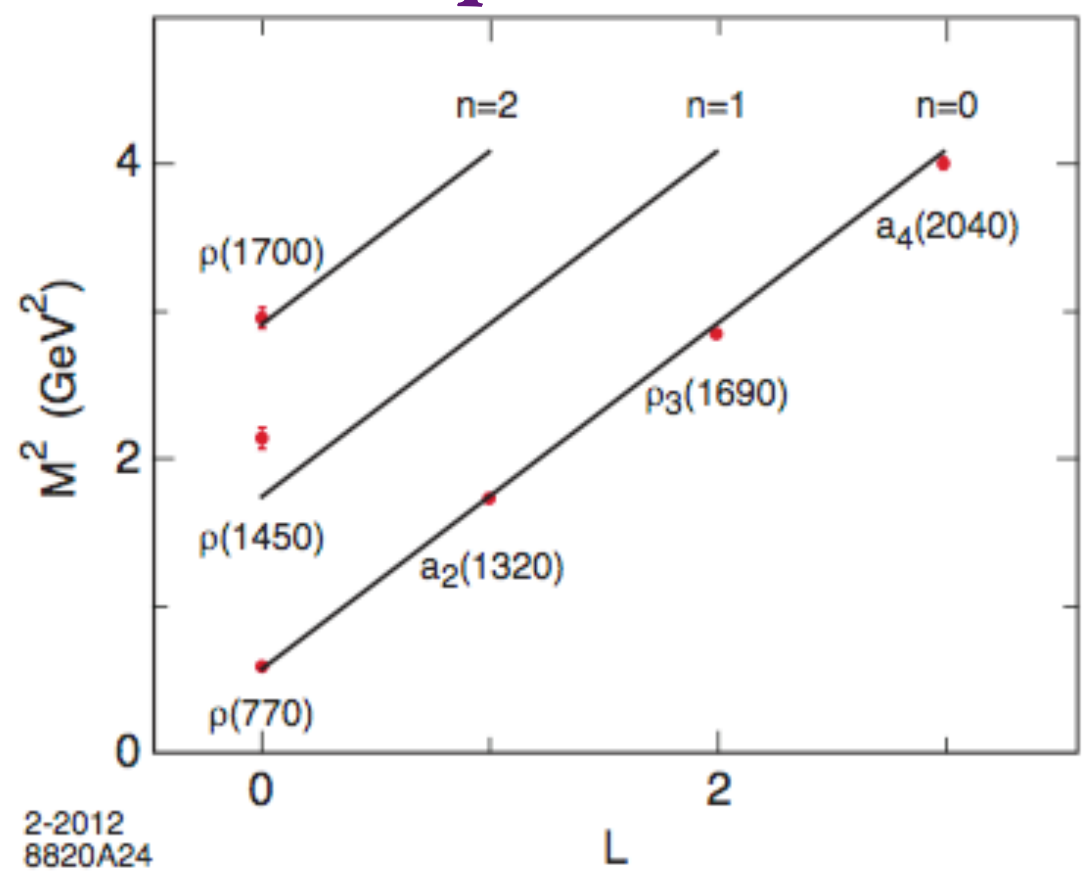
- $J = L + S$, $I = 1$ meson families $\boxed{\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)}$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$\boxed{m_q = 0}$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

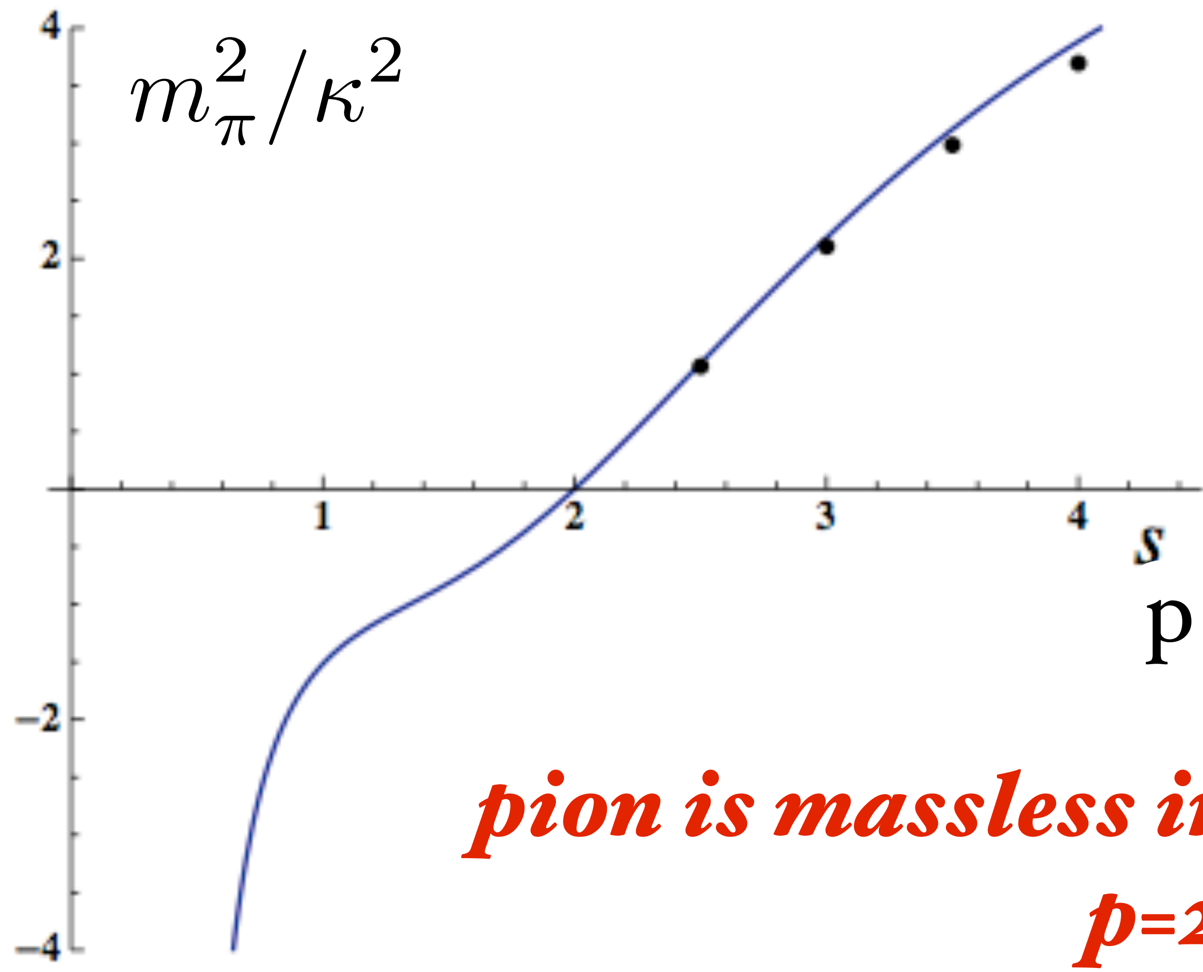
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$

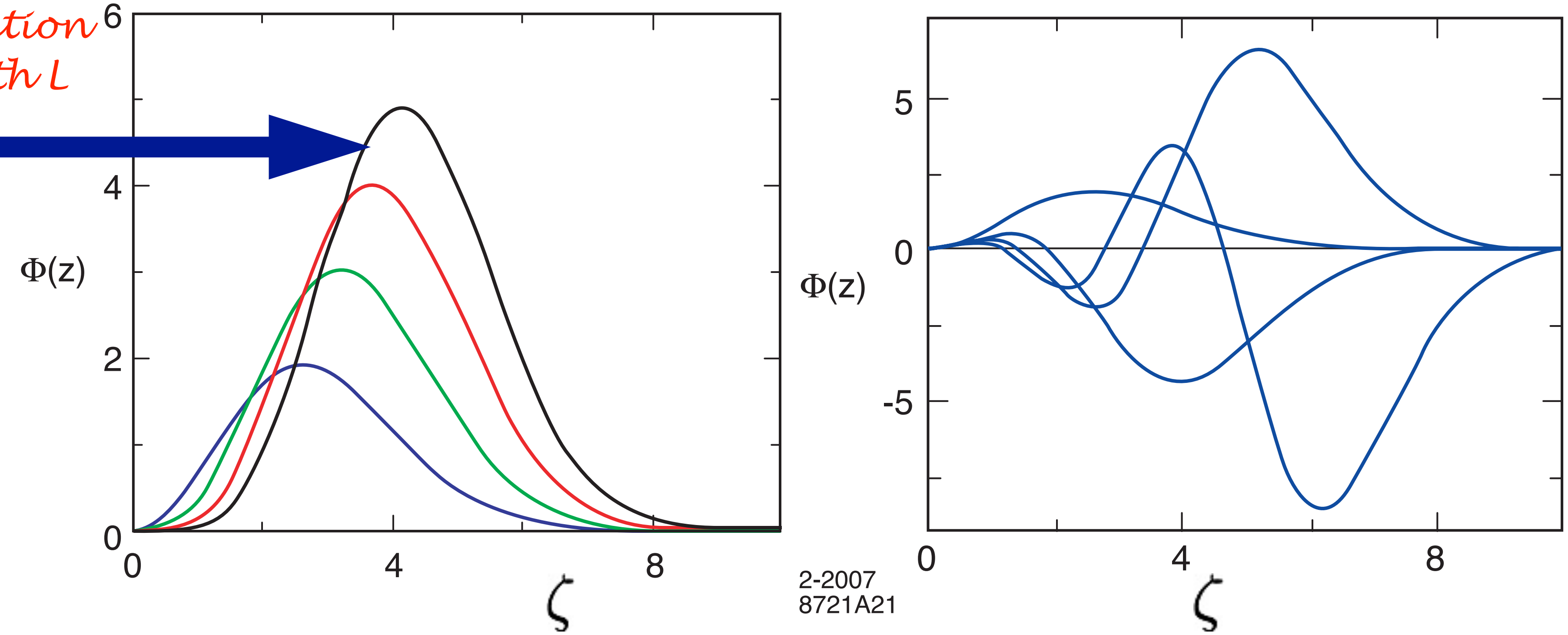


pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

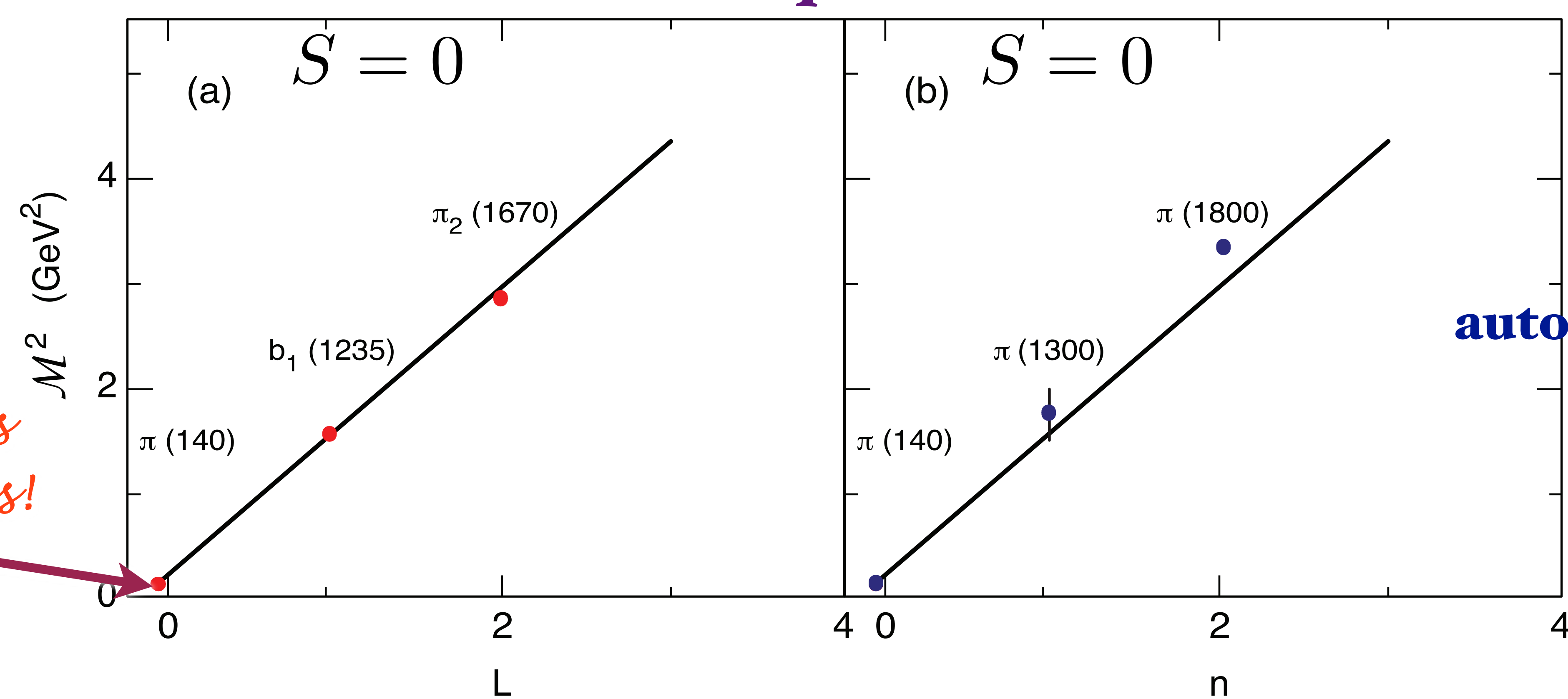
Quark separation increases with L



Soft Wall Model

Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!

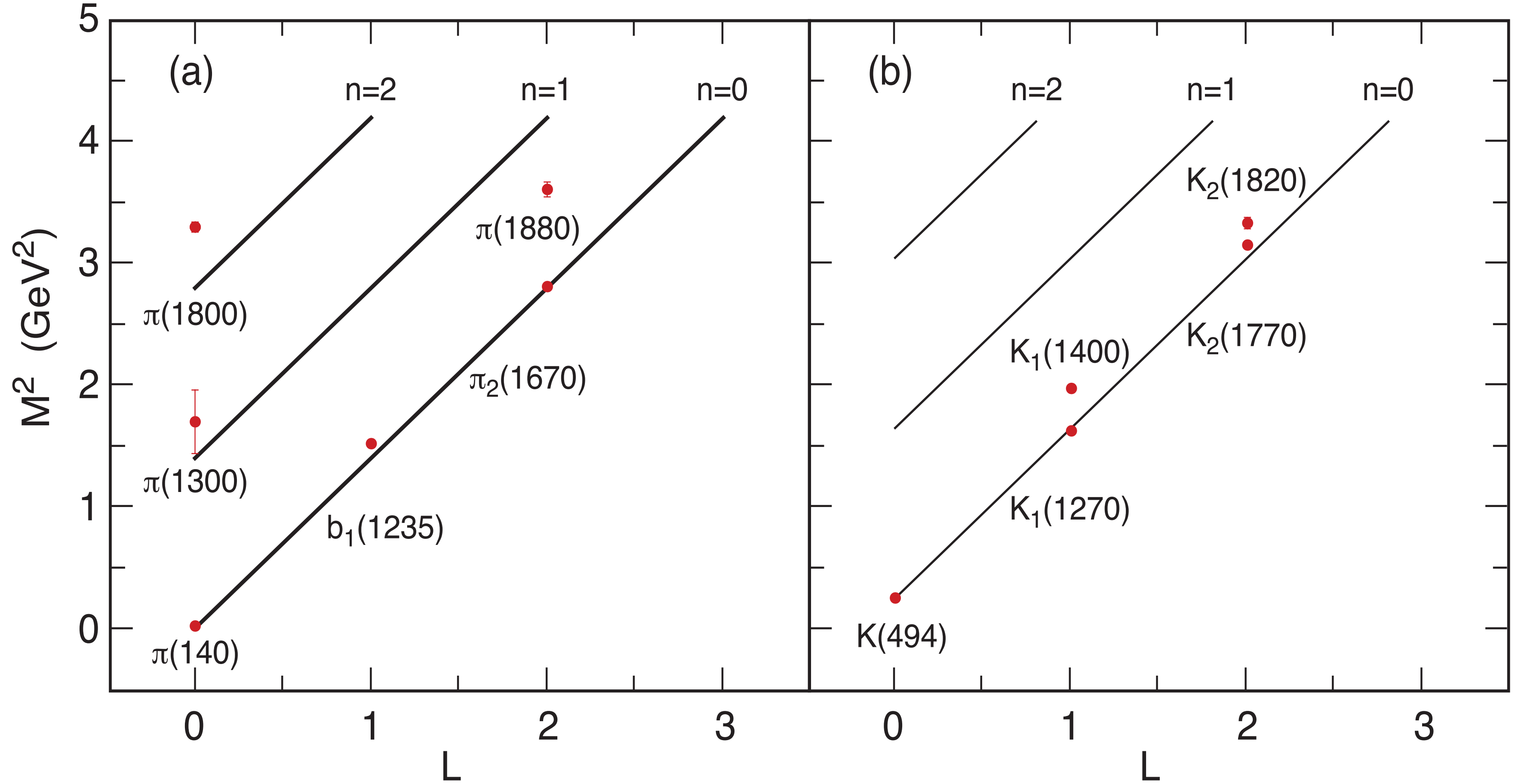


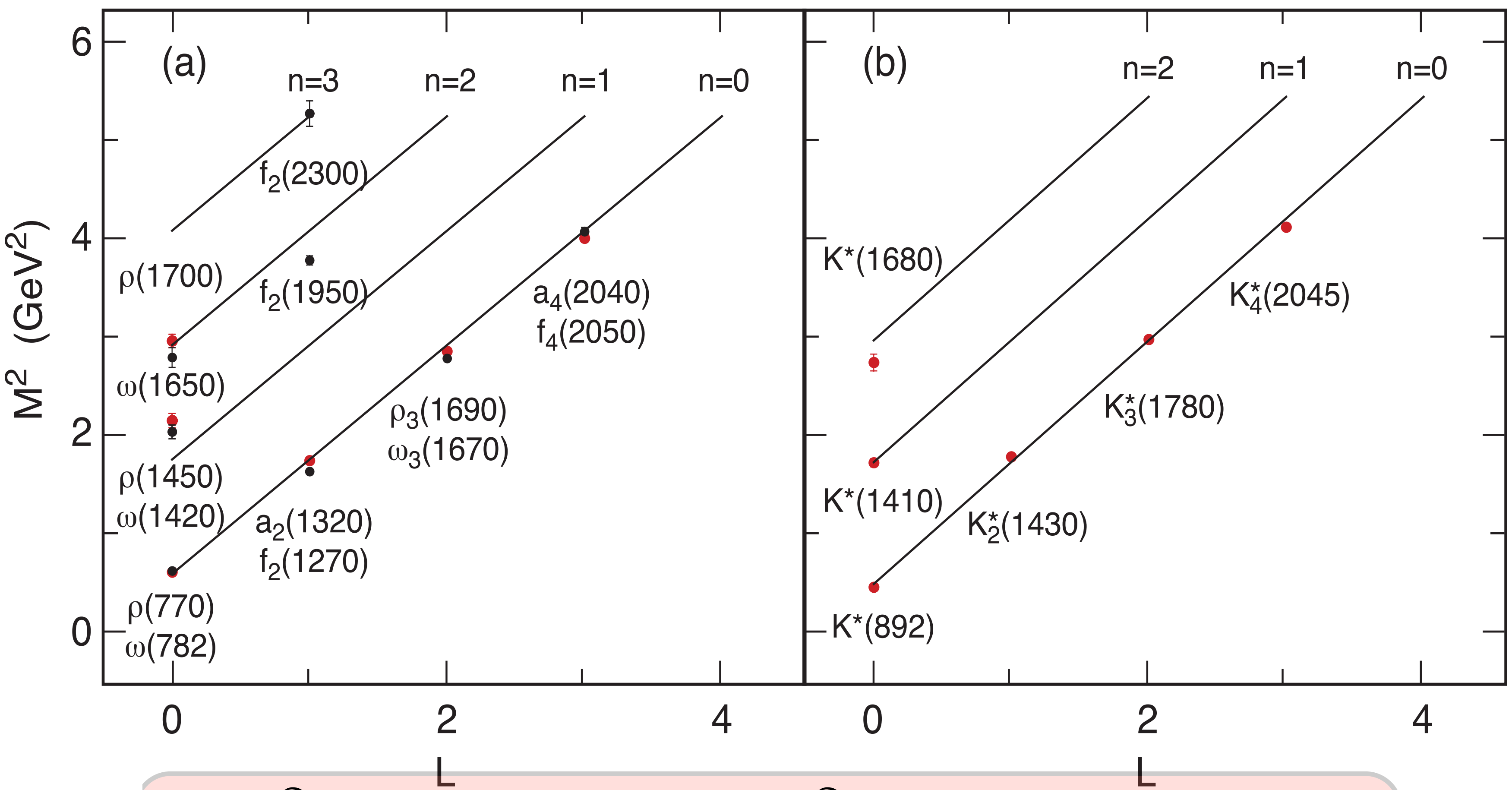
Pion has zero mass!

