Fermion Masses and Mixings and Dark Matter Constraints in a Model with Radiative Seesaw Mechanism

Antonio Enrique Cárcamo Hernández

Universidad Técnica Federico Santa María

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Based on: Nicolás Bernal, A. E. Cárcamo Hernández, Ivo de Medeiros Varzielas and Sergey Kovalenko, arxiv:hep-ph/1712.02792.

Overview

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The origin of fermion masses and mixings is not explained by the SM.

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	(0–0.13)×10^{-9}	0
e electron	0.000511	–1
ν_M middle neutrino*	(0.009–0.13)×10^{-9}	0
μ muon	0.106	–1
ν_H heaviest neutrino*	(0.04–0.14)×10^{-9}	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

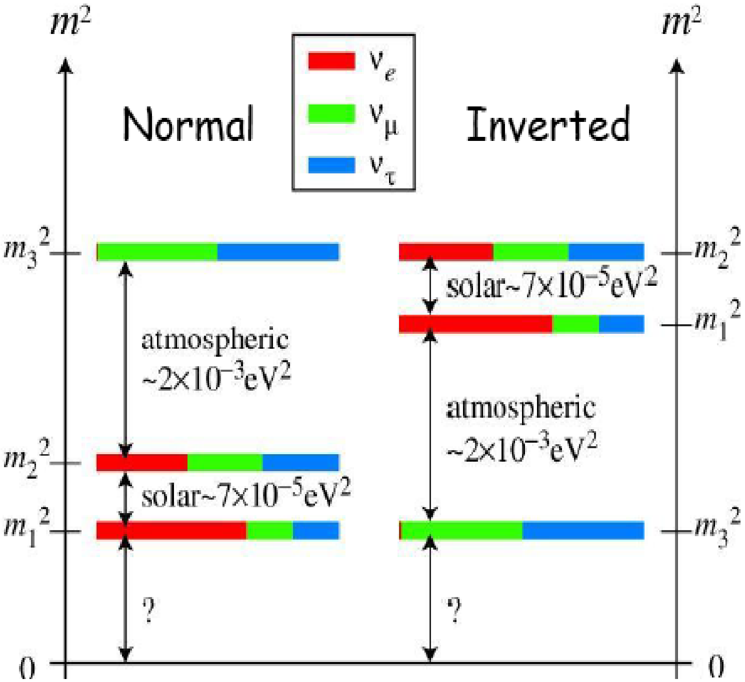
$$m_e \sim \lambda^9 m_t, \quad m_u \sim m_d \sim \lambda^8 m_t,$$

$$m_s \sim m_\mu \sim \lambda^5 m_t, \quad \lambda = 0.225,$$

$$m_c \sim \lambda^4 m_t, \quad m_b \sim m_\tau \sim \lambda^3 m_t,$$

$$\sin\theta_{12}^{(q)} \sim \lambda, \quad \sin\theta_{23}^{(q)} \sim \lambda^2, \quad \sin\theta_{13}^{(q)} \sim \lambda^4,$$

$$\sin\theta_{12}^{(l)} \sim \sqrt{\frac{1}{3}}, \quad \sin\theta_{23}^{(l)} \sim \sqrt{\frac{1}{2}}, \quad \sin\theta_{13}^{(l)} \sim \frac{\lambda}{\sqrt{2}}.$$


m^2
m^2
ν_e
ν_μ
ν_τ
Normal
Inverted
$m_3{}^2$
$m_2{}^2$
$m_1{}^2$
atmospheric
$\sim 2\times 10^{-3} eV^2$
solar$\sim 7\times 10^{-5} eV^2$
?
0
$m_2{}^2$
$m_1{}^2$
$m_3{}^2$
solar$\sim 7\times 10^{-5} eV^2$
atmospheric
$\sim 2\times 10^{-3} eV^2$
?
0

Some mechanisms to describe the SM charged fermion mass hierarchy are:

1. Spontaneously broken abelian symmetries as originally proposed by Froggatt and Nielsen in NPB, 1979.
2. Universal Seesaw mechanism as originally proposed by Davidson and Wali in PRL, 1987
3. Localization of the profiles of the fermionic zero modes in extradimensions as originally proposed by Dvali and Schifman in PLB, 2000.
4. Sequential loop supression mechanism as originally proposed by A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt in JHEP, 2017.

