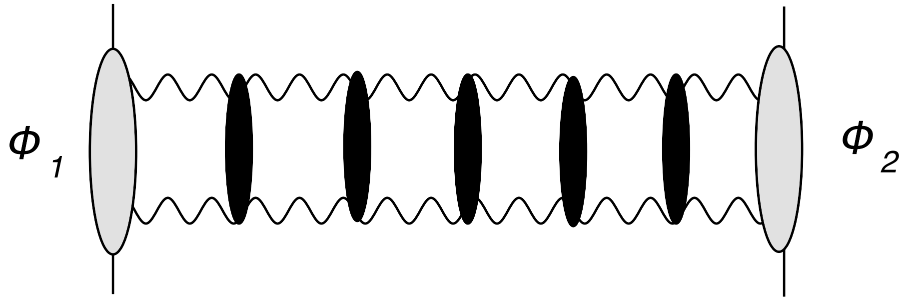
BFKL - The discrete Pomeron

Lipatov 1986;
Lipatov, Ross,Kowalski

- Introduction
- BFKL with infrared cutoff
- Results from HERA fits

Introduction

BFKL ladder:



$$A(s,t) = is \int \frac{d\omega}{2\pi i} (-s)^{\omega} \Phi_{\omega}(q^2)$$

$$\Phi_{\omega}(q^2) = \int \frac{d^2k d^2k'}{(2\pi)^6} \Phi_1(k,q) \Phi_{\omega}(k,k',q) \Phi_2(k',q)$$

$$\omega \Phi_{\omega}(k,k',q) = \frac{\delta^{(2)}(k-k')}{k^2 {k'}^2} + K_{BFKL} \otimes \Phi_{\omega}(k,k') - (\beta(k^2) + \beta((q-k)^2) \Phi_{\omega}(k,k',q)$$

leading order:

$$K(k,k',q) = g^2 \left(q^2 - \frac{k^2(q-{k'}^2) + k'^2(q-k)^2}{(k-k')^2} \right)$$

In leading order: scale invariant, infrared finite
In NLO: scale invariance lost (not: in N =4 SYM)

Many attempts to find experimental evidence
(forward jets, Mueller-Navelet, HERA data)

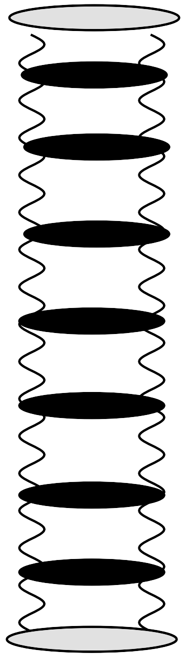
Building block for saturation, heavy ion physics