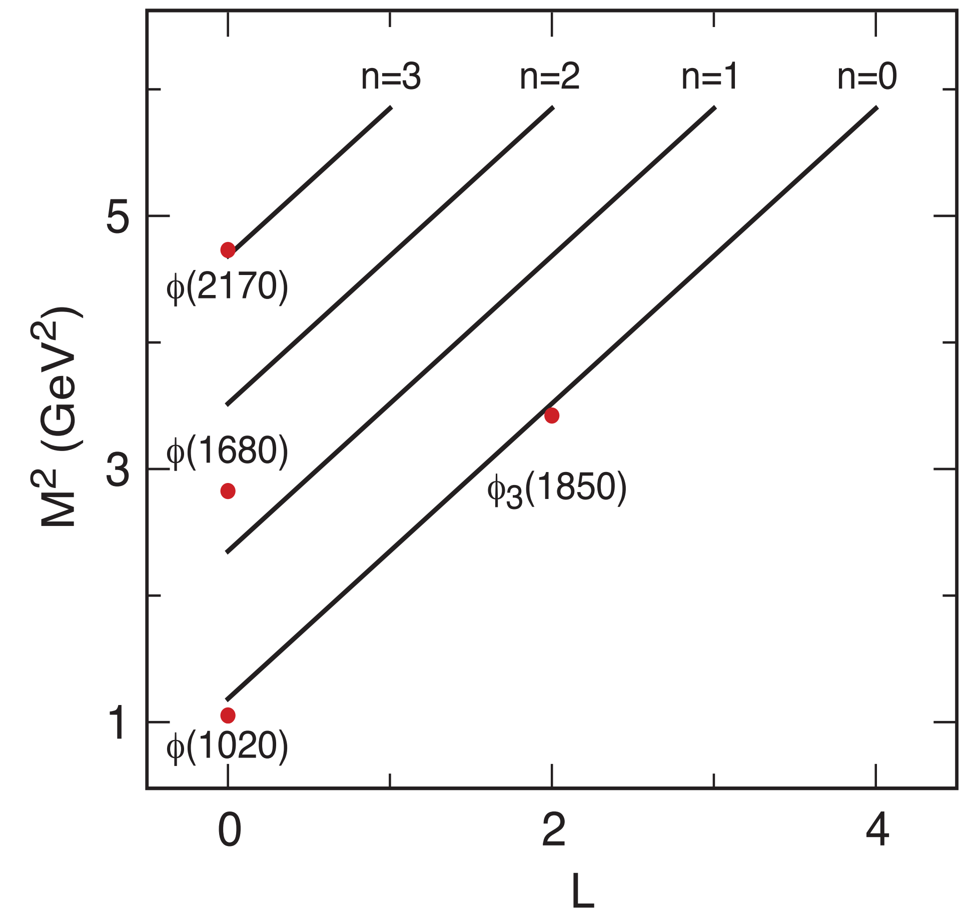
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

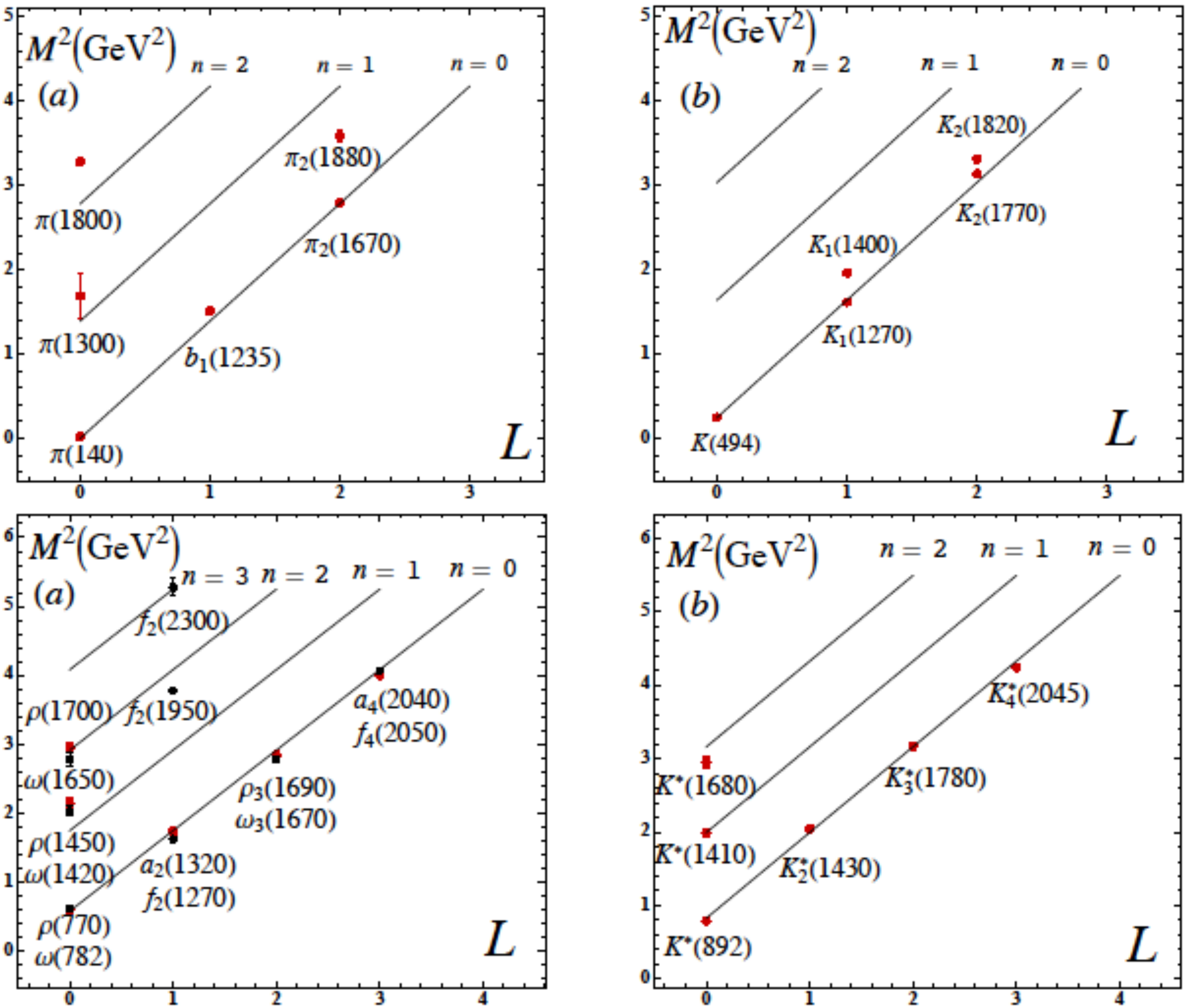
Equal Slope in n and L

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



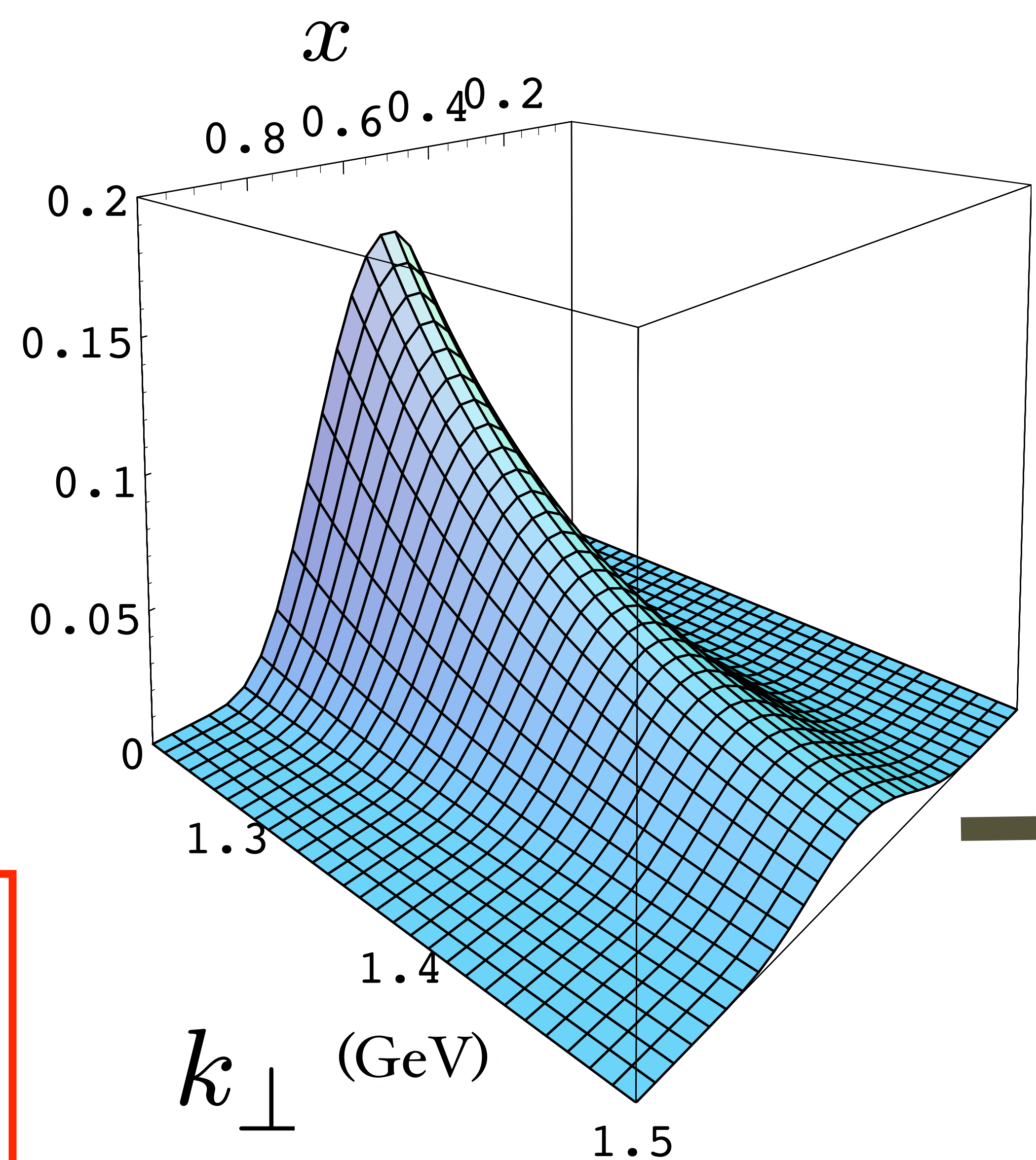
Effective mass from $m(p^2)$

Tandy, Roberts, et al

Prediction from AdS/QCD: Meson LFWF

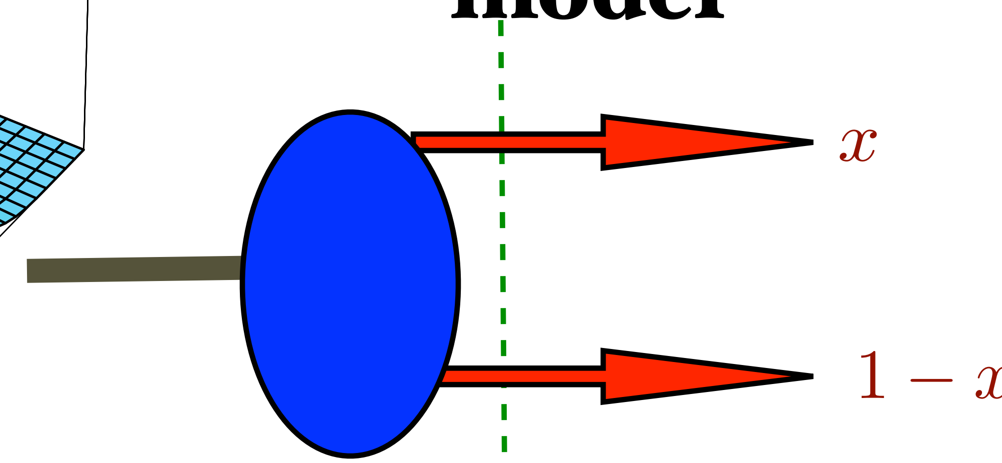
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

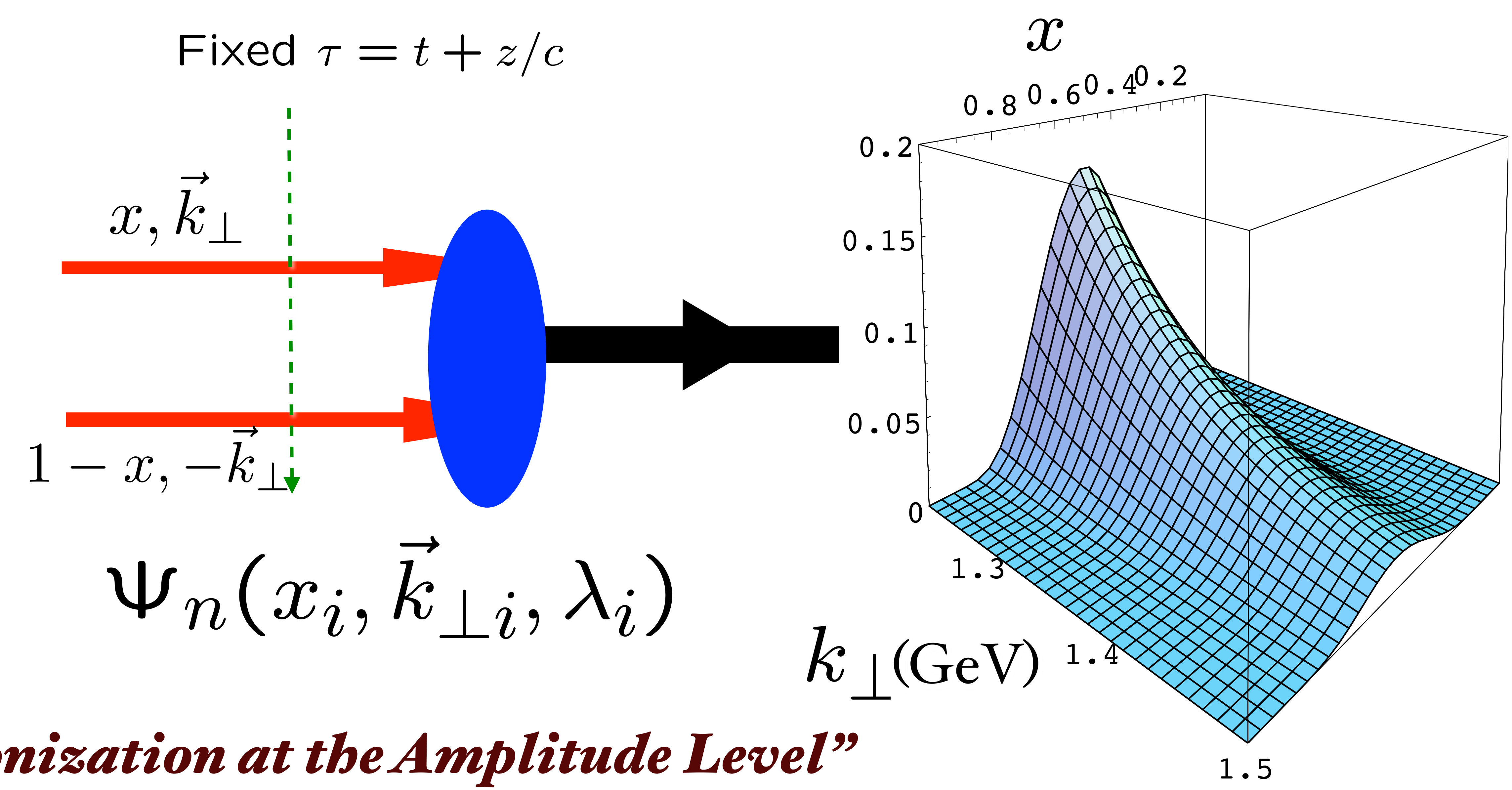
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

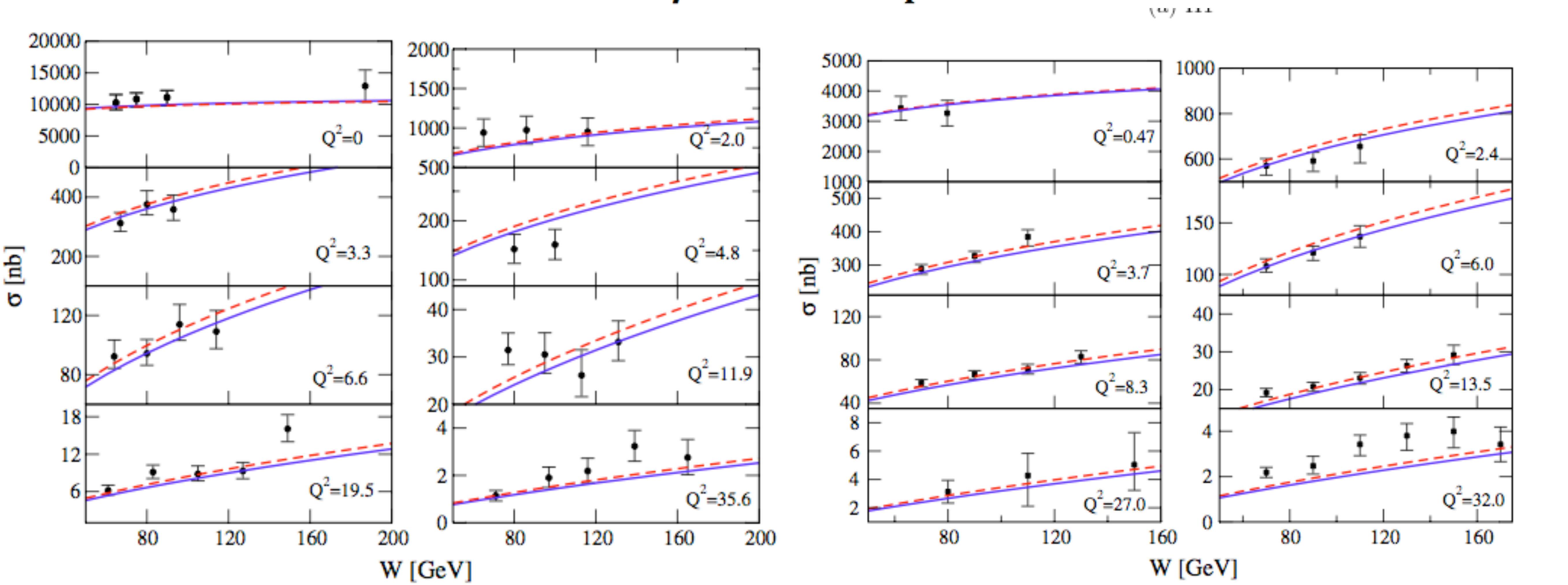
off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

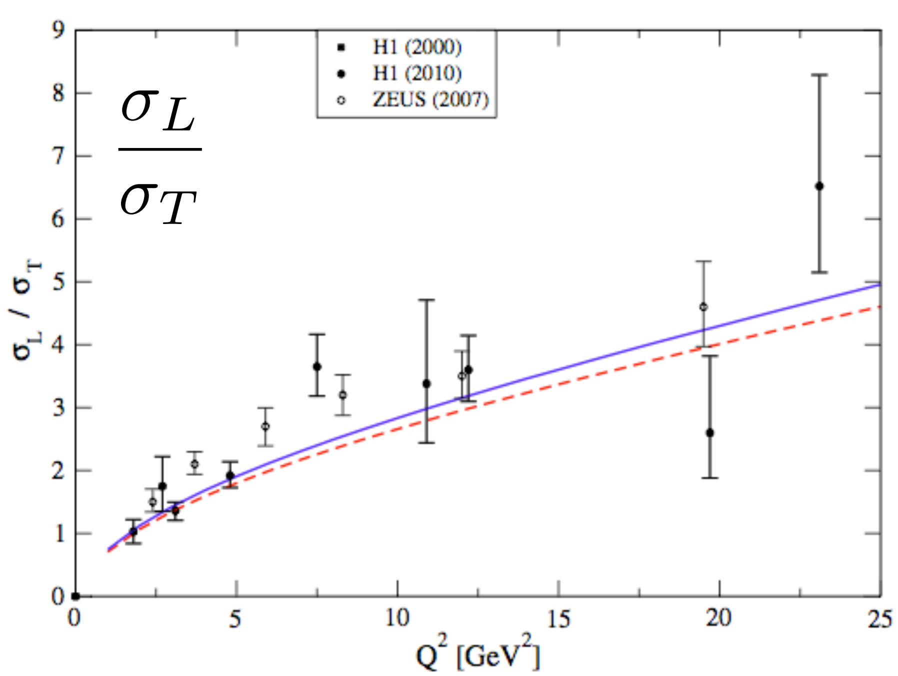
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