Several mechanisms to generate light active neutrino masses are: Weinberg Operator, type I seesaw, type II seesaw, type III seesaw, double seesaw, linear seesaw, inverse seesaw, radiative seesaw at one, two, three or four loop level. Some of these mechanisms were explained by Celso Nishi, Steve King, Juan Carlos Helo and Diego Restrepo in their talks.

The $\Delta(27)$ discrete group

$$
\begin{aligned}
\mathbf{3}\otimes\mathbf{3} &= \overline{\mathbf{3}}_{S_1}\oplus\overline{\mathbf{3}}_{S_2}\oplus\overline{\mathbf{3}}_{A}\\
\overline{\mathbf{3}}\otimes\overline{\mathbf{3}} &= \mathbf{3}_{S_1}\otimes\mathbf{3}_{S_2}\oplus\mathbf{3}_{A}\\
\mathbf{3}\otimes\overline{\mathbf{3}} &= \sum_{r=0}^{2}\mathbf{1}_{r,0}\oplus\sum_{r=0}^{2}\mathbf{1}_{r,1}\oplus\sum_{r=0}^{2}\mathbf{1}_{r,2}\\
\mathbf{1}_{k,\ell}\otimes\mathbf{1}_{k',\ell'} &= \mathbf{1}_{k+k' mod 3,\ell+\ell' mod 3} \qquad (1)\\
(\mathbf{3}\otimes\mathbf{3})_{\overline{\mathbf{3}}_{S_1}} &= (x_1y_1, x_2y_2, x_3y_3)\,,\\
(\mathbf{3}\otimes\mathbf{3})_{\overline{\mathbf{3}}_{S_2}} &= \frac{1}{2}(x_2y_3+x_3y_2, x_3y_1+x_1y_3, x_1y_2+x_2y_1)\,,\\
(\mathbf{3}\otimes\mathbf{3})_{\overline{\mathbf{3}}_{A}} &= \frac{1}{2}(x_2y_3-x_3y_2, x_3y_1-x_1y_3, x_1y_2-x_2y_1)\,,\\
\left(\mathbf{3}\otimes\overline{\mathbf{3}}\right)_{\mathbf{1}_{r,0}} &= x_1y_1+\omega^{2r}x_2y_2+\omega^{r}x_3y_3,\\
\left(\mathbf{3}\otimes\overline{\mathbf{3}}\right)_{\mathbf{1}_{r,1}} &= x_1y_2+\omega^{2r}x_2y_3+\omega^{r}x_3y_1,\\
\left(\mathbf{3}\otimes\overline{\mathbf{3}}\right)_{\mathbf{1}_{r,2}} &= x_1y_3+\omega^{2r}x_2y_1+\omega^{r}x_3y_2, \qquad (2)
\end{aligned}
$$

where $r=0,1,2$ and $\omega=e^{i\frac{2\pi}{3}}$.

The model

The model is an extension of the SM that incorporates the $\Delta(27) \times \mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_{12}$ discrete symmetry and a particle content extended with the SM singlets: scalars $\sigma_{1,2}$, $\eta_{1,2}$, χ, $\rho_{1,2}$, Φ, Ξ, φ and two right handed Majorana neutrinos $N_{1,2R}$. All the non-SM fields are charged under the above mentioned discrete symmetry.