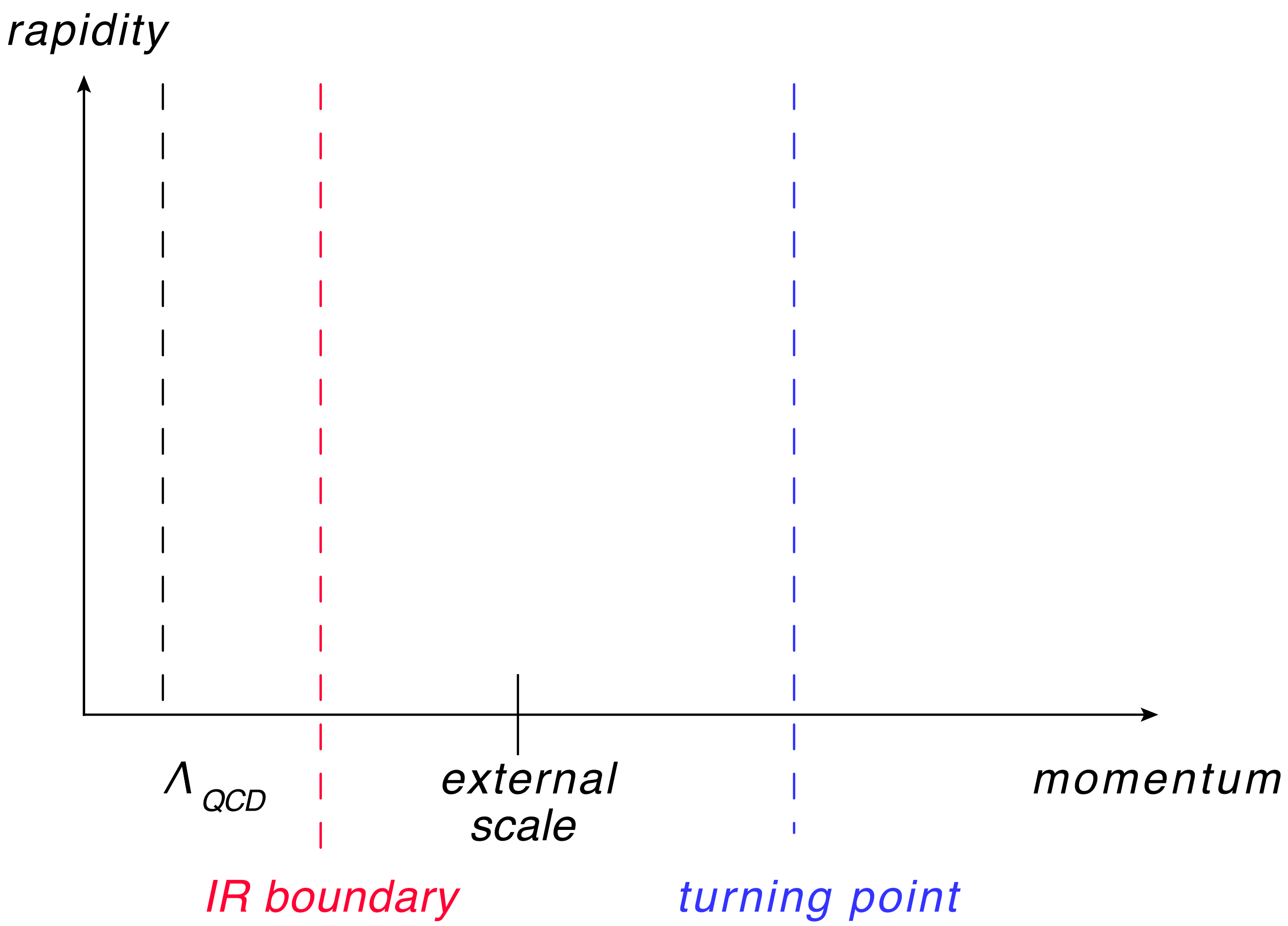
In NLO: QCD coupling becomes running

- scale invariance is lost
- BFKL kernel is modified

IR cutoff needed:

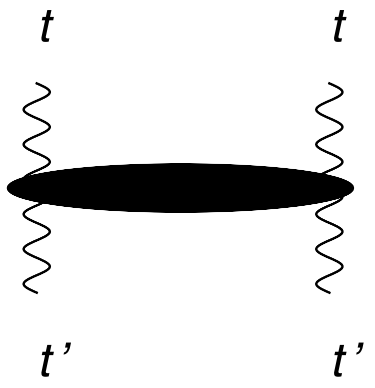
- impose boundary value at fixed scale (Lipatov; Lipatov, Kowalski, Ross)
- Infrared regulator: Higgs mass (Lipatov, Levin, Sidikov)
- IR cutoff, RG equations (JB, Contreras, Vacca)





The discrete Pomeron

BFKL equation (for n=0 only, forward direction):



$$\omega f_\omega(t) = \int dt' \sqrt{\bar\alpha(t)} K(t,t') \sqrt{\bar\alpha(t')} f_\omega(t') \qquad t = \frac{k^2}{\Lambda^2_{QCD}},\ \bar\alpha(t) = \frac{1}{\beta_0 t}$$

Ansatz:

$$f_\omega(t) = \sqrt{\frac{t}{2\pi\omega}} \int d\nu g_\omega(\nu) e^{it\nu}$$

Obtain:

$$i\omega\beta_0 \frac{\partial g_\omega(\nu)}{\partial\nu} = \chi_1(\nu) g_\omega(\nu),\ \ g_\omega(\nu) = \exp\left[\frac{1}{i\omega\beta_0}\int_0^\nu d\nu' \chi_1(\nu')\right]$$

