Collider Physics

— From basic knowledge

to new physics searches

The 5^{th} Chilean School of High Energy Physics
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Basics of Collider physics

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Physics at an e^+e^- Collider

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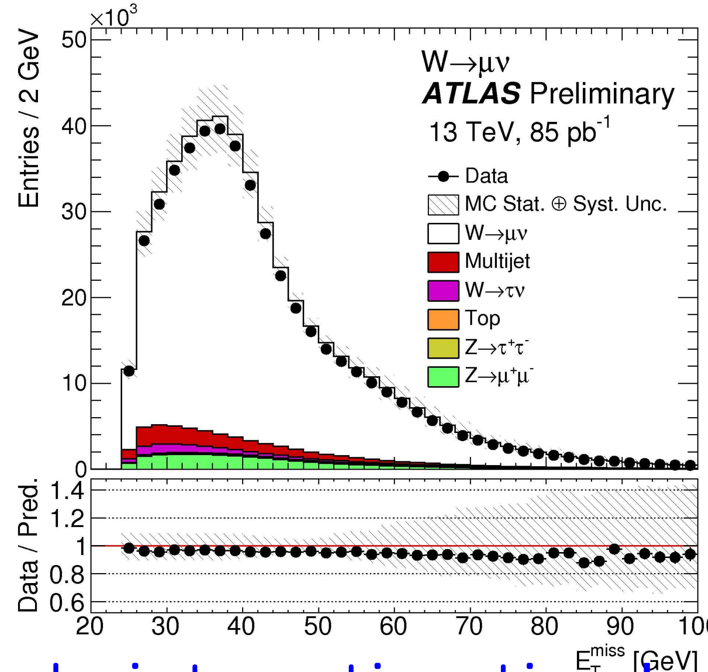
(and New Physics Searches)

Prelude: LHC Run-II is in mission!

June 3, 2015: Run-II started at
$E_{cm} = 6.5 \oplus 6.5 = 13$ TeV.
New era in science begun!

Reaching ≈ 50 fb^{-1}/expt,
LHC is now in winter break,
will resume next April.
Run-II: till the end of 2018.



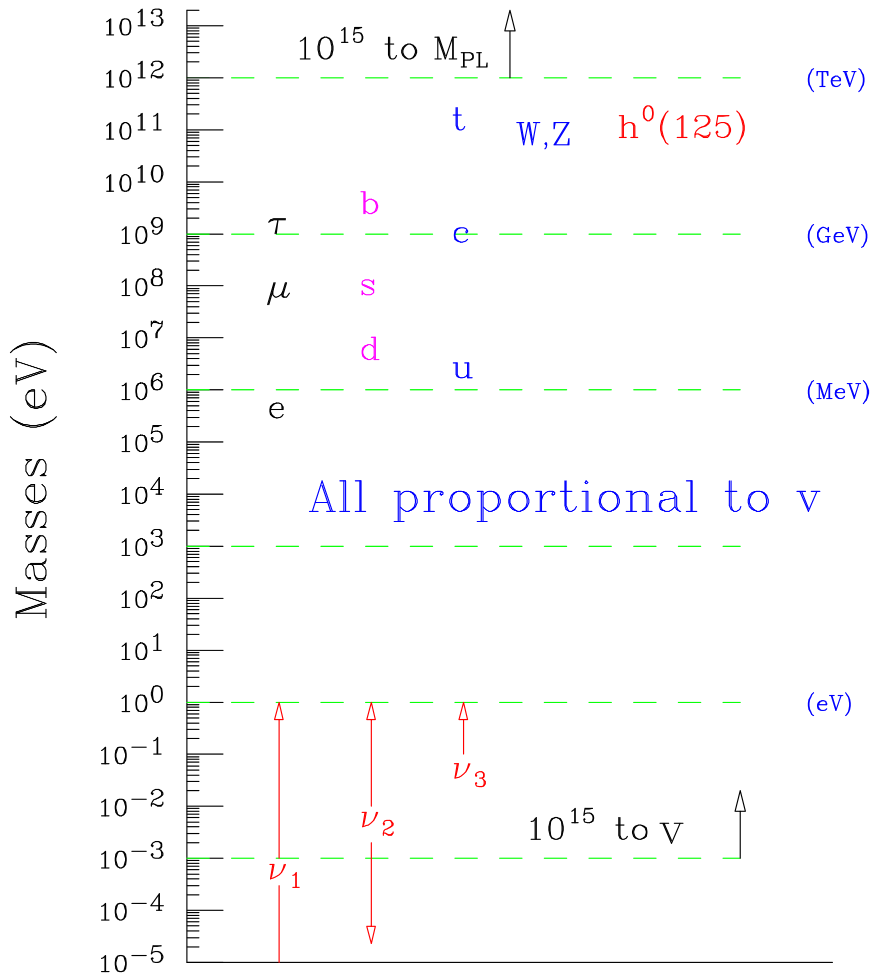
High Energy Physics IS at an extremely interesting time!

The completion of the Standard Model: With the discovery of the Higgs boson, for the first time ever, we have a consistent relativistic quantum-mechanical theory, weakly coupled, unitary, renormalizable, vacuum (quasi?) stable, valid up to an exponentially high scale!

Question: Where IS the next scale?

$\mathcal{O}(1$ TeV$)$? M_{GUT}? M_{Planck}?

Large spread of masses for elementary particles:

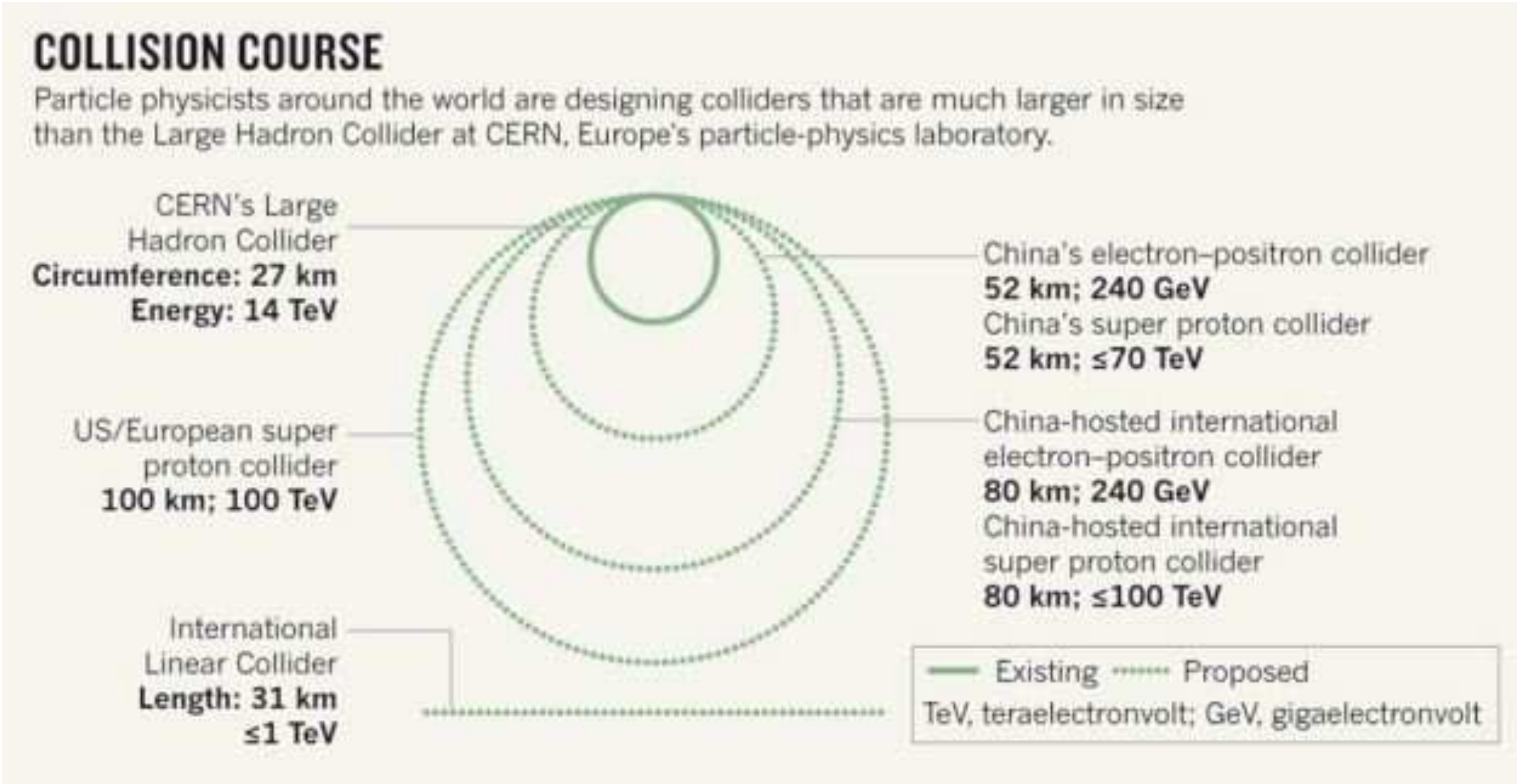


Large hierarchy: Electroweak scale $\Leftrightarrow$ M_{Planck}? Conceptual.

Little hierarchy: Electroweak scale $\Leftrightarrow$ Next scale at TeV? Observational.

Consult with the other excellent lectures.

That motivates us to the new energy frontier! *



- LHC (300 fb^{-1}), HL-LHC (3 ab^{-1}) lead to way: 2015–2030
- HE-LHC at 27 TeV, 15 ab^{-1} under consideration: start 2035–2040?
- ILC as a Higgs factory (250 GeV) and beyond: 2020–2030?
  (250/500/1000 GeV, 250/500/1000 fb^{-1}).
- FCC_{ee} (4×2.5 ab^{-1})/CEPC as a Higgs factory: 2028–2035?
- FCC_{hh}/SPPC/VLHC (100 TeV, 3 ab^{-1}) to the energy frontier: 2040?

