UNIVERSITY OF Southampton
School of Physics and Astronomy

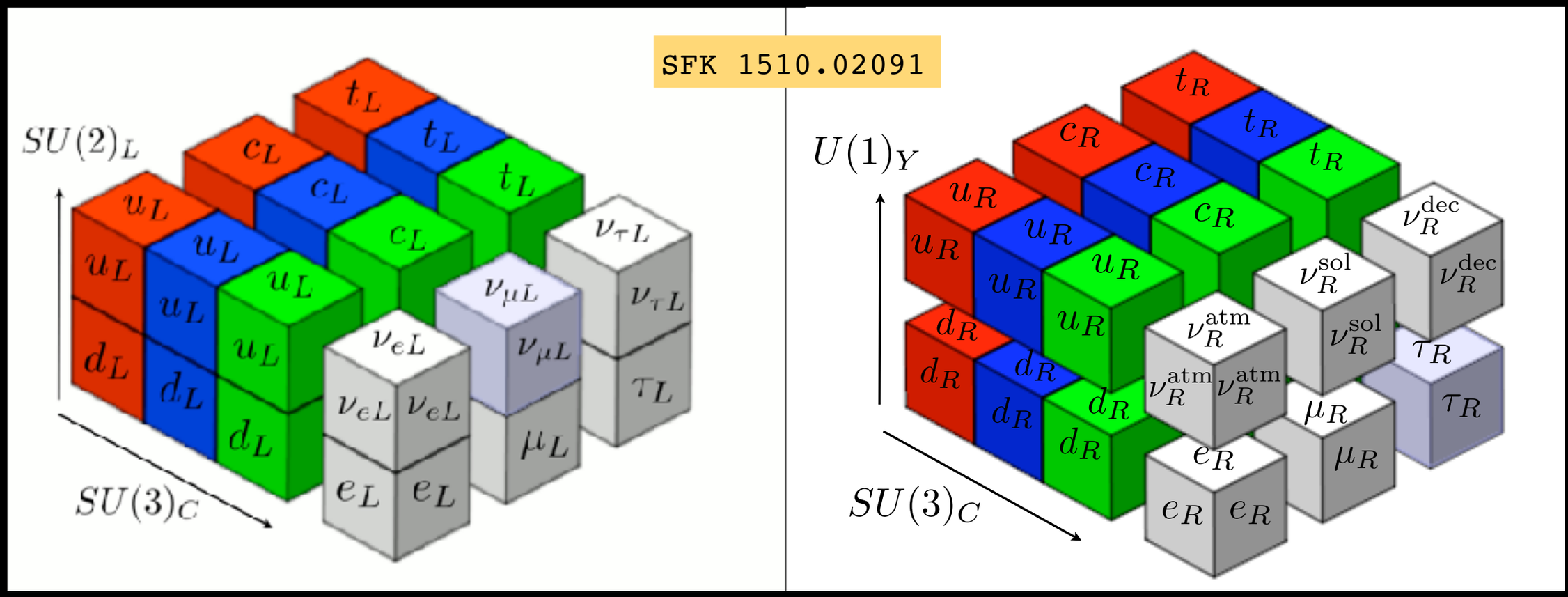
elusives invisiblesPlus
neutrinos, dark matter & dark energy physics

Grand Unified Theories of Flavour

Steve King

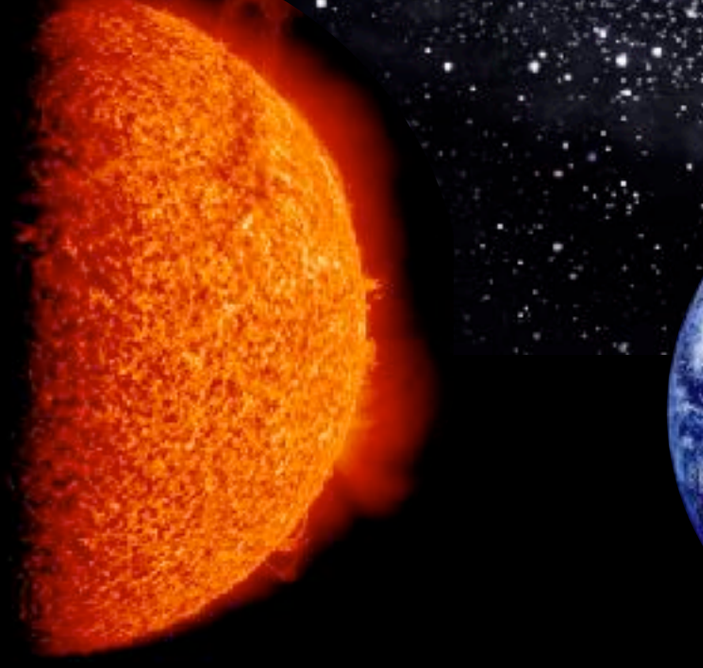
The Standard Model

SFK 1510.02091



leaves many questions unanswered...

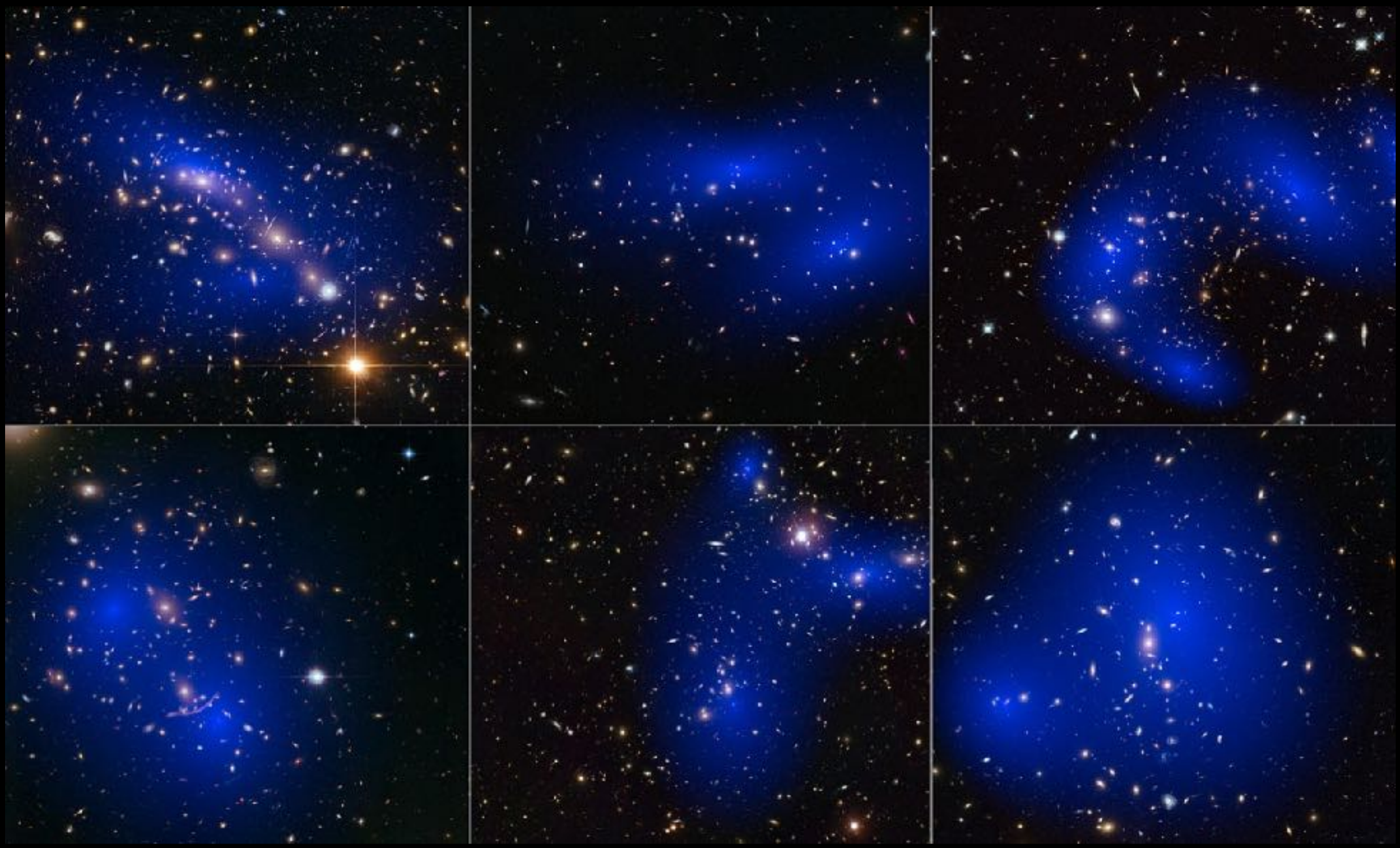
The baryon asymmetry of the Universe



$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} \approx 6 \times 10^{-10}$$