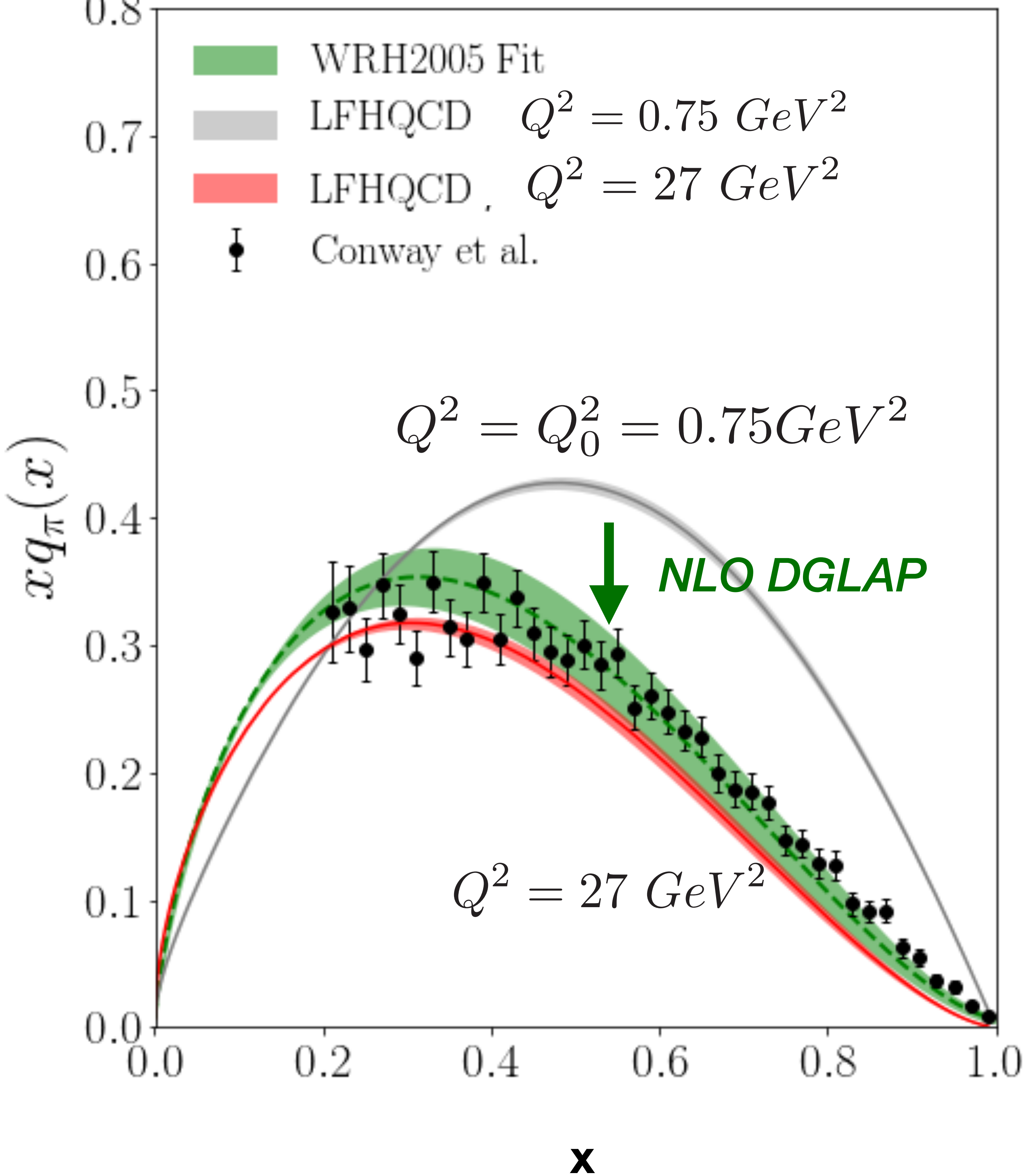
(a) H1 (b) ZEUS

J. R. Forshaw,
R. Sandapen

$$\gamma^* p \rightarrow \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$



T. Liu,
G. de Tèramond,
G. Dosch, A. Deur,
R.S. Sufian, sjb
(preliminary)

$$q_\pi(x, Q^2 < Q_0^2) = \int d^2\vec{k}_\perp |\psi_\pi(x, \vec{k}_\perp)|^2$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

"No parameters"

Start DGLAP evolution at transition scale Q^2_0

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f} \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau) >= i\frac{\partial}{\partial \tau}|\psi(\tau) >$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**

- **Finite range**

- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[\mathrm{H},\mathrm{D}] = \mathrm{i}\ \mathrm{H}, \quad [\mathrm{H}, \mathrm{K}] = 2\ \mathrm{i}\ \mathrm{D}, \quad [\mathrm{K}, \mathrm{D}] = -\ \mathrm{i}\ \mathrm{K}$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

$\boxed{\text{Consider } R_w = Q + wS;}$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$G_{22} = \Big(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$\boxed{\text{Identify } f - \tfrac{1}{2} = L_B \ , \quad w = \kappa^2}$$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

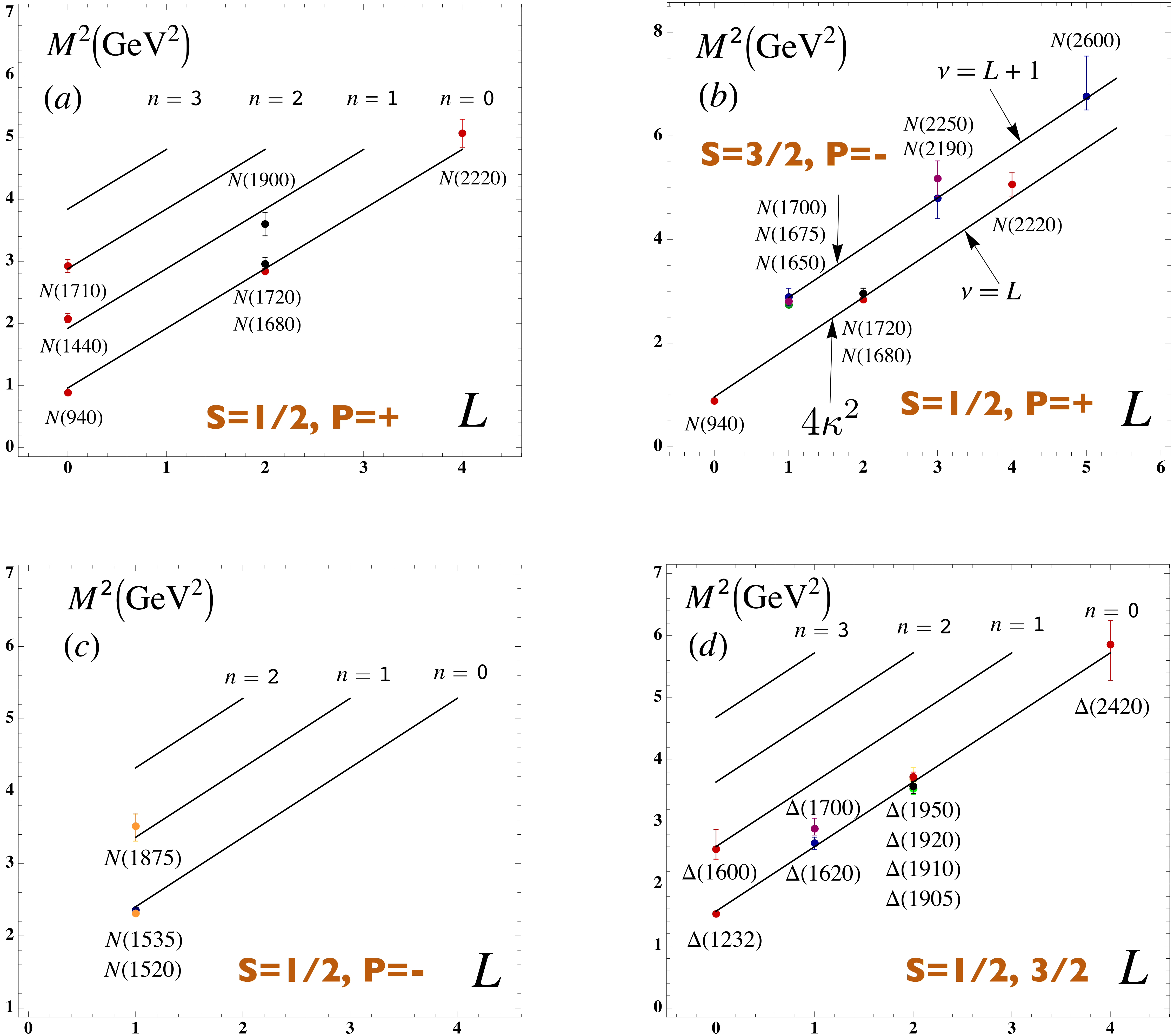
S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$



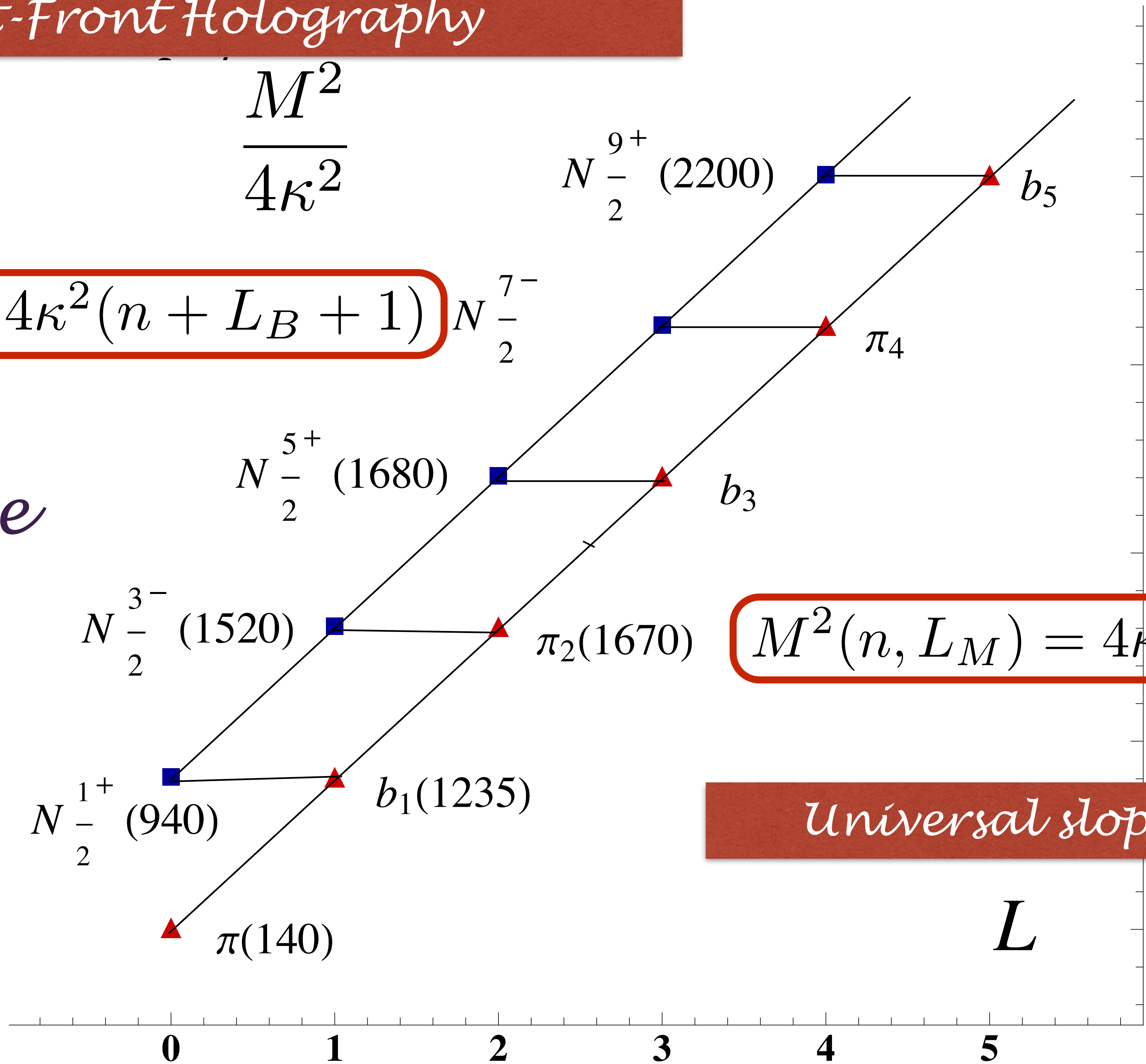
$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1720)
N(1680)
N(1440)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
ν = L + 1
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Superconformal Quantum Mechanics Light-Front Holography

de Tèramond, Dosch, Lorce, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

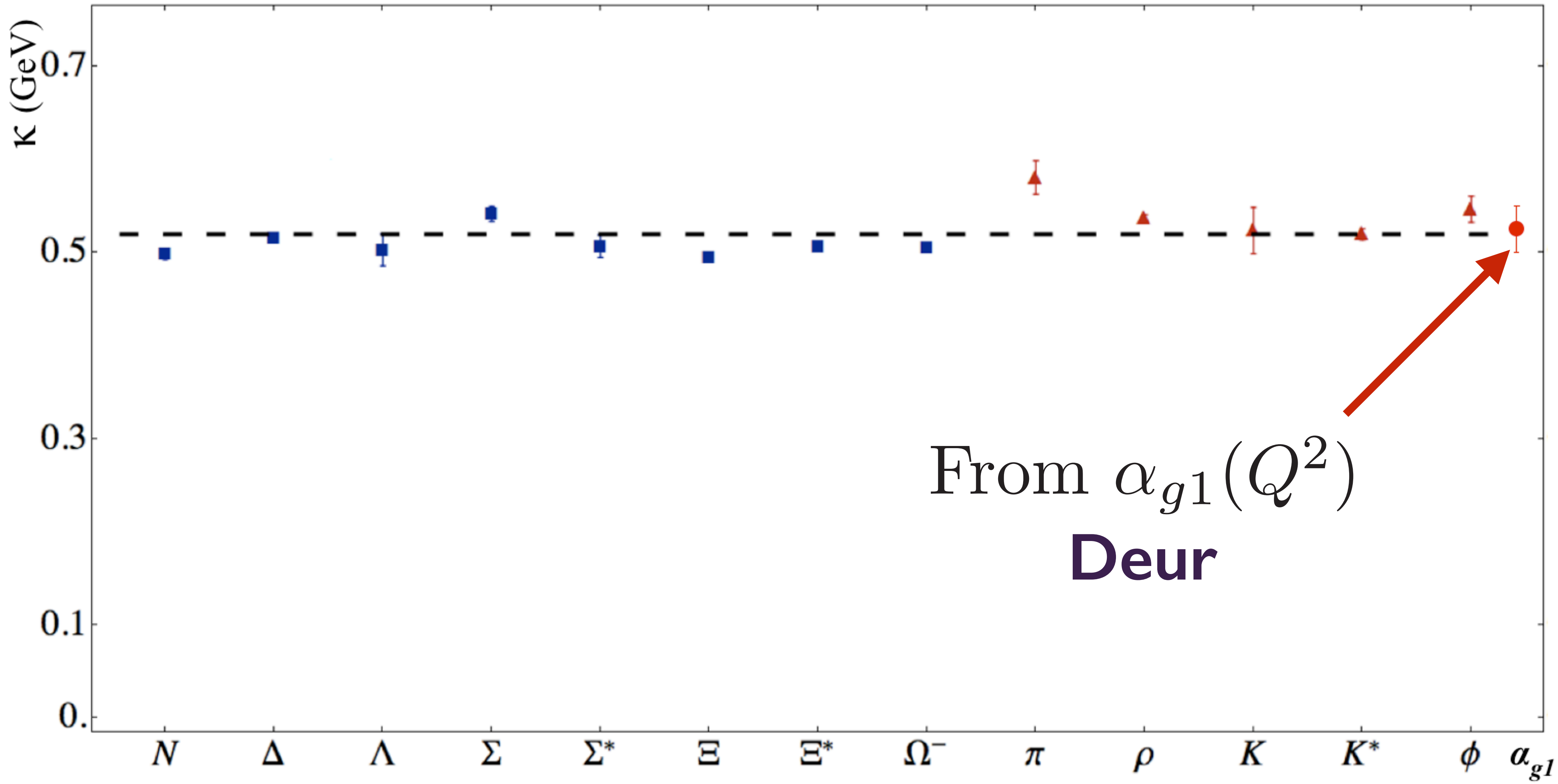
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Dosch, de Teramond, Lorce, sjb

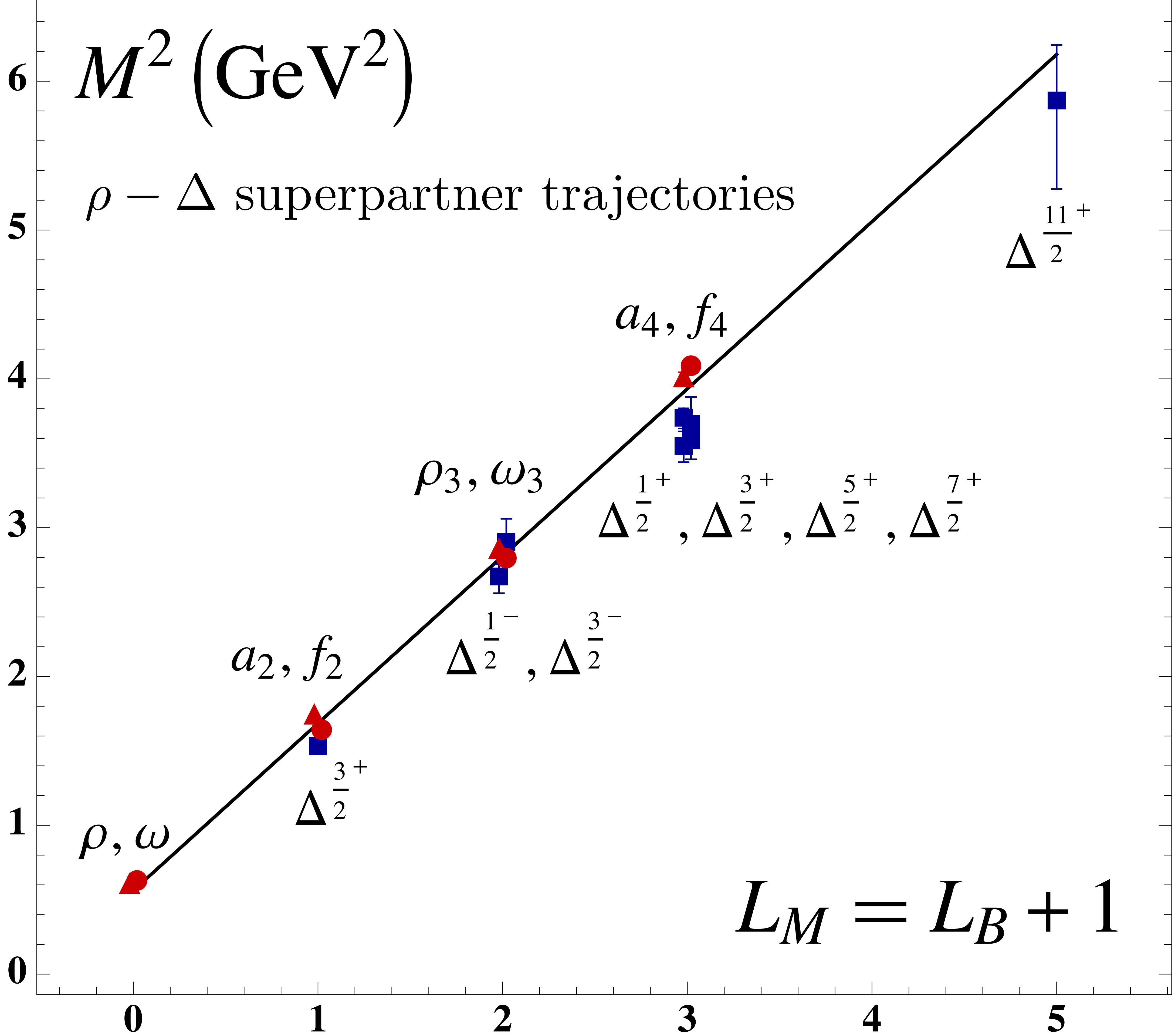
$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



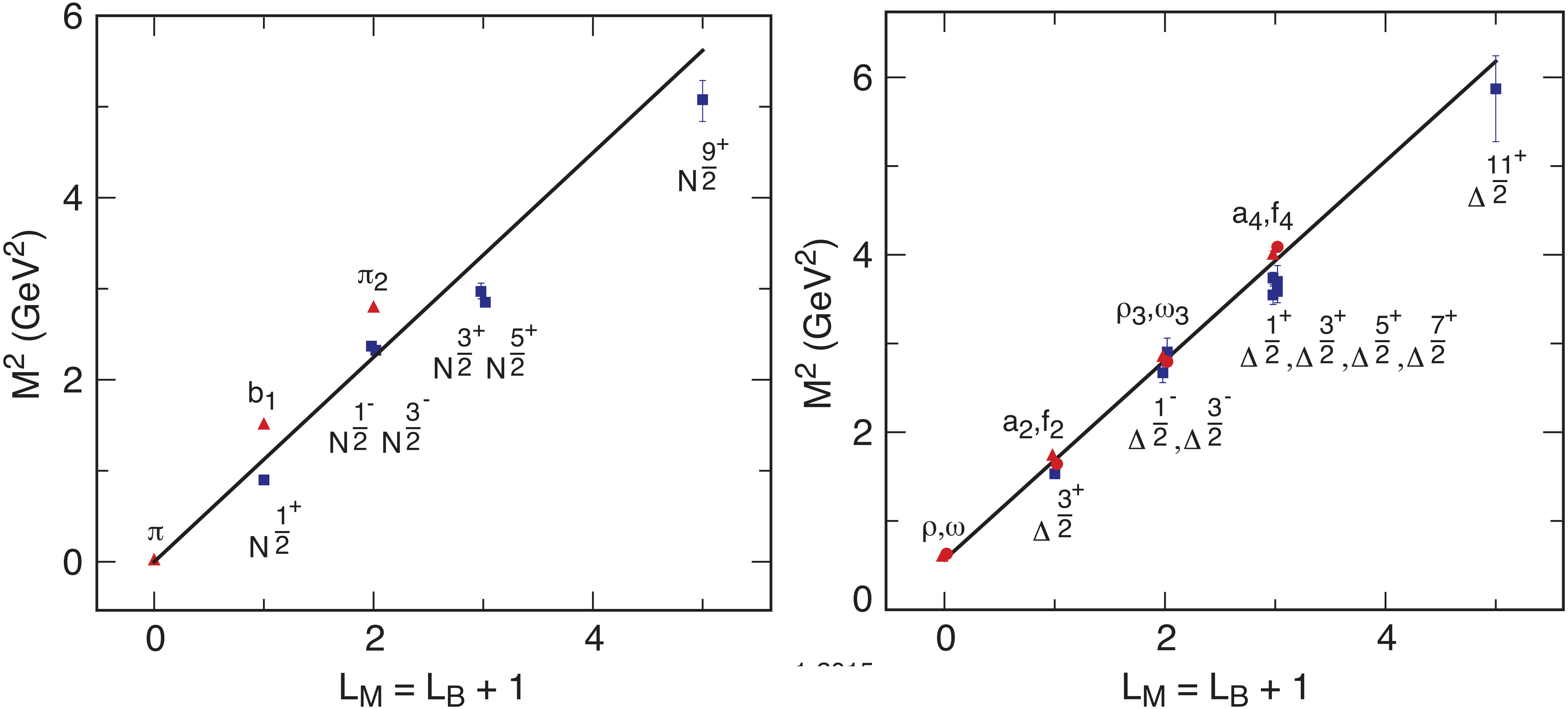
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics



