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High energy scattering in N=4 SYM

- Introduction: Motivation
- Multiregge limit
- Results

Introduction: Motivation

Conjecture (Maldacena 1997):
correspondence between quantum field theory and String theory

Favorite example: N=4 supersymmetric QFT dual to String theory in AdS_5

Hope: can be solved exactly (integrable)

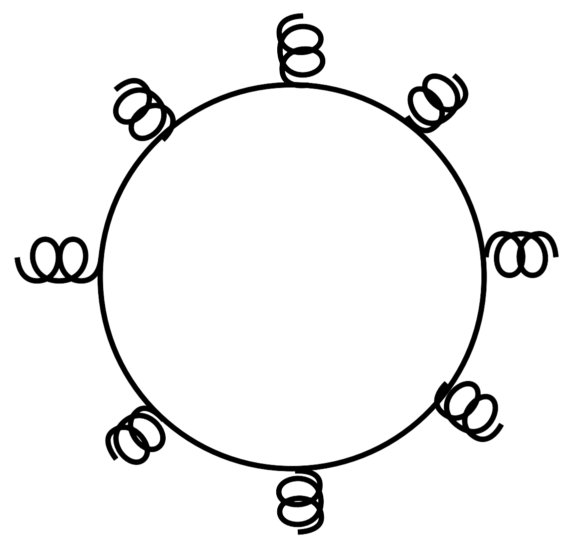
Dream: QCD, but dual analogue is not known exactly, needs modelling

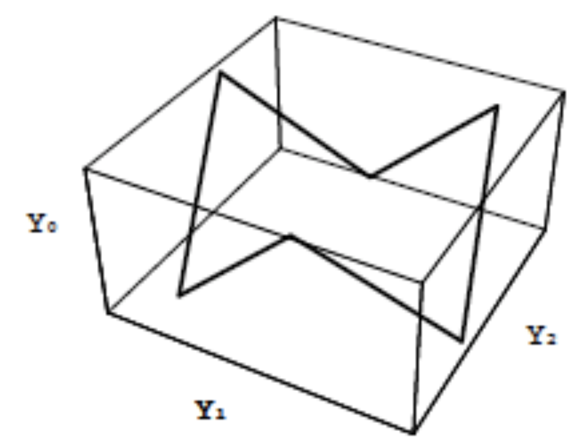
N=4 SYM : somewhat close to QCD, but has important differences
particle content, scale invariant, no running coupling --> 'Laboratory'

N=4 correspondence:
large N_c

N=4 SYM ⇄ *AdS_5 x S_5*

$g,\ N_c,\ \lambda = g^2 N_c$ $g^2 N_c = \frac{R^4}{l_s^4}$





λ small λ large

weak coupling:
perturbation theory

strong coupling:
semiclassical approximation,
minimal area of polygon

Perturbation theory and beyond:

exact scattering amplitudes in two, three,... loops,strong coupling limit
scattering amplitudes in multiregge limit

BDS conjecture for MHV amplitudes: Bern, Dixon, Smirnow.

$$tr(T^{a_1}...T^{a_n}) + noncycl.perm, \quad A_n = A_n^{tree} \cdot M_n(\epsilon)$$

$$\ln M_n = \sum_l a^l \Big[\Big(f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \Big) + C^{(l)} + E_n^{(l)}[\epsilon] \Big]$$

$$a = \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^\epsilon, \;\; d = 4 - 2\epsilon$$

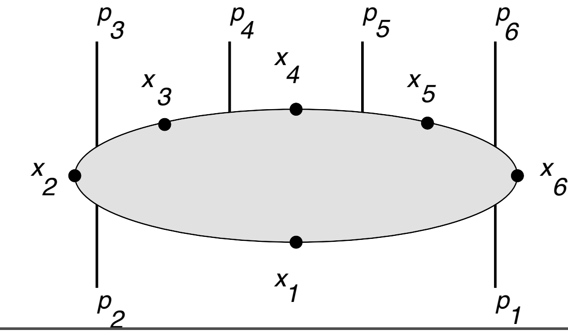
Corrections needed for n ≥ 6: conformal invariant remainder function $R^{(n)}$

$$M_n \sim \exp(\ln M_n^{BDS} + R^{(n)})$$

Correspondence: amplitude - Wilson loop (color non-singlet!).