heic1506 — Science Release

Dark Matter



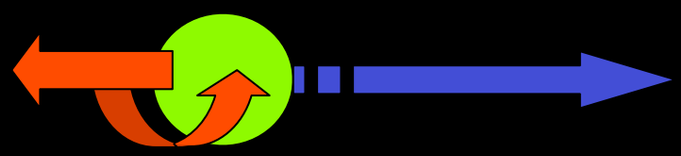
The origin of neutrino mass

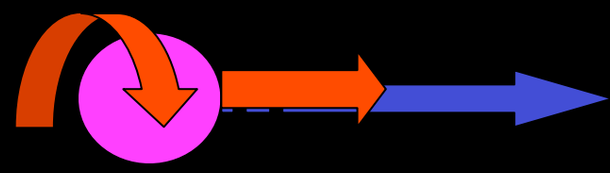
SFK 0712.1750

ν_L ν_R

Dirac

$m_{\rm D}\bar{\nu}_L\nu_R$

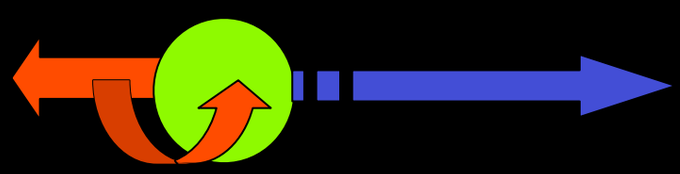


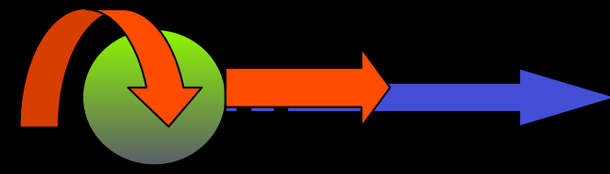


ν_L ν_L^c

Majorana

$m_{\rm L}\bar{\nu}_L\nu_L^c$

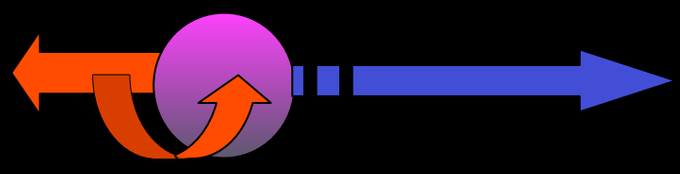


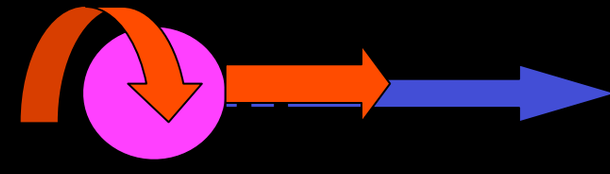


ν_R^c ν_R

Majorana

$M_{\rm R}\bar{\nu}_R^c\nu_R$





Seesaw mechanism

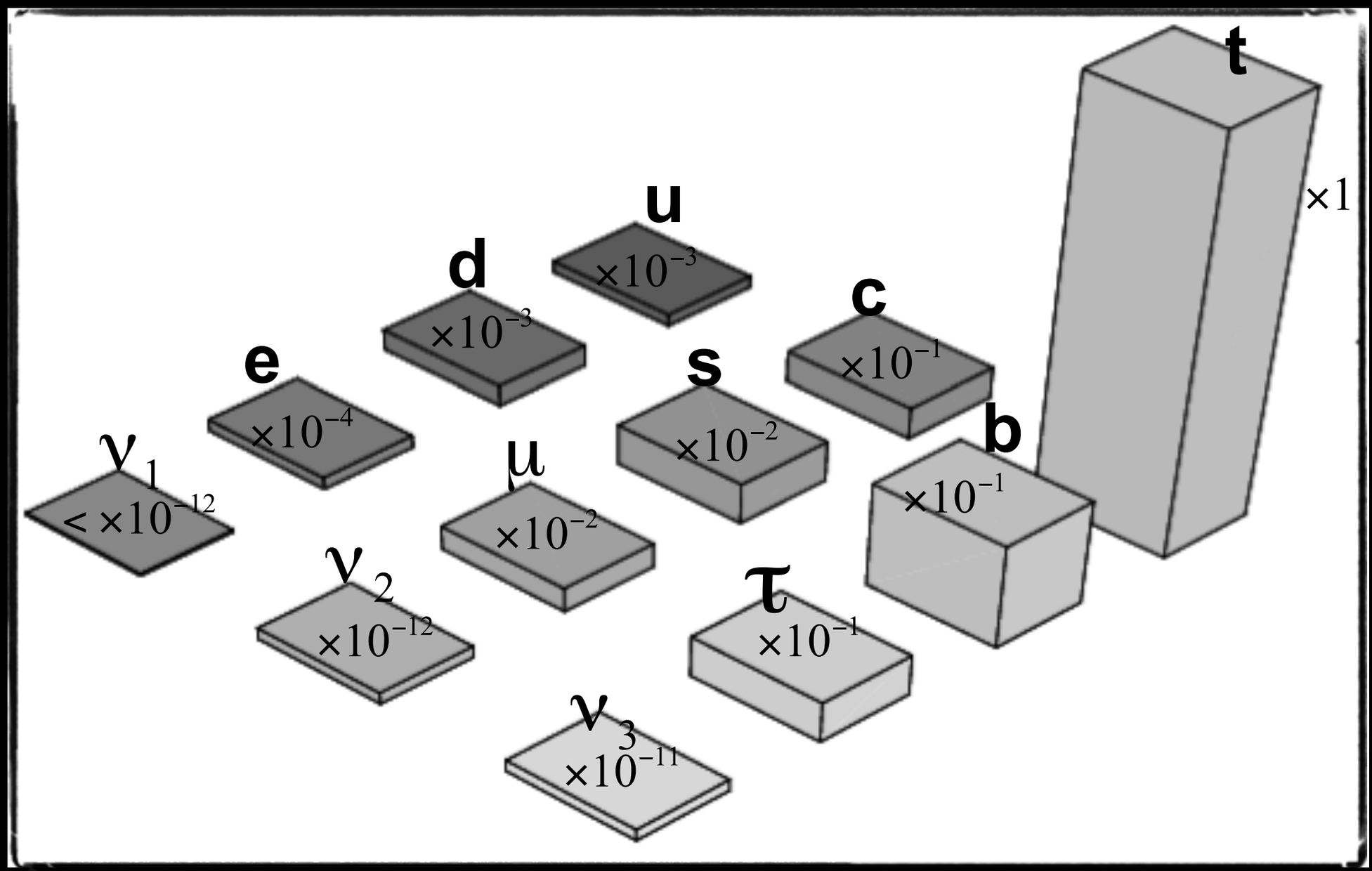
$$(\bar{\nu}_L \quad \bar{\nu}_R^c) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

$$M_R = \begin{pmatrix} M_{\rm atm} & 0 & 0 \\ 0 & M_{\rm sol} & 0 \\ 0 & 0 & [M_{\rm dec}] \end{pmatrix}$$