Dosch, de Teramond, sjb

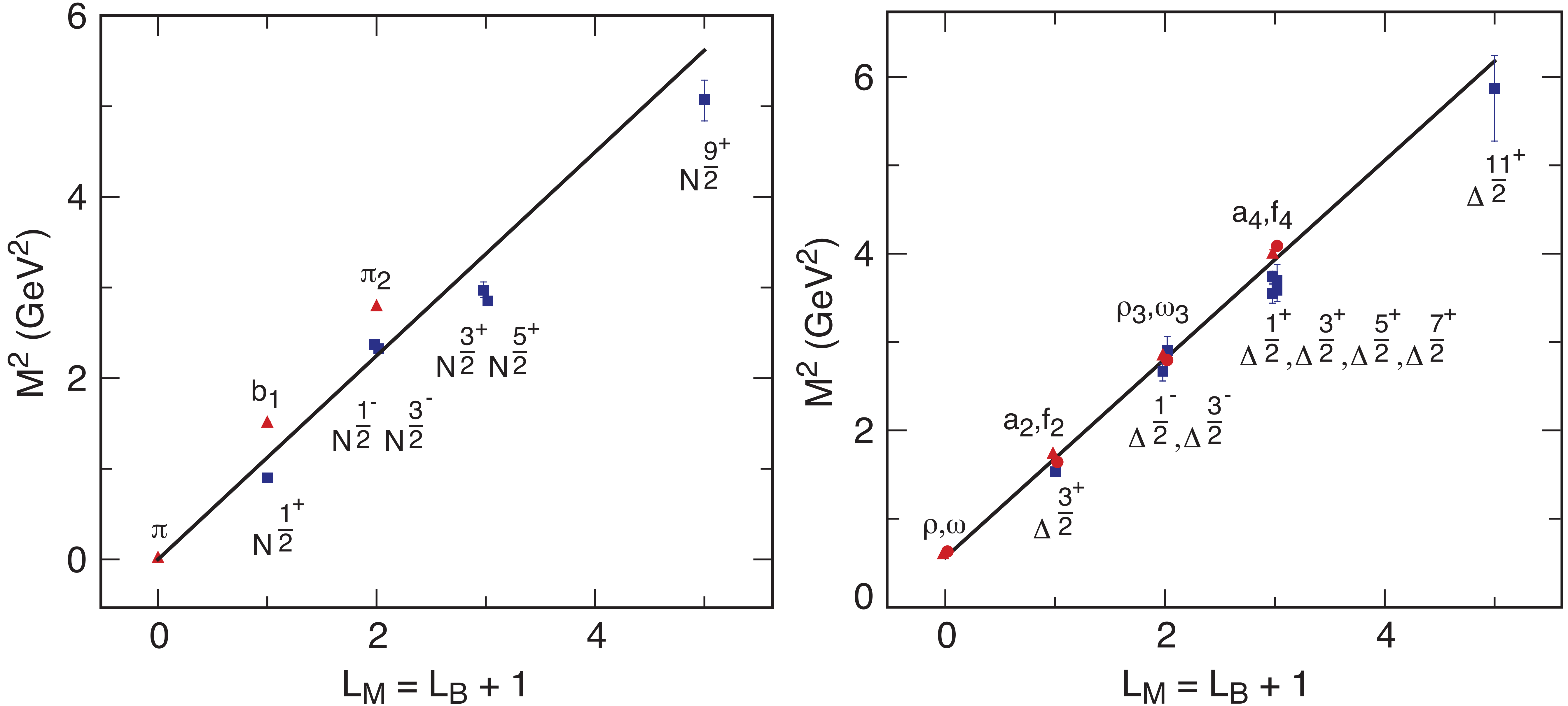
Solid line: κ = 0.53 GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

Solid line: κ = 0.53 GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

Universal Hadronic Features

Equal: Virial Theorem!

- ***Universal quark light-front kinetic energy***

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1+2n+L)$$

- ***Universal quark light-front potential energy***

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1+2n+L)$$

- ***Universal Constant Term***

$$\mathcal{M}^2_{spin} = 2\kappa^2(S+L-1+2n_{diquark})$$

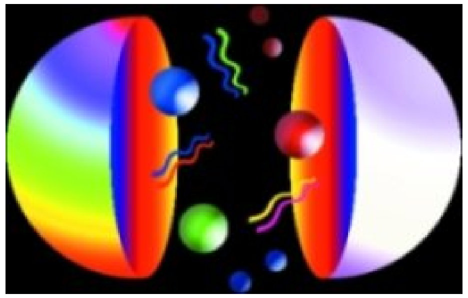
$$M^2 = \Delta\mathcal{M}^2_{LFKE} + \Delta\mathcal{M}^2_{LFPE} + \Delta\mathcal{M}^2_{spin}$$

$$+ < \sum_i \frac{m_i^2}{x_i} >$$

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 with same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$
- Proton spin carried by quark L^z

$$\boxed{< J^z >= \frac{1}{2}(S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2}(S_q^z = -\frac{1}{2}, L^z = 1) =< L^z >= \frac{1}{2}}$$

- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

Mesons and baryons have same κ !

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No vacuum condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$\boxed{J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0}$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$\begin{aligned} F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2, \end{aligned}$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$\begin{aligned} F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \end{aligned}$$

  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad ({F_1}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

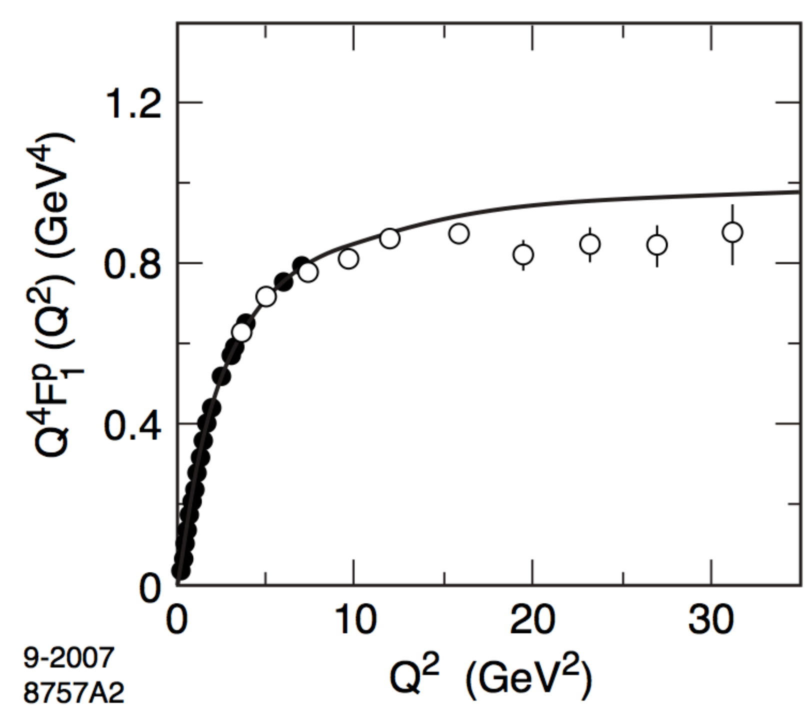
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} \, e^{-\kappa^2 z^2 x/(1-x)}$$

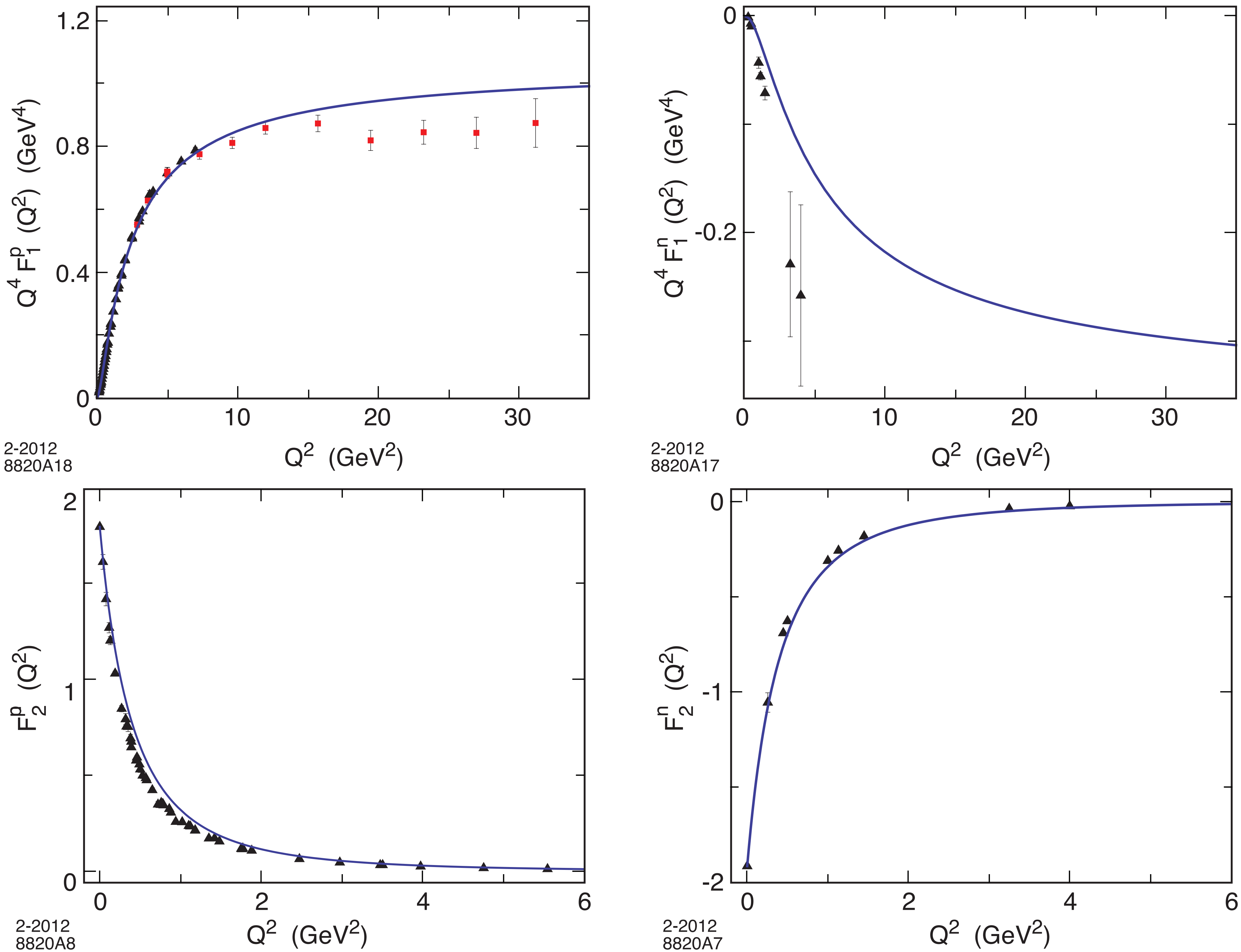
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

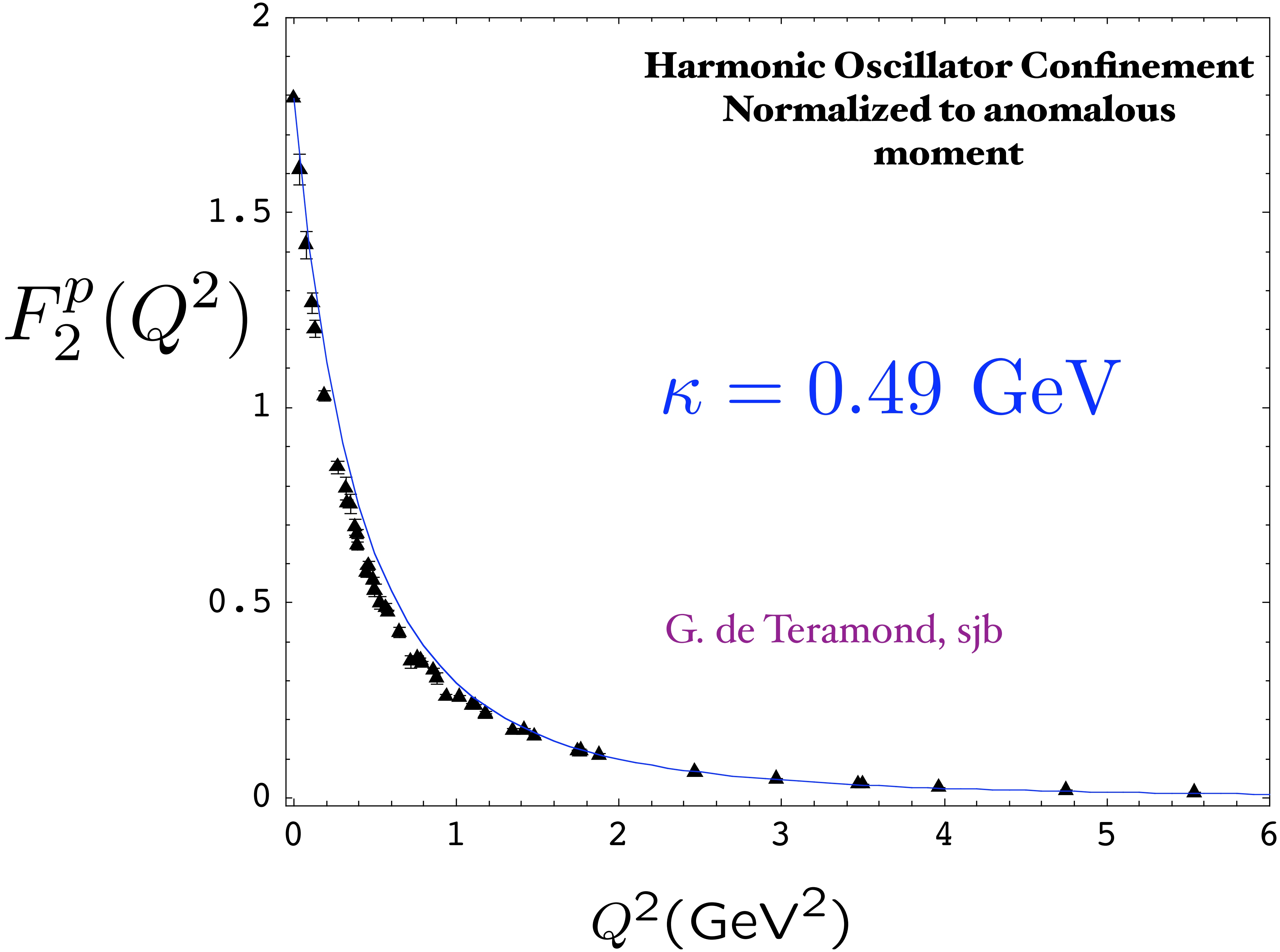


Using $SU(6)$ flavor symmetry and normalization to static quantities



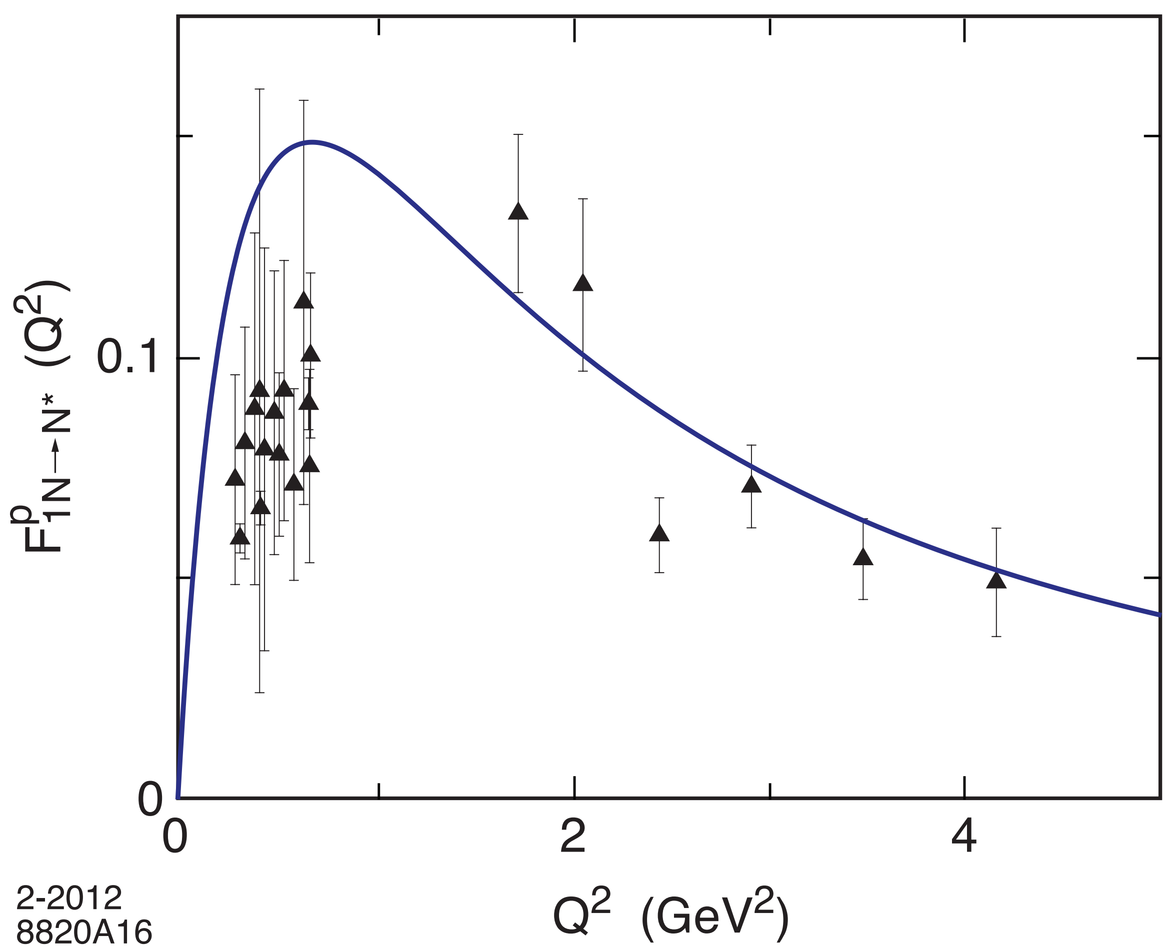
Spacelike Pauli Form Factor

From overlap of $L=1$ and $L=0$ LFWFs



Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_1{}^p_{N\to N^*}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\quad \left(F_1{}^p_{N\to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$