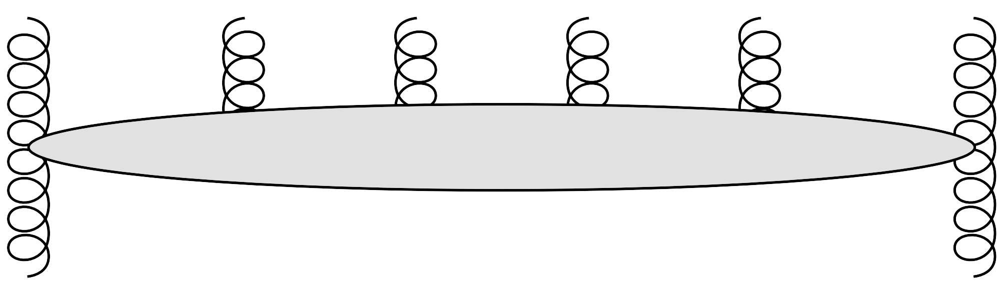
Remainder function depends upon anharmonic ratios, e.g. for n=6:

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, u_2 = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{41}^2}, u_3 = \frac{x_{26}^2 x_{35}^2}{x_{25}^2 x_{36}^2} = \frac{s_2 s}{s_{345} s_{456}}$$



Motivation: why multiregge limit?

- Historically, the Regge limit has played an important role
- structural insight (Regge factorization): test of 2,3, ... loop results
- Regge limit is of practical interest: want to know multi-leg partonic amplitudes at high energies:



$2 \to n$ gluon production

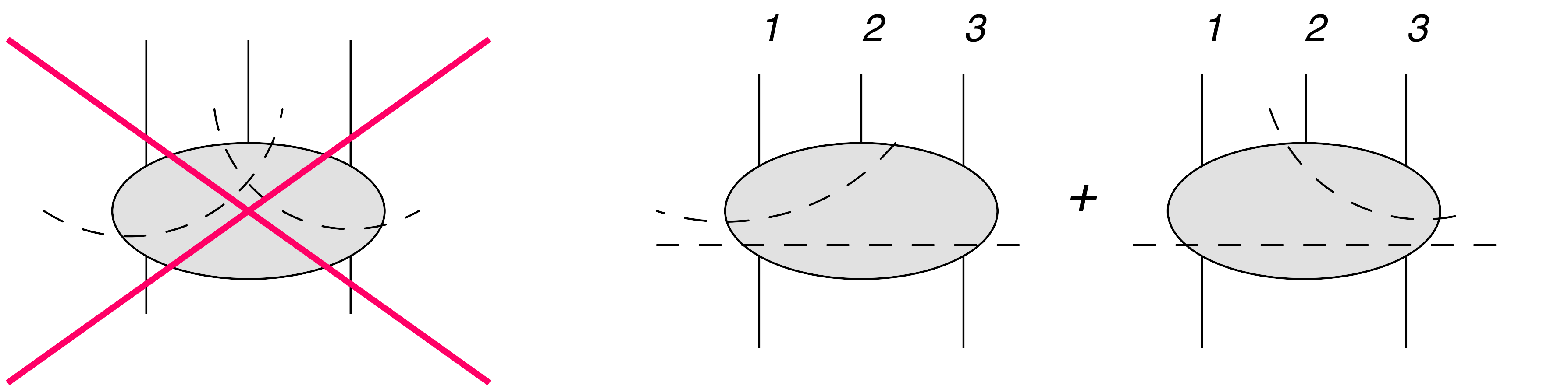
In the following:
compute scattering amplitudes for

$$2 \to 4,\ 2 \to 5, 2 \to 6,\ ...$$

Multiregge limit

Method: unitarity, analytic properties

Steinmann relations: no simultaneous singularities in overlapping channels



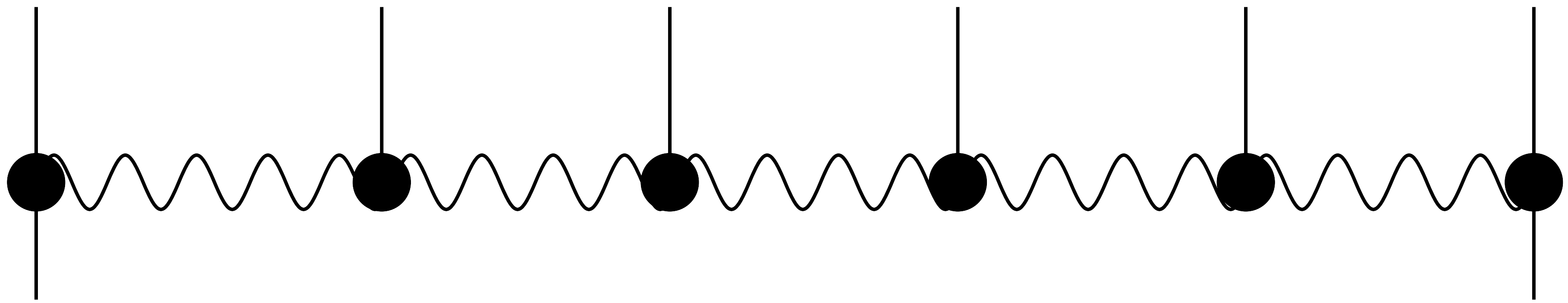
$$(-s_{12})^{\omega_1-\omega_2}(-s)^{\omega_2}F_L+(-s_{23})^{\omega_2-\omega_1}(-s)^{\omega_1}F_R$$