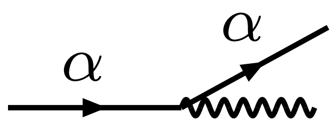
*Nature News (July, 2014)

I-A. Colliders and Detectors

(0). A Historical Count:

Rutherford's experiments were the first

to study matter structure:

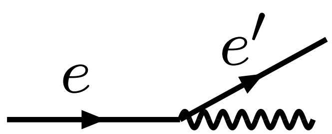


Gold foil target

discover the point-like nucleus:

$$\frac{d\sigma}{d\Omega} = \frac{(\alpha Z_1 Z_2)^2}{4E^2 \sin^4 \theta/2}$$

SLAC-MIT DIS experiments



Proton target

discover the point-like structure of the proton:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(\frac{F_1(x,Q^2)}{m_p} \sin^2 \frac{\theta}{2} + \frac{F_2(x,Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

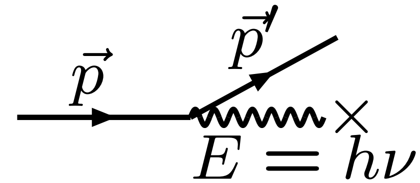
QCD parton model $\Rightarrow 2xF_1(x,Q^2) = F_2(x,Q^2) = \sum_i x f_i(x) e_i^2.$

Rutherford's legendary method continues to date!

(A). High-energy Colliders:

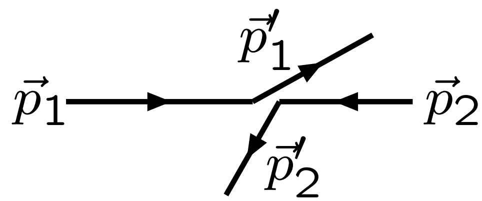
To study the deepest layers of matter,

we need the probes with highest energies.



Two parameters of importance:

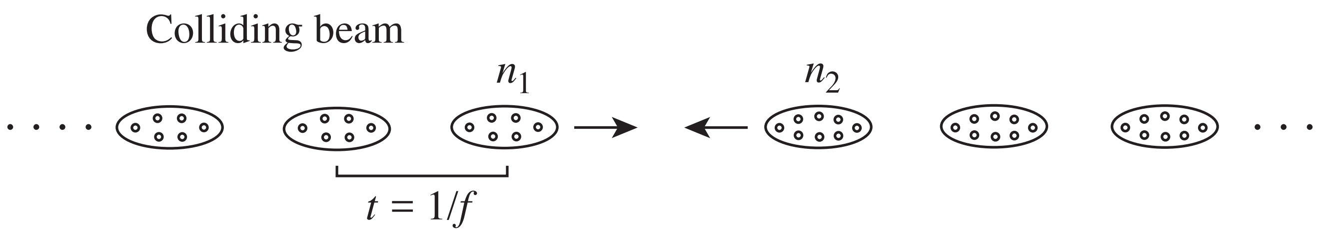
1. The energy:



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{p}_2 = 0. \end{cases}$$

2. The luminosity:



$$\mathcal{L} \propto f n_1 n_2 / a,$$

(a some beam transverse profile) in units of #particles/cm^2/s

$\Rightarrow$ 10^{33} cm^{-2}s^{-1} = 1 nb^{-1} s^{-1} $\approx$ 10 fb^{-1}/year.

Current and future high-energy colliders:

Hadron Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
LHC Run (I) II	(7,8) 13	(10^{32}) 10^{33}	0.01%	40	10.5	26.66
HL-LHC	14	7×10^{34}	0.013%	40	22	26.66
FCC$_{hh}$ (SppC)	100	1.2×10^{35}	0.01%	40	10	100

e^+e^- Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	polar.	L (km)
ILC	0.5–1	2.5×10^{34}	0.1%	3	80, 60%	14 – 33
CEPC	0.25–0.35	2×10^{34}	0.13%			50-100
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

(B). e^+e^- Colliders

The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,

$\Longrightarrow$ it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,

$\Longrightarrow$ the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties,

$\Longrightarrow$ the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol:

For $\sigma \approx 10$ pb $\Rightarrow 0.1$ Hz at 10^{34} cm^{-2}s^{-1}.

- Linear Collider: possible to achieve high degrees of beam polarizations,

$\Longrightarrow$ chiral couplings and other asymmetries can be effectively explored.

Disadvantages

- Large synchrotron radiation due to acceleration,

$$\Delta E \sim \frac{1}{R} \left(\frac{E}{m_e}\right)^4 .$$

Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.

CEPC/FCC$_{ee}$ Higgs Factory

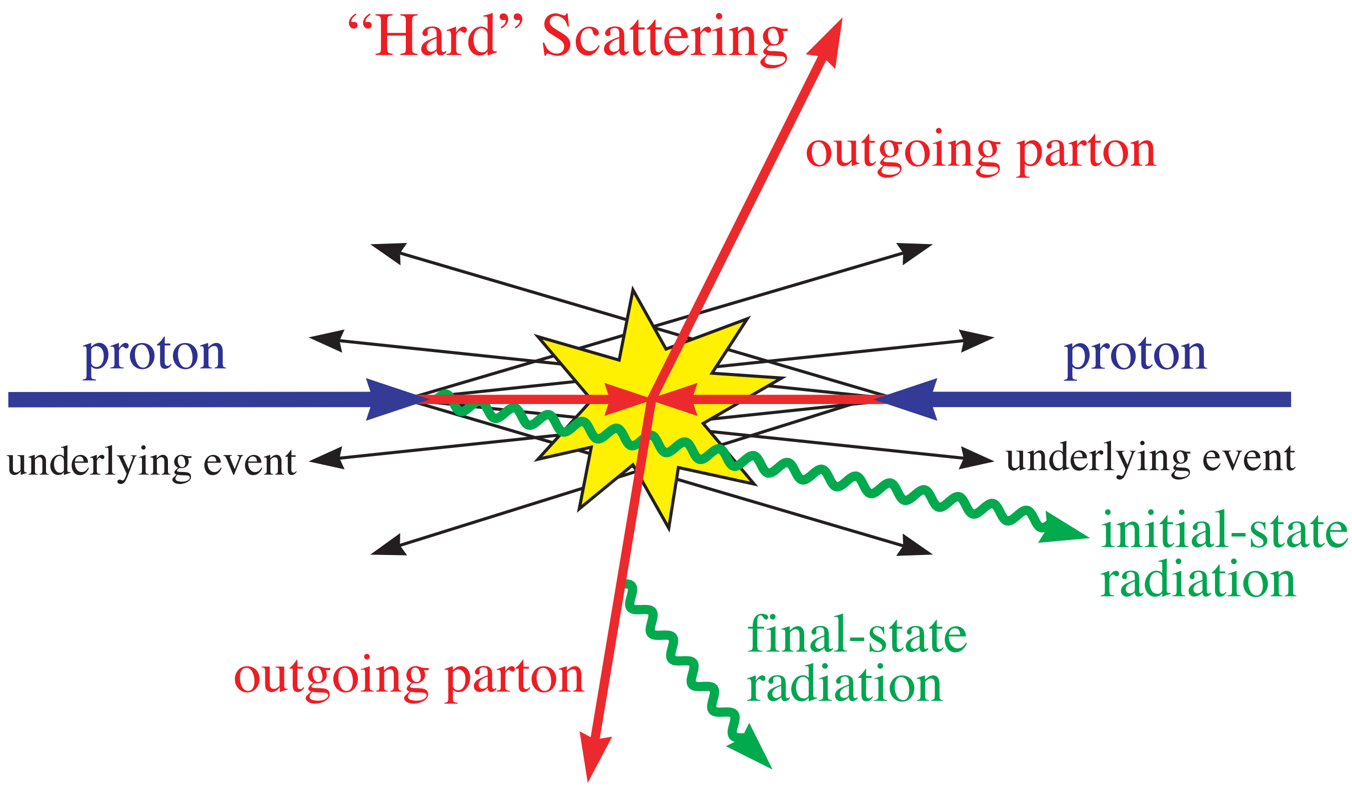
It has been discussed to build a circular e^+e^- collider

$$E_{cm} = 245 \text{ GeV}-350 \text{ GeV}$$

with multiple interaction points for very high luminosities.

(C). Hadron Colliders

LHC: the new high-energy frontier



Advantages

- Higher c.m. energy, thus higher energy threshold:

$\sqrt{S} = 14$ TeV: $\quad M^2_{new} \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.3\sqrt{S} \sim 4$ TeV.

- Higher luminosity: 10^{34}/cm^2/s $\Rightarrow$ 100 fb^{-1}/yr.

  Annual yield: 1B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...
- Multiple (strong, electroweak) channels:

  $q\bar{q}'$, gg, qg, $b\bar{b} \to$ colored; $Q = 0, \pm 1$; $J = 0, 1, 2$ states;

  WW, WZ, ZZ, $\gamma\gamma \to I_W = 0, 1, 2$; $Q = 0, \pm 1, \pm 2$; $J = 0, 1, 2$ states.

Disadvantages

- Initial state unknown:

  colliding partons unknown on event-by-event basis;

  parton c.m. energy unknown: $E^2_{cm} \equiv s = x_1 x_2 S$;

  parton c.m. frame unknown.

