High energy scattering in N=4 SYM

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Introduction: Motivation

Conjecture (Maldacena 1997):
correspondence between quantum field theory and String theory

Favorite example: N=4 supersymmetric QFT dual to String theory in AdS_5

Hope: can be solved exactly (integrable)

Dream: QCD, but dual analogue is not known exactly, needs modelling

N=4 SYM: somewhat close to QCD, but has important differences
particle content, scale invariant, no running coupling --> ‘Laboratory’
N=4 correspondence:
large $N_c$

\[ g, N_c, \lambda = g^2 N_c \]

**Weak coupling:**
perturbation theory

**Strong coupling:**
semiclassical approximation,
minimal area of polygon

Perturbation theory and beyond:

exact scattering amplitudes in two, three,... loops,strong coupling limit
scattering amplitudes in multiregge limit
BDS conjecture for MHV amplitudes: Bern, Dixon, Smirnow.

\[ tr(T^{a_1} \cdots T^{a_n}) + \text{noncycl.perm}, \quad A_n = A_n^{\text{tree}} \cdot M_n(\epsilon) \]

\[ \ln M_n = \sum_l a^l \left[ \left( f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \right) + C^{(l)} + E_n^{(l)}[\epsilon] \right] \]

\[ a = \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^\epsilon, \quad d = 4 - 2\epsilon \]

Corrections needed for \( n \geq 6 \): conformal invariant remainder function \( R^{(n)} \)

\[ M_n \sim \exp(\ln M_n^{BDS} + R^{(n)}) \]

Correspondence: amplitude - Wilson loop \( \) (color non-singlet!).

Remainder function depends upon anharmonic ratios, e.g. for \( n=6 \):

\[ u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, \quad u_2 = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{41}^2}, \quad u_3 = \frac{x_{26}^2 x_{35}^2}{x_{25}^2 x_{36}^2} = \frac{s_2 s}{s_{345} s_{456}} \]
Motivation: why multiregge limit?

- Historically, the Regge limit has played an important role
- Structural insight (Regge factorization): test of 2, 3, ... loop results
- Regge limit is of practical interest: want to know multi-leg partonic amplitudes at high energies:

\[ 2 \rightarrow n \text{ gluon production} \]

In the following:
compute scattering amplitudes for

\[ 2 \rightarrow 4, \ 2 \rightarrow 5, \ 2 \rightarrow 6, \ ... \]
Multiregge limit

Method: unitarity, analytic properties

Steinmann relations: no simultaneous singularities in overlapping channels

Each term: energy factors times real partial wave.
Allows analytic continuation into different kinematic regions

\[ (-s_{12})^{\omega_1-\omega_2} (-s)^{\omega_2} F_L + (-s_{23})^{\omega_2-\omega_1} (-s)^{\omega_1} F_R \]

\(2 \rightarrow 4\) five terms
\(2 \rightarrow 5\) fourteen terms
\(2 \rightarrow 6\) forty two terms
...
(Catalan numbers)
Results (Summary)

Regge poles: (BDS)

\[ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \]

Signatured amplitudes: factorization

Large $\mathbb{N}_c$ (planar approximation): in some kinematic regions factorization is broken, unphysical singularities appear

BDS: not correct in just these regions

--- need remainder function, correction to BDS formula: Regge cuts

Friday, 29. December 17
The 2 --> 4 amplitude:

Regge cut cancels the unphysical singularity of the Regge pole. Appears only in the kinematic region where

\[ s, s_{23} > 0, s_{123} < 0, s_{234} < 0 \]

BDS formula contains one loop part of the Regge cut.

In the large N_c (planar) approximation:
Regge cuts are needed for consistency reasons.
The 2--> 5 amplitude:

The same story:

• in certain kinematic regions Regge pole contribution does not factorize but generates unphysical singularities
• Regge cuts compensate these unwanted terms
• BDS contains Regge cuts in one loop.
The $2 \rightarrow 6$ case:

Novel features:

- repetition of cuts (beginning of exponentiation?)
- new cut: three reggeon cut $\rightarrow$ spin chain, integrability
Summary (ongoing work)

n-point scattering amplitudes, remainder functions = corrections to BDS multi-Regge limit

in large $N_c$: Regge poles alone are inconsistent (cf. Veneziano model)

remainder functions= Regge cuts (beginning of cuts contained in BDS)

integrable spin chain: $2\to4$, $2\to5$ two sites

$2\to6$, $2\to7$: three sites

a step towards an exact solution of $N=4$ SYM