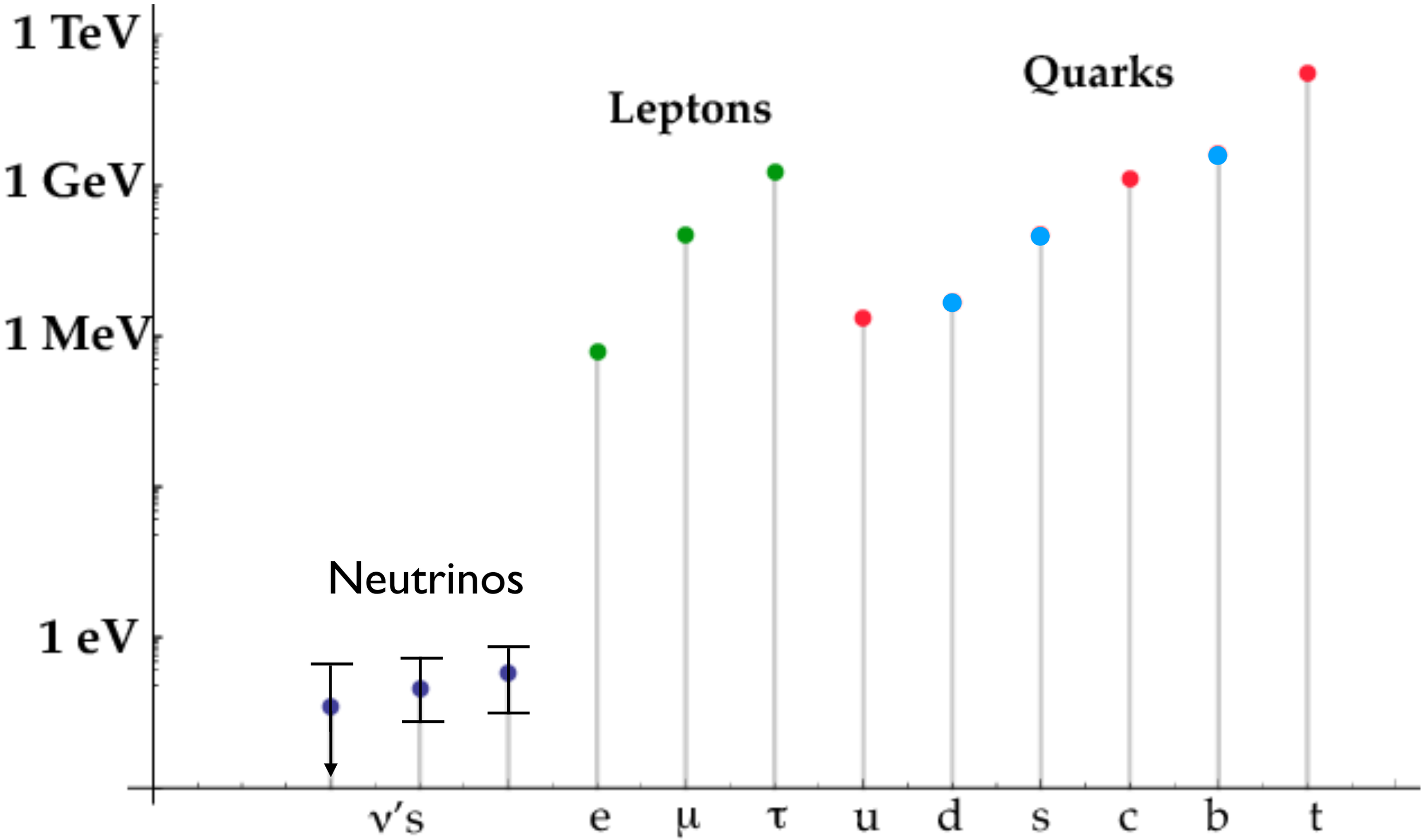
$$m^D = \begin{pmatrix} m^D_{e,\rm atm} & m^D_{e,\rm sol} & [m^D_{e,\rm dec}] \\ m^D_{\mu,\rm atm} & m^D_{\mu,\rm sol} & [m^D_{\mu,\rm dec}] \\ m^D_{\tau,\rm atm} & m^D_{\tau,\rm sol} & [m^D_{\tau,\rm dec}] \end{pmatrix}$$

$$m^\nu = m^D M_R^{-1} m^{D\,T} \longrightarrow \begin{pmatrix} m^\nu & 0 \\ 0 & M_R \end{pmatrix}$$

Flavour


t
×1
u
×10⁻³
d
×10⁻³
c
×10⁻¹
e
×10⁻⁴
s
×10⁻²
b
×10⁻¹
ν1
< ×10⁻¹²
μ
×10⁻²
ν2
×10⁻¹²
τ
×10⁻¹
ν3
×10⁻¹¹