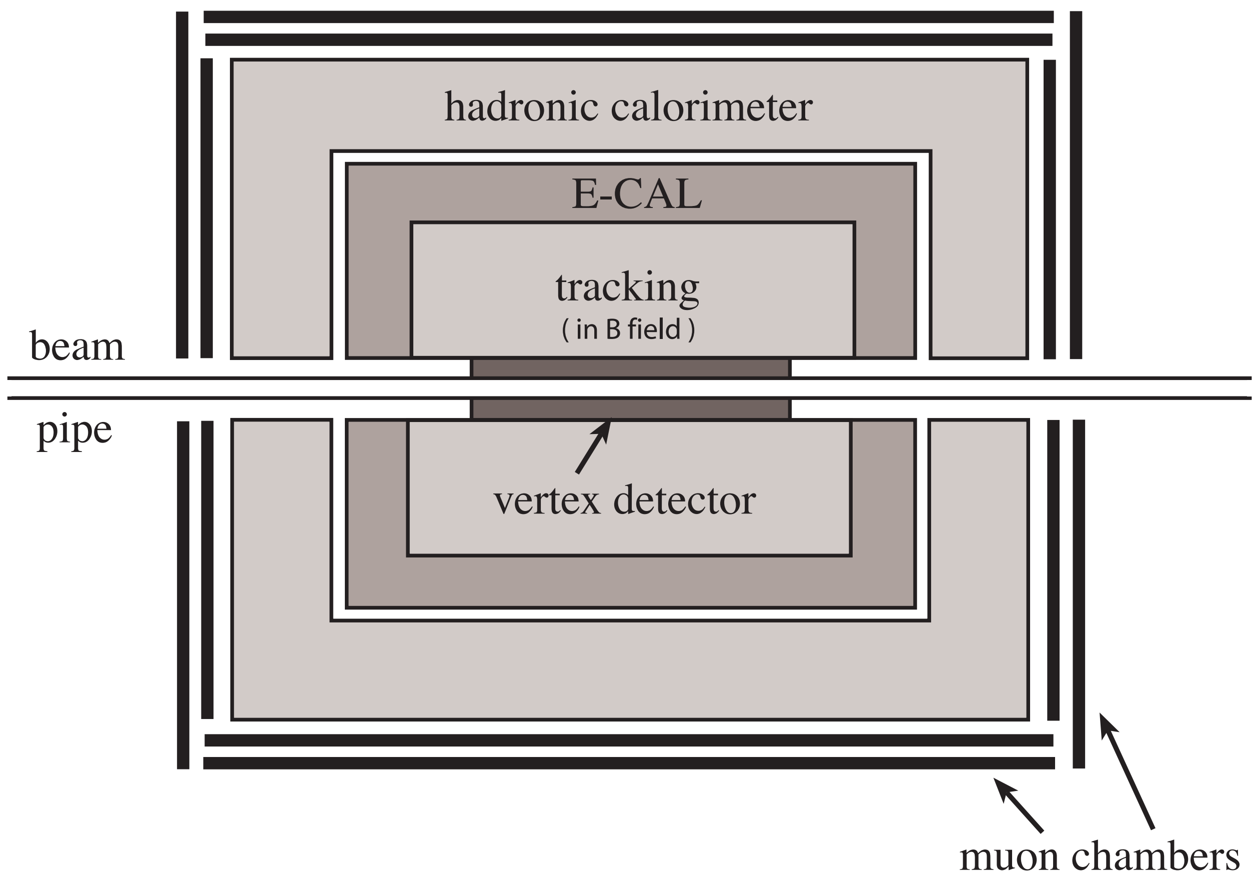
  $\Rightarrow$ largely rely on final state reconstruction.
- The large rate turns to a hostile environment:

  $\Rightarrow$ Severe backgrounds!

Our primary job !

(D). Particle Detection:

The detector complex:
Utilize the strong and electromagnetic interactions between detector materials and produced particles.



What we "see" as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c\ \tau)\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

- stable particles directly "seen":

$$p,\ \bar{p},\ e^{\pm},\ \gamma$$

- quasi-stable particles of a life-time $\tau \geq 10^{-10}$ s also directly "seen":

$$n, \Lambda, K_L^0, ...,\ \mu^{\pm},\ \pi^{\pm}, K^{\pm}...$$

- a life-time $\tau \sim 10^{-12}$ s may display a secondary decay vertex, "vertex-tagged particles":

$$B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}...$$

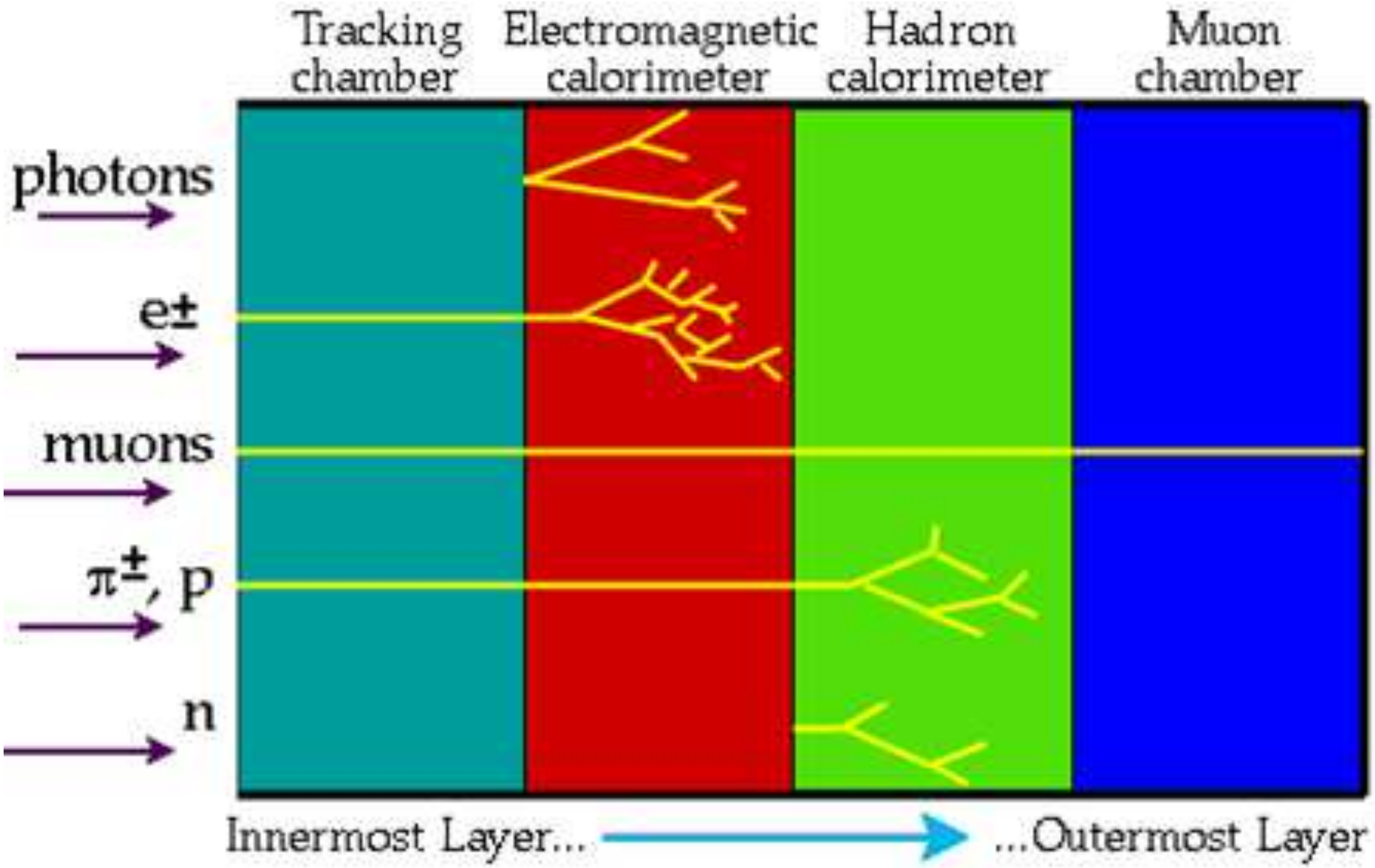
- short-lived not "directly seen", but "reconstructable":

$$\pi^0,\ \rho^{0,\pm}...\ ,\quad Z, W^{\pm}, t, H...$$

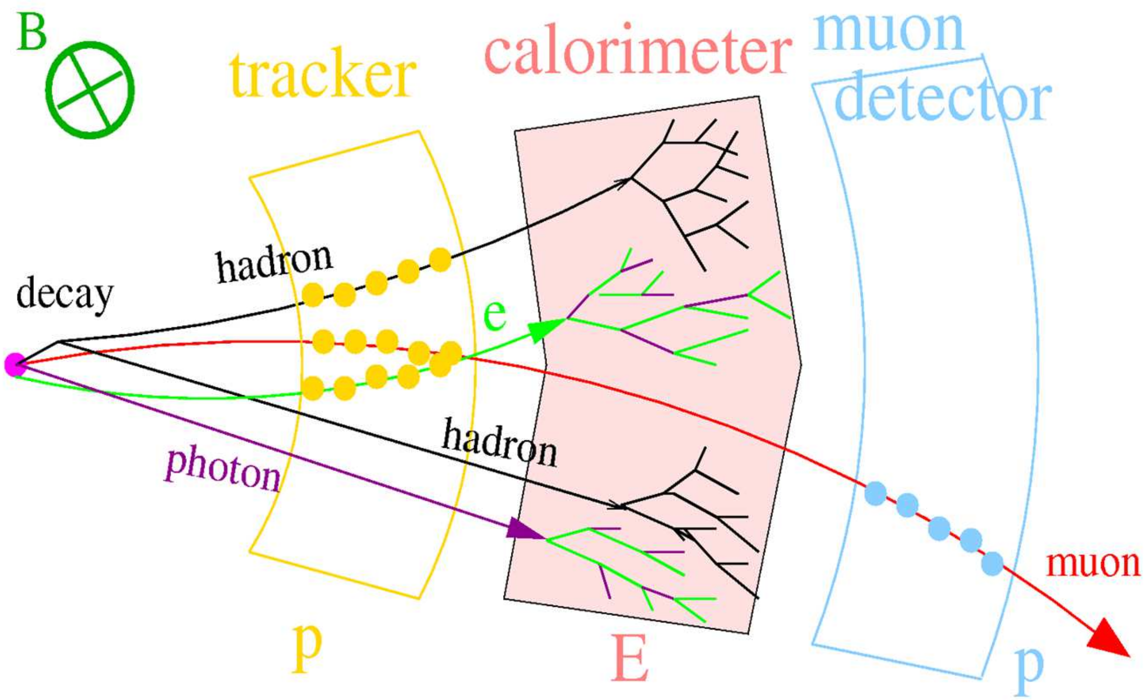
- missing particles are weakly-interacting and neutral:

$$\nu,\ \tilde{\chi}^0, G_{KK}...$$

† For stable and quasi-stable particles of a life-time $\tau \geq 10^{-10} - 10^{-12}$ s, they show up as



