

The role played by QCD running in $0\nu\beta\beta$

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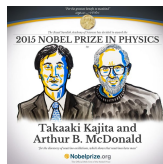
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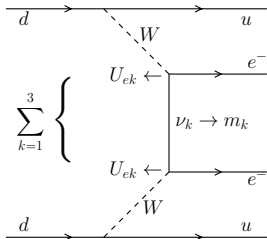
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LHC Era*

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- Neutrino oscillation \Rightarrow
Neutrinos have masses
- Open question: Dirac or Majorana?
- Is Lepton Number conserved in nature?



Introduction II



$$\langle m_\nu \rangle = \sum_j U_{ej}^2 m_j \equiv m_{ee}$$

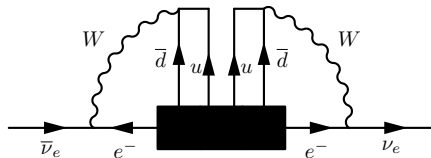
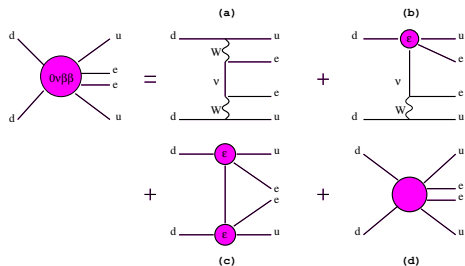


Figure: Black Box Theorem [Schechter & Valle, 1982]

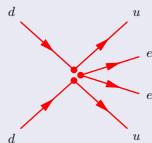
- Left-Right Symmetry
- R-Parity Violating Supersymmetry
- Leptoquarks
- Extradimensions
- etc.



Picture taken from:
J. Phys. G **39, 124007 (2012)**

Short-Range Mechanisms (SRM)

Effective Lagrangian



$$\mathcal{L}_{\text{eff}}^{0\nu\beta\beta} = \frac{G_F^2}{2m_p} \sum_i C_i^{XY}(\mu) \cdot \mathcal{O}_i(\mu), \quad (1)$$

with the operator basis:

$$\mathcal{O}_1^{XY} = 4(\bar{u}P_X d)(\bar{u}P_Y d) j, \quad (2)$$

$$\mathcal{O}_2^{XX} = 4(\bar{u}\sigma^{\mu\nu}P_X d)(\bar{u}\sigma_{\mu\nu}P_X d) j, \quad (3)$$

$$\mathcal{O}_3^{XY} = 4(\bar{u}\gamma^\mu P_X d)(\bar{u}\gamma_\mu P_Y d) j, \quad (4)$$

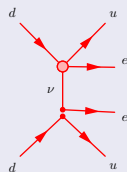
$$\mathcal{O}_4^{XY} = 4(\bar{u}\gamma^\mu P_X d)(\bar{u}\sigma_{\mu\nu}P_Y d) j^\nu, \quad (5)$$

$$\mathcal{O}_5^{XY} = 4(\bar{u}\gamma^\mu P_X d)(\bar{u}P_Y d) j_\mu \quad (6)$$

$$j = \bar{e}(1 \pm \gamma_5)e^c, \quad j_\mu = \bar{e}\gamma_\mu\gamma_5e^c.$$

Long-Range Mechanisms (LRM)

Effective Lagrangian



$$\mathcal{L}_{\text{eff}}^{d=6} = \frac{G_F}{\sqrt{2}} \left(j^\mu J_\mu^\dagger + \sum_i C_i^X(\mu) \mathcal{O}_i^{(6)X}(\mu) \right) \quad (7)$$

$$\mathcal{O}_1^{(6)X} = \bar{u} P_X d \cdot \bar{e} P_R \nu^C, \quad (8)$$

$$\mathcal{O}_2^{(6)X} = \bar{u} \sigma^{\mu\nu} P_X d \cdot \bar{e} \sigma^{\mu\nu} P_R \nu^C, \quad (9)$$

$$\mathcal{O}_3^{(6)X} = \bar{u} \gamma_\mu P_X d \cdot \bar{e} \gamma^\mu P_R \nu^C \quad (10)$$

$$j^\mu = \bar{e} \gamma^\mu (1 - \gamma_5) \nu, \quad J_\mu = \bar{d} \gamma^\mu (1 - \gamma_5) u$$