For small t oscillatory behavior,
unknown phase

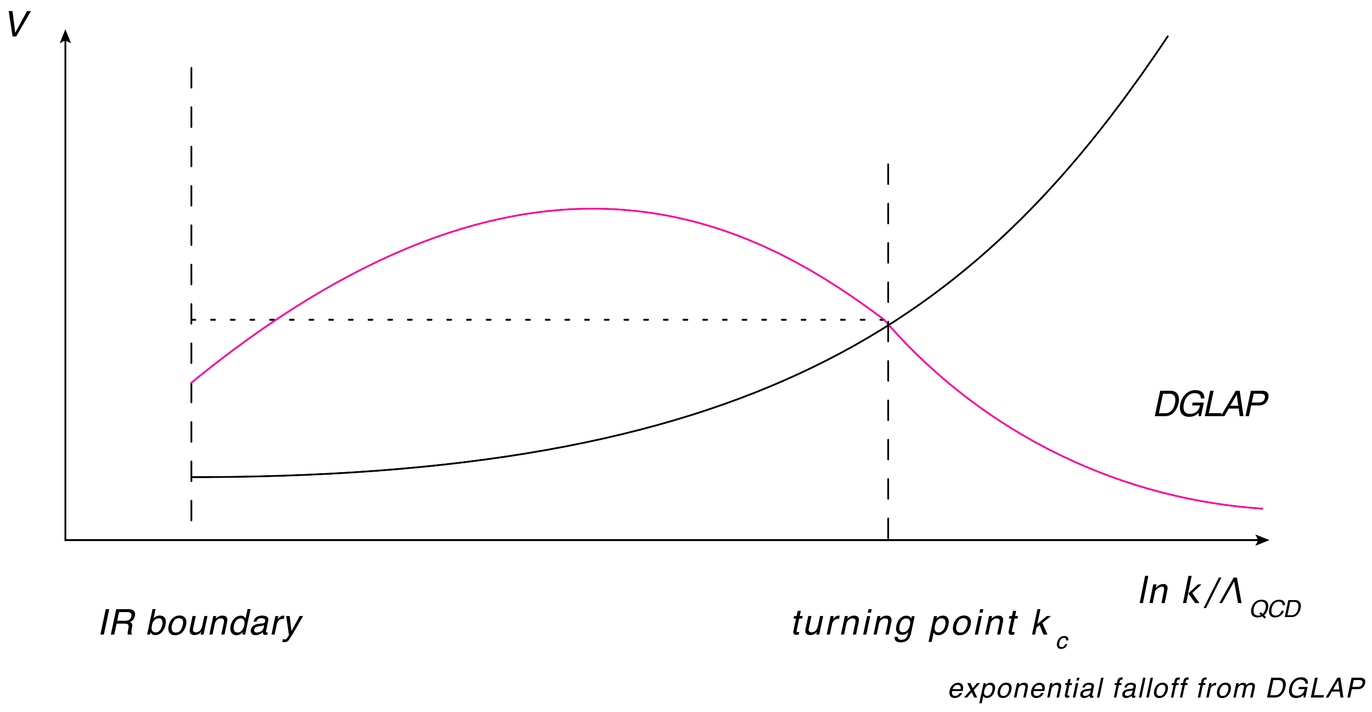
Eigenvalue condition, turning point:

$$\omega = \bar\alpha(t)\chi_1(\nu(t)) \qquad \omega = \bar\alpha(t_c)\chi_1(0):$$