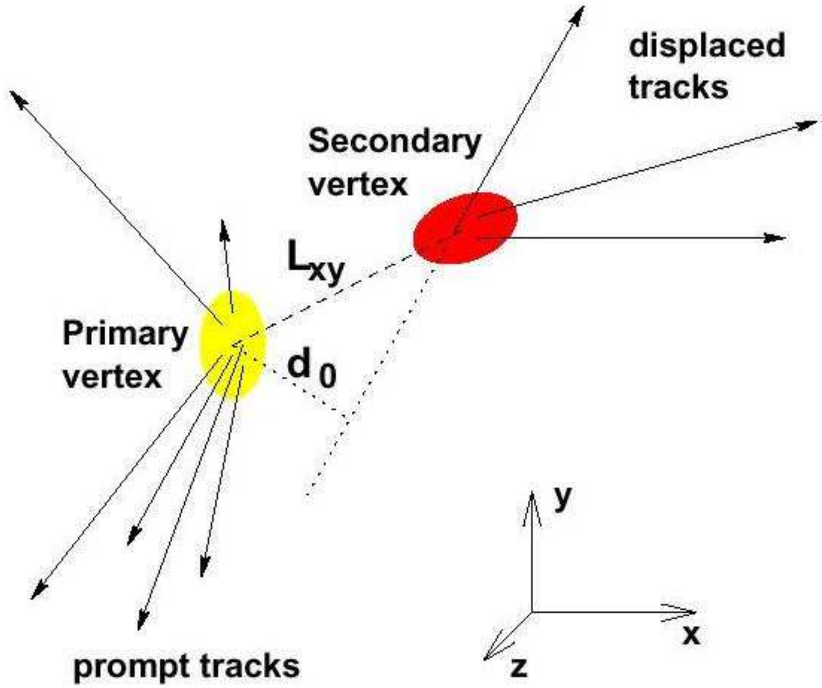
A closer look:



Theorists should know:

$$\begin{aligned}
\text{For charged tracks:} \quad \Delta p/p &\propto p, \\
\text{typical resolution:} \quad &\sim p/(10^4 \text{ GeV}). \\
\text{For calorimetry:} \quad \Delta E/E &\propto \frac{1}{\sqrt{E}}, \\
\text{typical resolution:} \quad &\sim (10\%_{ecal},\ 50\%_{hcal})/\sqrt{E/\text{GeV}}
\end{aligned}$$

† For vertex-tagged particles $\tau \approx 10^{-12}$ s,
heavy flavor tagging: the secondary vertex:



Typical resolution: $d_0 \sim 30 - 50\ \mu$m or so

⇒ Better have two (non-collinear) charged tracks for a secondary vertex;

Or use the "impact parameter" w.r.t. the primary vertex.

For theorists: just multiply a "tagging efficiency":

$$\epsilon_b \sim 70\%; \quad \epsilon_c \sim 40\%; \quad \epsilon_\tau \sim 40\%.$$

† For short-lived particles: $\tau < 10^{-12}$ s or so,
make use of final state kinematics to reconstruct the resonance.

† For missing particles:
make use of energy-momentum conservation to deduce their existence.

$$p_1^i + p_2^i = \sum_f^{obs.} p_f \; + \; p_{miss}.$$

But in hadron collisions, the longitudinal momenta unknown,
thus transverse direction only:

$$0 = \sum_f^{obs.} \vec{p}_{f\ T} \; + \; \vec{p}_{miss\ T}.$$

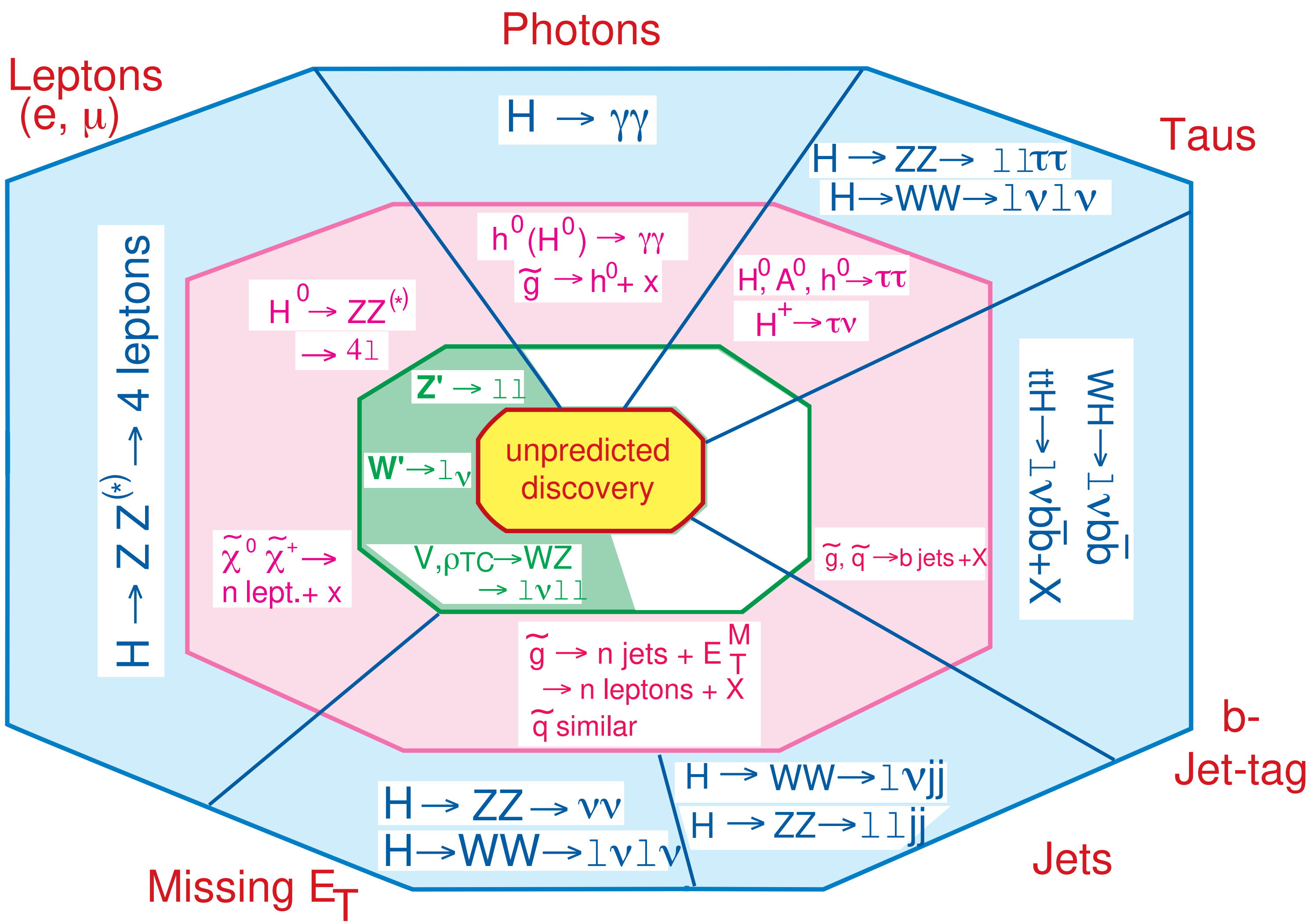
often called "missing p_T" ($\not{p}_T$) or (conventionally) "missing E_T" ($\not{E}_T$).

Note: "missing E_T" (MET) is *conceptually* ill-defined!
It is only sensible for massless particles: $\not{E}_T = \sqrt{\vec{p}^{\,2}_{miss\ T} + m^2}$.

What we "see" for the SM particles (no universality!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b +$ 2 jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^\pm \to \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+ \ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$
the Higgs boson					
$h^0 \to b\bar{b}$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$\to ZZ^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$
$\to WW^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$

How to search for new particles?



y98014_416dPauss rd

Homework:

Exercise 1.1: For a π^0, μ^-, or a τ^- respectively, calculate its decay length for $E = 10$ GeV.

Exercise 1.2: An event was identified to have a $\mu^+\mu^-$ pair, along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider both an e^+e^- and a hadron collider.

Exercise 1.3: Electron and muon measurements: Estimate the relative errors of energy-momentum measurements for an electron by an electromagnetic calorimetry ($\Delta E/E$) and for a muon by tracking ($\Delta p/p$) at energies of $E = 50$ GeV and 500 GeV, respectively.

Exercise 1.4: A 125 GeV Higgs boson will have a production cross section of 20 pb at the 14 TeV LHC. How many events per year do you expect to produce for the Higgs boson with an instantaneous luminosity 10^{33}/cm^2/s? Do you expect it to be easy to observe and why?

