

Regge poles and Watson-Sommerfeld Regge formula in QM

Although the Big Challenge is in QFT, the importance of Regge works cannot be underestimated.

Understanding how the Watson-Sommerfeld-Regge formula is modified by the presence of monopoles will shed considerable light on how to extend the GRIBOV-FROISSART formula in QFT when defects (such as monopoles) are present

STARTING POINTS

Schrodinger equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V(\mathbf{r}) \psi = E \psi$$

$$\Psi(r, \theta, \varphi, t) = \frac{e^{-iEt}}{r} \psi(r) Y_l^m(\theta, \varphi)$$

$$K^2 \psi = \left[-\frac{d^2}{dr^2} + \frac{l^2 - \frac{1}{4}}{r^2} + V \right] \psi;$$

$$(\vec{l})^2 Y_l^m = -l(l+1) Y_l^m$$

$$E = \frac{K^2}{2M}; \quad V(r) = \int_{M_0}^{+\infty} \sigma(\rho) \frac{\exp[-\rho r]}{r} d\rho; \quad M_0 > 0;$$

$\vec{l} \Rightarrow$ orbital angular momentum generators \Rightarrow

Y_l^m are the usual spherical harmonics \Rightarrow

$\boxed{l = l + \frac{1}{2}}$ \Rightarrow This 'obvious relation' is the link between the radial and the angular part in the

Scattering problem

Watson-Sommerfeld-Regge formula

$$f(\theta, k) = \frac{1}{2ki} \sum_{l=0}^{+\infty} P_l(\cos\theta) \left\{ \exp\left[2i\delta_{\lambda(l)}(k)\right] - 1 \right\}; \quad \lambda(l) = l + \frac{1}{2}$$

Standard formula for the scattering amplitude from short range potential where $\delta_l(k)$ is the phase shift and $P_l(\cos\theta)$ are Legendre Polynomial (from the spherical harmonics)

Note that without the 'link' $\lambda = \lambda(l)$ we would be unable to evaluate the sum

Regge: the radial Schrodinger equation (which determines the phase-shift $\delta_\lambda(k)$) has nice properties also for complex values of λ

The S-matrix element $S(\lambda, k) = \exp[2i\delta_\lambda(k)]$ can be analytically

The S-matrix element $S(l, k) = \exp[i\delta_l(k)]$ can be analytically continued for complex values of l to a meromorphic function of l : in QM for short range potential the only singularities are poles (and there is only a finite number of them)

The poles of $S(l, k)$ as function of l are Regge poles

The positions of these poles depend on the energy

"moving poles" \rightarrow Since the link is trivial the analytic properties of $S(l, k)$ as function of l are the same as function of l

Asymptotic behavior and Regge poles

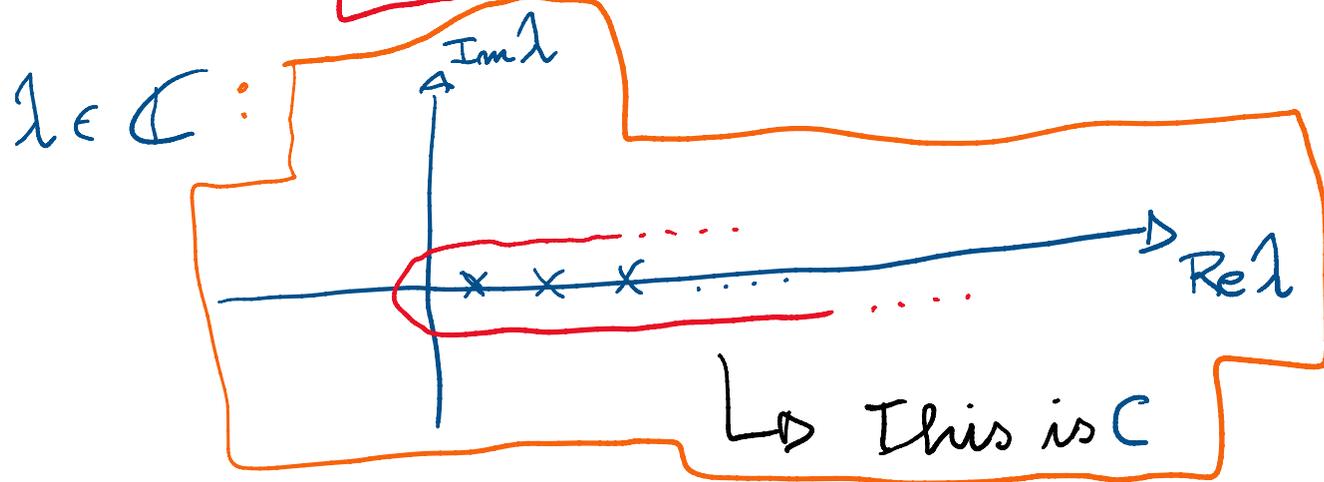
Since the phase shift can be extended to a meromorphic function of l and l is an analytic function of l