The full symmetry $\mathcal{G}$ of the model exhibits the following spontaneous breaking:

$$\begin{array}{c} \mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times \Delta(27) \times \mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_{12} \\ \Downarrow \Lambda \\ SU(3)_C \times SU(2)_L \times U(1)_Y \times \mathbb{Z}_2 \\ \Downarrow v \\ SU(3)_C \times U(1)_Q \times \mathbb{Z}_2 , \end{array} \tag{3}$$

	ϕ	σ_1	σ_2	η_1	η_2	χ	ρ_1	ρ_2	Φ	Ξ	φ
$\Delta(27)$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{1,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{1,1}$	$\mathbf{1}_{0,1}$	$\mathbf{1}_{2,2}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}_{0,0}$
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	1
$\mathbb{Z}_8$	0	-1	-1	-1	-1	0	0	0	0	-4	0
$\mathbb{Z}_{12}$	3	0	-1	-3	-2	6	-3	-2	0	0	0

Table: Scalar assignments under $\Delta(27) \times \mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_{12}$. The $\mathbb{Z}_N$ charges, q, shown in the additive notation so that the group element is $\omega = e^{2\pi i\, q/N}$.

	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	l_L	l_{1R}	l_{2R}	l_{3R}	N_{1R}	N_{2R}
$\Delta(27)$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{1,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\overline{\mathbf{3}}$	$\mathbf{1}_{2,2}$	$\mathbf{1}_{2,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$	$\mathbf{1}_{0,0}$
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$\mathbb{Z}_8$	-2	-1	0	2	1	0	2	1	0	0	3	1	0	0	4
$\mathbb{Z}_{12}$	-3	-3	-3	4	2	0	0	6	0	3	4	4	6	0	0

Table: The same as in Table 1 but for fermions.

$$
\begin{aligned}
\mathcal{L}_Y^{(Q)} &= y_{33}^{(U)}\overline{q}_{3L}\widetilde{\phi}u_{3R} + y_{23}^{(U)}\overline{q}_{2L}\widetilde{\phi}u_{3R}\frac{\sigma_1}{\Lambda} + y_{13}^{(U)}\overline{q}_{1L}\widetilde{\phi}u_{3R}\frac{\sigma_1^2}{\Lambda^2} \\
&+ y_{22}^{(U)}\overline{q}_{2L}\widetilde{\phi}u_{2R}\frac{\sigma_2^2}{\Lambda^2} + y_{11}^{(U)}\overline{q}_{1L}\widetilde{\phi}u_{1R}\frac{\sigma_2^4}{\Lambda^4} + y_{33}^{(D)}\overline{q}_{3L}\phi d_{3R}\frac{\chi^3}{\Lambda^3} \qquad (4 \\
&+ y_{22}^{(D)}\overline{q}_{2L}\phi d_{2R}\frac{\eta_1^2\chi^3}{\Lambda^5} + y_{12}^{(D)}\overline{q}_{1L}\phi d_{2R}\frac{\eta_2^3\chi^3}{\Lambda^6} + y_{11}^{(D)}\overline{q}_{1L}\phi d_{1R}\frac{\eta_1^4\chi^3}{\Lambda^7} + h.c
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_Y^{(l)} &= y_{33}^{(l)}\left(\bar{l}_L\phi\Phi\right)_{\mathbf{1}_{0,1}} l_{3R}\frac{\rho_1^2}{\Lambda^3} + y_{13}^{(l)}\left(\bar{l}_L\phi\Phi\right)_{\mathbf{1}_{0,0}} l_{3R}\frac{\rho_2^3}{\Lambda^4} \\
&+ y_{22}^{(l)}\left(\bar{l}_L\phi\Phi\right)_{\mathbf{1}_{0,2}} l_{2R}\frac{\rho_2^2\sigma_1}{\Lambda^4} + y_{11}^{(l)}\left(\bar{l}_L\phi\Phi\right)_{\mathbf{1}_{0,0}} l_{1R}\frac{\rho_2^2\sigma_1^3}{\Lambda^6} + h.c, \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_Y^{(\nu)} &= y_1^{(\nu)}\left(\bar{l}_L\widetilde{\phi}\Phi\right)_{\mathbf{1}_{0,0}} N_{1R}\frac{\varphi}{\Lambda^2} + y_2^{(\nu)}\left(\bar{l}_L\widetilde{\phi}\Xi\right)_{\mathbf{1}_{0,0}} N_{2R}\frac{\varphi}{\Lambda^2} \\
&+ m_{N_{1R}}\overline{N}_{1R}N_{1R}^C + m_{N_{2R}}\overline{N}_{2R}N_{2R}^C + h.c, \qquad (6)
\end{aligned}
$$