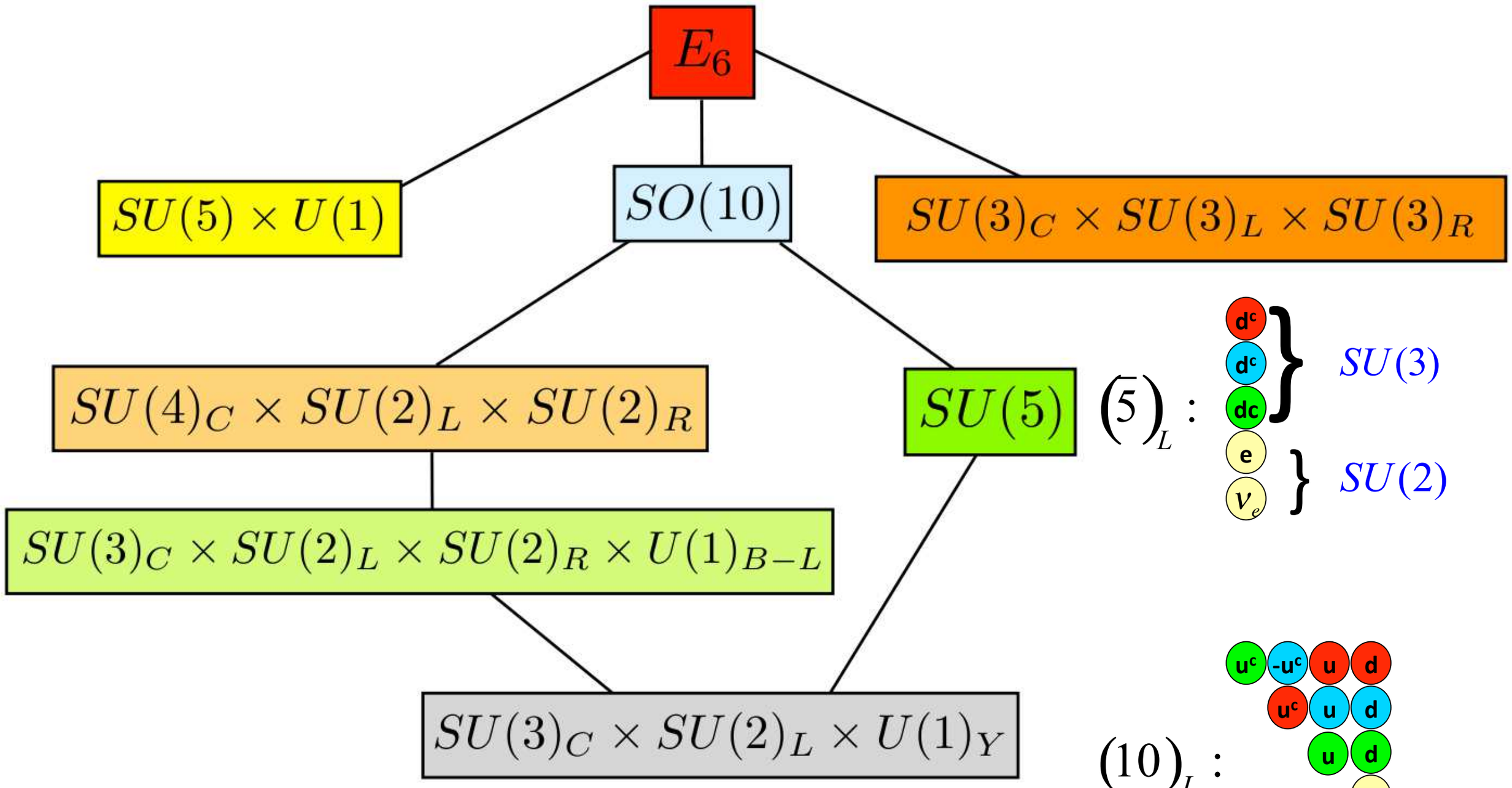
Masses



Mixing and CP

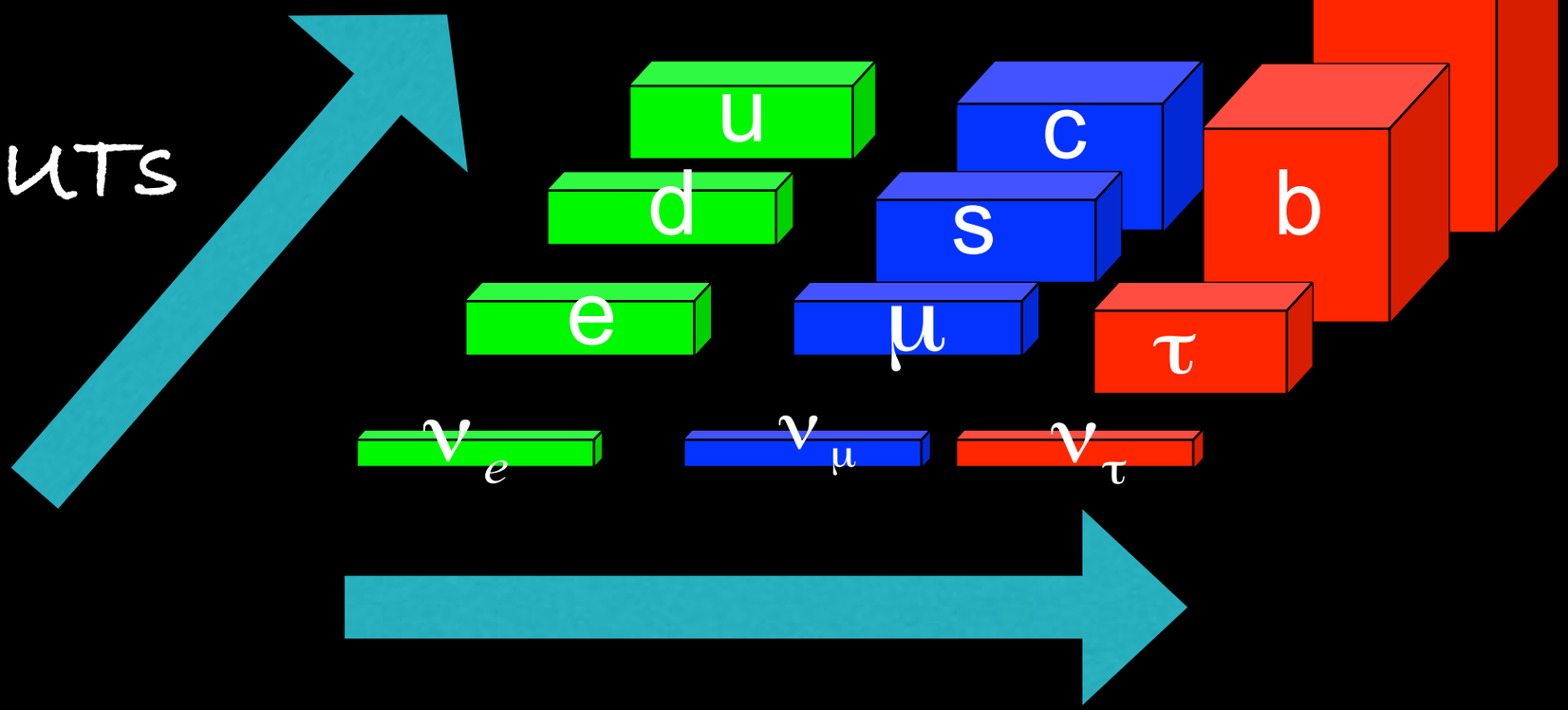
	θ_{12}	θ_{23}	θ_{13}	δ
Quarks	13° $\pm 0.1^\circ$	2.4° $\pm 0.1^\circ$	0.2° $\pm 0.05^\circ$	70° $\pm 5^\circ$
Leptons	34° $\pm 1^\circ$	45° $41^\circ \pm 1^\circ$ $50^\circ \pm 1^\circ$	8.5° $\pm 0.15^\circ$	-90° $\pm 50^\circ$

GUTs


E_6
$SU(5) \times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_C \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$
$(\bar{5})_L$:
d^c
d^c
dc
$SU(3)$
e
ν_e
$SU(2)$
$(10)_L$:
u^c
$-u^c$
u
d
u^c
u
d
u
d
e^c