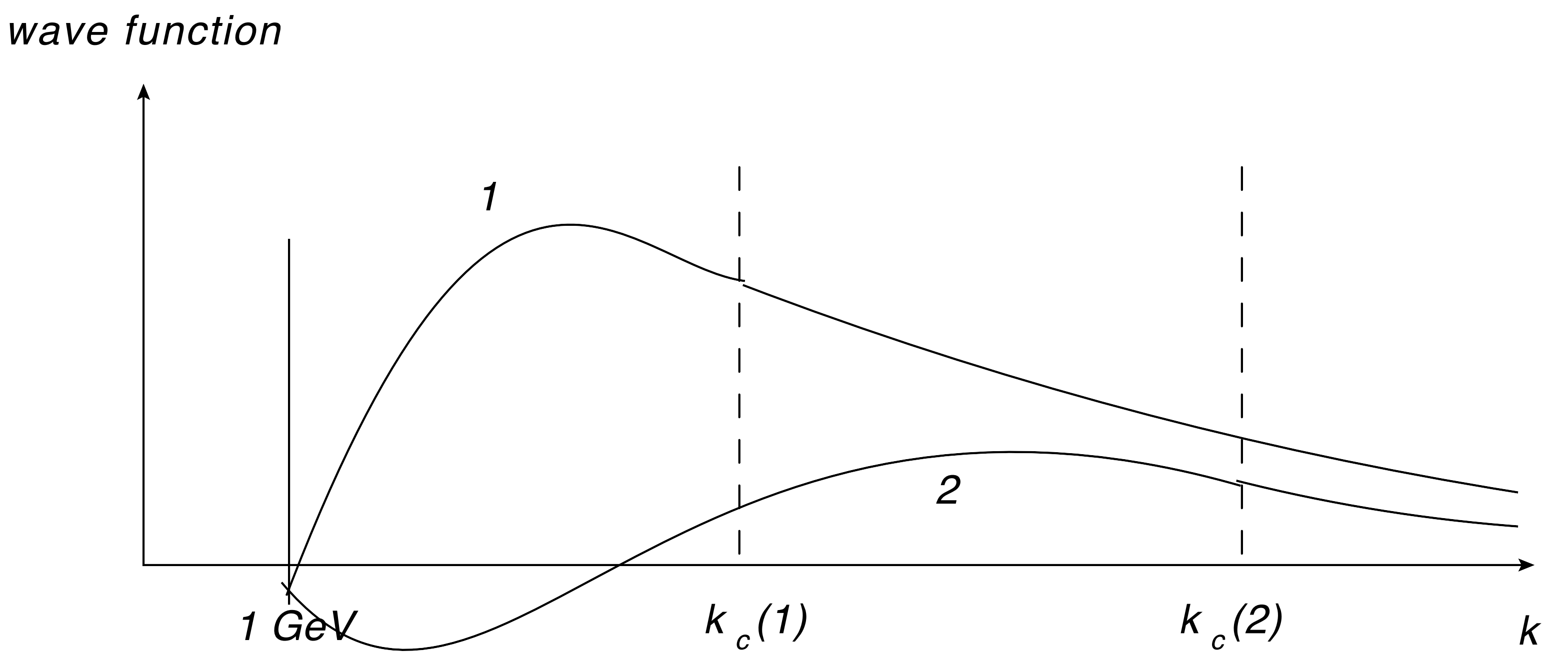
$$t < t_c : \nu(t) = real, \text{ oscillatory behaviour}$$
$$t > t_c : \nu(t) = imaginary, \text{ exponential falloff}$$

Near turning point: Airy functions (similar to 1-dim S-equation)

For illustration: one-dimensional Schrödinger equation:



BFKL picture:



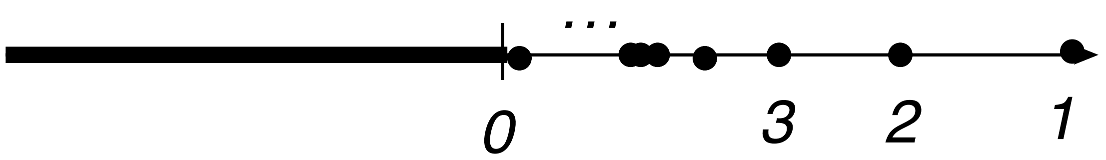
turning point moves into UV region

Results

Kowalski, Lipatov, Ross

Numerical evaluation: fit to HERA data

$$x < 0.001, Q^2 > 6\ GeV^2$$



eigenvalues accumulate at zero

$$\omega_n = \frac{A}{n+B}, \text{with } A = 0.52,\ \ B = 1.62$$

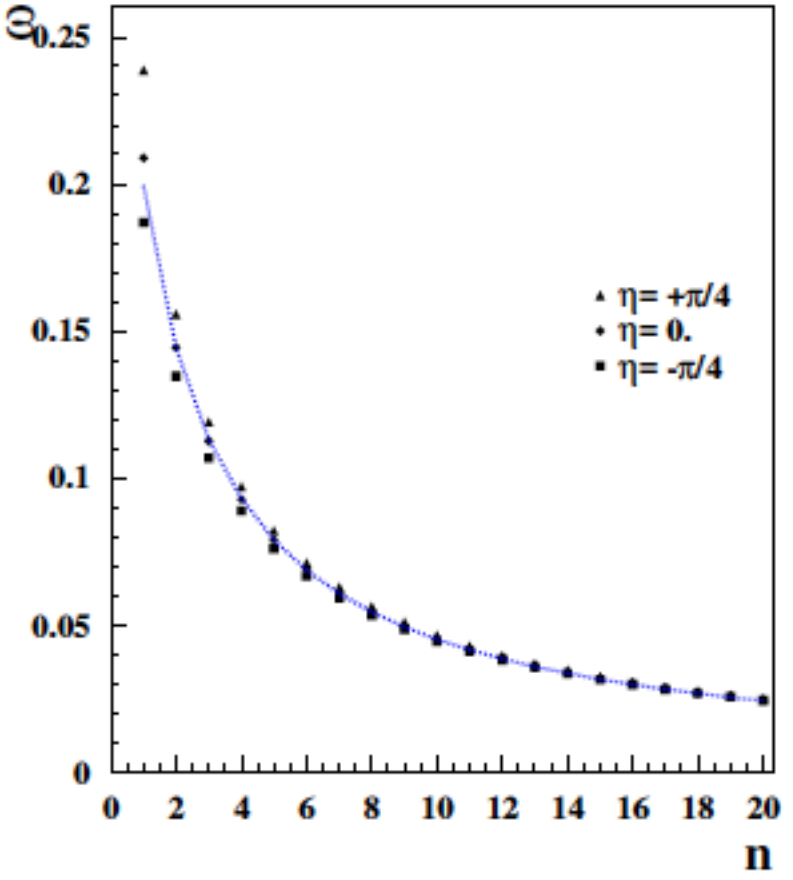
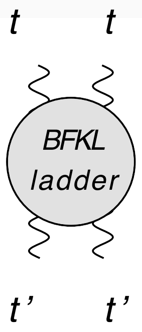