We assume the following VEV pattern for the $\Delta(27)$ triplet SM singlet scalars

$$\langle\Phi\rangle = v_{\Phi}\ (1,0,0)\ , \qquad \langle\Xi\rangle = v_{\Xi}\ (1,1,1)\ , \tag{7}$$

which is consistent with the scalar potential minimization equations for a large region of parameter space as shown in detail in (Medeiros, King, Luhn, Neder, PLB, 2017).

Since fermion mass and mixing pattern arises from the $\Delta(27) \times \mathbb{Z}_8 \times \mathbb{Z}_{12}$ symmetry breaking, we set:

$$v_{\sigma_1} \sim v_{\sigma_2} \sim \lambda^2\Lambda < v_{\eta_1} \sim v_{\eta_2} \sim v_{\rho_1} \sim v_{\rho_2} \sim v_{\chi} \sim v_{\Phi} \sim v_{\Xi} \sim \lambda\Lambda\,. \tag{8}$$

being $\lambda = 0.225$. The model cutoff scale Λ can be thought of as the scale of the UV completion of the model, e.g. the masses of Froggatt-Nielsen messenger fields.

Quark masses and mixing.

The quark mass matrices take the form:

$$
\begin{aligned}
M_U &= \begin{pmatrix} a_1^{(U)}\lambda^8 & 0 & a_4^{(U)}\lambda^4 \\ 0 & a_2^{(U)}\lambda^4 & a_5^{(U)}\lambda^2 \\ 0 & 0 & a_3^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \\
M_D &= \begin{pmatrix} a_1^{(D)}\lambda^7 & a_4^{(D)}\lambda^6 & 0 \\ 0 & a_2^{(D)}\lambda^5 & 0 \\ 0 & 0 & a_3^{(D)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}},
\end{aligned}
\tag{9}
$$

where $a_k^{(U)}$ $(k = 1, \cdots, 5)$, and $a_k^{(D)}$ $(k = 1, \cdots, 4)$ are $\mathcal{O}(1)$ parameters. Here $\lambda = 0.225$ is the Wolfenstein parameter and $v = 246$ GeV the scale of electroweak symmetry breaking.

The obtained values for the physical quark mass spectrum are consistent with their experimental data, starting from the following benchmark point:

$$\begin{aligned} a_1^{(U)} &\simeq 1.266, \quad a_2^{(U)} \simeq 1.430, \quad a_3^{(U)} \simeq 0.989, \\ a_4^{(U)} &\simeq -0.510 - 1.262i, \quad a_5^{(U)} \simeq 0.806, \quad a_1^{(D)} \simeq 0.550, \\ a_2^{(D)} &\simeq 0.554, \quad a_3^{(D)} \simeq 1.411, \quad a_4^{(D)} \simeq 0.565. \end{aligned} \tag{10}$$

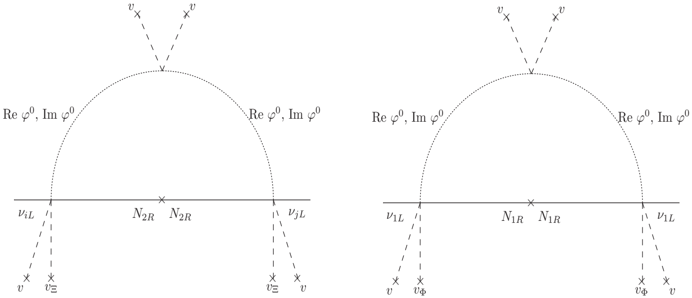
Observable	Model value	Experimental value
m_u [MeV]	1.47	$1.45^{+0.56}_{-0.45}$
m_c [MeV]	641	635 ± 86
m_t [GeV]	172	$172.1 \pm 0.6 \pm 0.9$
m_d [MeV]	2.8	$2.9^{+0.5}_{-0.4}$
m_s [MeV]	57.5	$57.7^{+16.8}_{-15.7}$
m_b [GeV]	2.81	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}^{(q)}$	0.225	0.225
$\sin\theta_{23}^{(q)}$	0.0414	0.0414
$\sin\theta_{13}^{(q)}$	0.00355	0.00355
δ	68°	68°

