NNLO classical solutions for Lipatov's effective action for reggeized gluons

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Effective action

The Lipatov effective action is gauge invariant and written in the covariant form in terms of gluon field v as

$$S_{eff} = -\int d^4 x \left(\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \right.$$
$$\left. + \frac{1}{N} tr \left[(A_+(v_+) - A_+) \partial_i^2 A^+_a + (A_-(v_-) - A_-) \partial_i^2 A^-_a \right] - L_{quark} \right), \quad (1)$$

where

$$A_\pm(v_\pm) = \frac{1}{g} \partial_\pm O(x^\pm, v_\pm); \;\; O(x^\pm, v_\pm) = P\, e^{g \int_{-\infty}^{x^\pm} dx'^\pm v_\pm}$$

and

$$L_{quark} = \bar{\psi}(i\gamma^\nu \partial_\nu - m - ig\gamma^\nu T^a v_{a\nu})\psi$$

There are additional kinematical constraints for the reggeon fields

$$\partial_- A_+ = \partial_+ A_- = 0\,,$$

[1] L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995); Phys. Rept. **286**, (1997) 131;

We consider the inclusion of quarks separately later. In the framework with an external source of the color charge introduced, keeping only gluon field depending terms in the Eq. (1) action, we rewrite Eq. (1) as

$$S_{eff} = -\int d^4 x \left(\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + v_- J^-(v_-) + v_+ J^+(v_+) \right) ,$$

Under variation on the gluon fields these currents reproduce the Lipatov's induced currents

$$\delta \left(v_\pm J^\pm(v_\pm) \right) = (\delta v_\pm) j^{ind}_\mp(v_\pm) = (\delta v_\pm) j^\pm(v_\pm) ,$$

The classical equations of motion for the gluon field v_μ which arose from the Eq. (**??**) action are the following:

$$(D_\mu G^{\mu\nu})_a = \partial_\mu G^{\mu\nu}_a + g f_{abc} v^b_\mu G^{c\,\mu\nu} = j^+_a \delta^{\nu+} + j^-_a \delta^{\nu-} \qquad (2)$$

The light-cone gauge $v^a_- = 0$. The topic of this report is the search of classical solutions for gluon fields in the form of an expansion in powers of the coupling constant g:

$$v^a_+ = \sum_{k=0}^{+\infty} g^k v^a_{+k}(A_+, A_-); \quad v^{ia} = \sum_{k=0}^{+\infty} g^k v^{ia}_k(A_+, A_-)$$

Considering Eq. (2) we obtain the following equations for the different field components. We see that from the 4 equations of motion we have obtained 3 unknown fields under investigation. The last equation is transformed into the transversality condition of the effective vertex.

$$-\partial_-[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^+_{k-1}$$

$$\Box v^j_k - \partial^j[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^j_{k-1}$$

$$\Box v_{+k} - \partial_+[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^-_{k-1}$$

Where $\bar{j}^+_{k-1}$, $\bar{j}^-_{k-1}$, $\bar{j}^i_{k-1}$ are functions of $v^\nu_{k-1}(A_+, A_-)$, ..., $v^\nu_0(A_+, A_-)$, A_+ and A_-.

We have

$$-\partial_-[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}^+_{k-1}$$

$$\Box v_k^j - \partial^j[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}^j_{k-1}$$

$$\Box v_{+k} - \partial_+[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}^-_{k-1}$$

The condition is equivalent to the consistency of the solutions

$$\partial_\mu \bar{j}^\mu_{k-1} = \Box[\partial_i v_k^i + \partial_- v_{+k}] - \Box[\partial_i v_k^i + \partial_- v_{+k}] = 0$$

Classical solutions for v_k when performing $\partial_\mu \bar{j}^\mu_{k-1} = 0$ are self-consistent and have the form

$$v_k^j = \Box^{-1}\left[\bar{j}^j_{k-1} - \partial^j \partial_-^{-1} \bar{j}^+_{k-1}\right],$$

$$v_{+k} = \Box^{-1}\left[\bar{j}^-_{k-1} - \partial_+ \partial_-^{-1} \bar{j}^+_{k-1}\right]$$