Each term: energy factors times real partial wave.
Allows analytic continuation into different kinematic regions

$2 \to 4$ five terms
$2 \to 5$ fourteen terms
$2 \to 6$ forty two terms
... (Catalan numbers)

Results (Summary)

Regge poles: (BDS)



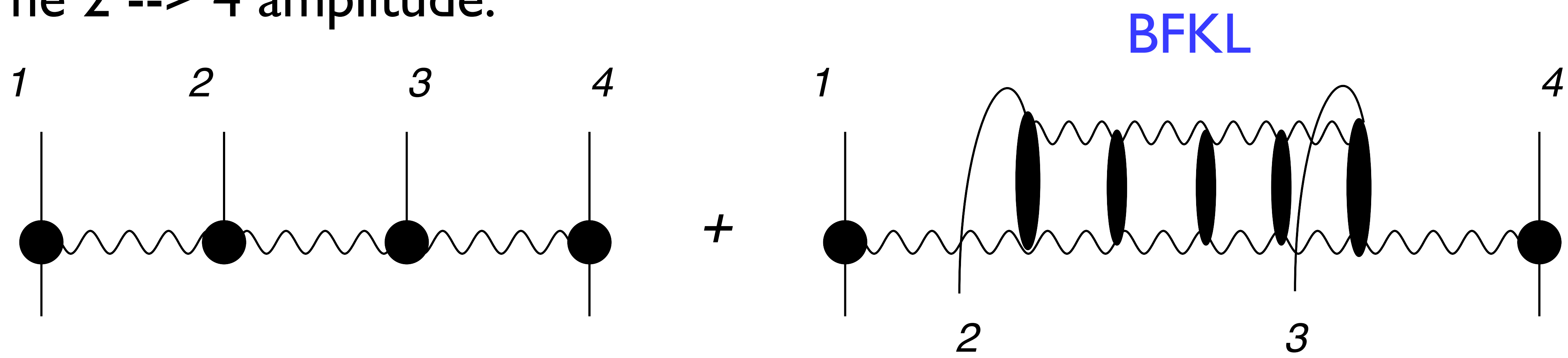
Signatured amplitudes: factorization

Large N_c (planar approximation):
in some kinematic regions factorization is broken, unphysical singularities appear

BDS: not correct in just these regions

---> need remainder function, correction to BDS formula: Regge cuts

The 2 --> 4 amplitude:



Regge cut cancels the unphysical singularity of the Regge pole. Appears only in the kinematic region where

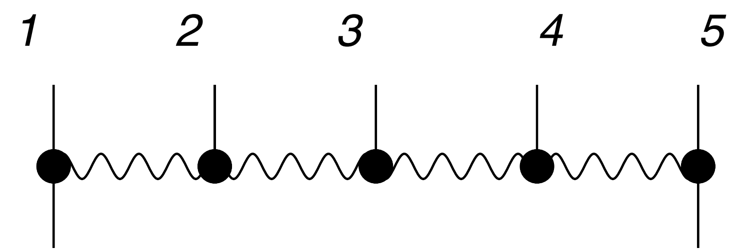
$$s, s_{23} > 0, s_{123} < 0, s_{234} < 0$$

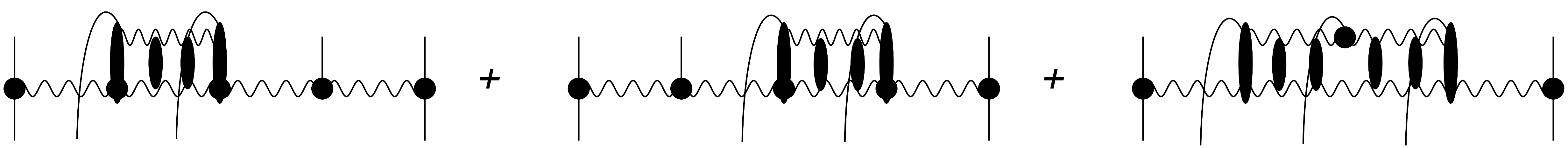
BDS formula contains one loop part of the Regge cut.

In the large N_c (planar) approximation:

Regge cuts are needed for consistency reasons.