Lepton masses and mixing.

The charged lepton mass matrix is:

$$M_l = \begin{pmatrix} a_1^{(l)}\lambda^9 & 0 & a_4^{(l)}\lambda^4 \\ 0 & a_2^{(l)}\lambda^5 & 0 \\ 0 & 0 & a_3^{(l)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \qquad (11)$$

where $a_k^{(l)}$ $(k = 1, \cdots, 4)$ are $\mathcal{O}(1)$ dimensionless parameters.
The contribution from the charged lepton sector to the PMNS matrix, $U^{(l)}$ consists in a rotation by a single non-vanishing angle $\theta_{13}^{(l)}$ which depends crucially on $a_4^{(l)}$.



The light active neutrino mass matrix has the form

$$M_\nu = \begin{pmatrix} A_1^{(\nu)} & A_2^{(\nu)} & A_2^{(\nu)} \\ A_2^{(\nu)} & A_2^{(\nu)} & A_2^{(\nu)} \\ A_2^{(\nu)} & A_2^{(\nu)} & A_2^{(\nu)} \end{pmatrix}, \qquad (12)$$

The $\mathcal{O}(1)$ dimensionless couplings $a_i^{(l)}$ $(i = 1, \cdots, 4)$ determine the charged lepton masses, the reactor mixing parameter $\sin^2\theta_{13} \neq 0$ and the deviation $\sin^2\theta_{23} - 1/2 \neq 0$, which are correlated:

$$\sin^2\theta_{23} = \frac{1}{2\left(1 - \sin^2\theta_{13}\right)}\,. \tag{13}$$

For the case of inverted neutrino mass hierarchy we find the following best fit result

$$\begin{aligned} a_1^{(l)} &\simeq 1.936, \quad a_2^{(l)} \simeq 1.025, \quad a_3^{(l)} \simeq 0.864, \quad a_4^{(l)} \simeq 0.813, \\ \left|A_1^{(\nu)}\right| &\simeq 69.7\,\mathrm{meV}, \quad A_2^{(\nu)} \simeq 20.6\,\mathrm{meV}, \quad \arg\left[A_1^{(\nu)}\right] \simeq -58.26^\circ. \end{aligned} \tag{14}$$

The obtained numerical values given above for the neutrino parameters $\left|A_1^{(\nu)}\right|$, $A_2^{(\nu)}$ and $\arg\left[A_1^{(\nu)}\right]$ can be obtained from the following benchmark point:

$$\begin{aligned} m_{N_1} &= 500\,\text{GeV}, \quad m_{N_2} = 2\text{ TeV}, \quad m_{Re\varphi} = 900\text{ GeV}, \quad m_{Im\varphi} = 600\,\text{GeV}, \\ \Lambda &= 2.41\times 10^5\text{ TeV}, \quad |y_{1\nu}| = 1.12, \quad y_{2\nu} = 0.61, \quad \arg[y_{1\nu}] \simeq -37.4^\circ\,. \end{aligned}$$