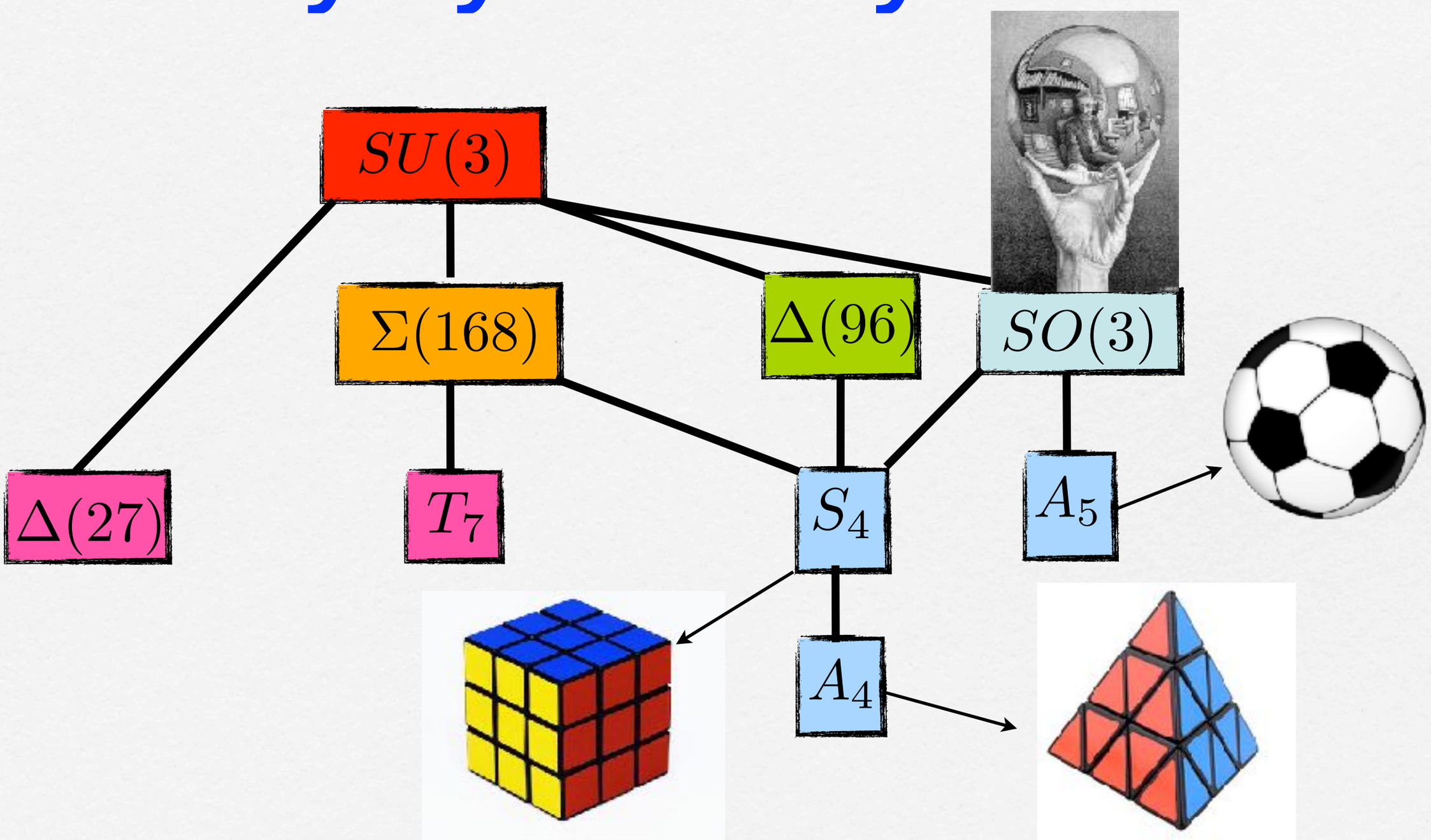
In this talk we consider

GUTs of Flavour



…as a possible answer to the above questions

Family Symmetry



Grand Unified Theories of Flavour

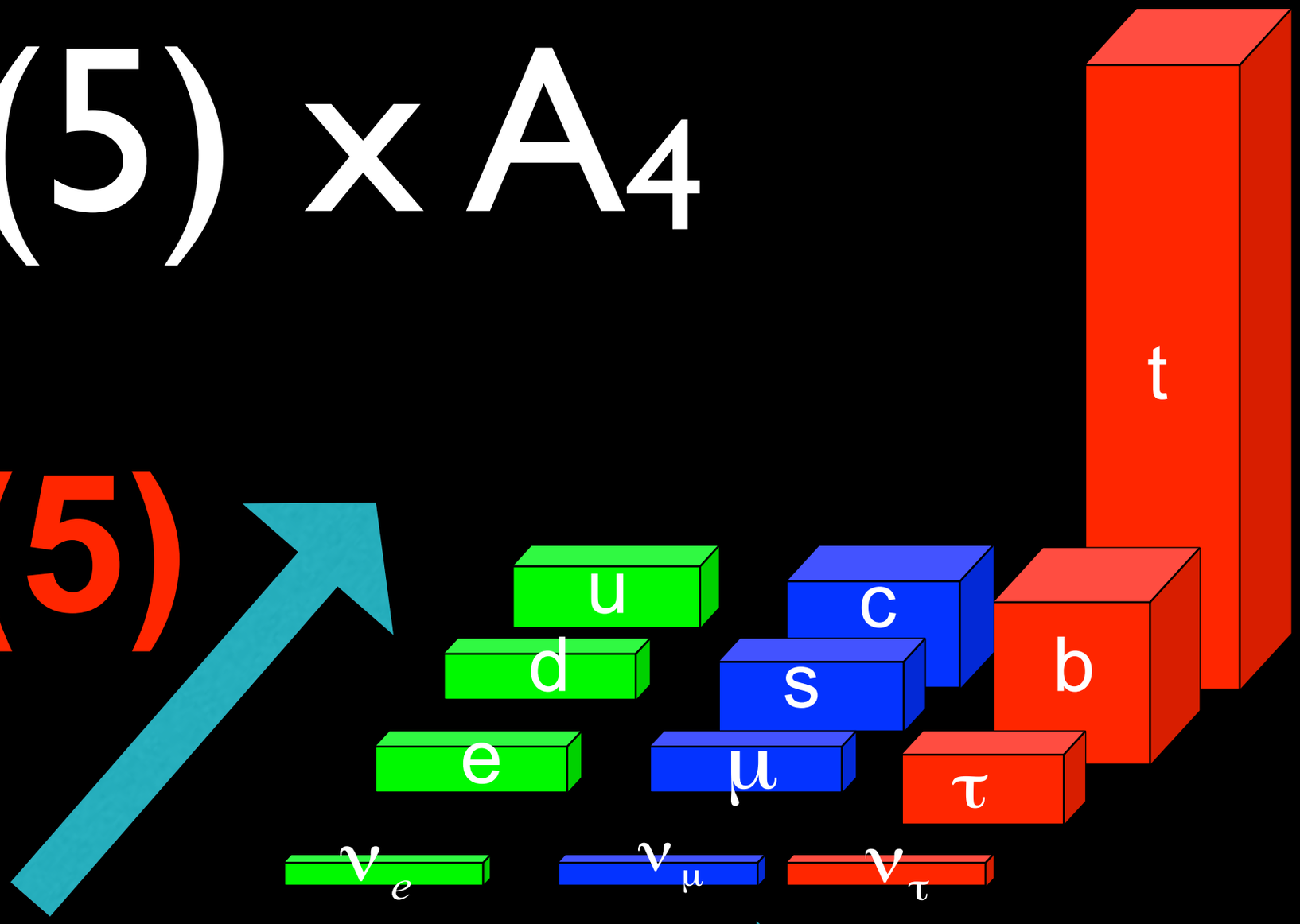
$G_{\rm GUT}$ / $G_{\rm FAM}$	$SU(2)_L \times U(1)_Y$	$SU(5)$	PS	$SO(10)$
S_3	[29]			[150]
A_4	[36, 49, 51, 62, 151–154]	[155–158]	[66, 159, 160]	
T'		[161]		
S_4	[31, 49, 51, 154, 163]	[164, 165]	[162]	[166]
A_5	[51, 169]	[170]		
T_7	[171, 172]			
$\Delta(27)$	[173]			[174]
$\Delta(96)$	[175, 176]	[177]		[178]
D_N	[179]			
Q_N	[180]			
other	[181]	[182]	[183]	

For references see review SFK 1701.04413

In this talk only one example:

$SU(5) \times A_4$

SU(5)



F ~ (3,5*)

T_i ~ (1,10)

A_4

Björkeroth, de Anda, de Medeiros Varzielas, SFK 1503.03306, 1505.05504

Field	Representation				
	A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
F	3	$\bar{5}$	0	0	1
T_1	1	10	5	0	1
T_2	1	10	7	0	1
T_3	1	10	0	0	1
N_1^c	1	1	7	3	1
N_2^c	1	1	8	3	1
Γ	1	1	0	3	1
H_5	1	5	0	0	0
$H_{\bar{5}}$	1	$\bar{5}$	2	0	0
H_{24}	$1'$	24	3	0	0
Λ_{24}	$1'$	24	0	0	0
H_{45}	1	45	4	0	2
$H_{\overline{45}}$	1	$\overline{45}$	5	0	0
ξ	1	1	2	0	0
θ_1	1	1	1	3	0
θ_2	1	1	1	4	0
ϕ_e	3	1	0	0	0
ϕ_μ	3	1	3	0	0
ϕ_τ	3	1	7	0	0
ϕ_1	3	1	3	2	0
ϕ_2	3	1	1	3	0
ϕ_3	3	1	3	1	0
ϕ_4	3	1	2	1	0
ϕ_5	3	1	6	2	0
ϕ_6	3	1	5	2	0

Quarks and Leptons

Higgs

Flavons

Fermion Messengers

Higgs Messengers

Field	Representation				
	A_4	SU(5)	$\mathbb{Z}_9$	$\mathbb{Z}_6$	$\mathbb{Z}_4^R$
X_1	1	$\bar{5}$	7	0	1
X_2	1	5	2	0	1
X_3	1	$\bar{5}$	6	0	1
X_4	1	5	3	0	1
X_5	$1''$	$\bar{5}$	3	0	1
X_6	$1'$	5	6	0	1
X_7	1	$\bar{5}$	2	0	1
X_8	$1''$	5	7	0	1
X_9	$1'$	$\bar{5}$	0	0	1
X_{10}	$1'$	5	0	0	1
X_{11}	1	$\bar{5}$	1	3	1
X_{12}	1	5	7	5	1
X_{13}	1	$\bar{5}$	2	3	1
X_{14}	1	5	6	5	1
Σ_1	1	$\bar{5}$	7	0	2
Σ_2	1	5	2	0	0
Σ_3	1	$\bar{5}$	5	0	2
Σ_4	1	5	4	0	0
Σ_5	1	$\bar{5}$	3	0	2
Σ_6	1	5	6	0	0
Σ_7	1	$\bar{5}$	1	0	2
Σ_8	1	5	8	0	0
Σ_9	1	$\bar{5}$	8	0	2
Σ_{10}	1	5	1	0	0
Σ_{11}	1	$\bar{5}$	6	0	2
Σ_{12}	1	5	3	0	0
Σ_{13}	1	$\bar{5}$	4	0	2
Σ_{14}	1	5	5	0	0
Σ_{15}	1	$\bar{5}$	2	0	2
Σ_{16}	1	5	7	0	0

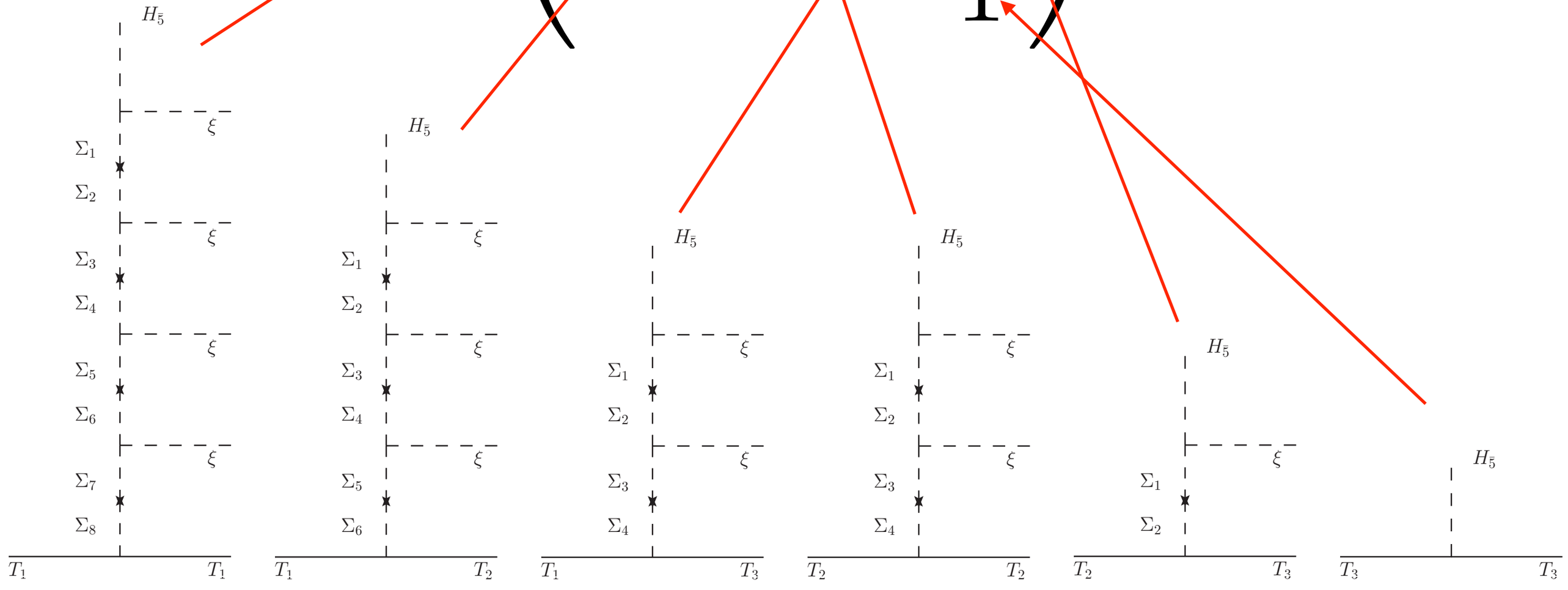
Up-type quarks

Froggatt-Nielsen

$$Y^u_{ij} \sim \begin{pmatrix} \tilde{\xi}^4 & \tilde{\xi}^3 & \tilde{\xi}^2 \\ \tilde{\xi}^2 & \tilde{\xi} & 1 \end{pmatrix}$$