Figure 2: Eigenvalues ω_n determined in NLO for three fixed non-perturbative phases, The dotted line shows a simple parametrisation described in the text.

Fit needs 10 poles

$$G(t,t';\omega) = \sum_{n=1} \frac{f_{\omega_n}(t) f^*_{\omega_n}(t')}{\omega - \omega_n} + \int_{-\infty}^{0} d\omega' \frac{f_{-|\omega'|}(t) f^*_{-|\omega'|}(t')}{\omega - \omega'}$$



Eigenfunctions:

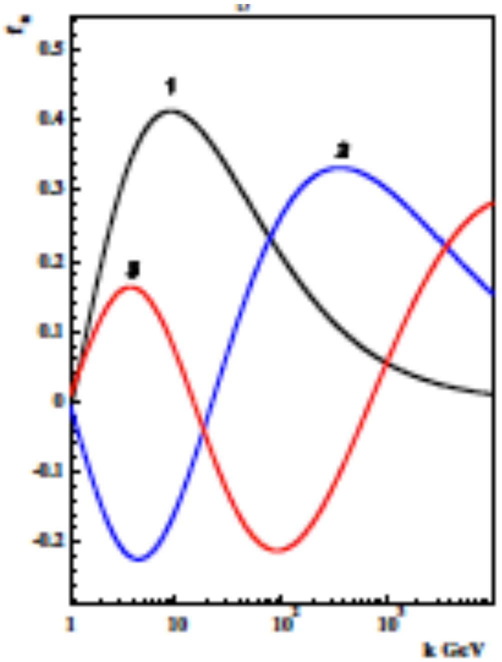


Fig. 12 Eigenfunctions 1, 2, 3, and 7, 8 and 9 in the k region accessible to experiments. The eigenfunctions are plotted with the η_n phases given by the AB fit, performed with the b value of the proton impact factor

First three wave functions:nodes

$$k_c(4) = 2100\text{TeV}$$
$$k_c(3) = 260\text{TeV}$$
$$k_c(2) = 3.3\text{TeV}$$
$$k_c(1) = 50\text{GeV}$$

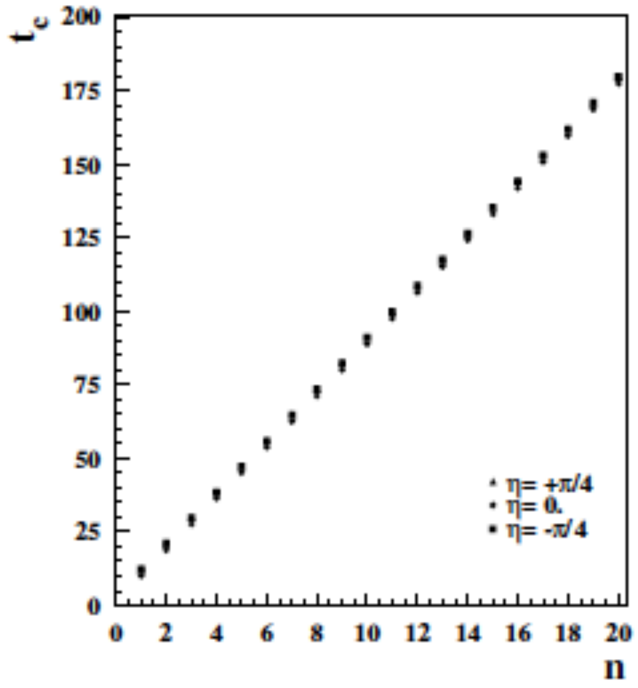


Figure 3: The critical momenta t_c determined in NLO for three fixed non-perturbative phases, η_n. $t_c = \ln k_c^2/\Lambda_{QCD}^2$ with $\Lambda_{QCD} = 275$ MeV.