Half-life in the SRM and LRM

Applying standard nuclear theory methods, one finds for the half-life:

Half-life

$$\left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} = G_1 \left| \sum_{i=1}^3 C_i(\mu_0) \mathcal{M}_i \right|^2 + G_2 \left| \sum_{i=4}^5 C_i(\mu_0) \mathcal{M}_i \right|^2 \quad (11)$$

Here, $G_1 = G_{01}$ and $G_2 = (m_e R)^2 G_{09}/8$ are phase space factors in the convention of [Doi et al., 1985], and $\mathcal{M}_i = \langle A_f | \mathcal{O}_i^h | A_i \rangle$ are the nuclear matrix elements defined in Ref. [Päs et al., 2001].

Half-life

$$\left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} = G_{0k} |C_i(\mu_0) \cdot ME|^2 \quad (12)$$

SRM ν/s LRM

SRM

Elementary quark-level

$(uudddd)$

$$dd \rightarrow uu + 2e^-$$

Hadronic level

$$nn \rightarrow pp + 2e^-$$

LRM

Elementary quark-level

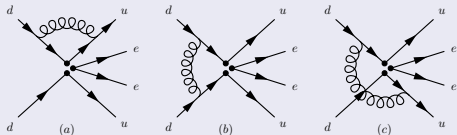
$$d \rightarrow u + e^- \nu$$

Hadronic level

$$nn \rightarrow pp + 2e^-$$

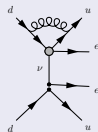
QCD corrections:

Short-Range Mechanisms

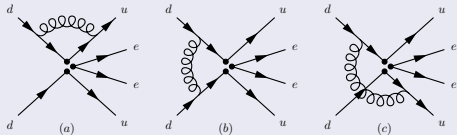


QCD corrections:

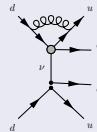
Long-Range Mechanisms



QCD corrections: Short-Range Mechanisms



QCD corrections: Long-Range Mechanisms



$$T_{\alpha\beta}^a T_{\gamma\rho}^a = -\frac{1}{2N} \delta_{\alpha\beta} \delta_{\gamma\rho} + \frac{1}{2} \delta_{\alpha\rho} \delta_{\beta\gamma} \quad (13)$$

Color Mismatch effect.

Renormalization Group Equations

[Buras, 1998] :

RGE of WC

$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T \vec{C}(\mu), \quad (14)$$

$$\hat{\gamma}(\alpha_s) = -2\alpha_s \frac{\partial \hat{Z}_1(\alpha_s)}{\partial \alpha_s}, \quad (15)$$

$$\vec{C}(\mu) = \hat{U}(\mu, \Lambda) \cdot \vec{C}(\Lambda). \quad (16)$$

Results: SRM

Without QCD

$$\left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} = G_1 \left| \sum_{i=1}^3 C_i(\mu_0) \mathcal{M}_i \right|^2 + G_2 \left| \sum_{i=4}^5 C_i(\mu_0) \mathcal{M}_i \right|^2 \quad (17)$$

With QCD

$$\begin{aligned} \left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} &= G_1 \left| \beta_1^{XX} C_1^{XX}(\Lambda) + \beta_1^{LR} C_1^{LR}(\Lambda) + \beta_2^{XX} C_2^{XX}(\Lambda) + \right. \\ &\quad \left. + \beta_3^{XX} C_3^{XX}(\Lambda) + \beta_3^{LR} C_3^{LR}(\Lambda) \right|^2 \\ &+ G_2 \left| \beta_4^{XX} C_4^{RR}(\Lambda) + \beta_4^{LR} C_4^{LR}(\Lambda) + \beta_5^{XX} C_5^{RR}(\Lambda) + \beta_5^{LR} C_5^{LR}(\Lambda) \right|^2, \end{aligned}$$

where

$$\beta_1^{XX} = \mathcal{M}_1 \overbrace{U_{11}^{XX}}^1 + \mathcal{M}_2 \overbrace{U_{21}^{XX}}^0$$

${}^A\text{X}$	\mathcal{M}_1	\mathcal{M}_2	$\mathcal{M}_3^{(+)}$	$\mathcal{M}_3^{(-)}$	$ \mathcal{M}_4 $	$ \mathcal{M}_5 $
${}^{76}\text{Ge}$	9.0	-1.6×10^3	1.3×10^2	2.1×10^2	$ 1.9 \times 10^2 $	$ 1.9 \times 10^1 $
${}^{136}\text{Xe}$	4.5	-8.5×10^2	6.9×10^1	1.1×10^2	$ 9.6 \times 10^1 $	$ 9.3 $

Table: The numerical values of the nuclear matrix elements \mathcal{M}_i ; taken from Ref. [Deppisch et al., 2012].

