To the memory of L.N.Lipatov (1940-2017)

The picture is from M.Shifman's live journal.

On Lipatov's effective action for reggeized gluons

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Based on:
S.Bondarenko, L.Lipatov and A.Prygarin,
Eur. Phys. J. C **77**, no. 8, 527 (2017) arXiv:1706.00278 (hep-ph);
S. Bondarenko, L. Lipatov, S. Pozdnyakov and A. Prygarin,
Eur. Phys. J. C **77**, no. 9, 630 (2017), arXiv:1708.05183 (hep-th).

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Effective action setup

- Construction of the Gauge-invariant action for the gluon-reggeon interactions local in rapidity interval $(y_0 - \eta, y_0 + \eta)$ similar to the Gribov's RFT for the calculation of the production amplitude in the multi-Regge kinematics.

  L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995); Phys. Rept. **286**, (1997) 131.

$$S_{eff} = -\int d^4 x \left(\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + tr \left[\left(A_+(\mathsf{v}_+) - A_+ \right) j^+_{reg} + \left(A_-(\mathsf{v}_-) - A_- \right) j^-_{reg} \right] \right)$$

$$A_\pm(\mathsf{v}_\pm) = \frac{1}{g} \partial_\pm O(x^\pm, \mathsf{v}_\pm) = \mathsf{v}_\pm O(x^\pm, \mathsf{v}_\pm) \,,$$

$$j^\pm_{reg\, a} = \frac{1}{C(R)} \partial_i^2 A^\pm_a \,, \;\; \partial_- A_+ = \partial_+ A_- = 0 \,,$$

where $C(R)$ is the eigenvalue of Casimir operator in the representation R, $C(R) = N$ in the case of adjoint representation

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Effective action: diagrammatic approach

- In simplest case:

$$O_x = P\, e^{g \int_{-\infty}^{x^+} dx'^+ \, v_+(x'^+)}, \quad O_x^T = P\, e^{g \int_{x^+}^{\infty} dx'^+ \, v_+(x'^+)}.$$

are usual ordered exponentials;

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- correspondingly there are new gluon-reggeon vertices in the action which can be used for the diagrammatic construction of the amplitudes with gluon and reggeon fields involved.

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- Publications:

L. N. Lipatov, Nucl. Phys. Proc. Suppl. **99A**, 175 (2001); M. A. Braun and M. I. Vyazovsky, Eur. Phys. J. C **51**, 103 (2007); M. A. Braun, L. N. Lipatov, M. Y. Salykin and M. I. Vyazovsky, Eur. Phys. J. C **71**, 1639 (2011); M. Hentschinski and A. Sabio Vera, Phys. Rev. D **85**, 056006 (2012); G. Chachamis, M. Hentschinski, J. D. Madrigal Martnez and A. Sabio Vera and etc.

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Effective action approach

- Another form of the action:

$$S_{eff} = - \int d^4 x \left(\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + tr \left[\mathsf{v}_+ J^+(\mathsf{v}_+) - A_+ j^+_{reg} + \mathsf{v}_- J^-(\mathsf{v}_-) - A_- j^-_{reg} \right] \right)$$

$$J^{\pm}_a(\mathsf{v}_{\pm}) = \frac{1}{C(R)} O(x^{\pm}, \mathsf{v}_{\pm}) \, \partial_i^2 A^{\pm}_a$$

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$$J^{\pm}_a(\mathrm{v}_\pm) = \frac{1}{C(R)} O(x^{\pm}, \mathrm{v}_\pm)\, \partial_i^2 A_a^{\pm}$$

- How we can interpret the action? Consider, for example, the transition amplitude of two t-channel "gluons" A_-, A_+ in the finite (small) interval of rapidity:

$$M_{-+} \propto \left(A_- \vec{\partial}^{\,2}_{\perp} \right) G(\mathrm{v}_{cl}) \left(\overleftarrow{\partial}^{\,2}_{\perp} A_+ \right),$$

where, correspondingly, there are the following boundary conditions to LO precision for the interacting classical gluon fields:

$$\mathrm{v}_{+\,cl}(y_0 + \eta) = A_+ , \quad \mathrm{v}_{-\,cl}(y_0 - \eta) = A_-$$

Effective action approach

- We consider the Lipatov's effective action as gauge invariant non-linear action which describes interaction of the gluons with Reggeons (new degrees of freedom).