Observable	Model value	Experimental value $\pm\,1\sigma$	2σ range	3σ range
m_e [MeV]	0.487	0.487	0.487	0.487
m_μ [MeV]	102.8	102.8 ± 0.0003	102.8 ± 0.0006	102.8 ± 0.0009
m_τ [GeV]	1.75	1.75 ± 0.0003	1.75 ± 0.0006	1.75 ± 0.0009
Δm_{21}^2 [10^{-5}eV^2] (IH)	7.56	7.56 ± 0.19	$7.20-7.95$	$7.05-8.14$
Δm_{13}^2 [10^{-3}eV^2] (IH)	2.49	2.49 ± 0.04	$2.41-2.57$	$2.37-2.61$
δ [°] (IH)	281.6	259^{+47}_{-41}	$182-347$	$0-31$ and $142-360$
$\sin^2\theta_{12}$ (IH)	0.321	$0.321^{+0.018}_{-0.016}$	$0.289-0.359$	$0.273-0.379$
$\sin^2\theta_{23}$ (IH)	0.511	$0.596^{+0.017}_{-0.018}$	$0.404-0.456$ and $0.556-0.625$	$0.388-0.638$
$\sin^2\theta_{13}$ (IH)	0.0214	$0.0214^{+0.00082}_{-0.00085}$	$0.0197-0.0230$	$0.0189-0.0239$

Table: Model and experimental values of the charged lepton masses, neutrino mass squared splittings and leptonic mixing parameters for the inverted (IH) mass hierarchy

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Dark Matter Constraints.

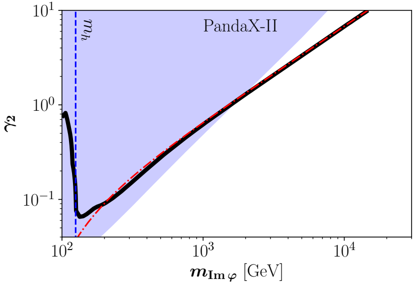

Figure: Scalar Dark Matter Scenario. Parameter space generating the observed DM relic abundance via the WIMP mechanism, using the full annihilation cross-section (thick black line) and the only the annihilation into Higgs bosons (thin red line). The light blue region is in tension with the latest PandaX-II results.

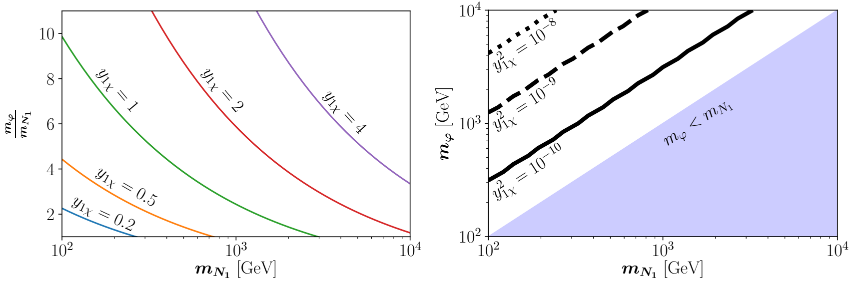

Figure: Fermionic Dark Matter Scenario. Effective coupling $y_{1\chi}$ needed in order to generate the observed relic abundance via the WIMP (left plot) and FIMP (right plot) mechanisms, assuming $m_\varphi \equiv m_{\mathrm{Re}\,\varphi} \sim m_{\mathrm{Im}\,\varphi}$.

Conclusions

- Fermion masses and mixings are successfully accounted for.
- For the quarks, the down sector parameters control the Cabibbo angle, and the up sector parameters control the remaining angles.
- For the leptons, the effective neutrino parameters that arise after radiative seesaw control the solar angle, and the charged lepton parameters control the reactor angle, which is also correlated to the deviation of the atmospheric angle from its maximal value.
- The model is only viable for inverted hierarchy and after fitting to the best-fit values of the solar and reactor angle, predicts $\sin^2\theta_{23} \simeq 0.51$, $\delta \simeq 281.6^\circ$ and $m_{ee} = 41.3$ meV.
- The breaking of $\Delta(27) \times \mathbb{Z}_8 \times \mathbb{Z}_{12}$ generates the non SM fermion masses as well as the observed pattern of SM fermion masses and mixings.
- The model addresses the flavour problem while providing a viable DM candidate (scalar or fermionic).

Acknowledgements

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