	With QCD	Without QCD	With QCD	Without QCD
A_X	$ C_1^{XX}(\Lambda) $	$ C_1^{XX} $	$ C_1^{LR,RL}(\Lambda) $	$ C_1^{LR,RL} $
^{76}Ge	3.8×10^{-10}	2.6×10^{-7}	9.1×10^{-9}	2.6×10^{-7}
^{136}Xe	2.6×10^{-10}	1.8×10^{-7}	6.1×10^{-9}	1.8×10^{-7}
A_X	$ C_2^{XX}(\Lambda) $	$ C_2^{XX} $	—	—
^{76}Ge	5.2×10^{-9}	1.4×10^{-9}	—	—
^{136}Xe	3.5×10^{-9}	9.4×10^{-10}	—	—
A_X	$ C_3^{XX}(\Lambda) $	$ C_3^{XX} $	$ C_3^{LR,RL}(\Lambda) $	$ C_3^{LR,RL} $
^{76}Ge	1.6×10^{-8}	1.1×10^{-8}	2.1×10^{-8}	1.8×10^{-8}
^{136}Xe	1.1×10^{-8}	7.4×10^{-9}	1.4×10^{-8}	1.2×10^{-8}
A_X	$ C_4^{XX}(\Lambda) $	$ C_4^{XX(0)} $	$ C_4^{LR,RL}(\Lambda) $	$ C_4^{LR,RL(0)} $
^{76}Ge	3.9×10^{-9}	1.2×10^{-8}	1.9×10^{-8}	1.2×10^{-8}
^{136}Xe	2.7×10^{-9}	7.9×10^{-9}	1.3×10^{-8}	7.9×10^{-9}
A_X	$ C_5^{XX}(\Lambda) $	$ C_5^{XX} $	$ C_5^{LR,RL}(\Lambda) $	$ C_5^{LR,RL} $
^{76}Ge	1.4×10^{-8}	1.2×10^{-7}	2.8×10^{-8}	1.2×10^{-7}
^{136}Xe	9.5×10^{-9}	8.2×10^{-8}	2.0×10^{-8}	8.2×10^{-8}

Table: Individual upper limits on Wilson Coefficients

Currently the best bounds are

$$\text{KamLAND-Zen} \quad : \quad T_{1/2}^{0\nu}({}^{136}\text{Xe}) = 1.07 \times 10^{26} \text{ ys (90\%C.L.)},$$

$$\text{GERDA Phase-II} \quad : \quad T_{1/2}^{0\nu}({}^{76}\text{Ge}) = 5.2 \times 10^{25} \text{ ys (90\%C.L.)}.$$

without QCD

$$\left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} = G_{0k} |C_i(\mu_0) \cdot ME|^2 \quad (18)$$

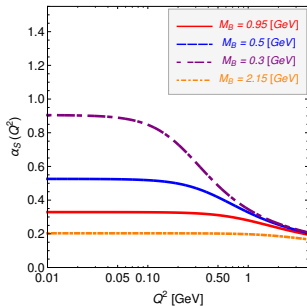
With QCD

$$\left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} = G_{0k} |U_i(\mu_0, \Lambda) C_i(\Lambda) \cdot ME|^2 \quad (19)$$

with

$$U_1 = 1.6, \quad U_2 = 0.6, \quad U_3 = 0.8$$

- The pQCD RGE results are valid above $\sim 1\text{GeV}$
- $0\nu\beta\beta \sim 100\text{MeV}$
- IR models review
[A. Deur et al., 2016]
- for example BPTH
 $A_\mu = a_\mu + B_\mu$
[Simonov, 2011]



$$\tilde{\alpha}_s(\mu^2) = \frac{\alpha_s(\lambda)}{1 + \beta_0 \frac{\alpha_s(\lambda)}{4\pi} \log \frac{\mu^2 + M_B^2}{\lambda^2}}$$