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Effective action approach

- We consider the Lipatov's effective action as gauge invariant non-linear action which describes interaction of the gluons with Reggeons (new degrees of freedom).

- How we can independently find the form of $J_a^{\pm}(v_{\pm})$ currents? We assume for this current that under a variation on the gluon fields these currents reproduce the Lipatov's induced currents

$$\delta\left(v_{\pm}\,J^{\pm}(v_{\pm})\right)\,=\,(\delta\,v_{\pm})\,j_{\mp}^{ind}(v_{\pm})\,=\,(\delta\,v_{\pm})\,j^{\pm}(v_{\pm})$$

which posseses a covariant conservation property:

$$\left(D_{\pm}\,j_{\mp}^{ind}(v_{\pm})\right)^{a}\,=\,\left(D_{\pm}\,j^{\pm}(v_{\pm})\right)^{a}\,=\,0$$

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- In fact, it is enough for the determination of $J_a^{\pm}(v_{\pm})$ currents's form.

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Effective action approach: LO equations of motion

- We look the classical solutons in the form of the following "perturbative" ansatz (light-cone gauge $v_- = 0$):

$$v_\perp = \Lambda[g\,A_+] + g\,\Lambda_1[g\,A_+] + \cdots\,,\ v_+ = A_+ + g\Phi[g\,A_+, v_\perp] + \cdots\,,\ \partial_-\,A_+ = 0$$

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- Equations of motion:

$$\partial_i\,\partial^i\,v_{a\,+} = -\,\partial_i^2 A_{a\,+}\,, \quad v_{a\,+} = A_{a\,+}\,,$$

and

$$\left(D_+\,(\partial_- v_i)\,\right)_a = 0\,, \quad v_i^b = U^{bc}\,(v_+)\,\rho_{c\,i}\left(x^-, x_\perp\right)\,, \quad D_+\,U\,(v_+) = 0$$

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- Equations of motion:

$$\partial_{i}\,\partial^{\,i}\,v_{a\,+} = -\,\partial_{i}^{2}A_{a\,+}\,, \quad v_{a\,+} = A_{a\,+}\,,$$

and

$$\left(D_{+}\,\left(\partial_{-}v_{i}\right)\right)_{a} = 0\,, \quad v_{i}^{\,b} = U^{bc}\left(v_{+}\right)\,\rho_{c\,i}\left(x^{-}, x_{\perp}\right)\,, \quad D_{+}\,U\left(v_{+}\right) = 0$$

- Self-consistency condition, induced current and definition of the $\rho_{c\,i}\left(x^{-}, x_{\perp}\right)$ function (third equation):

$$-\,\partial_{i}\,\partial_{-}\,v_{a}^{i} = j_{a}^{+}\,, \quad j_{a}^{+} = -\,U^{\,ab}\,\left(v_{+}\right)\,\partial_{i}\,\partial_{-}\,\rho_{a}^{i}$$

similarly to CGC approach.

Effective action approach: current´s form

- So, what is U^{ab} operator? From Lipatov´s effective action we have:

$$\delta\,(v_+\,J^+) = \delta\,tr[\,(\,v_{+x}\,O_x\,\partial_i^2\,A^+\,)\,] = -\,\delta v_+^a\,tr[\,T_a\,O\,T_b\,O^T\,]\,\left(\partial_i^2 A_b^+\right)$$

that in the case of adjoint representation gives:

$$\delta\,(v_+\,J^+) = \frac{1}{N}\,(\delta v_+^a)\,tr\,[\,f_a\,O\,f_b\,O^T\,]\,\left(\partial_i^2\,A_b^+\right) = \frac{1}{N}\,(\delta v_+^a)\,U^{a\,b}\,\left(\partial_i^2\,A_b^+\right)$$

that provides

$$U^{a\,b} = tr\,[\,f_a\,O\,f_b\,O^T\,]$$

and

$$\partial_i\,\partial_-\,\rho_a^i = -\,\frac{1}{N}\,\partial_\perp^2\,A_a^+\,,\quad \rho_a^i = \frac{1}{N}\,\partial_-^{-1}\,(\partial^i\,A_-^a)\,,$$

(connection with CGC approach).