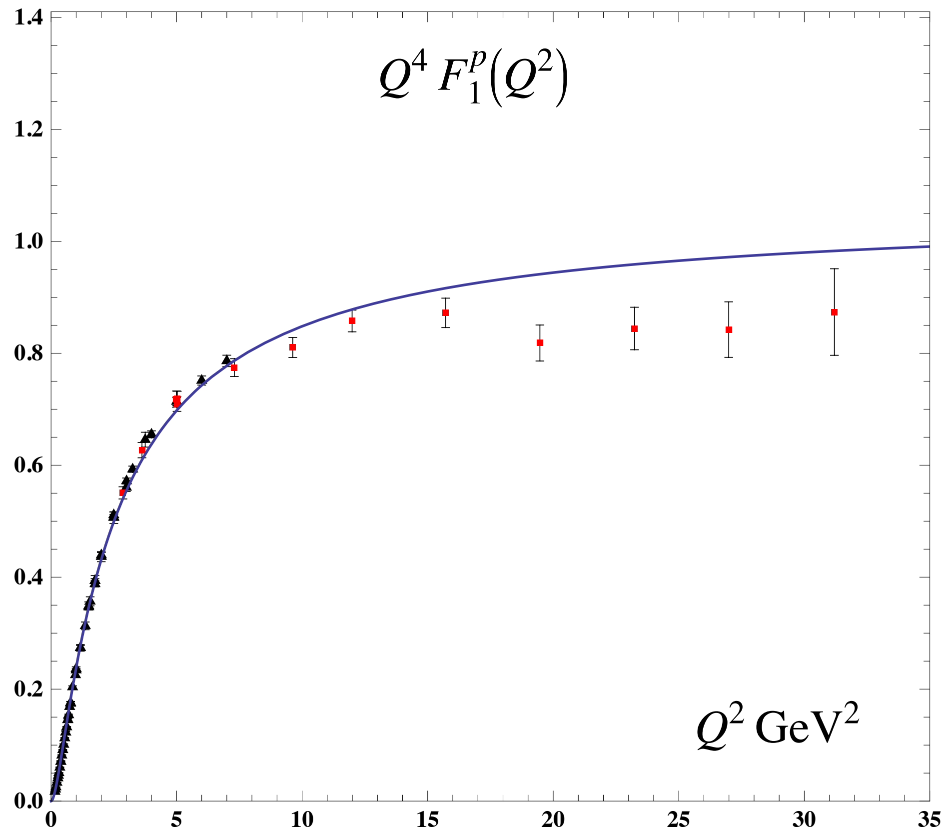
de Teramond, sjb

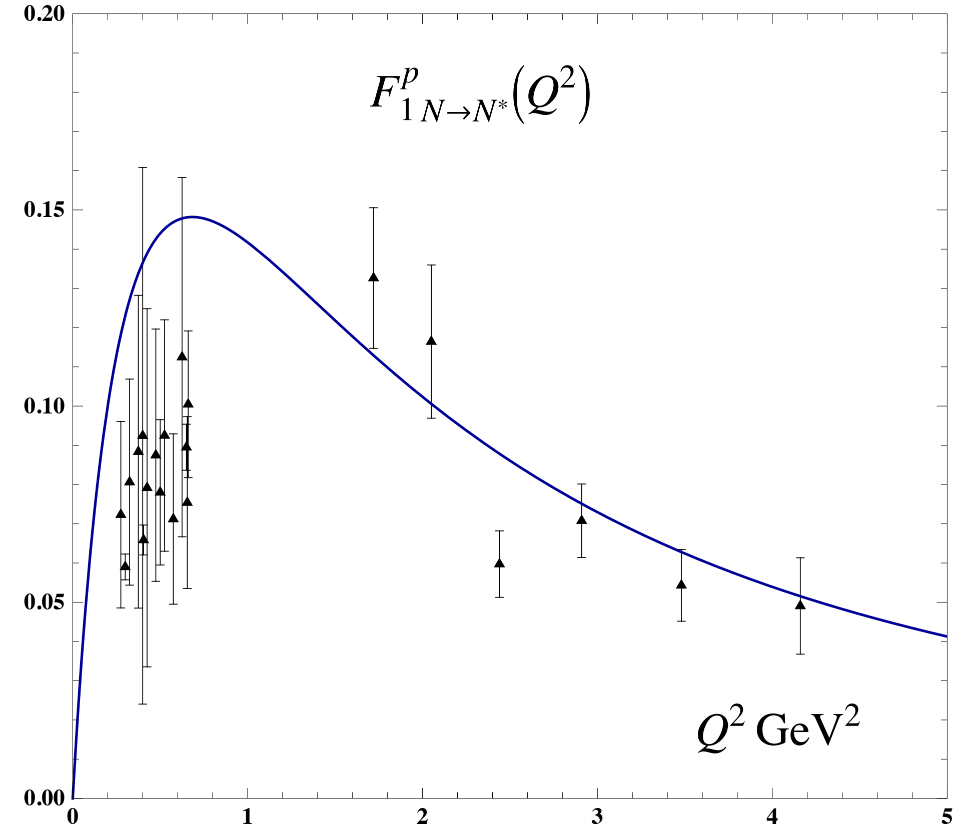
Consistent with counting rule, twist 3

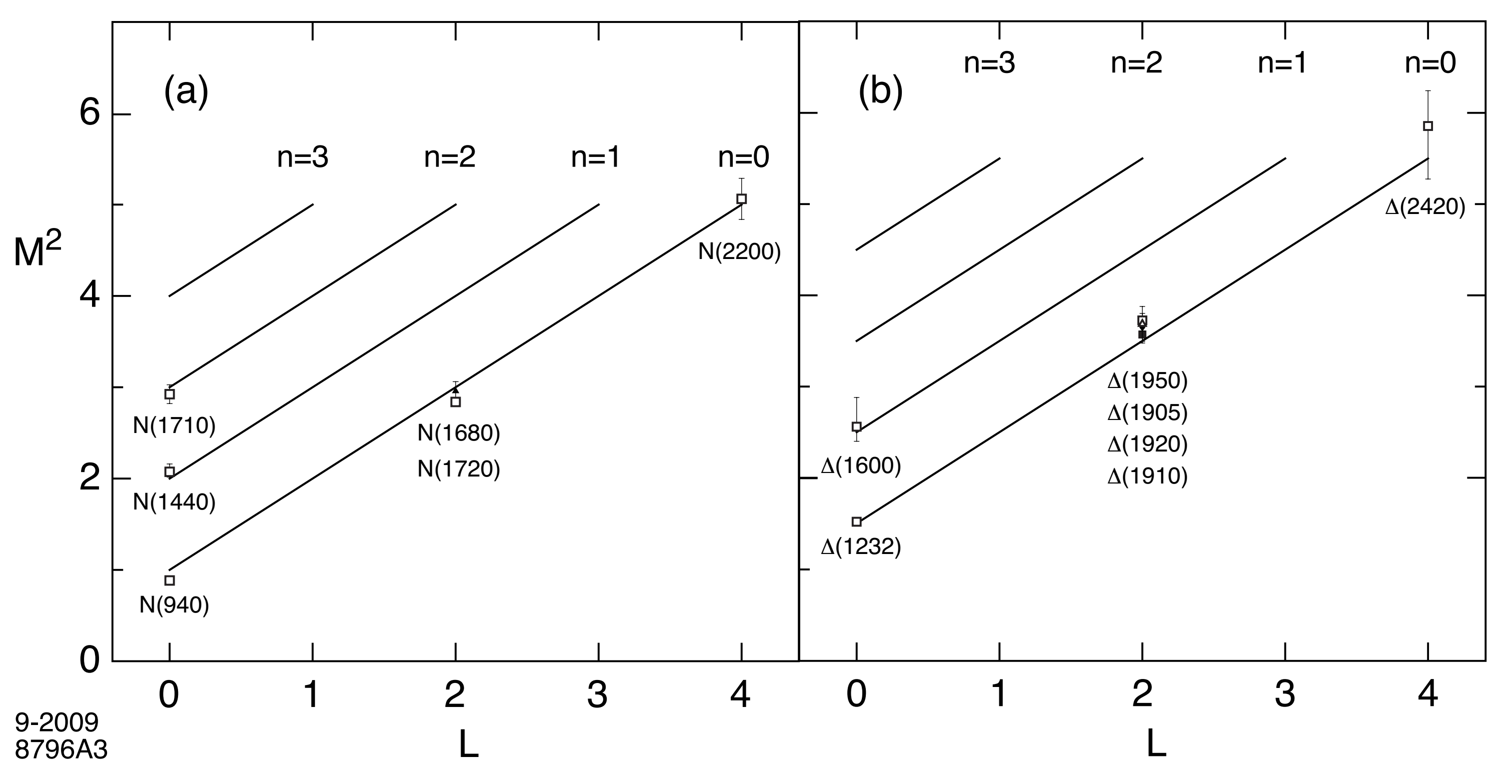
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

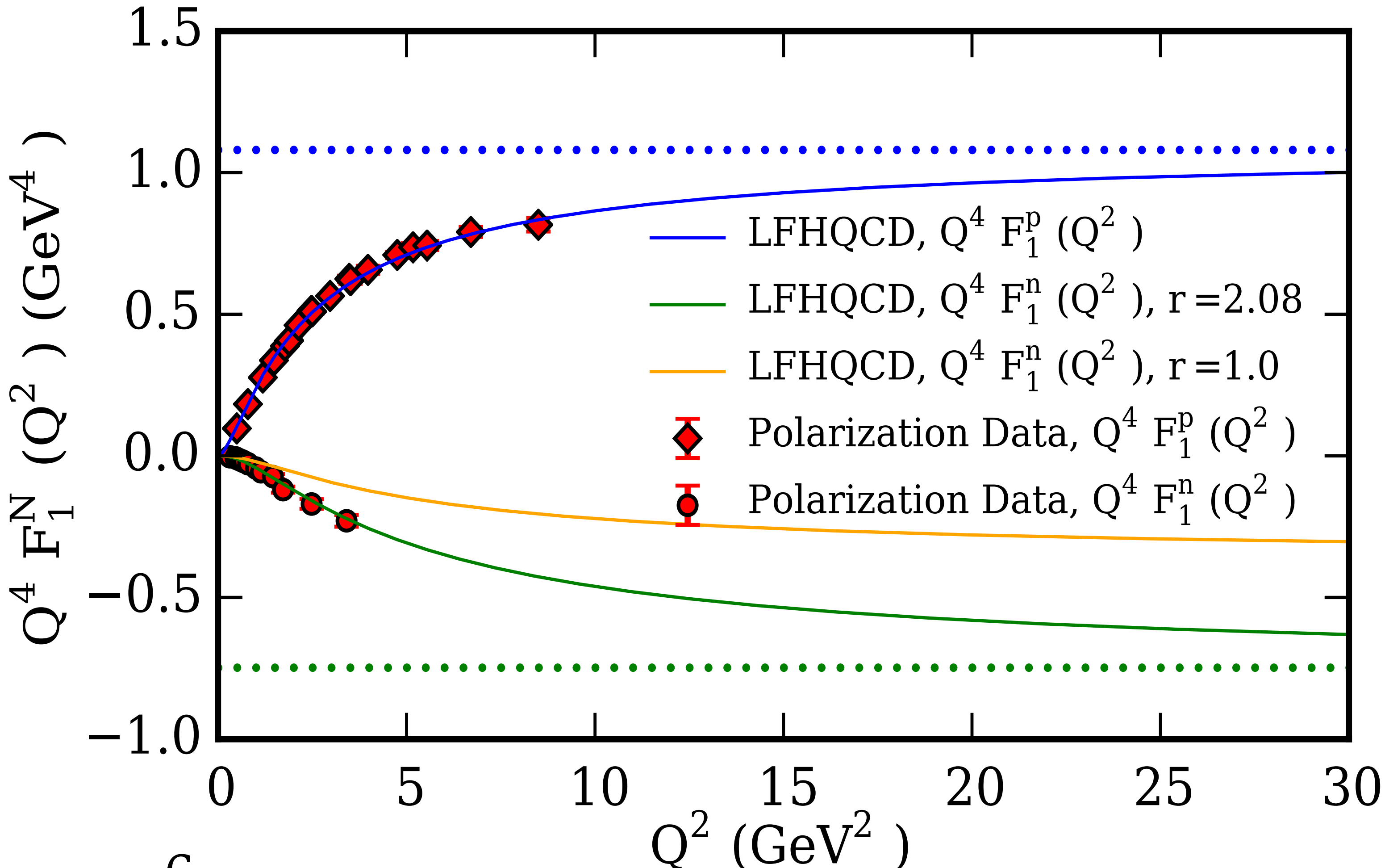
G. de Teramond & sjb







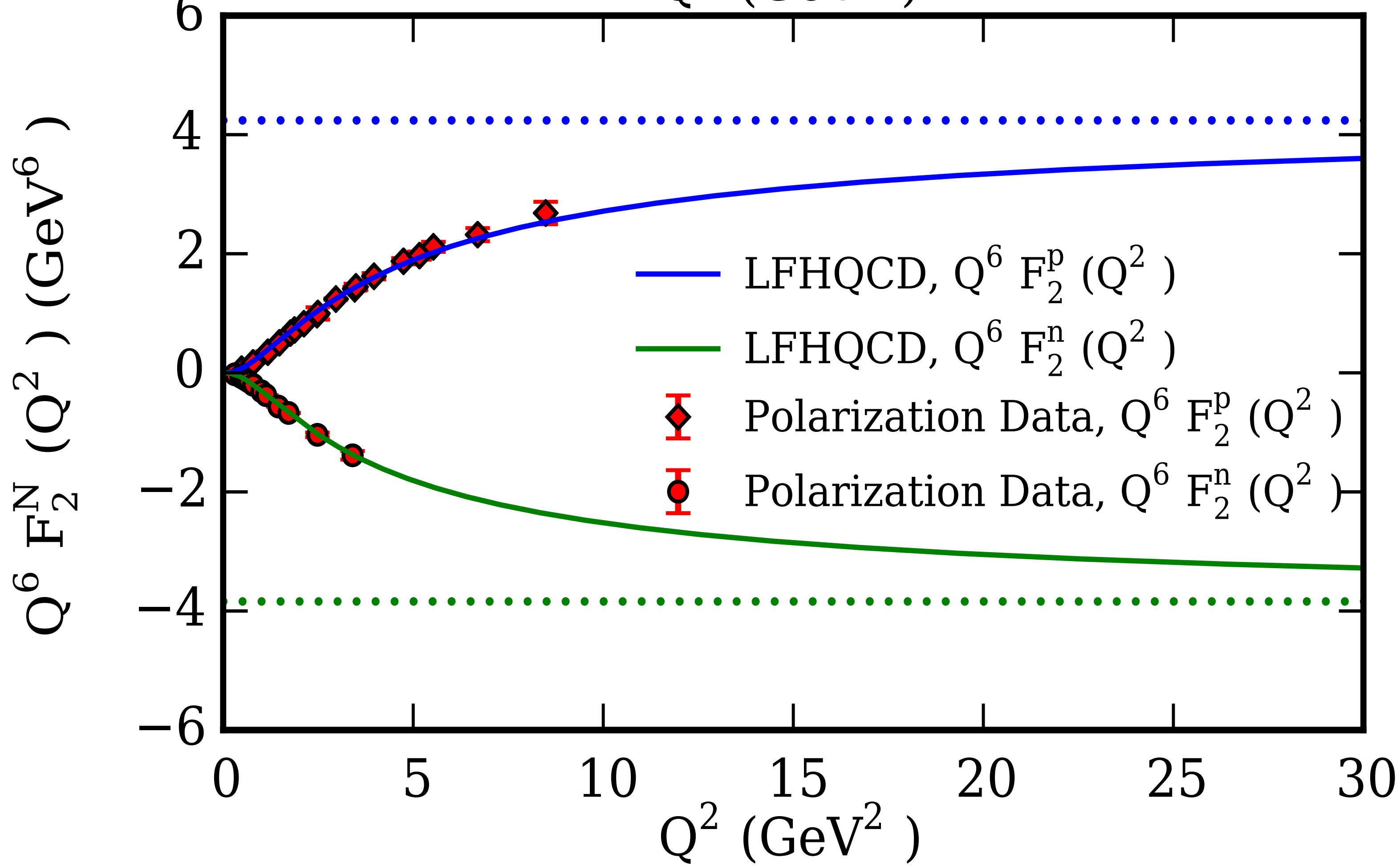
Sufian, de Teramond, Deur, Dosch, sjb



$Q^4 F_1^p(Q^2)$

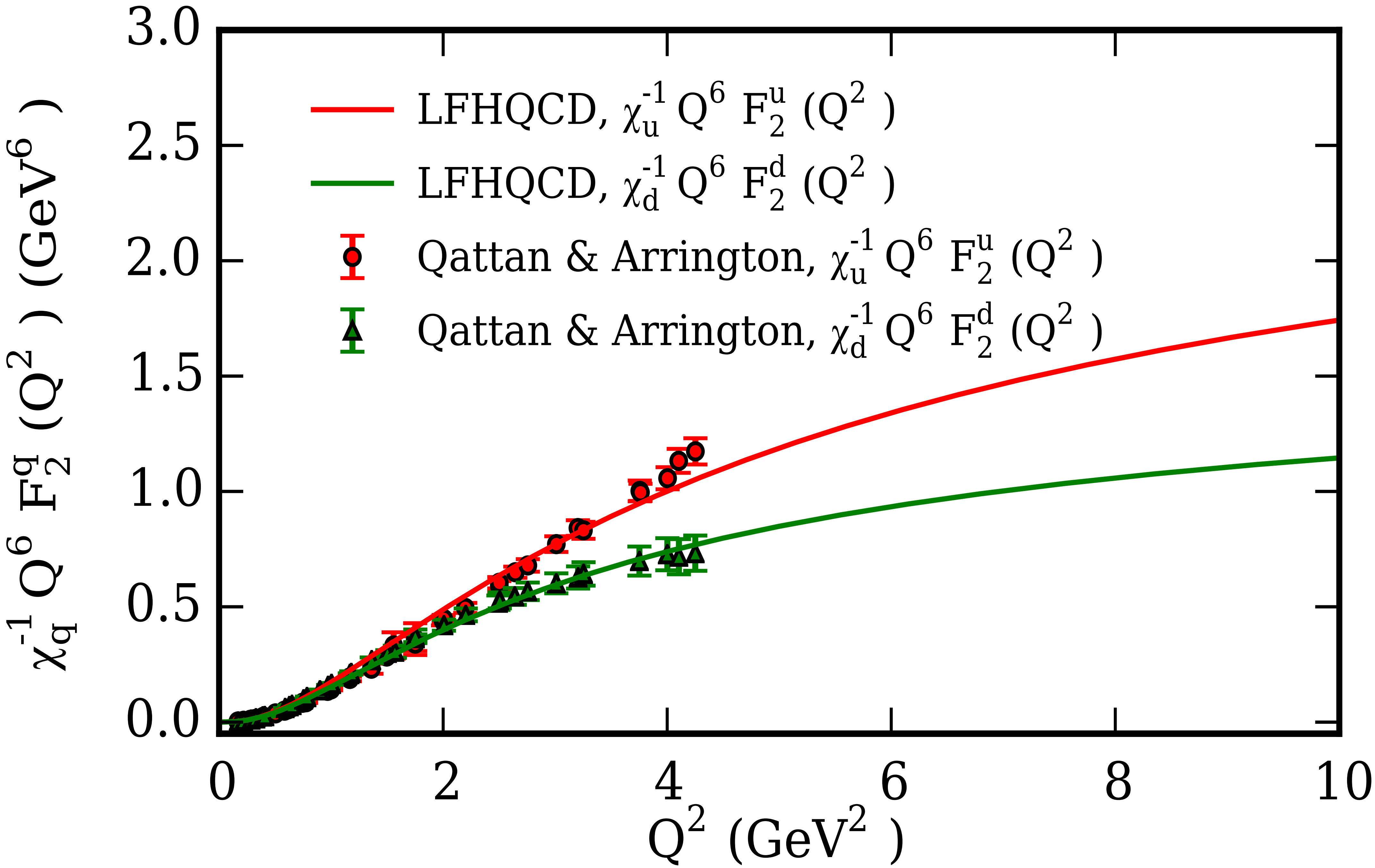
$Q^4 F_1^n(Q^2)$

Includes 5-quark Fock states



$Q^6 F_2^p(Q^2)$

$Q^6 F_2^n(Q^2)$



Flavor Dependence of $Q^6\ F_2(Q^2)$

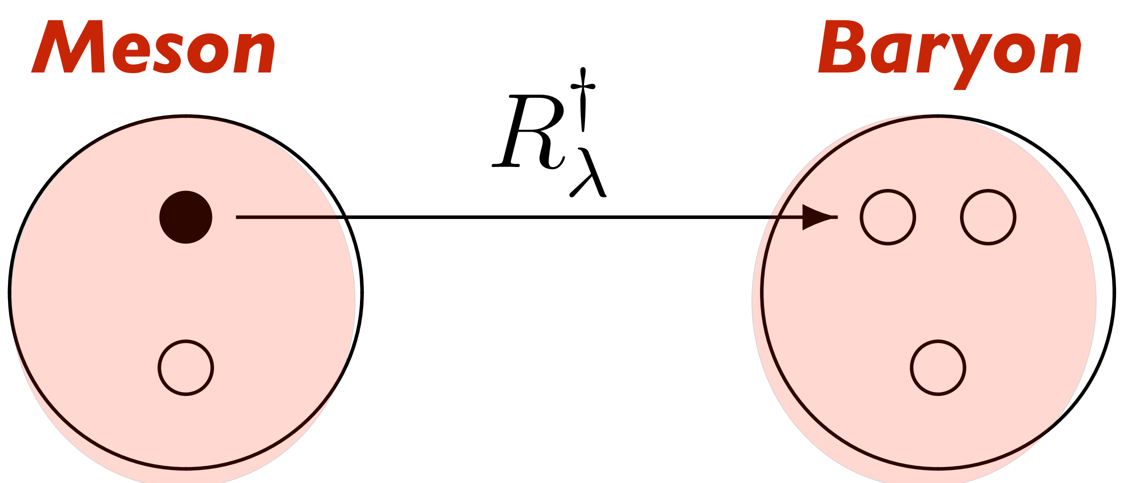
Sufian, de Teramond, Deur, Dosch, sjb

Guy de Tèramond, Hans Günter Dosch, sjb

Superconformal Algebra

2X2 Hadronic Multiplets

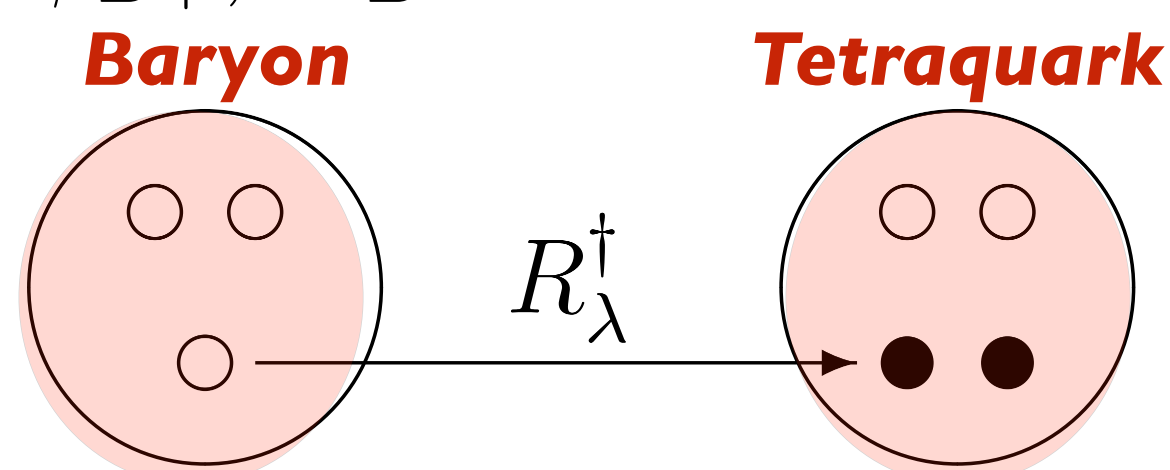
Bosons, Fermions with Equal Mass!