I-B. Basic Techniques and Tools for Collider Physics

(A). Scattering cross section

For a $2 \to n$ scattering process:

$$\sigma(ab \to 1+2+...n) = \frac{1}{2s} \overline{\sum} |\mathcal{M}|^2 \, dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \, \delta^4 \left(P - \sum_{i=1}^{n} p_i \right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i \right)^2,$$

where $\overline{\sum}|\mathcal{M}|^2$: dynamics (dimension $4-2n$);
dPS_n: kinematics (Lorentz invariant, dimension $2n-4$.)

For a $1 \to n$ decay process, the partial width in the rest frame:

$$\Gamma(a \to 1+2+...n) = \frac{1}{2M_a} \overline{\sum} |\mathcal{M}|^2 \, dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = (\sum_f \Gamma_f)^{-1}.$$

(B). Phase space and kinematics [*]

One-particle Final State $a + b \to 1$:

$$
\begin{aligned}
dPS_1 &\equiv (2\pi)\, \frac{d^3\vec{p}_1}{2E_1} \delta^4(P - p_1) \\
&\doteq \pi |\vec{p}_1| d\Omega_1 \delta^3(\vec{P} - \vec{p}_1) \\
&\doteq 2\pi\ \delta(s - m_1^2).
\end{aligned}
$$

where the first and second equal signs made use of the identities:

$$
|\vec{p}| d|\vec{p}| = E dE, \quad \frac{d^3\vec{p}}{2E} = \int d^4p\ \delta(p^2 - m^2).
$$

Kinematical relations:

$$
\begin{aligned}
\vec{P} &\equiv \vec{p}_a + \vec{p}_b = \vec{p}_1, \quad E_1^{cm} = \sqrt{s} \text{ in the c.m. frame,} \\
s &= (p_a + p_b)^2 = m_1^2.
\end{aligned}
$$

The "dimensinless phase-space volume" is $s(dPS_1) = 2\pi$.

[*]E.Byckling, K. Kajantie: Particle Kinemaitcs (1973).

Two-particle Final State $a + b \to 1 + 2$:

$$\begin{aligned}
dPS_2 &\equiv \frac{1}{(2\pi)^2}\ \delta^4\,(P - p_1 - p_2)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2} \\
&\doteq \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\ d\Omega_1 = \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\ d\cos\theta_1 d\phi_1 \\
&= \frac{1}{4\pi}\frac{1}{2}\ \lambda^{1/2}\left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2, \\
d\cos\theta_1 &= 2dx_1, \quad d\phi_1 = 2\pi dx_2, \quad 0 \le x_{1,2} \le 1,
\end{aligned}$$

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

$$|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}},\ \ E_1^{cm} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}},\ \ E_2^{cm} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}},$$

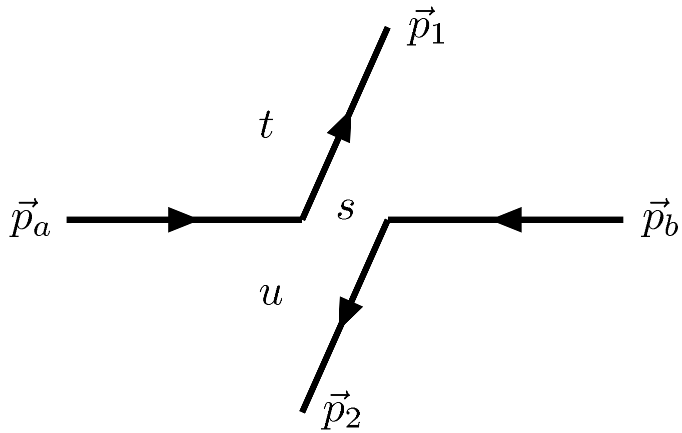
$$\lambda(x, y, z) = (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

$$\frac{dPS_2}{s\ dPS_1} \approx \frac{1}{(4\pi)^2}.$$

just like a "loop factor".

Consider a $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$,



the (Lorentz invariant) Mandelstam variables are defined as

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\ t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\ u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}), \\ & s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2. \end{aligned}$$

The two-body phase space can be thus written as

$$dPS_2 = \frac{1}{(4\pi)^2} \frac{dt\ d\phi_1}{s\ \lambda^{1/2}\left(1, m_a^2/s, m_b^2/s\right)}.$$

Exercise 2.1: Assume that $m_a = m_1$ and $m_b = m_2$. Show that

$$t = -2p_{cm}^2(1 - \cos\theta_{a1}^*),$$
$$u = -2p_{cm}^2(1 + \cos\theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s},$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.
Note: t is negative-definite; $t \to 0$ in the collinear limit.

Exercise 2.2: A particle of mass M decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed β_z? Compare the result with your expectation for the shape change for a basket ball.

Three-particle Final State $a+b \to 1+2+3$:

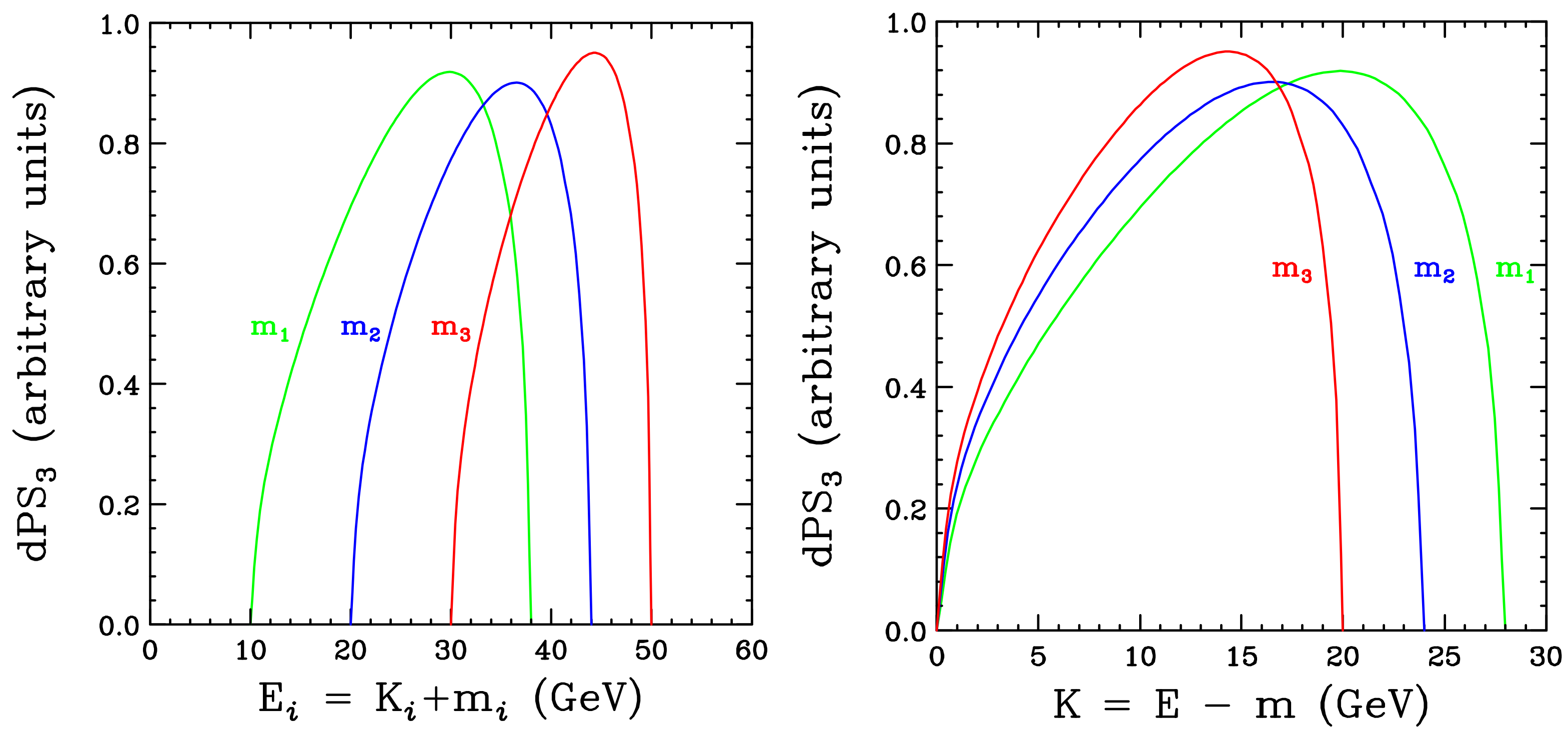
$$dPS_3 \equiv \frac{1}{(2\pi)^5}\,\delta^4(P-p_1-p_2-p_3)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_3}{2E_3}$$
$$\doteq \frac{|\vec{p}_1|^2\ d|\vec{p}_1|\ d\Omega_1}{(2\pi)^3\ 2E_1}\ \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_2^{(23)}|}{m_{23}}\ d\Omega_2$$
$$= \frac{1}{(4\pi)^3}\ \lambda^{1/2}\left(1, \frac{m_2^2}{m_{23}^2}, \frac{m_3^2}{m_{23}^2}\right)\ 2|\vec{p}_1|\ dE_1\ dx_2 dx_3 dx_4 dx_5.$$

$$d\cos\theta_{1,2} = 2dx_{2,4}, \quad d\phi_{1,2} = 2\pi dx_{3,5}, \quad 0 \le x_{2,3,4,5} \le 1,$$
$$|\vec{p}_1^{\,cm}|^2 = |\vec{p}_2^{\,cm} + \vec{p}_3^{\,cm}|^2 = (E_1^{cm})^2 - m_1^2,$$
$$m_{23}^2 = s - 2\sqrt{s}E_1^{cm} + m_1^2, \quad |\vec{p}_2^{\,23}| = |\vec{p}_3^{\,23}| = \frac{\lambda^{1/2}(m_{23}^2, m_2^2, m_3^2)}{2m_{23}},$$

The particle energy spectrum is not monochromatic.
The maximum value (the end-point) for particle 1 in c.m. frame is

$$E_1^{max} = \frac{s + m_1^2 - (m_2+m_3)^2}{2\sqrt{s}}, \quad m_1 \le E_1 \le E_1^{max},$$
$$|\vec{p}_1^{\,max}| = \frac{\lambda^{1/2}(s, m_1^2, (m_2+m_3)^2)}{2\sqrt{s}}, \quad 0 \le p_1 \le p_1^{max}.$$

With $m_i = 10,\ 20,\ 30,\ \sqrt{s} = 100$ GeV.