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Effective action approach: RFT calculus

- Bare Regge Field Theory (RFT) calculus: inserting classical solutions in the effective action

$$S_{eff}(A_{+},\, A_{-})\,=\,\Gamma\,=\,-\int d^{4}\,x\,\left(\,s_{1}[g,\,A_{+},\,A_{-}]\,+\,g\,s_{2}[g,\,A_{+},\,A_{-}]\,+\,\cdots\right)$$

we obtain effective action in terms of Reggeon fields A_{+} and A_{-}, this action can be considered as generating function of the Reggeon interactions vertices:

$$\Gamma\,=\,\sum_{n,m\,=\,0}\left(A_{+}^{a_1}\,\cdots\,A_{+}^{a_n}\,K_{b_1\,\cdots\,b_m}^{a_1\,\cdots\,a_n}\,A_{-}^{b_1}\,\cdots\,A_{-}^{b_m}\right)$$

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- This effective action consists a lot of parts of new Reggeon-Reggeon vertices. But, in general, the LO precision is not enough. The non-trivial results are revealed after the one-loop corrections calculation.

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Effective action approach: one loop expression

- We define:

$$v_i^a \to v_{i\,cl}^a + \varepsilon_i^a\,, \quad v_+^a \to v_{+\,cl}^a + \varepsilon_+^a$$

and expand the light-cone Lagrangian

$$L = -\frac{1}{4}\,F_{ij}^a\,F_{ij}^a + F_{i+}^a\,F_{i-}^a + \frac{1}{2}\,F_{+-}^a\,F_{+-}^a$$

around the non-trivial classical solutions.

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  around the non-trivial classical solutions.

- Effective current expansion:

$$v_+^a J_a^+(v_+) = -v_+^{a\,cl} O^{ab}(v_+^{cl}) \left(\partial_i \partial_- \rho_b^i\right) - \frac{1}{2} \left(\frac{\delta U^{b\,a}(v_+)}{\delta v_+^c} \right)_{v_+ = v_+^{cl}}^{xy} \left(\partial_i \partial_- \rho_a^i\right)_x \varepsilon_{+\,x}^b \varepsilon_{+\,y}^c + \dots$$

  where

$$U_x^{a\,b}(v_+) = U_x^{a\,b}(v_{+\,0}^{cl}) + g \left(U_1^{a\,b}(v_{+\,0}^{cl})\right)_{xy}^{c} \varepsilon_{+\,y}^c + \frac{1}{2} g^2 \left(U_2^{a\,b}(v_{+\,0}^{cl})\right)_{xyz}^{c\,d} \varepsilon_{+\,y}^c \varepsilon_{+\,z}^d + \dots$$

Effective action approach: one loop expression

- where

$$\left(U_1^{a\,b}\right)^{c}_{xy} = tr[\,f_a\,G^{+}_{xy}\,f_c\,O_y\,f_b\,O^T_x\,] + tr[\,f_c\,G^{+}_{yx}\,f_a\,O_x\,f_b\,O^T_y\,]$$

and

$$\begin{aligned}\left(U_2^{a\,b}\right)^{c\,d}_{xyz} &= tr[\,f_a\,G^{+}_{xy}\,f_c\,G^{+}_{yz}\,f_d\,O_z\,f_b\,O^T_x\,] + tr[\,f_a\,G^{+}_{xz}\,f_d\,G^{+}_{zy}\,f_c\,O_y\,f_b\,O^T_x\,] + \\ &+ tr[\,f_d\,G^{+}_{zx}\,f_a\,G^{+}_{xy}\,f_c\,O_y\,f_b\,O^T_z\,] + tr[\,f_c\,G^{+}_{yx}\,f_a\,G^{+}_{xz}\,f_d\,O_z\,f_b\,O^T_y\,] + \\ &+ tr[\,f_d\,G^{+}_{zy}\,f_c\,G^{+}_{yx}\,f_a\,O_x\,f_b\,O^T_z\,] + tr[\,f_c\,G^{+}_{yz}\,f_d\,G^{+}_{zx}\,f_a\,O_x\,f_b\,O^T_y\,]\end{aligned}$$

with

$$O_x = \delta^{a\,b} + g\int d^4y\,G^{+\,a\,a_1}_{xy}\,\left(v_+(y)\right)_{a_1\,b} = 1 + g\,G^{+}_{xy}\,v_{+y}$$

and Green's functions of D_+ operators

$$G^{+}_{xy} = G^{+\,0}_{xy} + g\,G^{+\,0}_{xz}\,v_{+z}\,G^{+}_{zy}$$

where

$$\partial_{+x}\,G^{+\,0}_{xy} = \delta_{x\,y}\,,\quad G^{+\,0}_{yx}\,\overleftarrow{\partial}_{+x} = -\delta_{x\,y}$$