$$\tilde{\xi} = \langle \xi \rangle / M \sim 0.1$$

With spontaneous CP violation all phases can be removed from the entire matrix



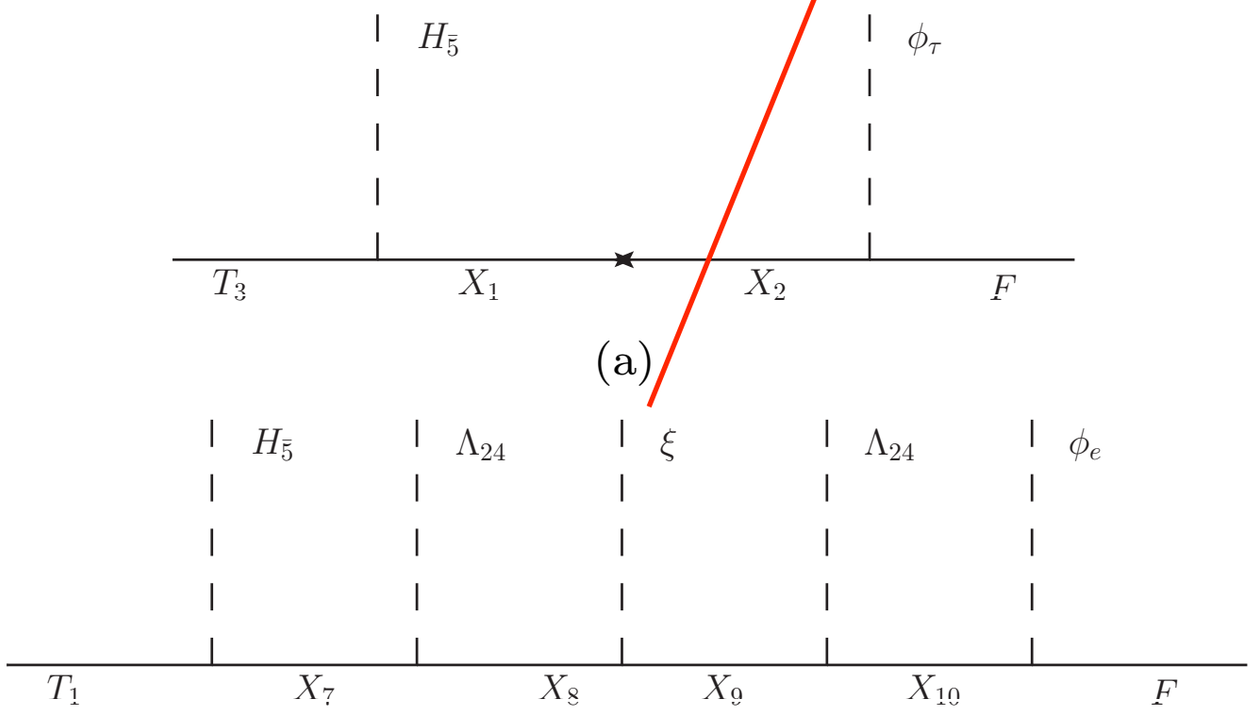
$$\langle \phi_e \rangle = v_e \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \langle \phi_\mu \rangle = v_\mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \langle \phi_\tau \rangle = v_\tau \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

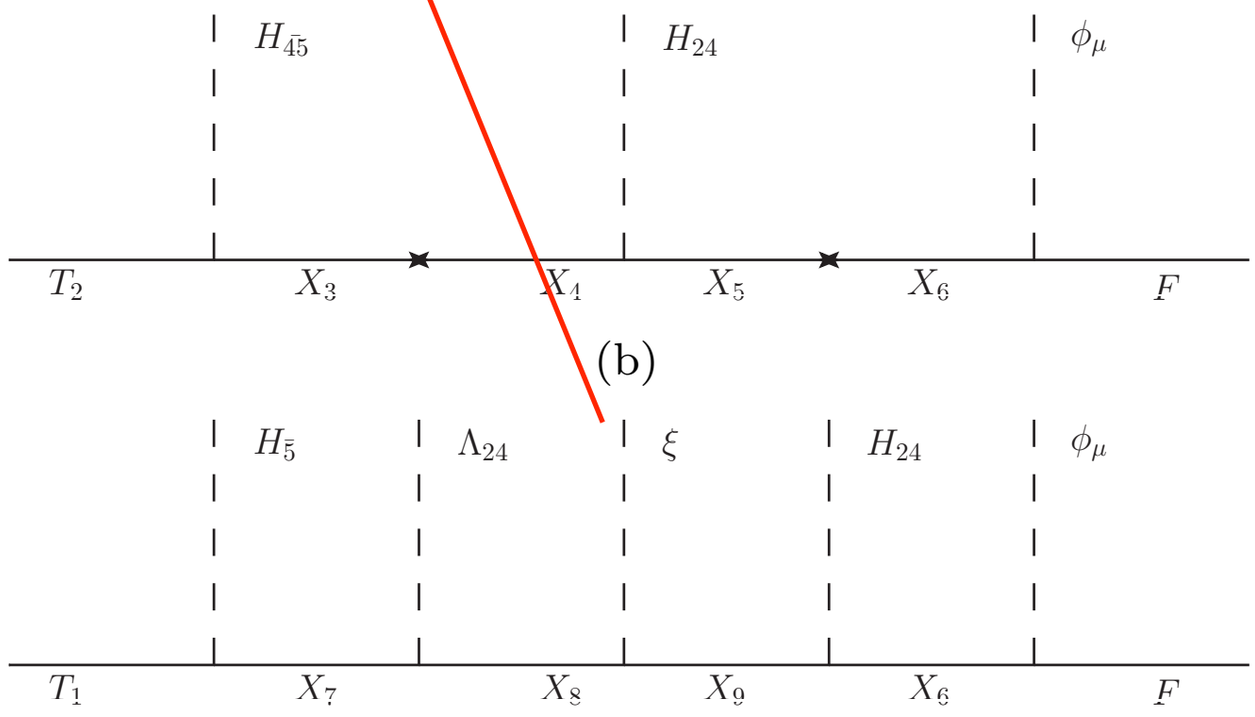
Down-type quarks

$$Y^d_{LR} \sim Y^e_{RL} \sim \begin{pmatrix} \dfrac{\langle \xi \rangle v_e}{v^2_{\Lambda_{24}}} & \dfrac{\langle \xi \rangle v_\mu}{v_{\Lambda_{24}} v_{H_{24}}} & 0 \\ 0 & \dfrac{v_{H_{24}} v_\mu}{M^2} & 0 \\ 0 & 0 & \dfrac{v_\tau}{M} \end{pmatrix}$$

Charged Leptons

With spontaneous CP violation all phases can be removed from the diagonal elements





