

Partial Wave expansion and W-S-R formulae with Dirac monopoles

$$f_\mu(k, x) = \frac{\exp(-i\pi\kappa)}{k} \left(\frac{1-x}{2}\right)^\kappa \sum_{l=0}^{+\infty} [2(l+\kappa)+1] P_l^{(2\kappa, 0)}(x) \{ \exp[2i\delta(\lambda(l), k)] - 1 \}$$

typical kinematical singularity of amplitudes of particles with spin

Jacobi polynomial

Now, the S-matrix element as function of λ has the same properties as in Regge papers

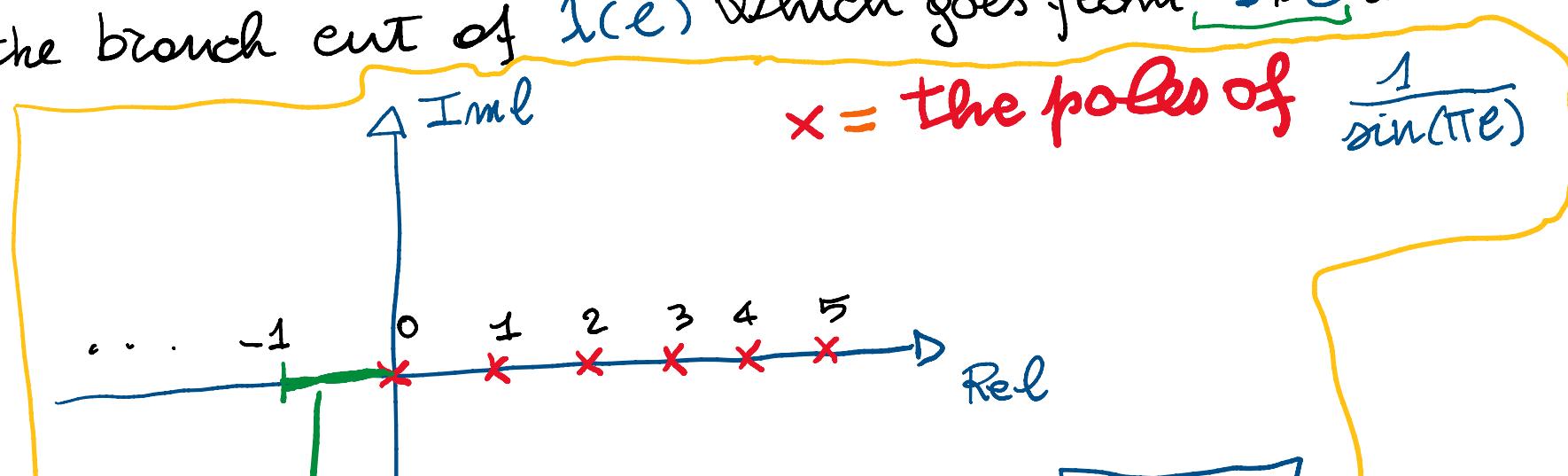
However when writing the W-S-R formula what are really important are the properties of $S(\lambda(l), k)$ as function of l and not as function of λ since the integral above sum is an integral over l and not over λ !

function of x ---
partial wave sum is an integral over ℓ and more...
Although in the usual case there is no difference,
with monopoles there is a difference

the $\mu = 1/2$ case: the fundamental Dirac monopole

to build the W-S-R formula one usually transforms
the sum into an integral using the factor $\frac{1}{\sin(\pi e)}$
as the poles give back the original sum.

However now some of these poles happen to be on
the branch cut of $\text{Im}e$ which goes from -1 to 0 on the real axis



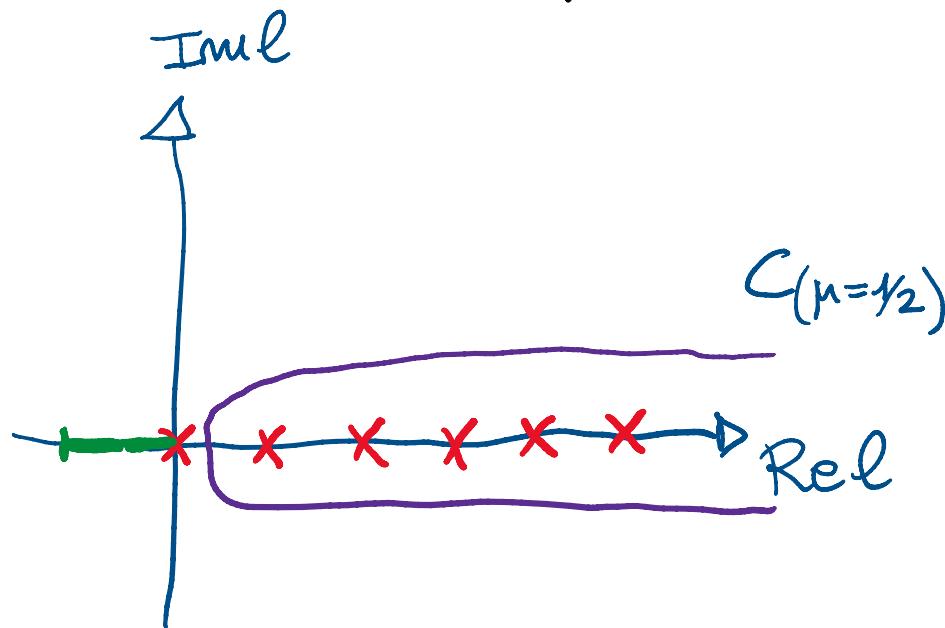
Re

$\lambda(l) = \sqrt{(l+\frac{1}{2})^2 - \mu^2}$

the cut of

therefore we have a 'bad' singularity: a cut which multiplies a pole (the first one)

this means that the c -circuit of Watson-Sommerfeld must exclude the first pole ($l=0$)