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Effective action approach: one loop expression

- Final expression for the one loop effective action:

$$\Gamma = \int d^4x \, \Big(L_{YM}(v_i^{cl}, v_+^{cl}) - v_{+\,cl}^{a} \, J_a^{+}(v_+^{cl}) - A_+^a \, (\, \partial_i^2 \, A_-^a \,) \Big) + \frac{\imath}{2} \, \ln \Big(1 + G(v^{cl}) \, M(v^{cl}) \Big)$$

which is functional of the reggeized gluon fields only.

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- The interaction of reggeized gluons A_+ and A_- is defined as effective vertex of interactions of reggeon fields in teh action:

$$\left(K_{x\,y}^{a\,b} \right)^{+\,-} = K_{x\,y}^{a\,b} = \left(\frac{\delta^2\,\Gamma}{\delta A_{+\,x}^{a}\;\delta A_{-\,y}^{b}} \right)_{A_+,\,A_- = 0},$$

we can call this vertex as interaction kernel as well. There are also other kernels, related to $< A_+\, A_+ >$ and $< A_-\, A_- >$ propagators, in the leading order the contributions to these kernels are zero:

$$\left(K_{xy}^{a\,b} \right)_0^{+\,+} = \left(K_{x\,y}^{a\,b} \right)_0^{-\,-} = 0$$

Effective action approach: interactions kernels

- The contributions to this kernel are provided by the different terms in the action which are linear with respect to A_+, A_- fields. For example, the variation of a logarithms gives:

$$-2\imath\, K_{x\,y}^{a\,b} = \left(\frac{\delta^2\, \ln\left(1 + G\, M\,\right)}{\delta A_{+\,x}^{a}\, \delta A_{-\,y}^{b}} \right)_{A_+,\, A_- = 0} =$$

$$= \left[\left(\frac{\delta^2\, G}{\delta A_{+\,x}^{a} \delta A_{-\,y}^{b}} M + \frac{\delta\, G}{\delta A_{+\,x}^{a}} \frac{\delta\, M}{\delta A_{-\,y}^{b}} + \frac{\delta\, G}{\delta A_{-\,y}^{b}} \frac{\delta\, M}{\delta A_{+\,x}^{a}} + G \frac{\delta^2\, M}{\delta A_{+\,x}^{a} \delta A_{-\,y}^{b}} \right) \left(1 + GM\right)^{-1} - \right.$$

$$\left. - \left(\frac{\delta\, G}{\delta A_{-\,y}^{b}} M + G\, \frac{\delta\, M}{\delta A_{-\,y}^{b}} \right) \left(1 + G\, M\right)^{-1} \left(\frac{\delta\, G}{\delta A_{+\,x}^{a}} M + G\, \frac{\delta\, M}{\delta A_{+\,x}^{a}} \right) \left(1 + G\, M\right)^{-1} \right]_{A=0}$$

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- Leading order contribution:

$$K^{a\,b}_{x y\,0} = -\,\delta^{a\,b}\,\delta_{x\,y}\,\partial^2_{i\,x} = \delta^{a\,b}\,\delta_{x\,y}\,\left(\partial_i\,\partial^i\right)_x$$

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Effective action approach: propagator

- For the kernel

$$K_{zw}^{bd} = \sum_{k=0} K_{zw\,k}^{bd}$$

the propagator is defined as

$$\int d^4z \left(K_{xz}^{ab} \right)^{-+} \left(D_{zy}^{bc} \right)_{+-} = \delta^{ac}\,\delta_{xy}$$

or

$$\left(D_{xy}^{ac}\right)_{+-} = \left(D_{xy}^{ac}\right)_{0\,+-} - \int d^4z \int d^4w \left(D_{xz}^{ab}\right)_{0\,+-} \left(\left(K_{zw}^{bd} \right)^{-+} - \left(K_{zw}^{bd} \right)_0^{-+} \right) \left(D_{wy}^{dc} \right)_{+-}$$

where

$$\int d^4z\, K_{xz\,0}^{ab}\, D_{zy\,0}^{bc} = \delta^{ac}\,\delta_{xy}$$

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Effective action approach: propagator