(which is the angular label which defines the partial wave sum) $f(\theta, k)$ can be expressed as

$$f(\theta, k) = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l(k)$$

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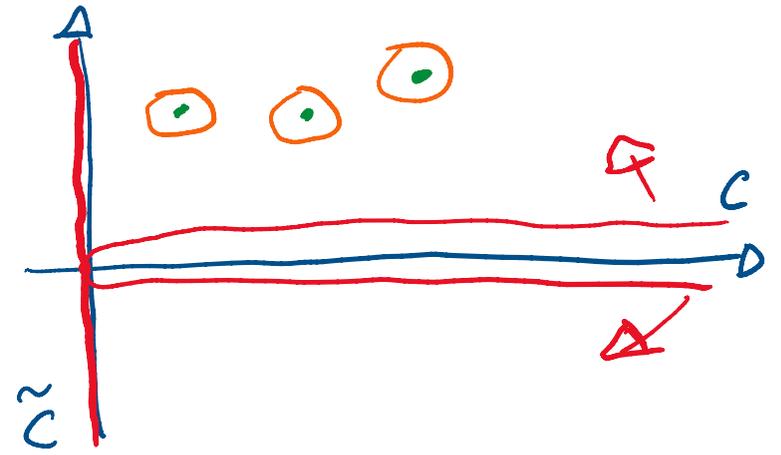
$$f(\theta, \kappa) = \frac{1}{2\pi\kappa} \int_C \frac{\lambda d\lambda}{\cos(\pi\lambda)} P_{\lambda-1/2}(-\cos\theta) \{S(\lambda, \kappa) - 1\}$$



Regge: one can open the circuit C to the circuits

$$\tilde{C} = \{ \text{Re } \lambda = 0 \iff \text{Re } \lambda = -1/2 \}$$

and in doing so one picks up (a finite number of) Regge poles: the final



... + 1 - scattering amplitude reads:

Regge formula for the scattering amplitude reads:

$$f(\theta, \kappa) = \frac{1}{2\pi\kappa} \int_{-\infty}^{+\infty} \frac{\lambda d\lambda}{\cos(\pi\lambda)} P_{i\lambda-1/2}(-\cos\theta) [S(i\lambda, \kappa) - 1] - \frac{i}{\kappa} \sum_{j=1}^N R_j P_{\alpha_j}(-\cos\theta)$$

B.I. \equiv background integral

sum over Regge poles

Interesting region from QFT perspective

$\cos\theta \rightarrow$ large and complex $\Rightarrow P_{i\lambda-1/2}(-\cos\theta)$ is small

compared to the sum over Regge poles since the real part of the label of the Legendre polynomial is small

thus the asymptotic behavior is:

$$f(\theta, \kappa) \xrightarrow{\text{large and complex}} (-\cos\theta)^{\alpha_j^*(\kappa)} \approx_{\text{QFT}} S^{\alpha(t)}$$

$$f(\theta, k) \xrightarrow{\text{large } |k|} (-k^{\alpha_j}) \quad \approx \text{QFT}$$

where $\alpha_j^*(k)$ is the Regge pole with the biggest positive real part

Importance of this result

This result (extended to QFT by GRIBOV, FROISSART, LIPATOV, Mandelstam, Gell-mann, Chew, Frautschi, ...) had a huge impact on high energy physics allowing a very predictive phenomenological approach in terms of few parameters

A key technical result is that the background integral is negligible with respect to Regge poles

($P_{i, \lambda - 1/2}$ with λ real in the B.I.)

$n \quad D \dots T$

($\gamma_{i2-\frac{1}{2}}$ with λ max in the v. l.)

from QFT perspective this is especially relevant
in the I. R. region.