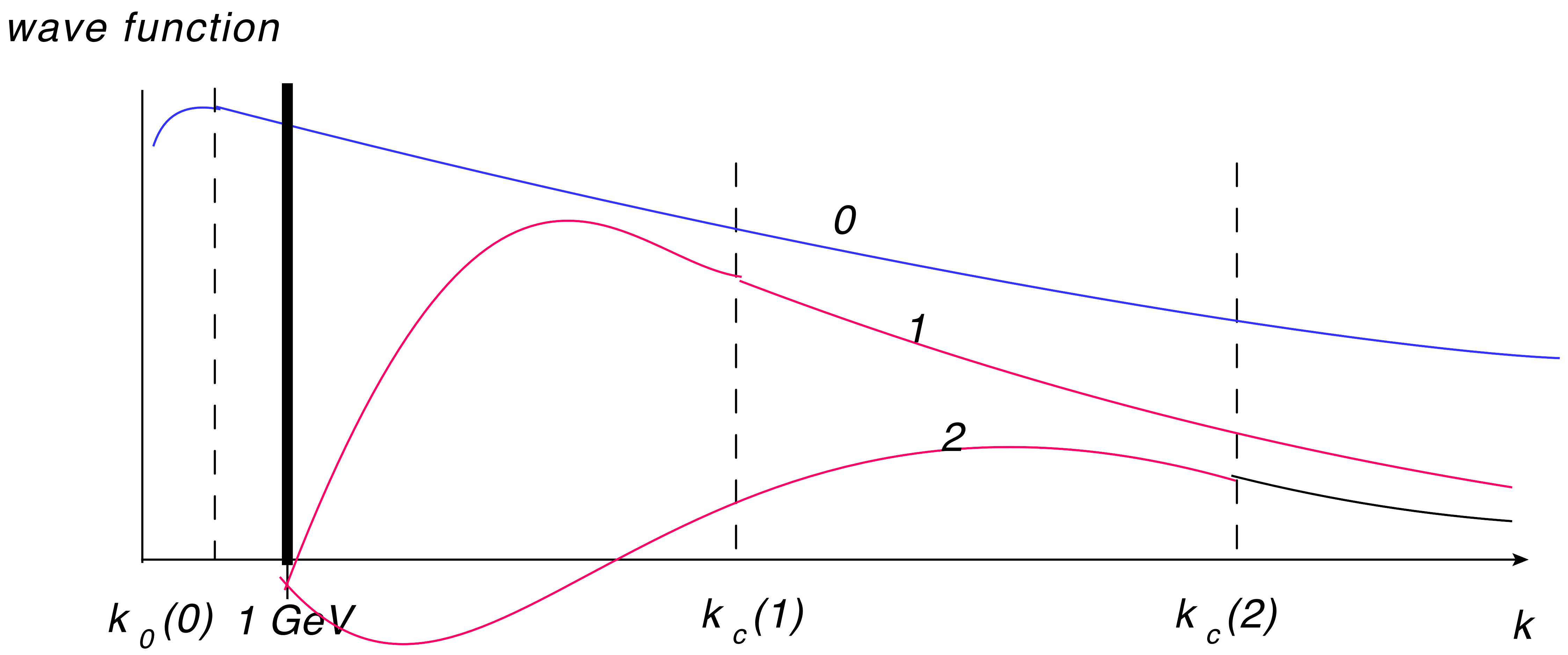
rapid growth of k_c

Quantum mechanics: connection between small and large momenta

Pecularities:

- leading pole decouples
- there should be a ground state, not seen in the fit

Possible picture:



What do we learn from this first numerical application of the discrete Pomeron:

- BFKL needs IR cutoff : spectrum becomes quite different:
  infinite sum of discrete poles
- details of the discrete spectrum are sensitive to large momentum region
- leading eigenvalue close to nonperturbative region
  (still needs ‘unitarization’)
- open questions: nonforward direction, dependence on IR cutoff,...

- QCD BFKL Pomeron is not so far from the nonperturbative Pomeron

- still needs unitarization