- The NLO kernel (well known of course):

$$-2\,\imath\,K^{a\,b}_{x\,y\,1}\,=\,\frac{\imath\,g^2\,N}{4\,\pi}\,\partial^2_{i\,x}\left(\int\frac{dp_-}{p_-}\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{d^2k_\perp}{(2\pi)^2}\,\frac{k^2_\perp}{p^2_\perp\,\left(\,p_\perp\,-\,k_\perp\,\right)^2}\,e^{-\imath\,k_i\,(x_i\,-\,y_i)}\right)$$

provides the reggeized gluons propagator equation after the Fourier transfrom:

$$\tilde{D}^{ab}(p_\perp,\,p_-)\,=\,\frac{\delta^{ab}}{p^2_\perp}\,-\,\frac{g^2\,N}{32\,\pi^3}\int\frac{dk^{'}_-}{k^{'}_-}\int d^2k_\perp\,\frac{p^2_\perp}{k^2_\perp\,\left(\,p_\perp\,-\,k_\perp\,\right)^2}\,\tilde{D}^{ab}(p_\perp,\,k^{'}_-)\,,$$

here we used

$$D^{ab}_0(x,y)\,=\,\delta^{ab}\int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-\imath\,p\,(x\,-\,y)}}{p^2_\perp}$$

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Effective action approach: propagator

- Introduce rapidity variable $y = \frac{1}{2} \, ln \, (\Lambda \, k_-)$, take into account the physical cut-off of the rapidity related with particles cluster size η, integrate on k_- variable:

$$\tilde{D}^{ab}(p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} - \frac{g^2 \, N}{16 \, \pi^3} \int_0^\eta d\eta' \int d^2 k_\perp \frac{p_\perp^2}{k_\perp^2 \, (p_\perp - k_\perp)^2} \tilde{D}^{ab}(p_\perp, \eta')$$

with

$$\epsilon(p_\perp^2) = - \frac{\alpha_s \, N}{4 \, \pi^2} \int d^2 k_\perp \frac{p_\perp^2}{k_\perp^2 \, (p_\perp - k_\perp)^2}$$

as intercept of the propagator of reggeized gluons. Rewriting this equation as the differential one:

$$\frac{\partial \, \tilde{D}^{ab}(p_\perp, \eta)}{\partial \, \eta} = \tilde{D}^{ab}(p_\perp, \eta) \, \epsilon(p_\perp^2)$$

we obtain the final expression for the propagator:

$$\tilde{D}^{ab}(p_\perp, Y) = \frac{\delta^{ab}}{p_\perp^2} \left(\frac{s}{s_0} \right)^{\epsilon(p_\perp^2)}$$

with some rapidity interval $0 < \eta < Y = \ln(s/s_0)$ of the problem of interest introduced.

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- Additional source of the corrections are the Regge Field Theory (RFT) loop's contributions. Lipatov's effective action can be considered as QCD variant of RFT and provides the base for these RFT calculations;

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- Formulated as RFT, the Lipatov's effective action provides self-consistent approach for the calculations of the complex vertices (kernels) of Reggeon-Reggeon interactions and correspondingly corrections to the reggeized gluons propagator or any other correlations functions of interests;

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- The calculations of the productions amplitudes in the quasi-multi-Regge kinematics can be done in the Lipatov's effective action frameworks as well, similarly to the done (work in progress);

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- Lipatov's effective action provides systematical calculations of unitarity corrections to the scattering amplitudes at high energy in the framework of the regular QFT methods;
- The perturbation theory is based on the knowledge of the classical solutions of equations of motion (see next talk) and loops contributions to the effective action;
- Additional source of the corrections are the Regge Field Theory (RFT) loop's contributions. Lipatov's effective action can be considered as QCD variant of RFT and provides the base for these RFT calculations;
- Formulated as RFT, the Lipatov's effective action provides self-consistent approach for the calculations of the complex vertices (kernels) of Reggeon-Reggeon interactions and correspondingly corrections to the reggeized gluons propagator or any other correlations functions of interests;
- The calculations of the productions amplitudes in the quasi-multi-Regge kinematics can be done in the Lipatov's effective action frameworks as well, similarly to the done (work in progress);
- There are theoretical questions related to the form of the Lipatov's action (possible corrections to it), integrability properties and etc.

Thanks

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