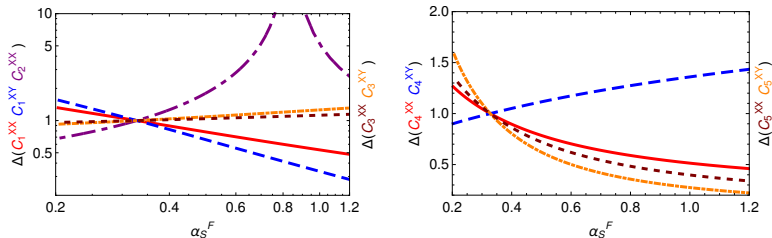


Figure: Relative change of the limit on the short-range coefficients with respect to the “frozen” value of α_S at low energies. Here α_S^F represents $\tilde{\alpha}_S(Q^2)$ for $Q^2 \leq 0.1 \text{ GeV}^2$. $\Delta(C_i^{AB})$ is calculated normalizing with respect to the value of the coefficient derived without “freezing” and using an $\alpha_S(1 \text{ GeV}^2) \simeq 0.32$.

$$\Delta(C_i^{XY}) = C_i^{XY}(\alpha_S^F) / C_i^{XY}(\alpha_S(1 \text{ GeV})) \quad (20)$$

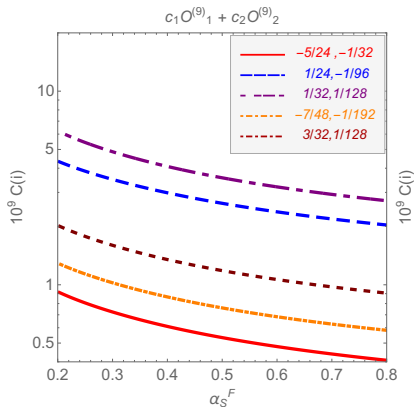


Figure: Limits on the five linearly independent short-range combinations of $c_1 O_1^{(9)} + c_2 O_2^{(9)}$ that appear in the tree-level decomposition of the $0\nu\beta\beta$ decay operator

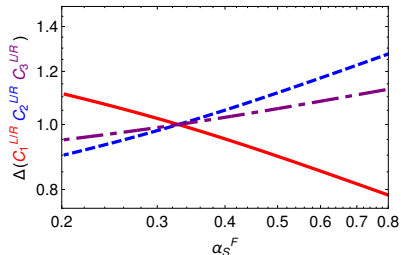


Figure: Relative change of the limit on the long-range coefficients with respect to the “frozen” value of α_S at low energies. Note the change of scale with respect to the short-range figure.

Summary and Conclusions

- We have calculated QCD running to the complete set of Lorentz-invariant operators for the short-range (SR) part and for the long-range part of the NDBD amplitude.
- We showed that the QCD corrections are indeed important in the SR principally because of the operator mixing
- We showed that in the long-range part the QCD correction is not as important as in the short-range part
- We analysed the IR behaviour of QCD running. With the exception of C_2^{XX} (in the SRM), the Wilson coefficients depend only moderately on the exact value of α_S^F and it seems we can extract reliable limits on these coefficients from $0\nu\beta\beta$

Thanks for your attention!

Back Up #1

A non-trivial example: R_p SUSY

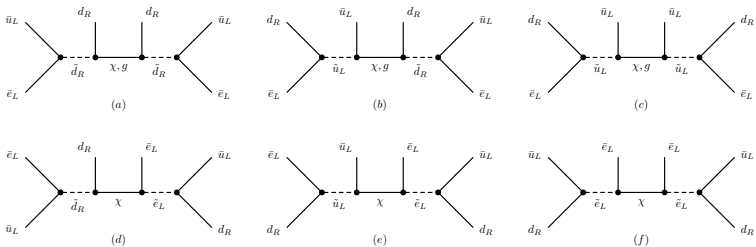


Figure: The six different Feynman diagrams in R-parity violating supersymmetry that contribute to $0\nu\beta\beta$ decay.