Neutrinos with Littlest Seesaw

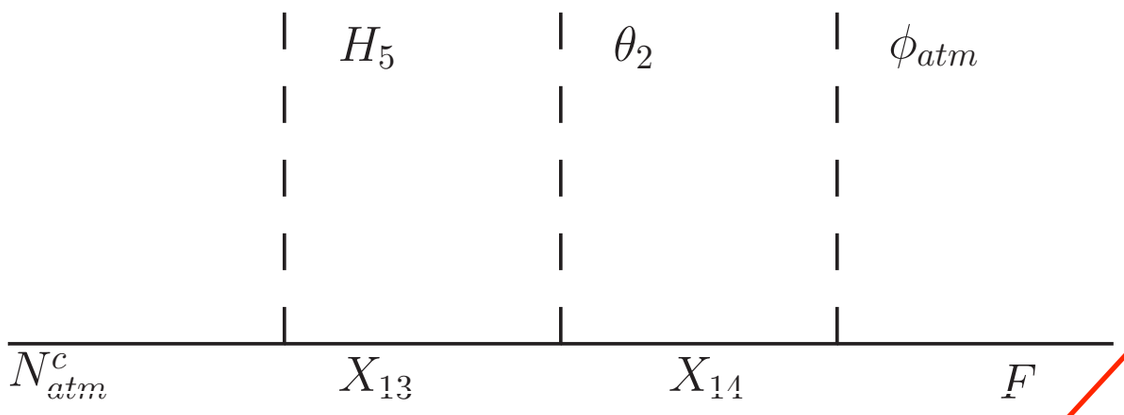
$$\langle \phi_{\rm atm} \rangle = v_{\rm atm} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\rm sol} \rangle = v_{\rm sol} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

Neutrino mass matrix involves only one relative phase

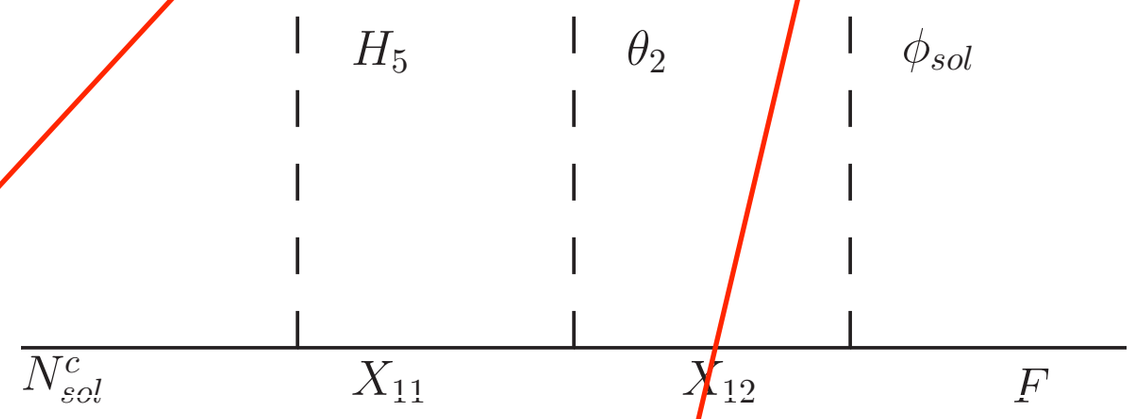
$$m_D = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}$$

Leptogenesis needs $M_{atm} \sim 4.10^{10}$ GeV

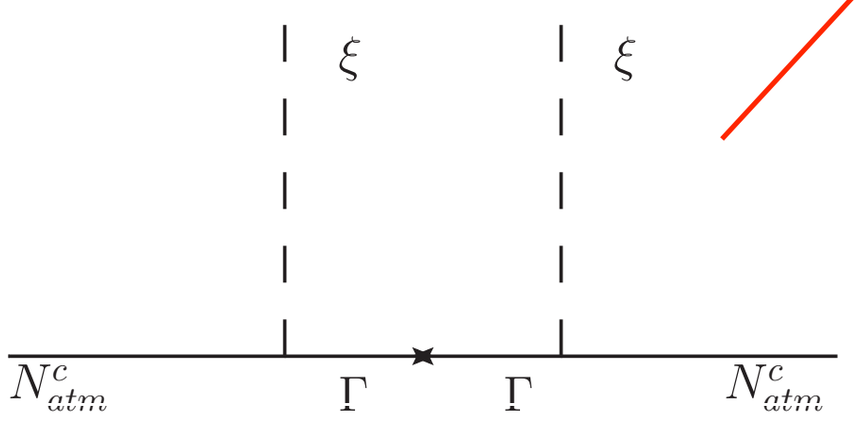
$$M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$



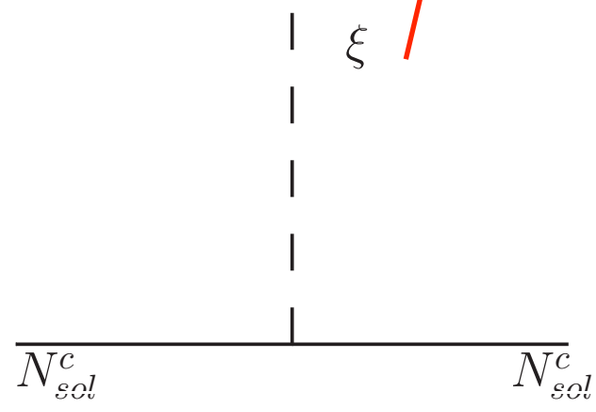
(a)



(b)



(c)



(d)

$$Y^u = \begin{pmatrix} u_{11}|\tilde{\xi}^4| & u_{12}|\tilde{\xi}^3| & u_{13}|\tilde{\xi}^2| \\ u_{12}|\tilde{\xi}^3| & u_{22}|\tilde{\xi}^2| & u_{23}|\tilde{\xi}| \\ u_{13}|\tilde{\xi}^2| & u_{23}|\tilde{\xi}| & u_{33} \end{pmatrix}$$

Up matrix has small mixing and no phases

$$Y^d = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{4} d_{11} \frac{|v_\xi v_e|}{|v_{\Lambda_{24}}|^2} & d_{12} \frac{|v_\xi v_\mu|}{|v_{\Lambda_{24}} v_{H_{24}}|} e^{i\zeta} & 0 \\ 0 & 2 d_{22} \frac{|v_{H_{24}} v_\mu|}{M^2} & 0 \\ 0 & 0 & d_{33} \frac{|v_\tau|}{M} \end{pmatrix}$$

Down matrix gives Cabibbo mixing and CP phase

Solves the strong CP problem: arg det ($Y^u Y^d$)=0

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$$m_a = \frac{|a|^2}{M_{atm}} \qquad m_b = \frac{|b|^2}{M_{sol}}$$

Littlest seesaw matrix controlled by ma, mb, eta

$$Y^e = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{9} d_{11} \frac{|v_\xi v_e|}{|v_{\Lambda_{24}}|^2} & 0 & 0 \\ d_{12} \frac{|v_\xi v_\mu|}{|v_{\Lambda_{24}} v_{H_{24}}|} e^{i\zeta} & 9 d_{22} \frac{|v_{H_{24}} v_\mu|}{M^2} & 0 \\ 0 & 0 & d_{33} \frac{|v_\tau|}{M}. \end{pmatrix}$$

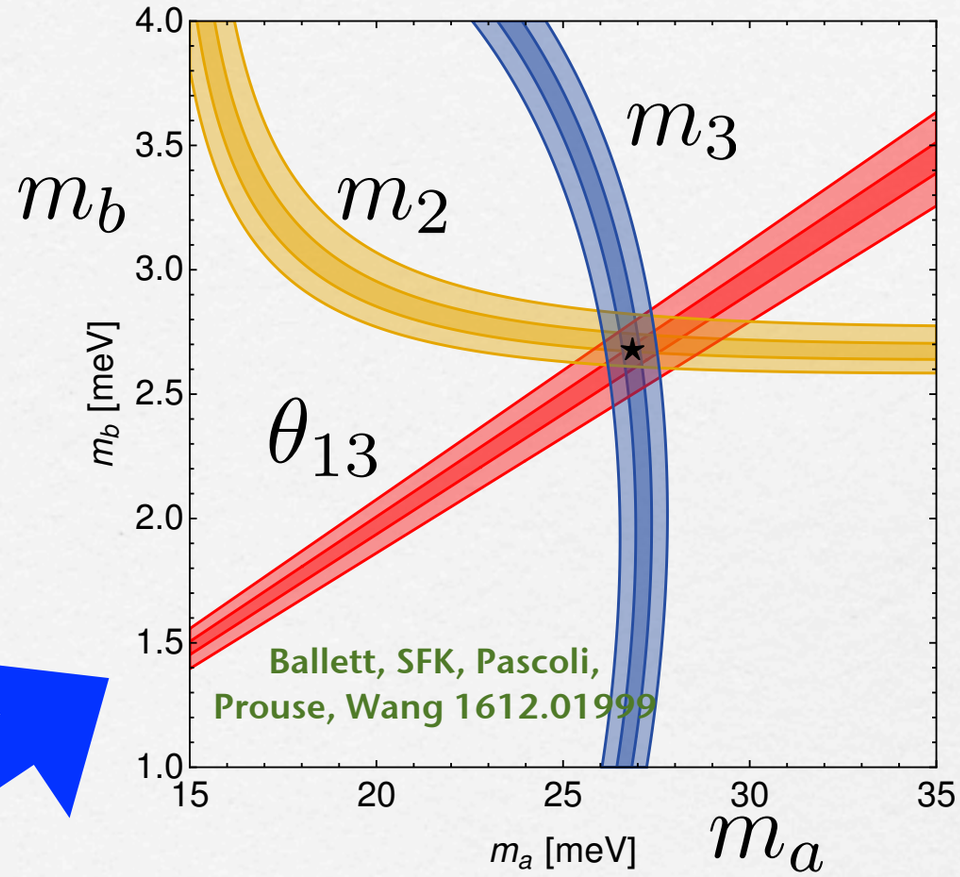
No LH charged lepton mixing to leading order