More intuitive to work out the end-point for the kinetic energy,
– recall the direct neutrino mass bound in β-decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$

In general, the 3-body phase space boundaries are non-trivial.
That leads to the "Dalitz Plots".

One practically useful formula is:

`Exercise 2.3:` A particle of mass M decays to 3 particles $M \to abc$.
Show that the phase space element can be expressed as

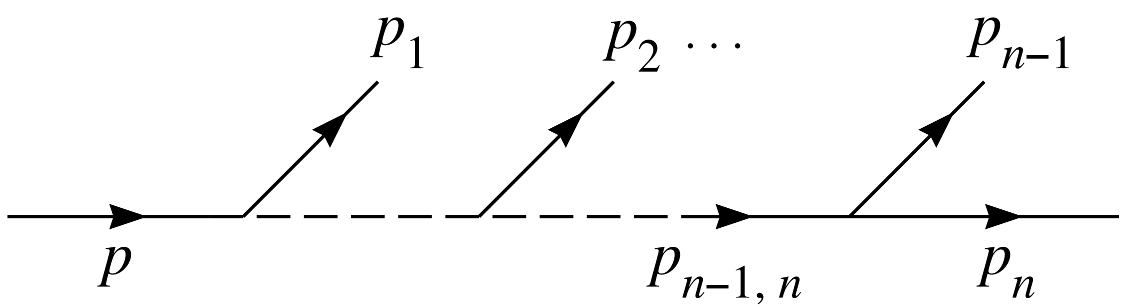
$$dPS_3 = \frac{1}{2^7 \pi^3} \; M^2 dx_a dx_b.$$

$$x_i = \frac{2E_i}{M}, \;\; (i = a, b, c, \; \sum_i x_i = 2).$$

where the integration limits for $m_a = m_b = m_c = 0$ are

$$0 \le x_a \le 1, \quad 1 - x_a \le x_b \le 1.$$

Recursion relation $P \to 1 + 2 + 3 ... + n$:



$$dPS_n(P; p_1, ..., p_n) \;=\; dPS_{n-1}(P; p_1, ..., p_{n-1,n})$$
$$dPS_2(p_{n-1,n}; p_{n-1}, p_n)\frac{dm^2_{n-1,n}}{2\pi}.$$

For instance,

$$dPS_3 = dPS_2(i)\ \frac{dm^2_{prop}}{2\pi}\ dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an s-channel particle propagation.

Breit-Wigner Resonance, the Narrow Width Approximation

An unstable particle of mass M and total width Γ_V, the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

Consider an intermediate state V^*

$$a \to bV^* \to b\ p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2=(m_1+m_2)^2}^{(m_*^{max})^2=(m_a-m_b)^2} dm_*^2.$$

Variable change

$$\tan\theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over θ

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

In the limit

$$(m_1 + m_2) + \Gamma_V \ll M_V \ll m_a - m_b - \Gamma_V,$$
$$\theta^{min} = \tan^{-1}\frac{(m_1 + m_2)^2 - M_V^2}{\Gamma_V M_V} \to -\pi,$$
$$\theta^{max} = \tan^{-1}\frac{(m_a - m_b)^2 - M_V^2}{\Gamma_V M_V} \to 0,$$

then the Narrow Width Approximation

$$\frac{1}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\ \delta(m_*^2 - M_V^2).$$

Exercise 2.4: Consider a three-body decay of a top quark, $t \to bW^* \to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

$$\Gamma(t \to bW^* \to b\ e\nu) \approx \Gamma(t \to bW) \cdot BR(W \to e\nu).$$

(C). Matrix element: The dynamics

Properties of scattering amplitudes $T(s,t,u)$

• Analyticity: A scattering amplitude is analytical except:
simple poles (corresponding to single particle states, bound states etc.);
branch cuts (corresponding to thresholds).

• Crossing symmetry: A scattering amplitude for a $2 \to 2$ process is symmetric among the s-, t-, u-channels.

• Unitarity:
S-matrix unitarity leads to :

$$-i(T - T^\dagger) = TT^\dagger$$

Partial wave expansion for $a+b \to 1+2$:

$$\mathcal{M}(s,t) = 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d^J_{\mu\mu'}(\cos\theta)$$
$$a_J(s) = \frac{1}{32\pi}\int_{-1}^{1} \mathcal{M}(s,t)\ d^J_{\mu\mu'}(\cos\theta) d\cos\theta.$$

where $\mu = s_a - s_b,\ \mu' = s_1 - s_2,\ M = \max(|\mu|, |\mu'|)$.

By Optical Theorem: $\sigma = \frac{1}{s}\mathrm{Im}\mathcal{M}(\theta=0) = \frac{16\pi}{s}\sum_{J=M}^{\infty}(2J+1)|a_J(s)|^2$.

The partial wave amplitude have the properties:

(a). partial wave unitarity: $\mathrm{Im}(a_J) \geq |a_J|^2$, or $|\mathrm{Re}(a_J)| \leq 1/2$,

(b). kinematical thresholds: $a_J(s) \propto \beta_i^{l_i}\ \beta_f^{l_f}\quad (J = L + S)$.

$\Rightarrow$ well-known behavior: $\sigma \propto \beta_f^{2l_f+1}$.

`Exercise 2.5: Appreciate the properties (a) and (b) by explicitly calculating the helicity amplitudes for`

$$e^-_L e^+_R \to \gamma^* \to H^- H^+, \quad e^-_L e^+_{L,R} \to \gamma^* \to \mu^-_L \mu^+_R, \quad H^- H^+ \to G^* \to H^- H^+.$$

(D). Calculational Tools

Traditional "Trace" Techniques: (Good for simple processes)

* You should be good at this — QFT course!

With algebraic symbolic manipulations:

* REDUCE, FORM, MATHEMATICA, MAPLE ...

Helicity Techniques: (Necessary for multiple particles)

More suitable for direct numerical evaluations.

- $*$ Hagiwara-Zeppenfeld: best for massless particles... (NPB, 1986)
- $*$ CalCul Method (by T.T. Wu et al., Parke-Mangano: Phys. Report);
- $*$ New techniques in loop calculations
  (by Z.Bern, L.Dixon, W. Giele, N. Glover, K.Melnikov, F. Petriello ...)
- $*$ "Twisters" (string theory motivated organization)
  (by Britto, F.Chachazo, B.Feng, E.Witten ...)

Exercise 2.6: Calculate the squared matrix element for $\overline{\sum}|\mathcal{M}(f\bar{f} \to ZZ)|^2$**,**
in terms of s, t, u**, in whatever technique you like.**

Much more recent efforts:

- $*$ Nima Arkani-Hamed et al. (2015–2017, new formalism.)

Calculational packages:

• Monte Carlo packages for phase space integration:

(1) VEGAS by P. LePage: adaptive important-sampling MC
http://en.wikipedia.org/wiki/Monte-Carlo_integration

(2) SAMPLE, RAINBOW, MISER ... (Rarely used.)

• Automated software for matrix elements:

(1) REDUCE — an interactive program designed for general algebraic computations, including to evaluate Dirac algebra, an old-time program,
http://www.uni-koeln.de/REDUCE;
http://reduce-algebra.com. (Rarely used.)

(2) FORM by Jos Vermaseren: A program for large scale symbolic manipulation, evaluate fermion traces automatically,
and perform loop calculations,s commercially available at
http://www.nikhef.nl/ form

(3) FeynCalc and FeynArts: Mathematica packages for algebraic calculations in elementary particle physics.
http://www.feyncalc.org;
http://www.feynarts.de

(4) MadGraph: Helicity amplitude method for tree-level matrix elements available upon request or
http://madgraph.hep.uiuc.edu

- Automated evaluation of cross sections:

(1) MadGraph/MadEvent and MadSUSY:
Generate Fortran codes on-line! http://madgraph.hep.uiuc.edu
(Now allows you to input new models.)

(2) CompHEP/CalHEP: computer program for calculation of elementary particle processes in Standard Model and beyond. CompHEP has a built-in numeric interpreter. So this version permits to make numeric calculation without additional Fortran/C compiler. It is convenient for more or less simple calculations.

— It allows your own construction of a Lagrangian model!

http://theory.npi.msu.su/k̃ryukov
(Now allows you to input new models.)

(3) GRACE and GRACE SUSY: squared matrix elements (Japan)
http://minami-home.kek.jp

(4) AlpGen: higher-order tree-level SM matrix elements (M. Mangano ...):
http://mlm.home.cern.ch/mlm/alpgen/

(5) SHERPA (F. Krauss et al.): (Gaining popularity)
Generate Fortran codes on-line! Merging with MC generators (see next).
http://www.sherpa-mc.de/

(6) Pandora by M. Peskin:
C++ based package for e^+e^-, including beam effects.
http://www-sldnt.slac.stanford.edu/nld/new/Docs/Generators/PANDORA.htm
The program pandora is a general-purpose parton-level event generator which includes beamstrahlung, initial state radiation, and full treatment of polarization effects. (An interface to PYTHIA that produces fully hadronized events is possible.)