$\phi_M, \; L_B + 1$ $\quad$ $\psi_{B+}, \; L_B$

$R_\lambda^\dagger \; \bar{q} \to [qq]$

$\bar{3}_C \to \bar{3}_C$



$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

$\psi_{B-}, \; L_B + 1$ $\quad$ $\phi_T, \; L_B$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

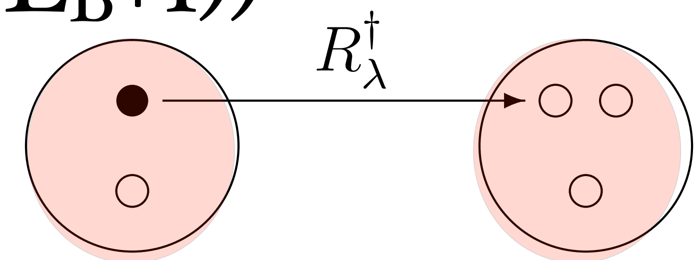
de Tèramond, Dosch, Lorce, sjb

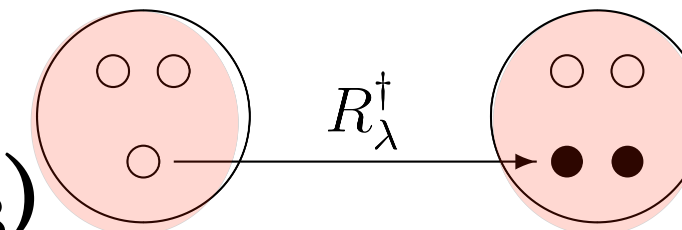
Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

- quark-antiquark meson ($L_M = L_{B+1}$))
- quark-diquark baryon (L_B)
- quark-diquark baryon (L_{B+1})
- diquark-antidiquark tetraquark ($L_T = L_B$)
- Universal Regge slopes $\lambda = \kappa^2$





Same Twist!

$$M^2_H/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{contribution\ from\ 2\text{-}dim\ light\text{-}front\ harmonic\ oscillator} + \overbrace{2(L_H + s) + 2\chi}^{contribution\ from\ AdS\ and\ superconformal\ algebra} + < \sum_i \frac{m_i^2}{x_i} >$$

$\chi(mesons) = -1$ $\chi(baryons, tetraquarks) = +1$

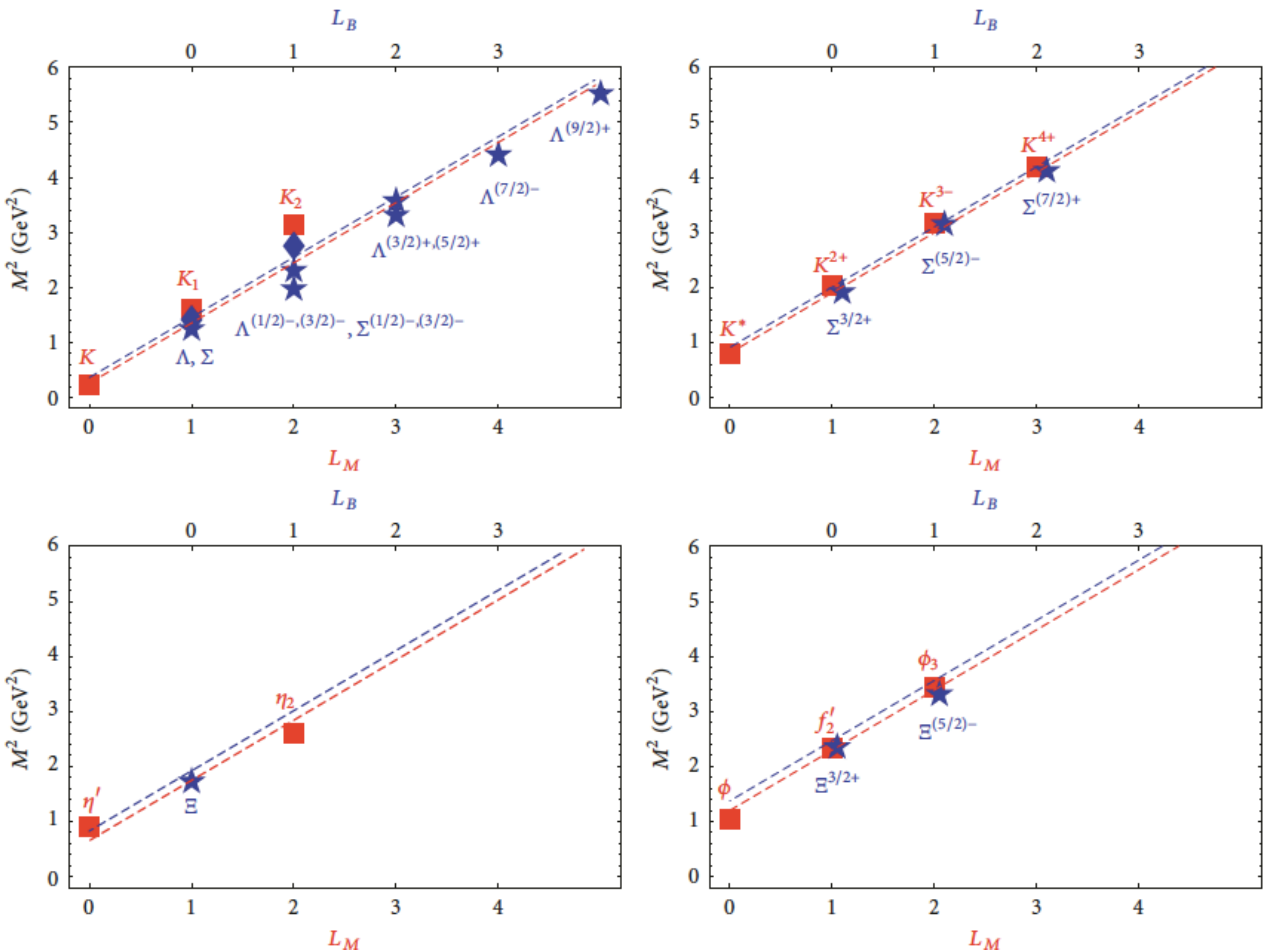
Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$\sigma(500)$
$\bar{q}q$	2^{-+}	$\pi_2(1670),\ \eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	—
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
							1^{+-}	$b_1(1235)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar{u}\bar{d}]$	2^{--}	—
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar{u}\bar{d}]$	3^{++}	—
$\bar{q}s$	0^-	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	1^+	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^+	$K_0^*(1430)$
$\bar{q}s$	2^-	$K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][\bar{s}\bar{q}]$	1^-	—
$\bar{s}q$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	1^-	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	2^+	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar{u}\bar{d}]$	1^+	$K_1(1400)$
$\bar{s}q$	3^-	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar{u}\bar{d}]$	2^-	—
$\bar{s}q$	4^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar{u}\bar{d}]$	3^+	—
$\bar{s}s$	0^{-+}	$\eta(550),\ \eta'(958)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	—
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
								$a_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar{s}\bar{q}]$	2^{--}	—
$\bar{s}s$	2^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar{s}\bar{q}]$	1^+	$K_1(1650)$

New Organization of the Hadron Spectrum

M. Nielsen

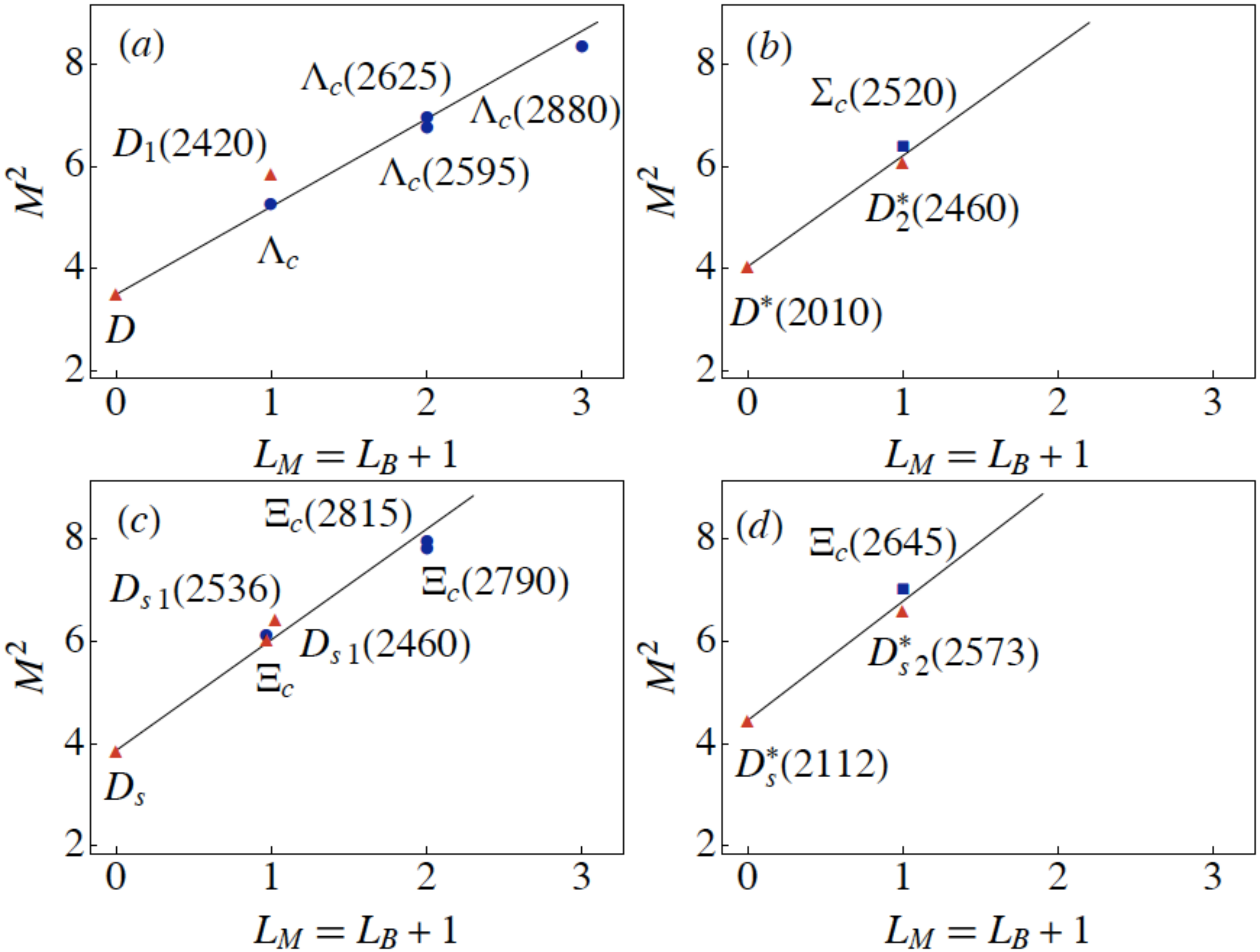
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

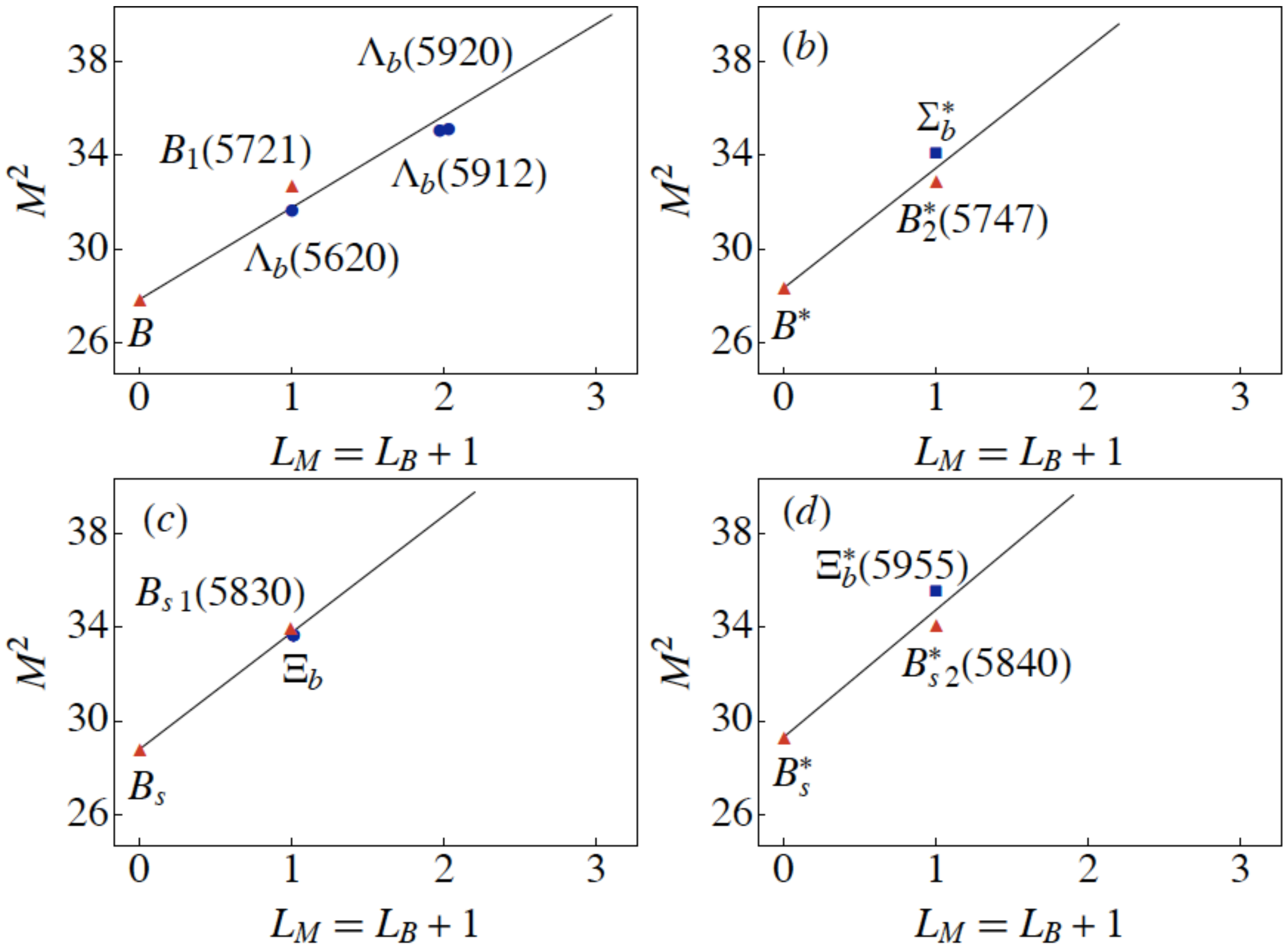
Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry *de Tèramond, Dosch, Lorce, sjb*

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$

$$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2\left[\sigma([\{qq\}N) + \sigma(qN)\right] - \left[\sigma(qN) + \sigma(\bar{q}N)\right] = \left[\sigma(\{qq\}N) + \sigma(\{qq\}N)\right]$$

Candidates $f_0(980) I = 0, J^P = 0^+$, partner of proton

$a_1(1260) I = 0, J^P = 1^+$, partner of $\Delta(1233)$

Test twist=4, power-law fall-off of form factors

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

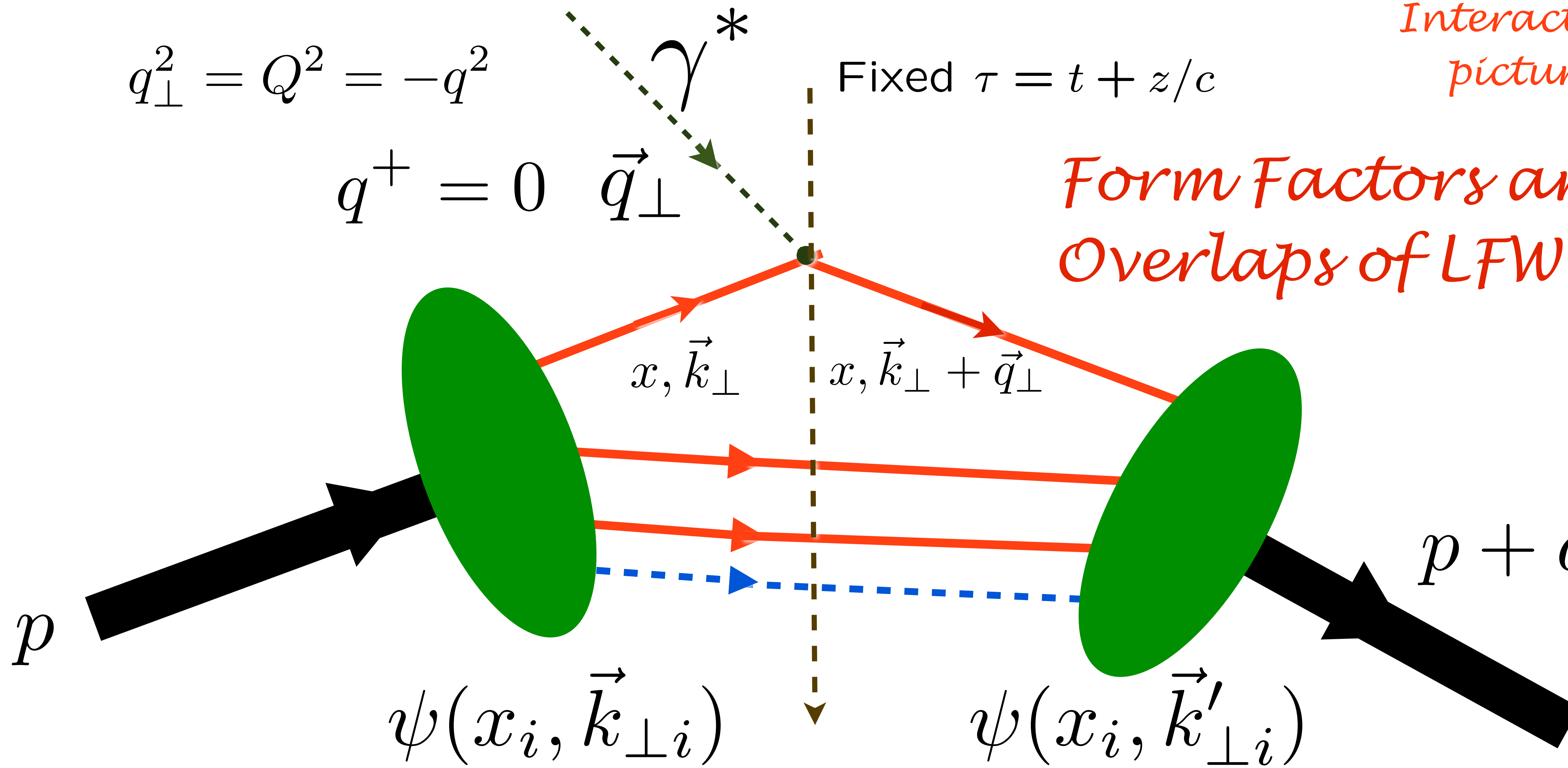
Interaction picture

$$q_\perp^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_\perp$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Exact LF Formula for Pauli Form Factor