Littlest Seesaw Model

King

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i2\pi/3} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

Only two parameters now: m_a, m_b



Very predictive: 7 outputs!!

m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	$\delta_{\rm CP}$ (°)	m_1 (meV)	m_2 (meV)	m_3 (meV)
26.57	2.684	$\frac{2\pi}{3}$	34.3	8.67	45.8	-86.7	0	8.59	49.8
Value	from	[25]	$33.48^{+0.78}_{-0.75}$	$8.50^{+0.20}_{-0.21}$	$42.3^{+3.0}_{-1.6}$	-54^{+39}_{-70}	0	8.66±0.10	49.57±0.47

Summary of SU(5)x A_4

- Explains quark mass hierarchies, mixing angles and the CP phase
- Spontaneously broken CP, arg det YuYd=0 solves strong CP prob
- Z_9 flavour symmetry fixes the phase η to be 2pi/3
- Littlest Seesaw predictions (2 parameters give PMNS and m_i)
- Leptogenesis fixes $M_{atm} \sim 4.10^{10}$ GeV
- Renormalisable at GUT scale, SU(5)x A_4 breaking potential
- Vacuum alignments (1,0,0), (0,1,0), (0,0,1), (0,1,1), (1,3,1)
- Doublet-triplet splitting via the Missing Partner mechanism
- mu term is generated at the correct scale
- Proton decay is suppressed, TTTF forbidden by A_4
- The MSSM is reproduced with R-parity from discrete Z_4^R -> Z_2^R

The Minimal Realistic Version of the Supersymmetric Standard Model

M. Strassler 2011

Gluinos

squarks

2 Charginos
4 Neutralinos

5 higgs particles

sleptons/ sneutrinos

Higgs Field

Weak Nuclear Force
W^+ W^- Z^0

quarks

up-type

down-type

charged leptons

neutrinos

Electromagnetic Force
Photon

Strong Nuclear Force
Gluons (8 types)

Supersymmetry "Breaking"

LHC connection: SUSY SU(5)xA$_4$

SUSY breaking masses at M_{GUT}

$$m_F = m_{\tilde{D}_i^c} = m_{\tilde{L}_i} = m_{H_u} = m_{H_d}\,,$$
$$m_{T1} = m_{\tilde{Q}_1} = m_{\tilde{U}_1{}^c} = m_{\tilde{E}_1{}^c}\,,$$
$$m_{T2} = m_{\tilde{Q}_2} = m_{\tilde{U}_2{}^c} = m_{\tilde{E}_2{}^c}\,,$$
$$m_{T3} = m_{\tilde{Q}_3} = m_{\tilde{U}_3{}^c} = m_{\tilde{E}_3{}^c}\,.$$

To account for muon g-2

$$\Delta a_\mu \equiv a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = (28.8 \pm 8.0) \times 10^{-10}$$

and Dark Matter need non-universal gauginos

$$M_1 < M_2 \ll M_3$$

$$\frac{< F_\Phi >_{ij}}{M_{Planck}} \lambda_i \lambda_j$$

$$(24 \times 24)_{symm} = 1 + 24 + 75 + 200$$

Belyaev, King, Schaefers

	Benchmark:	BP4	BP5	BP6	
	$\tan\beta$	30	28	30	
	$\mathrm{sgn}(\mu)$	-	-	-	
Input at GUT scale	m_F	5000.0	6200.0	5000.0	[GeV]
	m_{T1}	5000.0	5700.0	5000.0	
	m_{T2}	200.0	280.0	200.0	
	m_{T3}	2995.0	5430.0	3005.0	
	M_1	250.0	250.0	250.0	
	M_2	400.0	550.0	500.0	
	M_3	2600.0	2945.0	2595.0	
	M_{h_1}	5000.0	6200.0	5000.0	
	M_{h_2}	5000.0	6200.0	5000.0	
	A_{tri}	-4000.0	-500.0	-4000.0	
Higgs	m_h	126.3	124.7	126.2	
	$m_{\tilde{\mu}_R}$	98.9	96.8	99.4	
Bino	$m_{\tilde{\chi}_1^0}$	91.7	89.0	92.0	
Higgsinos	$m_{\tilde{\chi}_2^0}$	266.9	303.3	302.2	
	$m_{\tilde{\chi}_3^0}$	-335.1	-327.8	-335.9	
Winos	$m_{\tilde{\chi}_4^0}$	376.8	458.9	430.4	
	$m_{\tilde{\chi}_1^\pm}$	267.4	303.7	302.8	
	$m_{\tilde{\chi}_2^\pm}$	378.2	459.0	430.7	
mu	μ	-313.0	-293.3	-314.3	
Constraints	$\mathrm{Br}(b\to s\gamma)$	3.43×10^{-4}	3.34×10^{-4}	3.43×10^{-4}	
	$\mathrm{Br}(B_s\to\mu^+\mu^-)$	3.01×10^{-9}	3.04×10^{-9}	3.01×10^{-9}	
	$\sigma^{\mathrm{DD\ SI}}$	6.72×10^{-11}	6.81×10^{-11}	6.52×10^{-11}	[pb]
	Ωh^2	9.67×10^{-2}	1.10×10^{-1}	1.03×10^{-1}	
	Δa_μ	2.17×10^{-9}	2.14×10^{-9}	2.16×10^{-9}	

Light m_{T2}

$M_1<M_2<<M_3$

Light RH smuon, neutralinos and charginos

All else >>TeV

Dark Matter

muon g-2

Belyaev, King, Schaefers

Charginos and neutralinos @ LHC

Quantity	Unit	Benchmark BP4	BP5	BP6
$r_{\max}$		0.88	0.12	0.13
$\sqrt{s}$	TeV	13	13	13
Analysis		ATLAS_CONF_2016_096	ATLAS_CONF_2016_096	ATLAS_CONF_2016_096
Signal Region		3LI	2LADF	3LI
Ref.		[68]	[68]	[68]
$\sigma_{\rm LO}$	pb	0.54	0.24	0.26
$\mathrm{BR}(\tilde{\chi}_2^0 \to h\,\tilde{\chi}_1^0)$	%	51.0	55.5	55.4
$\mathrm{BR}(\tilde{\chi}_2^0 \to Z\,\tilde{\chi}_1^0)$	%	30.5	30.2	30.1
$\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\mu}_R^\pm\,\mu^\mp)$	%	18.5	14.3	14.5
$\mathrm{BR}(\tilde{\chi}_1^\pm \to W^\pm\,\tilde{\chi}_1^0)$	%	99.4	99.5	99.5
$\mathrm{BR}(\tilde{\chi}_1^\pm \to \tilde{\mu}_R^\pm\,\nu_\mu)$	%	0.6	0.5	0.5
$\Delta m(\tilde{\chi}_1^\pm, \tilde{\mu}_R)$	GeV	168.5	207.0	203.4
$\Delta m(\tilde{\chi}_2^0, \tilde{\mu}_R)$	GeV	168.0	206.5	202.7
$\Delta m(\tilde{\mu}_R, \tilde{\chi}_1^0)$	GeV	7.2	7.8	7.5

Table 6: CheckMATE analysis results for the benchmarks of table 5 with full gaugino non-universality $M_1 < M_2 \ll M_3$.

Conclusion

- The Standard Model leaves many questions: BAU, Dark Matter, nu mass, flavour, unification...
- Grand Unified Theories of Flavour can solve them
- Supersymmetry needed for hierarchy, unification, Dark Matter, muon g-2, leads to LHC predictions