

Schrödinger equation with a Dirac monopole and short-range potential

Genetic considerations:

Due to the well-known "Spin-from-Isospin" Effect
(Skyrme, 't Hooft, Hasenfratz, Jackiw, Rebbi, Goldhaber)

One should expect that the scattering amplitude
(and the corresponding Watson-Sommerfeld-Regge formula)
with a monopole and short-range potential for a scalar
field should look like the scattering amplitude for
a particle with spin

$$\left\{ \begin{array}{c} |\Psi\rangle_{\text{scalar}} \\ \otimes \\ V(r) \end{array} \right\} \sim \left\{ \begin{array}{c} |\Psi\rangle_{\text{Fermion}} \\ \{V(r)\} \end{array} \right\}$$

monopole

\$\rightarrow\$ difference in the B.I.

$\{ V(r) \}$

This is almost true: crucial difference in the B.I.

Schrodinger equation: direct computation shows

$$\vec{A}_D = \frac{g}{\pi} \frac{(1 - \cos\theta)}{\sin\theta} \hat{e}_\varphi ; \quad V(r) = \int_{M_0}^{+\infty} \sigma(p) \frac{\exp[-p\pi]}{r} dp ; \quad M_0 > 0$$

$$-\frac{1}{2M} [(\vec{D})^2 - V(r)] \Psi(\vec{r}) = E \Psi(\vec{r}) ; \quad E = \frac{k^2}{2M} ;$$

$$\vec{D} = \vec{\partial} + e\vec{A}_D ; \quad \Psi(\vec{r}) = \frac{\psi_{kl}(r)}{r} d_{\mu m}^l(\theta, \varphi) ;$$

the 'only' effect
of the defect is to
modify the angular
momentum

then replacing the covariant derivative into the
kinetic term:

$$K^2 \psi_{kl} = \left[-\frac{d^2}{dr^2} + \frac{\lambda^2 - \frac{1}{4}}{r^2} + V \right] \psi_{kl} \quad \lambda = \lambda(l) ;$$

$\rightarrow l^2, (l)$ $n \dots , l^{(e)}$ $\dots l \rightarrow n : -l \leq n \leq l :$

$$\left(\vec{j}\right)^2 d_{\mu m}^{(l)} = -l(l+1) d_{\mu m}^{(l)}$$

$l = n - \mu ; l \geq \mu_j - l \leq \mu \leq l_j$

$$\vec{j} = \vec{l} - \mu \frac{\hat{r} + \hat{z}}{(1 + \cos\theta)}$$

$$j_\theta = \frac{1}{\sin\theta} \left\{ i D_\varphi + \mu(1 - \cos\theta) \right\}, j_\varphi = -i D_\theta, j_n = -M_j$$

$$\mu = \frac{e g}{c \hbar}; \quad l = \mu, \mu+1, \dots \Rightarrow \lambda(l) = \sqrt{(l + \frac{1}{2})^2 - M^2}$$

$$d_{\mu m}^{(l)}(\theta, \varphi) = N_{\mu m} \exp[i(\mu + m)\varphi] (1 - x)^{\frac{|m|+m}{2}} (1+x)^{\frac{|m|-m}{2}} P_{(l-m)}^{(\mu-m, \mu+m)}(x)$$

$$x = \cos\theta;$$

The $d_{\mu m}^{(l)}$ are the so-called Jacobi polynomials which replaces the Legendre Polynomial in the partial wave amplitude for particles with spin.

Good News.

The radial equation (which determines the phase shift $\delta(l, k)$) as function of the energy k and of the angular label l) is exactly the same as in the monopole-free case \Rightarrow $C_{n+l+1} \propto \Gamma_{n+l+1}$.

is exactly the same as in the usual case
⇒ the analytic properties of $S(\lambda, k)$ as function of λ
are exactly the same as in the usual case

However ...

due to the fact the angular momentum operator is now the sum of the orbital angular momentum and an extra term needed to compensate the lack of spherical symmetry of \vec{A}_D the link between the radial and angular part is not anymore analytic:

$$\lambda = l + \frac{1}{2} \Rightarrow \lambda = \left[(l + \frac{1}{2}) - m^2 \right]^{\frac{1}{2}}$$

without monopole with monopole

this fact will make a huge difference when writing the scattering amplitude

Resume:

Compared to the usual case, the presence of a Dirac monopoles:

- 1) modifies the centrifugal barrier ($\vec{l} \rightarrow \vec{j}$)
- 2) modifies the link between the radial and the angular Schrödinger equation ($\lambda = l + \frac{1}{2} \rightarrow \lambda = [(l + \frac{1}{2})^2 - m^2]^{1/2}$)
- 3) Replaces Legendre Polynomial with Jacobi polynomial

Important Remark:

What is important when doing the W-S-R transform
are the analytic properties of the phase shift as function
of l (which is the label which defines the partial wave sum)
The properties of the phase shift as function of λ

The properties of the phase shift as given in
are not fundamental. In the usual case they coincide
but the defect introduces a 'mismatch'