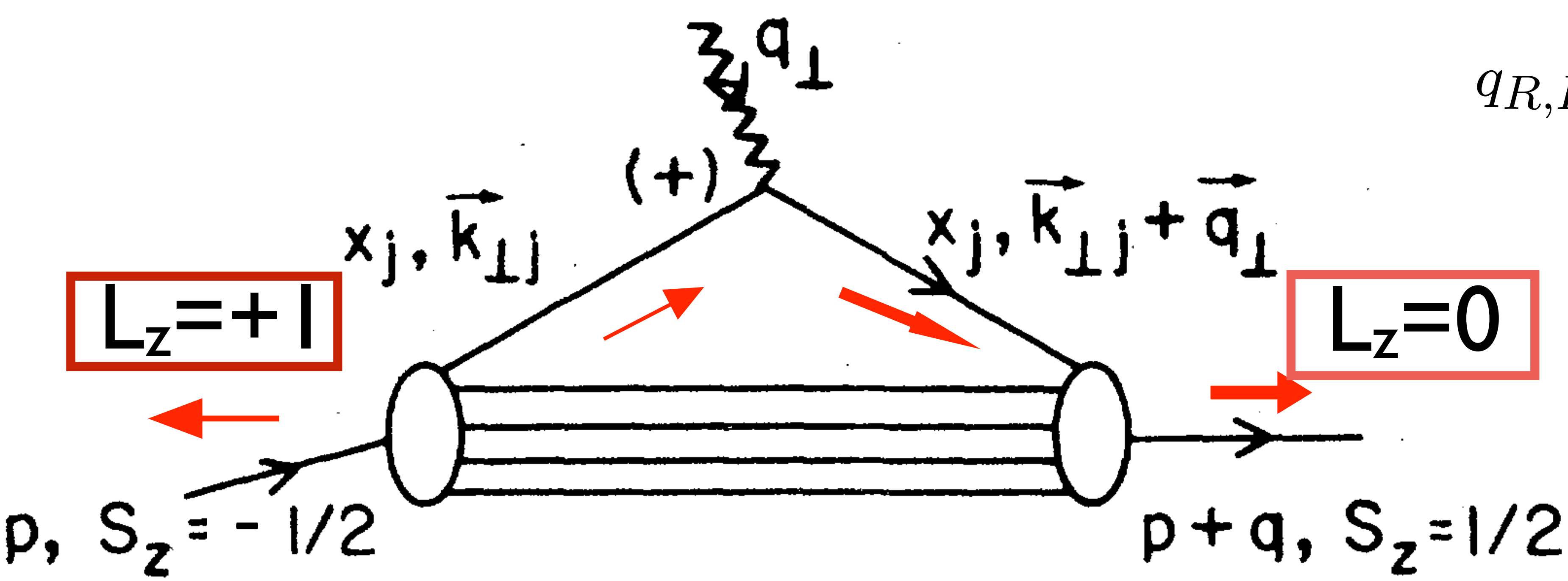
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Single-spin asymmetries

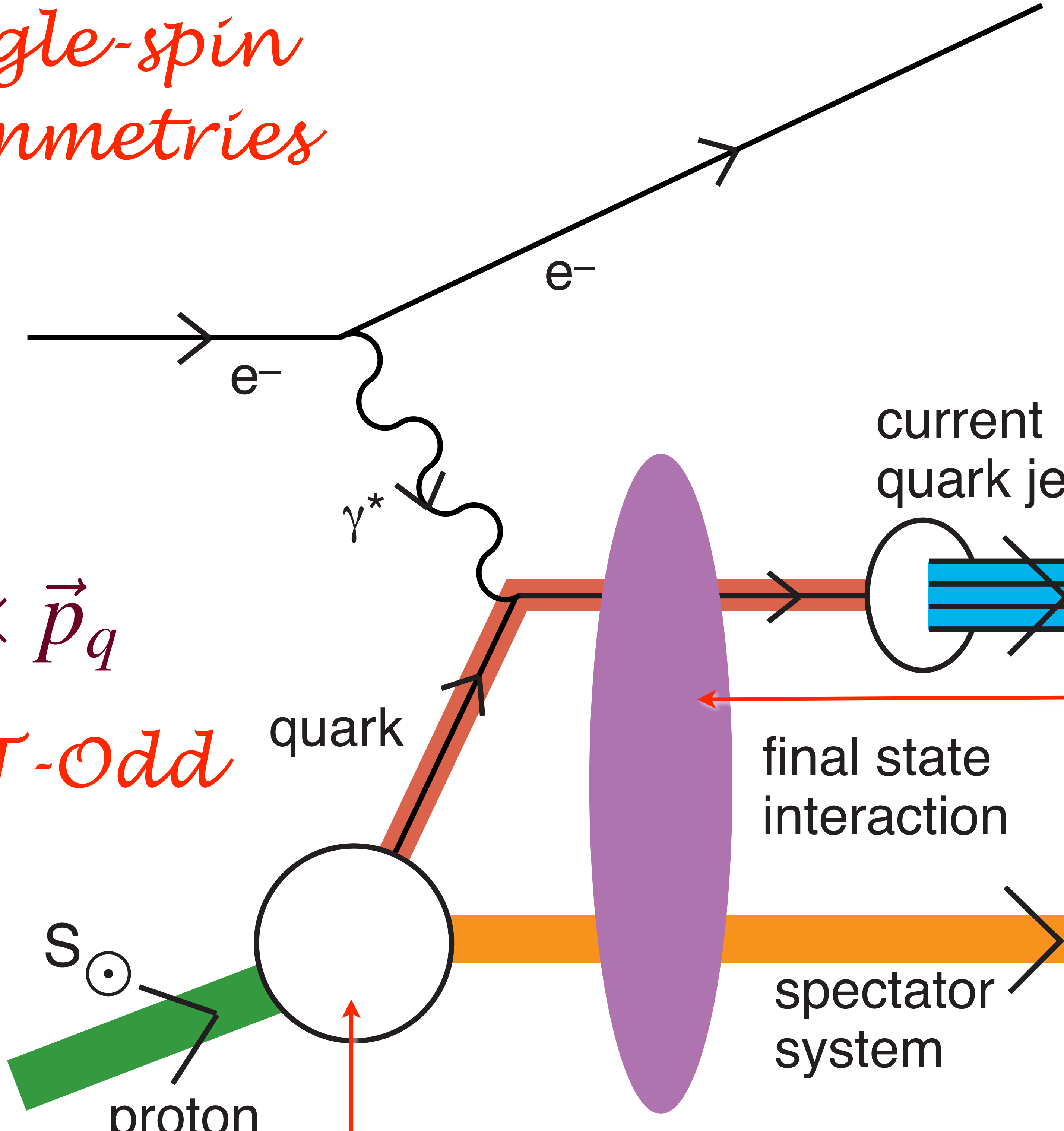
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



QCD S- and P-
Coulomb Phases
--Wilson Line

"Lensing Effect"

"Lensing" involves soft scales

Light-Front Wavefunction S and P- Waves!

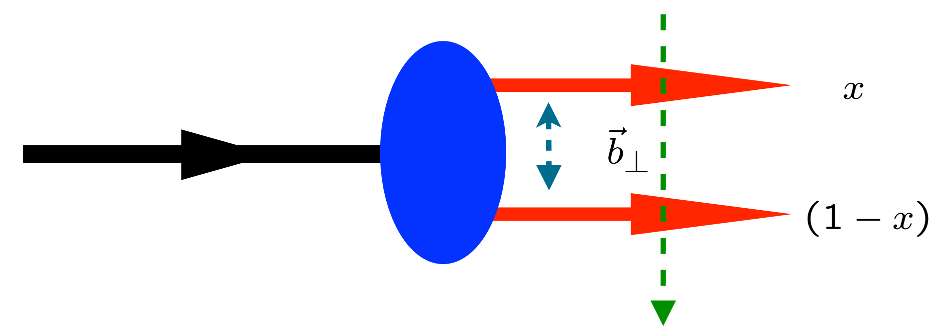
Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame*

Fixed $\tau = t + z/c$



- *Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon*
- *Light-Front Holography: AdS5 = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

$$\boxed{\zeta^2 = x(1-x)b_\perp^2}$$

- *Single fundamental hadronic mass scale* κ*: but retains the Conformal Invariance of the Action (dAFF)!*
- *Unique color-confining LF Potential!* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

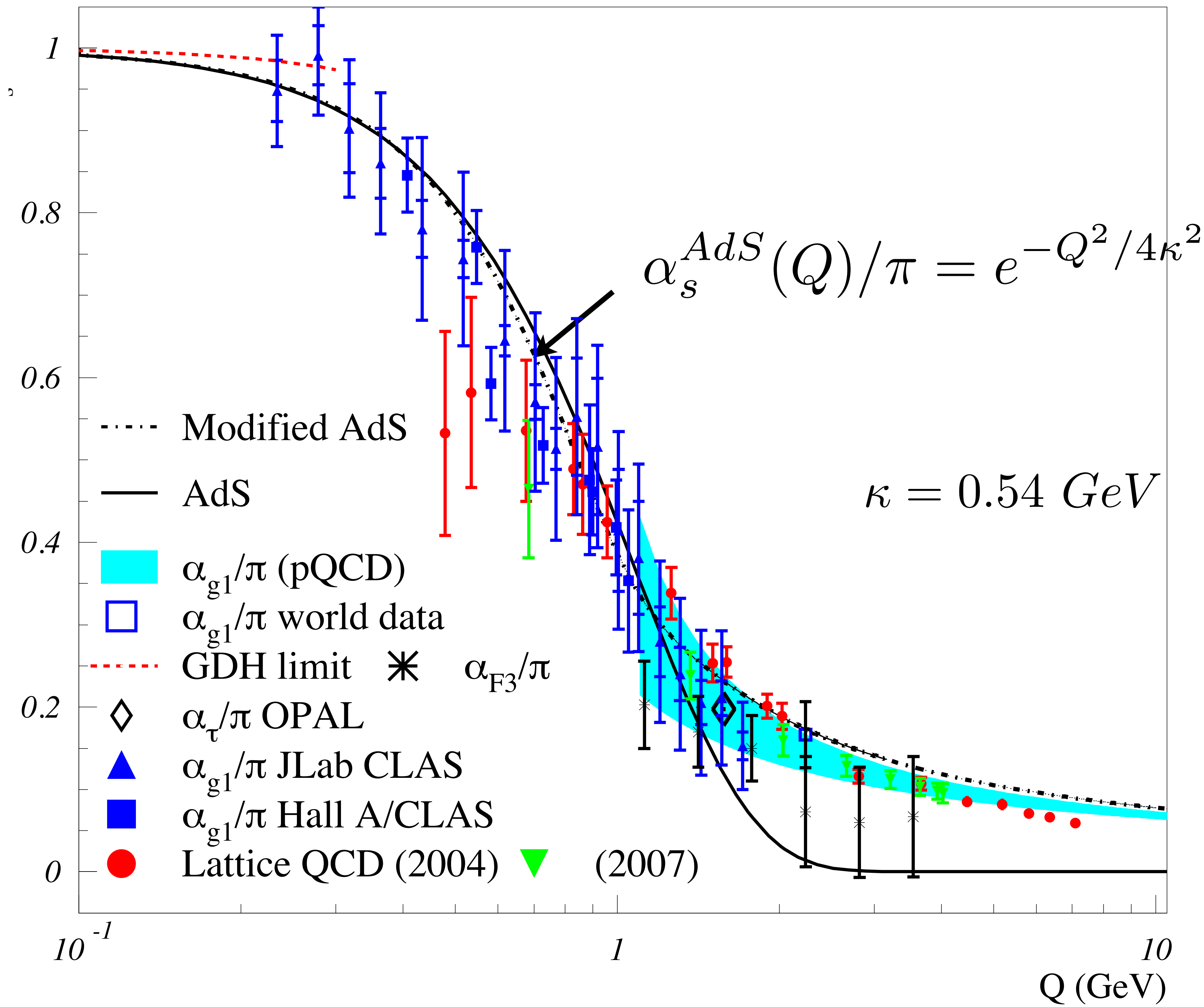
Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Analytic, defined at all scales, IR Fixed Point

$$\frac{\alpha_s(Q)}{\pi}$$

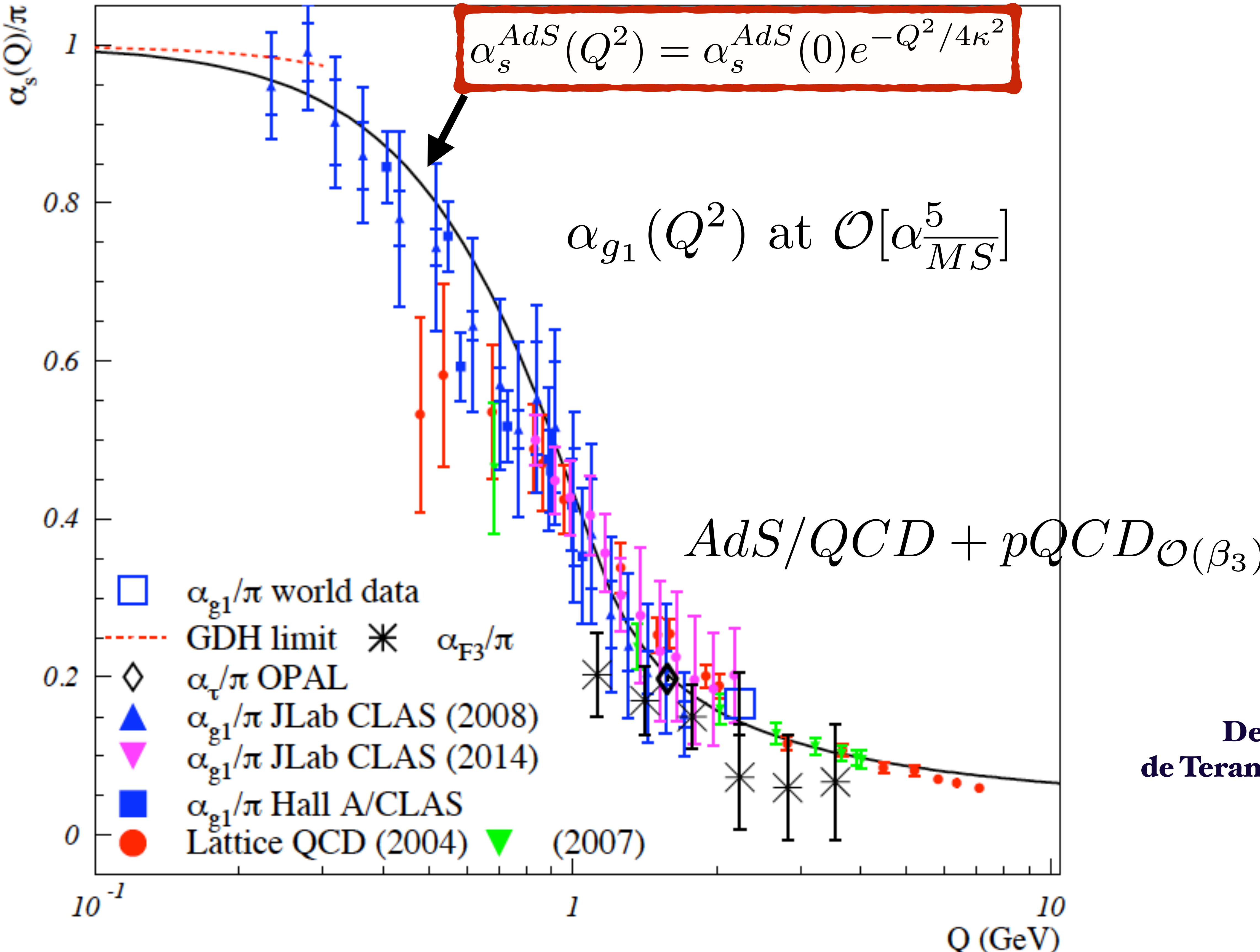


AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

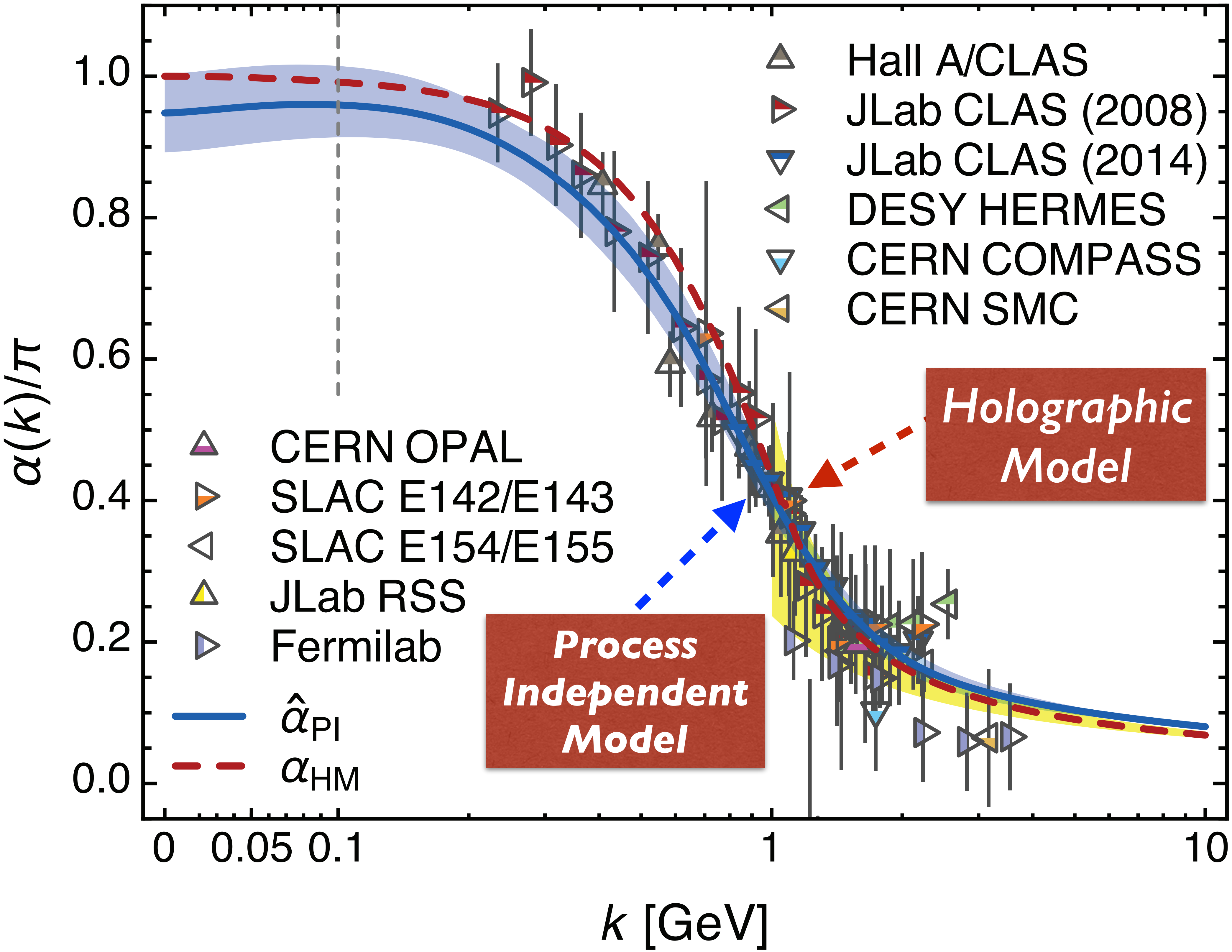
$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur, de Teramond, sjb



Process-independent strong running coupling

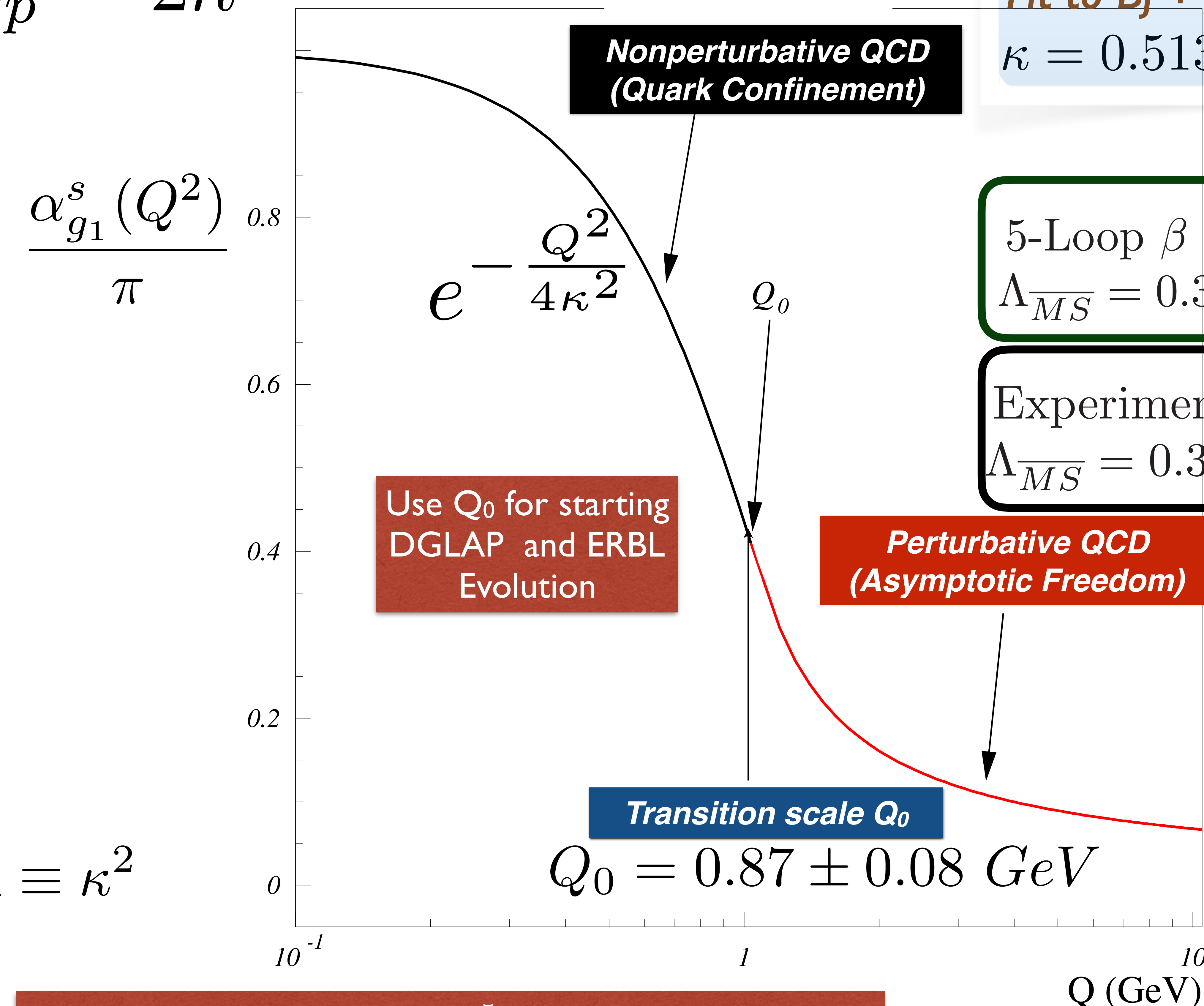
Daniele Binosi,[1] Cédric Mezrag,[2] Joannis Papavassiliou,[3] Craig D. Roberts,[2] and Jose Rodríguez-Quintero[4]

All-Scale QCD Coupling

Deur, de Tèramond, sjb

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$



5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

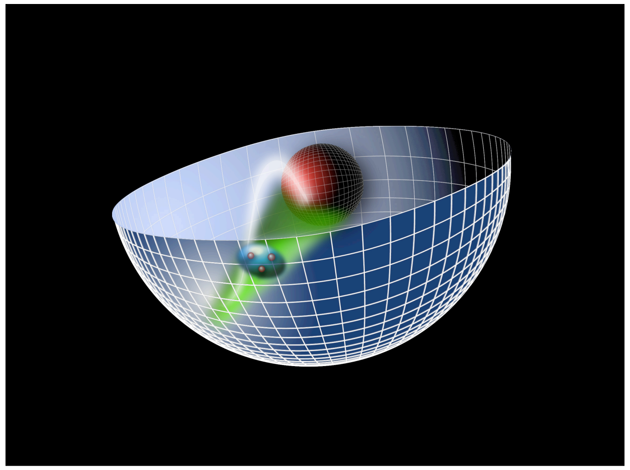
Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

$\lambda \equiv \kappa^2$

Reverse Dimensional Transmutation!