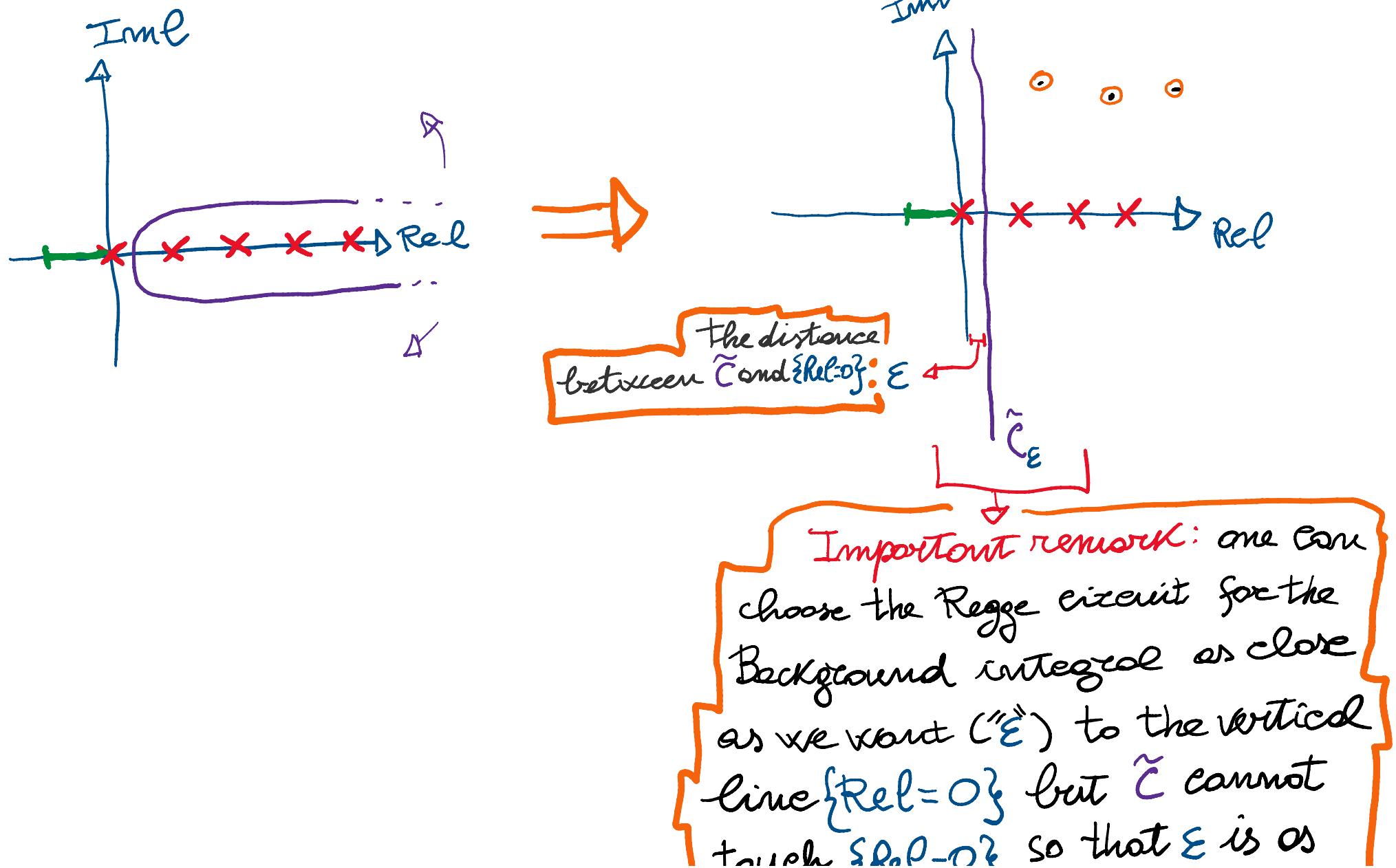
Important remark: We cannot push the next circuit $C_{(\mu=\frac{1}{2})}$ through a 'bad' singularity (such as a pole or cut) - if the cut \rightarrow would be slightly above or below the real axis then we could use the same circuit as Watson-Sommerfeld. However the presence of monopoles is precisely an obstruction to use the same circuits as W-S.