- Cross sections at NLO packages: (Gaining popularity)

(1) MC(at)NLO (B. Webber et al.):
http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
Combining a MC event generator with NLO calculations for QCD processes.

(2) MCFM (K. Ellis et al.):
http://mcfm.fnal.gov/
Parton-level, NLO processes for hadronic collisions.

(3) BlackHat (Z.Bern, L.Dixon, D.Kosover et al.):
http://blackhat.hepforge.org/
Parton-level, NLO processes to combine with Sherpa

- Numerical simulation packages: Monte Carlo Event Generators

  Reading: http://www.sherpa-mc.de/

(1) PYTHIA:

PYTHIA is a Monte Carlo program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between e^+, e^-, p and $\bar{p}$ in various combinations.

They contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.

— It can be combined with MadGraph and detector simulations.

http://www.thep.lu.se/ torbjorn/Pythia.html

Already made crucial contributions to Tevatron/LHC.

(2) HERWIG

HERWIG is a Monte Carlo program which simulates $pp, p\bar{p}$ interactions at high energies. It has the most sophisticated perturbative treatments, and possible NLO QCD matrix elements in parton showing.

http://hepwww.rl.ac.uk/theory/seymour/herwig/

(3) ISAJET

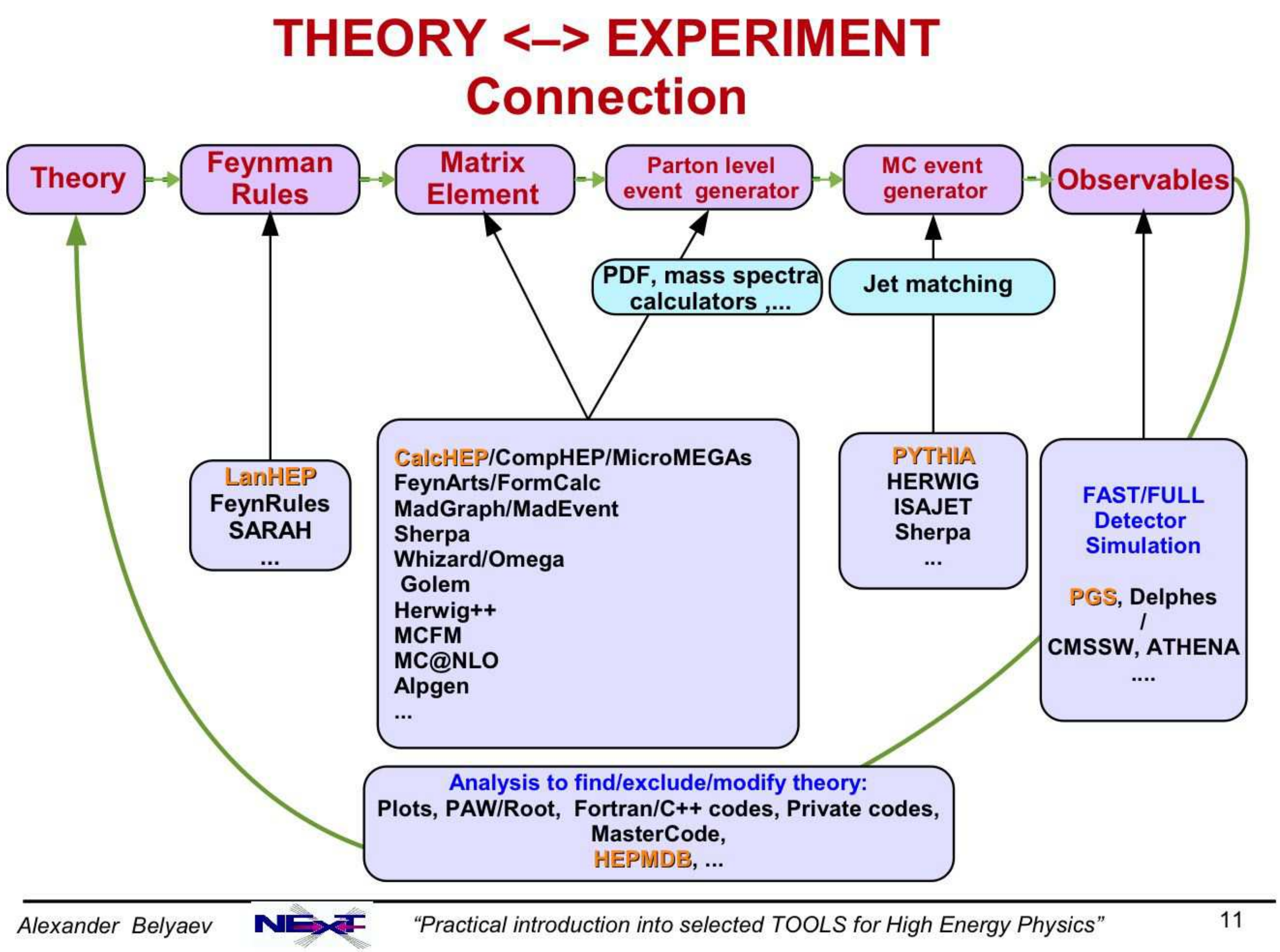
ISAJET is a Monte Carlo program which simulates $pp, \bar{p}p$, and ee interactions at high energies. It is largely obsolete.
ISASUSY option is still useful.
http://www.phy.bnl.gov/ isajet (Rarely used these days.)

• "Pretty Good Simulation" (PGS):
By John Conway: A simplified detector simulation,
mainly for theorists to estimate the detector effects.
http://www.physics.ucdavis.edu/ conway/research/software/pgs/pgs.html

PGS has been adopted for running with PYTHIA and MadGraph.
(but just a "toy".)

• DELPHES: A modular framework for fast simulation of a generic collider experiment.
http://arxiv.org/abs/1307.6346

Over all:


THEORY <–> EXPERIMENT
Connection
Theory
Feynman Rules
Matrix Element
Parton level event generator
MC event generator
Observables
PDF, mass spectra calculators ,...
Jet matching
LanHEP
FeynRules
SARAH
...
CalcHEP/CompHEP/MicroMEGAs
FeynArts/FormCalc
MadGraph/MadEvent
Sherpa
Whizard/Omega
Golem
Herwig++
MCFM
MC@NLO
Alpgen
...
PYTHIA
HERWIG
ISAJET
Sherpa
...
FAST/FULL Detector Simulation
PGS, Delphes
/
CMSSW, ATHENA
....
Analysis to find/exclude/modify theory:
Plots, PAW/Root, Fortran/C++ codes, Private codes,
MasterCode,
HEPMDB, ...
Alexander Belyaev
"Practical introduction into selected TOOLS for High Energy Physics"
11

II. Physics at an e^+e^- Collider

(A.) Simple Formalism

Event rate of a reaction:

$$R(s) = \sigma(s)\mathcal{L}, \quad \text{for constant } \mathcal{L}$$
$$= \mathcal{L} \int d\tau \frac{dL(s,\tau)}{d\tau}\, \sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}.$$

As for the differential production cross section of two-particle a, b,

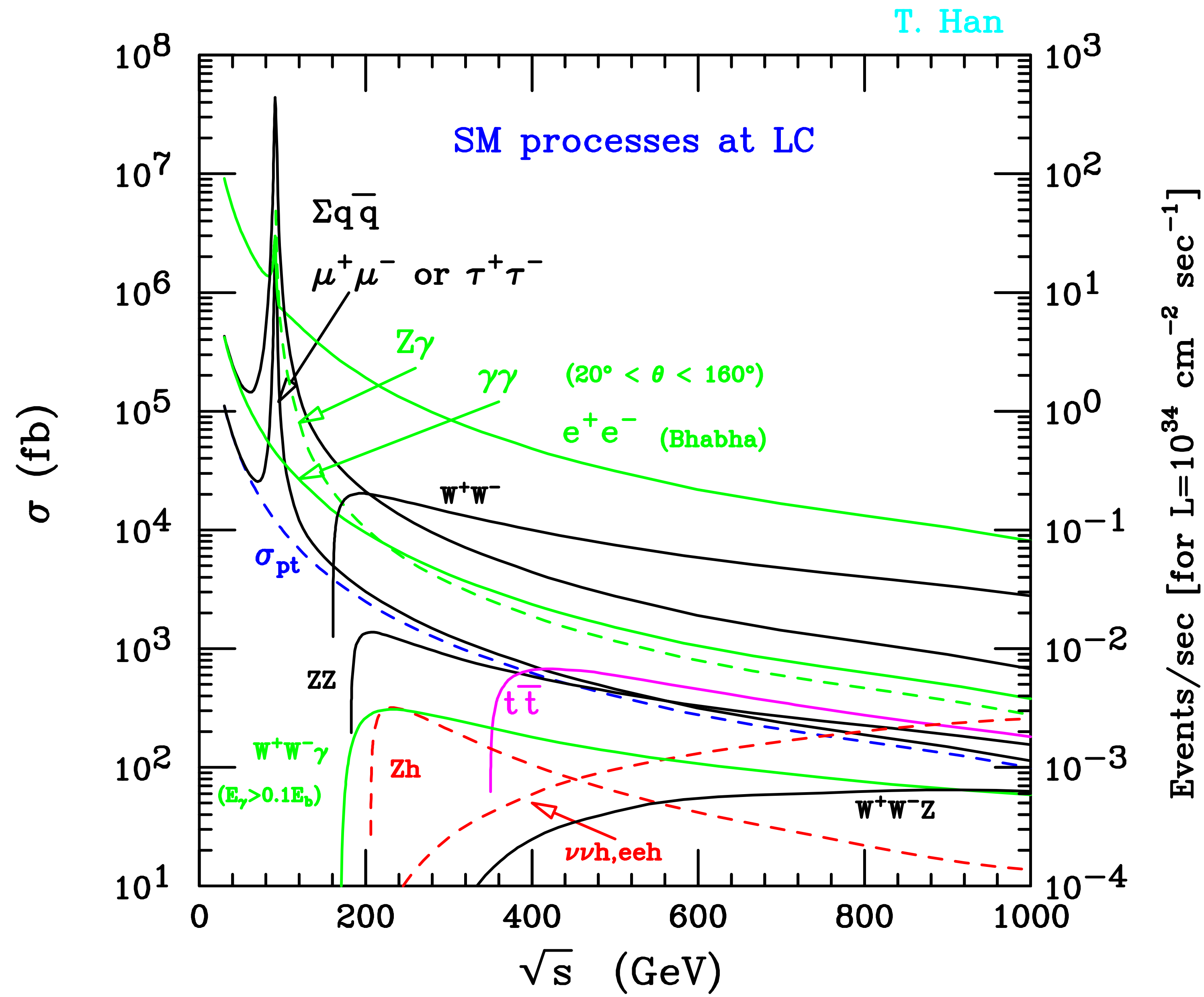
$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum}|\mathcal{M}|^2$$

where

• $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,

• $\overline{\sum|\mathcal{M}|^2}$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)