Inserting obtained classical gluon fields solutions in the S_{eff} action, we will obtain a action which will depend only on the reggeon fields, see [1], determining the LO RFT action of the approach. Formally, due to the presence of ordered path exponential in the solutions, the action will include all orders of perturbative terms which can be important for large $v_{+} \approx A_{+}$ in the processes where some large color charge is created. The expansion of these exponential must be supplemented by solution of equations of motion to corresponding orders, otherwise only part of the usual perturbative corrections will be accounted.
In general, the following expansion for the action exists:

$$S_{eff} = -\int d^4 x \left(s_1[g, A_{+}, A_{-}] + g\, s_2[g, A_{+}, A_{-}] + \cdots \right) .$$

The result of this action is that the effective action can be expanded in terms of reggeon fields A_- and A_+ as

$$\Gamma = \sum_{n,m=0} \left(A_+^{a_1} \cdots A_+^{a_n} K_{b_1 \cdots b_m}^{a_1 \cdots a_n} A_-^{b_1} \cdots A_-^{b_m} \right) ,$$

that determines this expression as functional of reggeon fields and provides effective vertices of the interactions of the reggeized gluons in the RFT calculus.

Substituting the LO of classical solutions into action gives us the full LO value of the kernel only for $n + m = 2$ and only a fraction of the contributions for more complex vertices.
NLO of classical solutions allows us to get a complete answer for $n + m = 3$.
With NNLO of classical solutions , we moved on to 4-reggeon interactions.

Results

[2] S. Bondarenko, L. Lipatov and A. Prygarin, Eur. Phys. J. C **77** (2017) no.8, 527.

$$v_{+0\ a} = A_{+\ a},$$

$$v_{i0\ a} = \rho_i^b(x_\perp, x^-)U_{ab},$$

$$v_{+1\ a} = -\frac{2}{g}\Box^{-1}\left((\partial_+\partial^i U_{ab})\rho_i^b\right),$$

$$v_{i1\ a} = -\Box^{-1}\left[\partial^j F_{ji\ a} + \frac{1}{g}\partial_i\left((\partial^j U_{ab})\rho_j^b\right) - \partial_i^{-1} j_{a1}^{+}\right].$$

$$U^{ab}(v_+) = -tr\left[T^a\left(Pe^{g\int_{-\infty}^{x^+} dx'^+ v_{+c}(x'^+,x^-,x_\perp)T^c}\right)T^b\left(Pe^{-g\int_{x^+}^{+\infty} dx'^+ v_{+d}(x'^+,x^-,x_\perp)T^d}\right)\right].$$

The new result is NNLO classical solution:

$$
\begin{aligned}
v_{+2\ a} = \Box^{-1}\partial_-^{-1}\Big[\frac{2}{g}\partial_+ j^+_{a1}
&+ f_{abc}\Big(2\partial_- v^b_{i0}\partial_+ v_1^{ic} + (\partial_i v_0^{i\ c})\partial_- v^b_{1+} + 2\partial_i(A^b_+\partial_- v_1^{i\ c})\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
v^i_{2a} = \Box^{-1}\Big[\partial_-^{-1}\partial^i\Big(L^+_{a2} - (\partial^j\rho^b_j)(\frac{1}{g^2}\partial_- U_{ab})\Big)
&- f_{abc}\Big(\frac{1}{g}v^b_{j0}\Big(\partial^j v_0^{i\ c} - \partial^i v_0^{j\ c}\Big) + v^b_{0j}(\partial^j v_1^{i\ c} - \partial^i v_1^{j\ c})\\
&- v_0^{i\ c} v^b_{1+} + 2A^b_+ v_1^{i\ c} +\\
&+ \partial_j(v_0^{j\ b} v_1^{i\ c} - v_0^{i\ b} v_1^{j\ c}) + f^{cde} v^b_{j0} v^j_{0d} v^i_{0e}\Big)\Big].
\end{aligned}
$$

Extending by including the quarks

We have added a part of the Lagrangian L_{quark}. We find only one correction of the order g^2 and ε^2. This addition will allow us to take into account the contribution with a quark loop in the gluon. Integrating out the fluctuation we obtain the following expression for the contribution to the action:

$$-\frac{g^2}{4} tr\Big(G_q^0(y,x)(\gamma^\nu v_{a\nu}(x))G_q^0(x,y)(\gamma^\mu v_\mu^a(y))\Big),$$

where

$$G_q^0(x,y) = (i\gamma_\nu \partial_x^\nu + m)\Delta_{xy}, \quad \Delta_{xy} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - m^2 + i0}$$