Back Up #2

A non-trivial example: R_p SUSY

\tilde{g} -exchange contribution:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\tilde{g}} &= \frac{G_F^2}{2m_p} (C_{\tilde{g}a} \mathcal{O}_a + C_{\tilde{g}b} \mathcal{O}_b + C_{\tilde{g}c} \mathcal{O}_c) = \\ &= \frac{G_F^2}{2m_p} \frac{1}{48} \left[(2C_{\tilde{g}a} + 2C_{\tilde{g}c} - 7C_{\tilde{g}b}) \mathcal{O}_1^{RR} - \frac{1}{4} (2C_{\tilde{g}a} + 2C_{\tilde{g}c} + C_{\tilde{g}b}) \mathcal{O}_2^{RR} \right].\end{aligned}\quad (21)$$

χ -exchange contribution:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\tilde{g}} &= \frac{G_F^2}{2m_p} \sum_{i=a\dots f} C_{\#i} \mathcal{O}_{\#i} = \\ &= \frac{G_F^2}{2m_p} \frac{1}{128} \left[4(C_b + C_c + C_a + 4C_f - 2C_d - 2C_e) \mathcal{O}_1^{RR} + \right. \\ &\quad \left. + (C_b - C_c - C_a) \mathcal{O}_2^{RR} \right].\end{aligned}\quad (22)$$

Back Up #3

A non-trivial example: R_p SUSY

$$C_{\tilde{g}c} = \frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m_{\tilde{u}_L}^4}, \quad C_{\tilde{g}a} = \frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m_{\tilde{d}_R}^4}, \quad C_{\tilde{g}b} = -\frac{\kappa_3}{m_{\tilde{g}}} \frac{1}{m_{\tilde{u}_L}^2 m_{\tilde{d}_R}^2}, \quad (23)$$

$$C_b = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(u)\epsilon_R(d)}{m_{\tilde{u}_L}^2 m_{\tilde{d}_R}^2}, \quad C_c = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L^2(u)}{m_{\tilde{u}_L}^4}, \quad C_a = \frac{\kappa_2}{m_\chi} \frac{\epsilon_R^2(d)}{m_{\tilde{d}_R}^4}, \quad (24)$$

$$C_f = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L^2(e)}{m_{\tilde{e}_L}^4}, \quad C_d = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(e)\epsilon_R(d)}{m_{\tilde{e}_L}^2 m_{\tilde{d}_R}^2}, \quad C_e = \frac{\kappa_2}{m_\chi} \frac{\epsilon_L(e)\epsilon_L(u)}{m_{\tilde{e}_L}^2 m_{\tilde{u}_L}^2}, \quad (25)$$

with

$$\kappa_2 = \lambda_{111}^{\prime 2} 4\pi\alpha_2 \frac{m_p}{G_F^2}, \quad \kappa_3 = \lambda_{111}^{\prime 2} 16\pi\alpha_s \frac{m_p}{G_F^2}, \quad (26)$$

$$\epsilon_L(\psi) = \tan\theta_W [T_3(\psi) - Q(\psi)], \quad \epsilon_R(\psi) = \tan\theta_W Q(\psi), \quad (27)$$

Back Up #4

A non-trivial example: R_p SUSY

$$\tilde{g} - \text{exchange} : \quad \lambda'_{111Ge} \leq 1.0 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{1\text{TeV}} \right)^2 \left(\frac{m_{\tilde{g}}}{1\text{TeV}} \right)^{1/2}, \quad (28)$$

$$\chi - \text{exchange} : \quad \lambda'_{111Ge} \leq 7.3 \times 10^{-1} \left(\frac{m_{\tilde{e}}}{1\text{TeV}} \right)^2 \left(\frac{m_{\tilde{\chi}}}{1\text{TeV}} \right)^{1/2} \quad (29)$$

$$\tilde{g} - \text{exchange} : \quad \lambda'_{111Ge} \leq 9.3 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{1\text{TeV}} \right)^2 \left(\frac{m_{\tilde{g}}}{1\text{TeV}} \right)^{1/2}, \quad (30)$$

$$\chi - \text{exchange} : \quad \lambda'_{111Ge} \leq 5.2 \left(\frac{m_{\tilde{e}}}{1\text{TeV}} \right)^2 \left(\frac{m_{\tilde{\chi}}}{1\text{TeV}} \right)^{1/2} \quad (31)$$

This is about ~ 10 (~ 7) weaker than the limits for gluino (neutralino) cases in Eqs. (28), (29) taking into account the QCD running.

Talk references



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