Second Important remark:

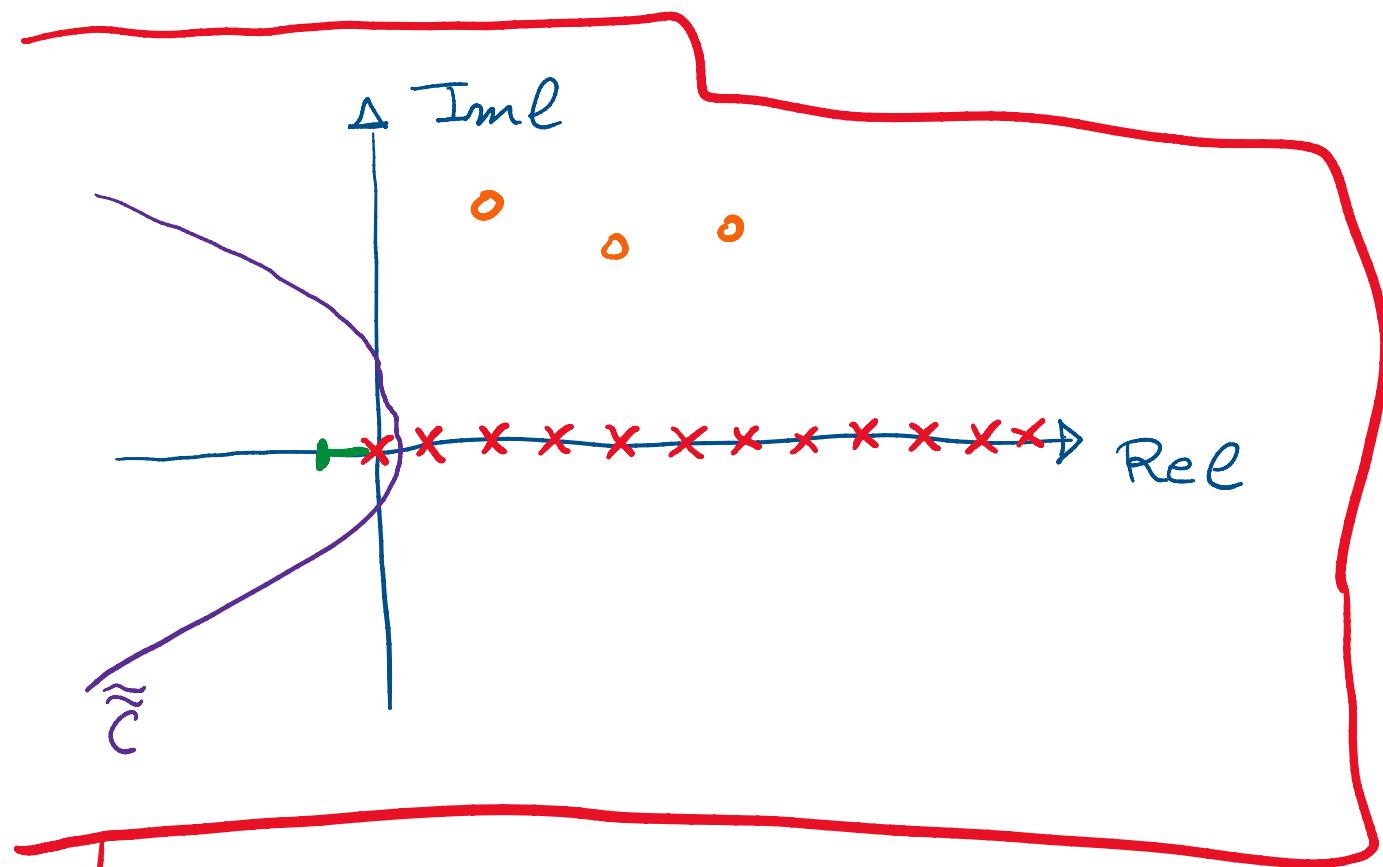
Usually, in QM one can prove that in the complex t -plane for short range potentials only Regge poles can appear. Indeed, it is commonly believed that branch cuts are peculiar features of QFT. Thus it is very interesting that topological defects introduce branch cut in the complex t -plane already at level of QM

ℓ -plane already at level of α^{ini}

Regge 'opening' with monopoles



we can ...
 touch $\{Re\ell=0\}$ so that ε is as
 small as we want but positive



useful remark: If one would choose the above circuit, it would not be possible to apply standard results of Regge theory to prove convergence of the Background integral

thus, all in all

$$f(k, x) \Big|_{k=\frac{1}{2}} = f_0 + \frac{1}{2\pi k} \int_{-\infty}^{\epsilon} \frac{[2(\ell+\frac{1}{2})+1]}{\sin(\pi\ell)} P_\ell^{(1,0)}(-x) [S(\alpha(\ell), k) - 1] +$$
$$+ \left(-\frac{i}{k}\right) \sum_{J=1}^N R_J P_{d_J}^{(1,0)}(-x)$$

Contribution from Regge poles

(which is the same as in the applications of Regge Theory to particles with spin **good nexus**)

this is the Background integral
and it discloses the
most important difference
with respect to the usual

with respect to the ~~the~~
case

Important remark

the analytic properties of the Jacobi polynomials
as function of the angular label l are the same
as the Legendre polynomials