The 2--> 5 amplitude:

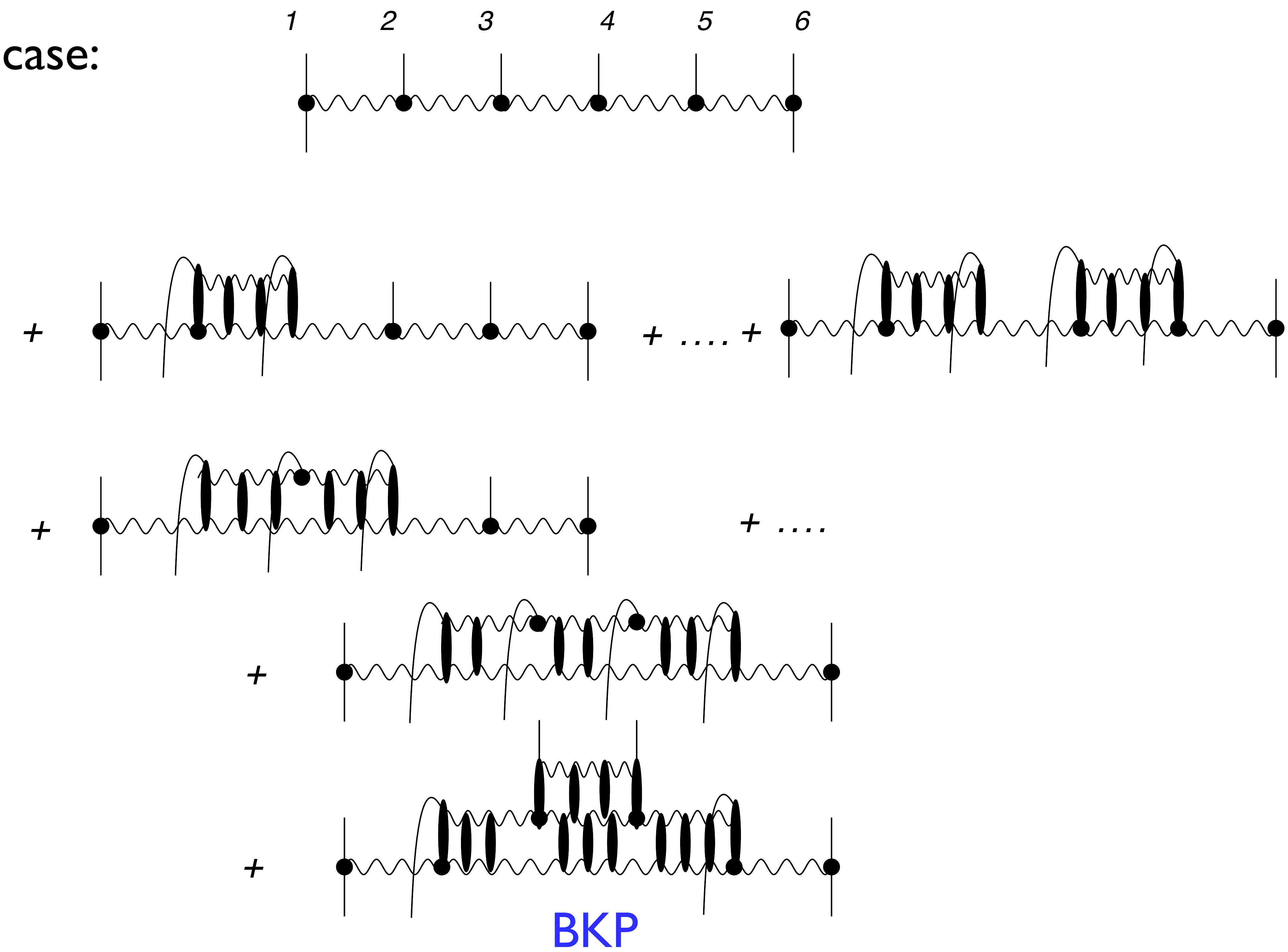




The same story:

- in certain kinematic regions Regge pole contribution does not factorize but generates unphysical singularities
- Regge cuts compensate these unwanted terms
- BDS contains Regge cuts in one loop.

The 2 -->6 case:



Novel features:

- repetition of cuts (beginning of exponentiation?)
- new cut: three reggeon cut--> spin chain, integrability

Summary (ongoing work)

n-point scattering amplitudes, remainder functions = corrections to BDS
multi-Regge limit

in large N_c: Regge poles alone are inconsistent (cf. Veneziano model)

remainder functions= Regge cuts (beginning of cuts contained in BDS)

integrable spin chain: 2-->4 ,2-->5 two sites
2-->6, 2-->7: three sites

a step towards an exact solution of N=4 SYM