Then after varying by $v_{a\rho}(z)$ we obtain the following contributions to the equation of motion

$$j^\rho_{quark\ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G_q^0(z,y)\gamma^\mu v_{a\ \mu}(y) G_q^0(y,z)\Big)$$

We verified, that condition $\partial_\rho j^\rho_{quark\ a} = 0$ is true. Then the classical solutions have additional contributions v^i_{q2} and v_{q+2}, respectively:

$$v^i_{q2a} = \Box^{-1}\left[j^i_{quark\ 0a} - \partial^i \partial_-^{-1} j^+_{quark\ 0a}\right],$$

$$v_{q+2a} = \Box^{-1}\left[j^-_{quark\ 0a} - \partial_+ \partial_-^{-1} j^+_{quark\ 0a}\right],$$

where

$$j^\rho_{quark\ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G^0_q(z,y)\gamma^\mu v_{a\ \mu}(y) G^0_q(y,z)\Big)$$

We want to estimate the contribution of quarks to the reggeon propagator

Inserting obtained classical gluon fields solutions in the Eq. (1) action, we will obtain a action which will depend only on the reggeon fields.
In order to calculate this action we need to know the components of the field strength tensor, with LO precision we have:

$$G^{a}_{+\,-\,\,0} = 0\,,\;\; G^{a}_{i\,+\,\,0} = \partial_i\, A^{a}_{+}\,,\;\; G^{a}_{i\,-\,\,0} = -\,\partial_-\, v^{a}_{i0}\,,\;\; G_{i\,j\,\,0} = 0\,,$$

and components of the field strength tensor with included quarks

$$G^{a}_{+\,-\,\,q2} = 0\,,\;\; G^{a}_{i\,+\,\,q2} = g^2\,\partial_i\, v^{a}_{q+2}\,,\;\; G^{a}_{i\,-\,\,q2} = -\,g^2\partial_-\, v^{a}_{qi2}\,,\;\; G_{i\,j\,\,q2} = 0\,,$$

that gives

$$\frac{1}{2}\, G_{\mu\,\nu\,\,0}\, G^{\mu\,\nu}_{q2} = g^2\,\left(\partial_-\, v^{a}_{i0}\right)\left(\partial_i\, v^{a}_{q+2}\right) + g^2\,\left(\partial_-\, v^{a}_{qi2}\right)\left(\partial_i\, A^{a}_{+}\right)\,.$$

Therefore, for the additional contributions to the effective action from the inclusion of quarks we obtain to LO:

$$S_{eff\ q2} = -\frac{g^2}{N} \int d^4x \Big[\partial_i \left(v^a_{q+2} \partial_i A^b_- \right) + N \left(\partial_- \left(v^b_{qi2} \partial_i A^a_+ \right) - v^b_{qi2} \left(\partial_i \partial_- A^a_+ \right) \right) \Big] = 0,$$

In addition, it is a direct contribution to the effective action:

$$-\frac{g^2}{4} \left(\frac{\delta^2\, tr \Big(G^0_q(y,x)(\gamma^\nu v_{a\nu}(x)) G^0_q(x,y)(\gamma^\mu v^a_\mu(y)) \Big)}{\delta A^a_{+\,x}\, \delta A^b_{-\,y}} \right)_{A_+,\, A_- = 0} .$$

After the calculations, we showed that this contribution is also zero.

Conclusion

1. Classical solutions of equations of motion in the effective action are written with precision NNLO
2. These results will allow us to calculate total interactions up to 4 reggeons and part of the contribution to a larger number of participants. They may also be of interest for use in the CGC and the Balitsky approach
3. Classical solutions of all orders have a common simple structure
4. We studied the inclusion of quarks in action. The contribution to the reggeon propogator was equal to zero on g^2, as expected.

1. S. Bondarenko, L. Lipatov and A. Prygarin, Eur. Phys. J. C 77 (2017) no.8, 527.
2. S. Bondarenko, L. Lipatov, S. Pozdnyakov, A. Prygarin, Eur. Phys. J. C 77 (2017) no.9, 630.
3. S. Bondarenko, S. Pozdnyakov "NNLO classical solutions for Lipatov's effective action for reggeized gluons" - work in progress.

Thank you for your attention