$\overline{MS}$ *scheme*

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\Big[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

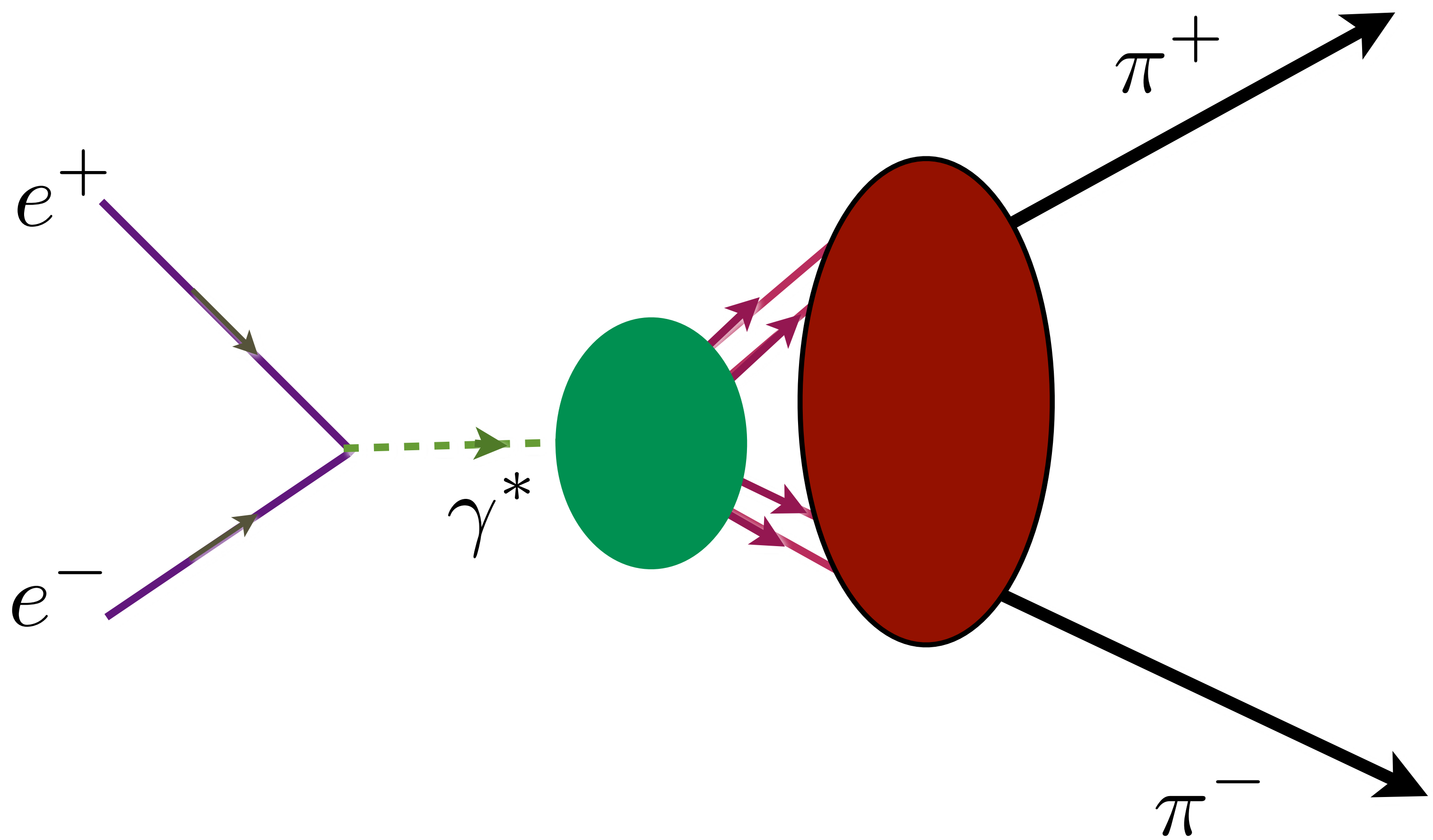
Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

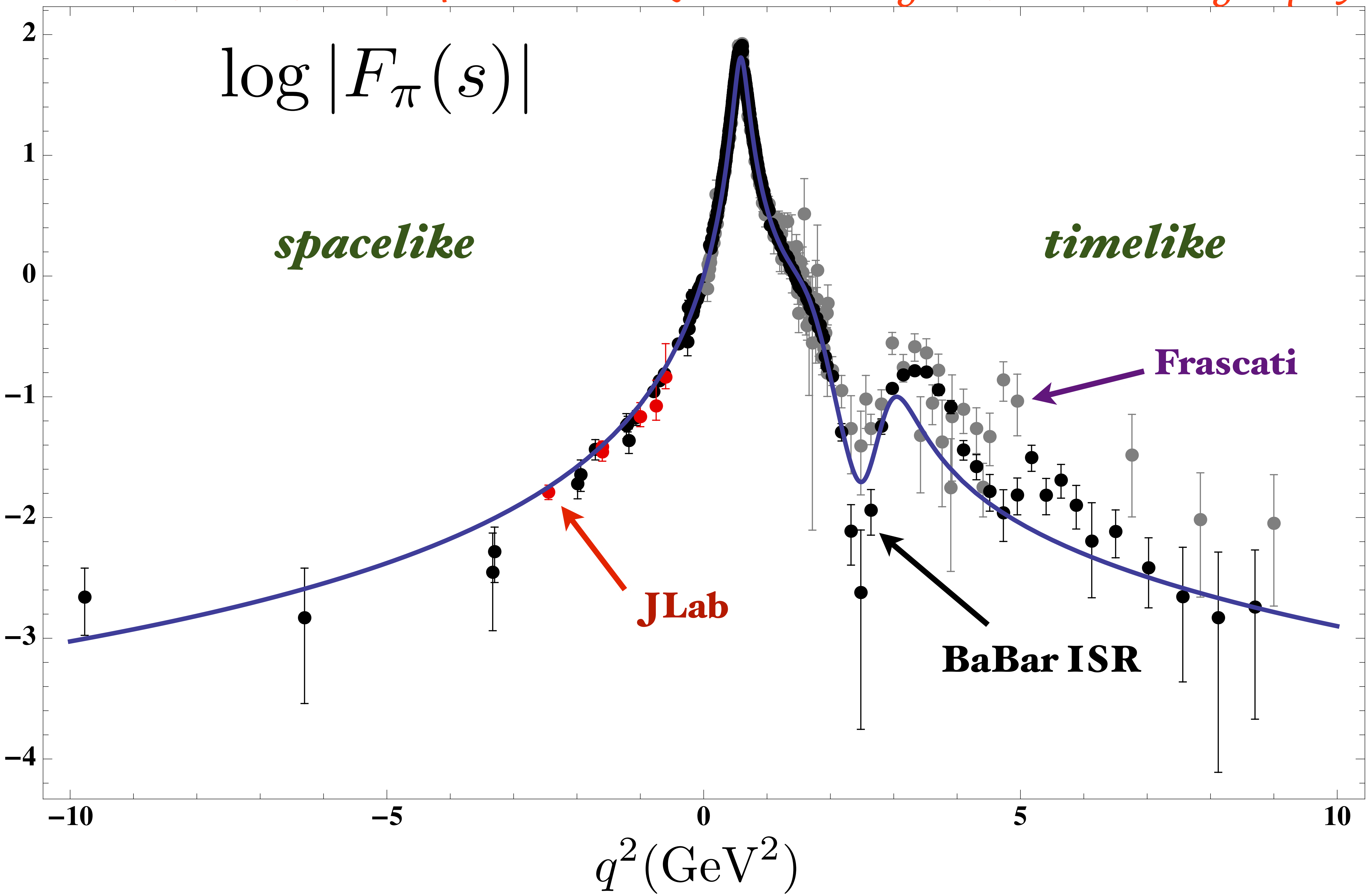
Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Pion Form Factor from AdS/QCD and Light-Front Holography



Superconformal Algebra

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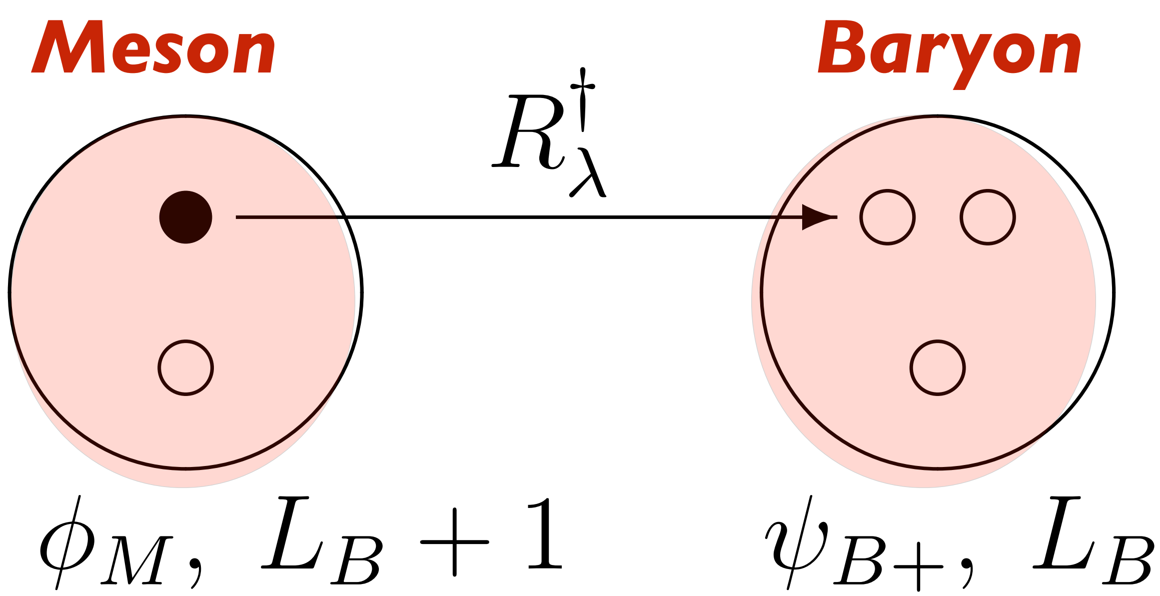
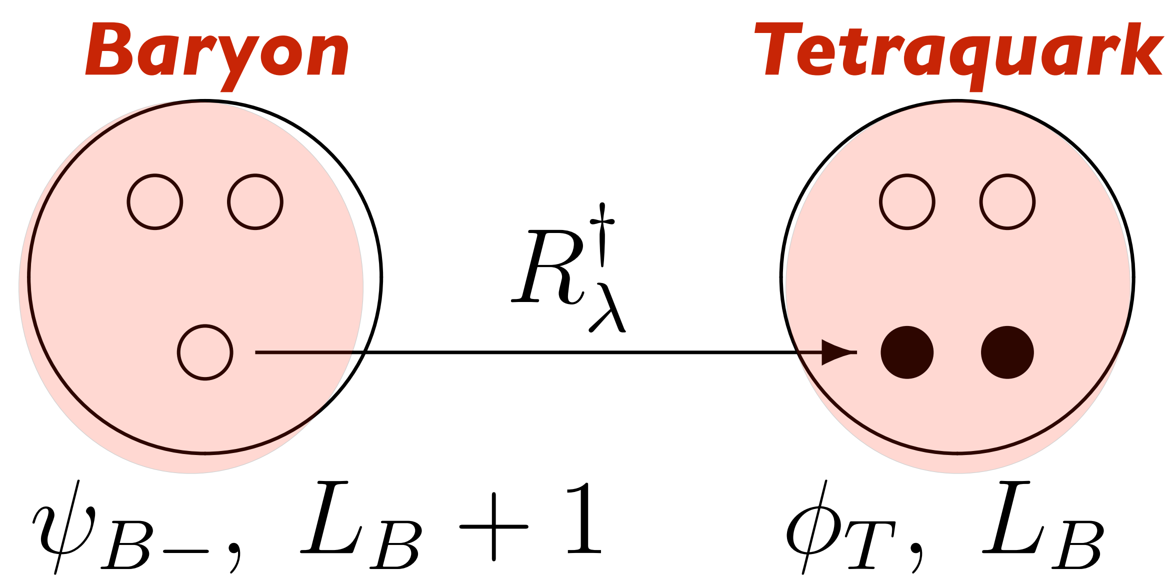
Bosons, Fermions with Equal Mass!



$$R^\dagger_\lambda \; \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$



$$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Proton: |u[ud]> Quark + Scalar Diquark

Equal Weight: L=0, L=1

Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*

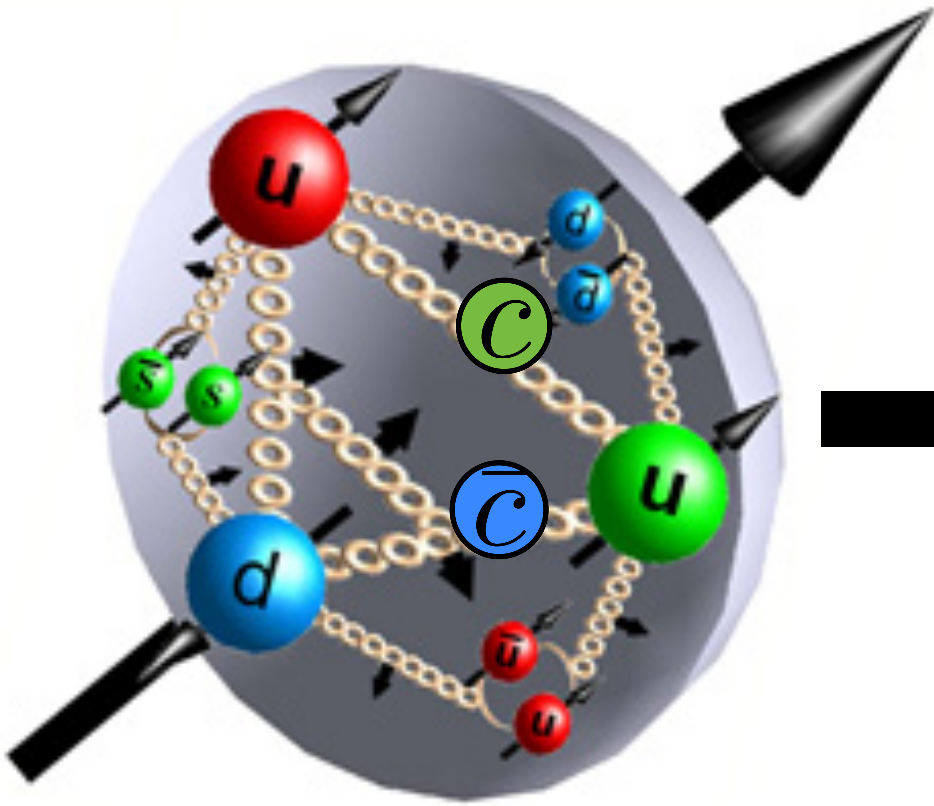
- *Causality: Information within causal horizon: Front Form*

- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*

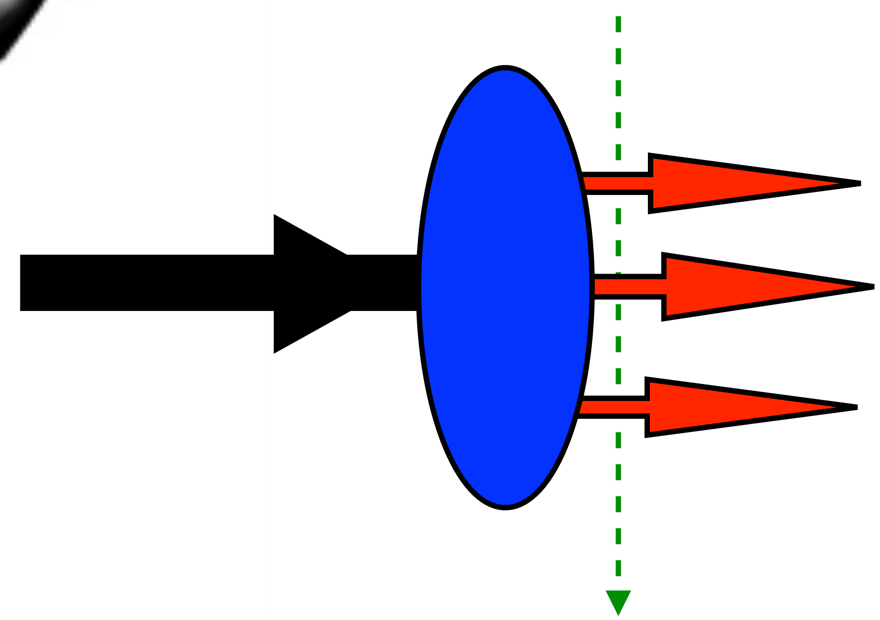
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*

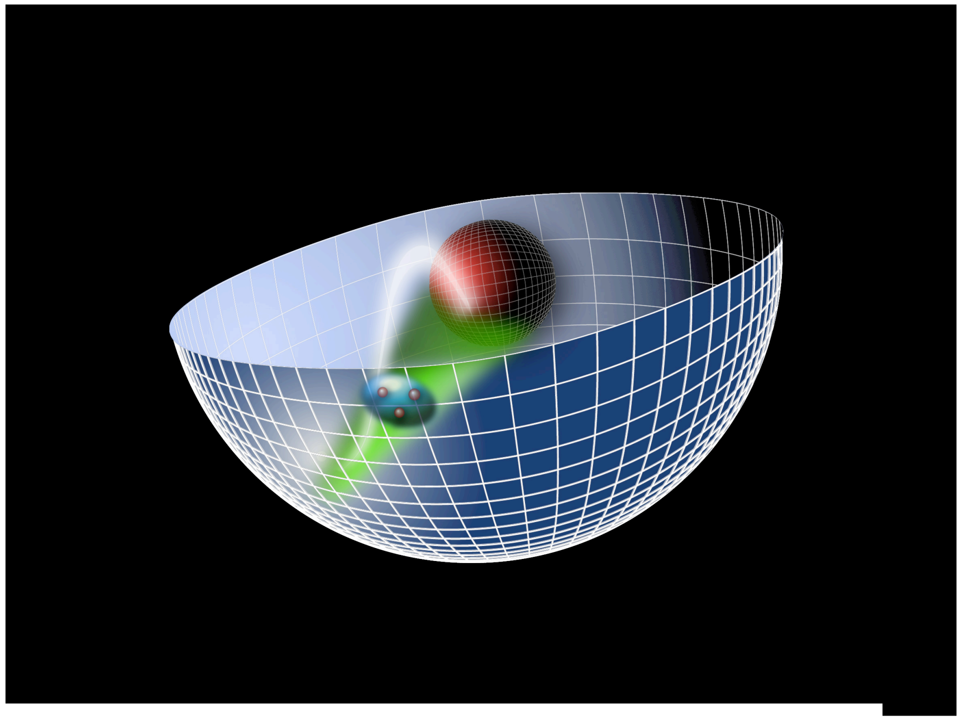
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

Supersymmetric Features of Hadron Physics and Properties of Quantum Chromodynamics from Light-Front Holography and Superconformal Algebra



Fixed $\tau = t + z/c$





Universidad Técnica Federico Santa María, Valparaiso, Chile
January 8-12, 2018

with Guy de Tèramond, Hans Günter Dosch, C. Lorcè, M. Nielsen, K. Chiu, R. S. Sufian, A. Deur

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

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