• unpolarized beams so that the azimuthal angle trivially integrated out, Total cross sections and event rates for SM processes:



(B). Resonant production: Breit-Wigner formula

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}$$

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \to V^* \to X) = \frac{4\pi(2j+1)\Gamma(V\to e^+e^-)\Gamma(V\to X)}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}\ \frac{s}{M_V^2},$$

If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2} \ \to\ \frac{\pi}{M_V\Gamma_V}\ \delta(s-M_V^2),$$

$$\sigma(e^+e^- \to V^* \to X) \ = \ \frac{2\pi^2(2j+1)\Gamma(V\to e^+e^-)BF(V\to X)}{M_V^2}\frac{dL(\hat{s}=M_V^2)}{d\sqrt{\hat{s}}}$$

`Exercise 3.1: sketch the derivation of these two formulas, assuming a Gaussian distribution for`

$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\ \Delta}\exp[\frac{-(\sqrt{\hat{s}}-\sqrt{s})^2}{2\Delta^2}].$$

Note: Away from resonance

For an s-channel or a finite-angle scattering:

$$\sigma \sim \frac{1}{s}.$$

For forward (co-linear) scattering:

$$\sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

(C). Fermion production:

Common processes: $e^-e^+ \to f\bar{f}$.
For most of the situations, the scattering matrix element can be casted into a $V \pm A$ chiral structure of the form (sometimes with the help of Fierz transformations)

$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta}\ [\bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1)]\ [\bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where $P_\mp = (1 \mp \gamma_5)/2$ are the L, R chirality projection operators, and $Q_{\alpha\beta}$ are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.
With this structure, the scattering matrix element squared:

$$\begin{aligned} \overline{\sum} |\mathcal{M}|^2 &= \frac{e^4}{s^2} \big[(|Q_{LL}|^2 + |Q_{RR}|^2)\ u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2)\ t_i t_j \\ &+ 2Re(Q^*_{LL} Q_{LR} + Q^*_{RR} Q_{RL}) m_f m_{\bar{f}} s \big], \end{aligned}$$

where $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$ and $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$.

`Exercise 3.2: Verify this formula.`

(D). Typical size of the cross sections:

- The simplest reaction

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact, $\sigma_{pt} \approx 100\ \text{fb}/(\sqrt{s}/\text{TeV})^2$ has become standard units to measure the size of cross sections.

- The Z resonance prominent (or other M_V),
- At the ILC $\sqrt{s} = 500$ GeV,

$$\sigma(e^+e^- \to e^+e^-) \sim 100\sigma_{pt} \sim 40\ \text{pb}.$$

(anglular cut dependent.)

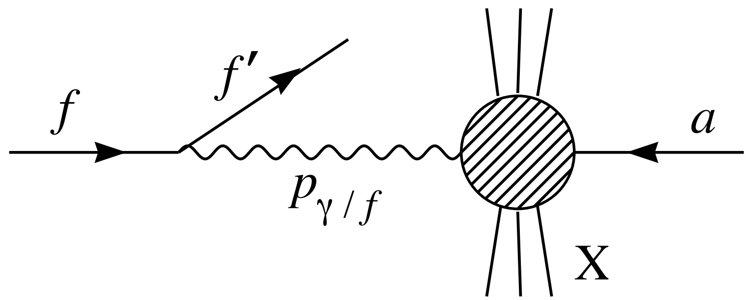
$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400\ \text{fb};$$
$$\sigma(u,d,s) \sim 9\sigma_{pt} \sim 3.6\ \text{pb};$$
$$\sigma(WW) \sim 20\sigma_{pt} \sim 8\ \text{pb}.$$

and

$$\sigma(ZH) \sim \sigma(WW \to H) \sim \sigma_{pt}/4 \sim 100\ \text{fb};$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40\ \text{fb}.$$

(E). Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+,\ \gamma^* e^- \to e^+e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

```
Exercise 3.3:  Try to derive this splitting function.
```

We see that:

- m_e enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^{\pm}, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

(F). Recoil mass technique:

One of the most important techniques, that distinguishes an e^+e^- collisions from hadronic collisions.

Consider a process:

$$e^+ + e^- \to V + X,$$

where V: a (bunch of) visible particle(s); X: unspecified.

Then:

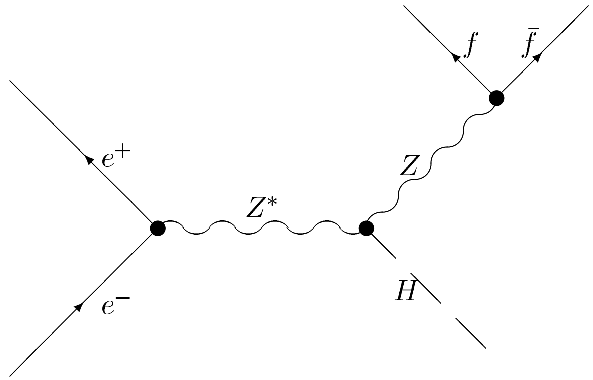
$$p_{e^+} + p_{e^-} = p_V + p_X, \ (p_{e^+} + p_{e^-} - p_V)^2 = p_X^2,$$
$$M_X^2 = (p_{e^+} + p_{e^-} - p_V)^2 = s + M_V^2 - 2\sqrt{s}E_V.$$

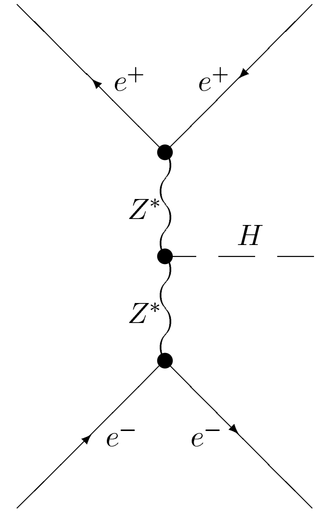
One thus obtain the “model-independent” inclusive measurements

a. mass of X by the recoil mass peak

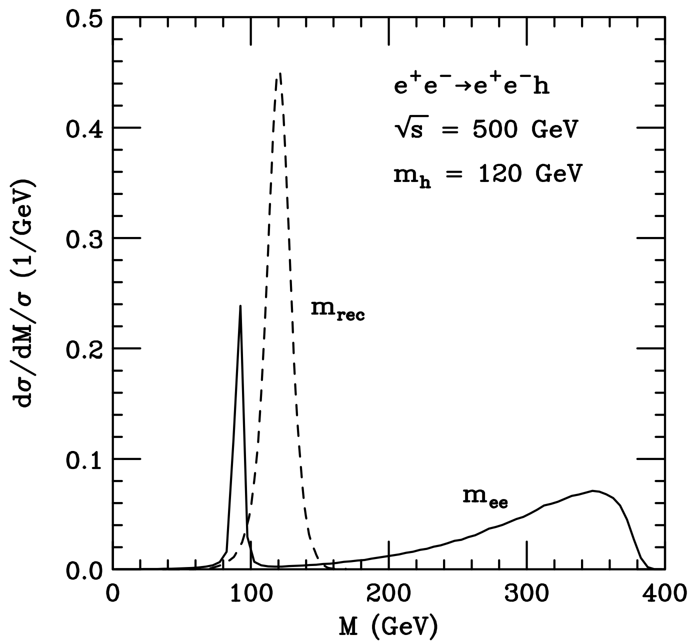
b. coupling of X by simple event-count at the peak

The key point for a Higgs factory: $e^+ + e^- \to f\bar{f} + h.$





Then: $$M_h^2 = (p_{e^+} + p_{e^-} - p_f - p_{\bar{f}})^2 = s + M_V^2 - 2\sqrt{s}E_{f\bar{f}}.$$



Model-independent, kinematical selection of signal events!

(G). Beam polarization:

One of the merits for an e^+e^- linear collider is the possible high polarization for both beams.
Consider first the longitudinal polarization along the beam line direction. Denote the average $e^\pm$ beam polarization by $P^L_\pm$, with $P^L_\pm = -1$ purely left-handed and $+1$ purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $\mathcal{M}_{\sigma_{e^-}\sigma_{e^+}}$:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{4}[(1-P^L_-)(1-P^L_+)|\mathcal{M}_{--}|^2 + (1-P^L_-)(1+P^L_+)|\mathcal{M}_{-+}|^2$$
$$+(1+P^L_-(1-P^L_+)|\mathcal{M}_{+-}|^2 + (1+P^L_-)(1+P^L_+)|\mathcal{M}_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral:
Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: *e.g.*, $W^\pm$ exchange ...

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.

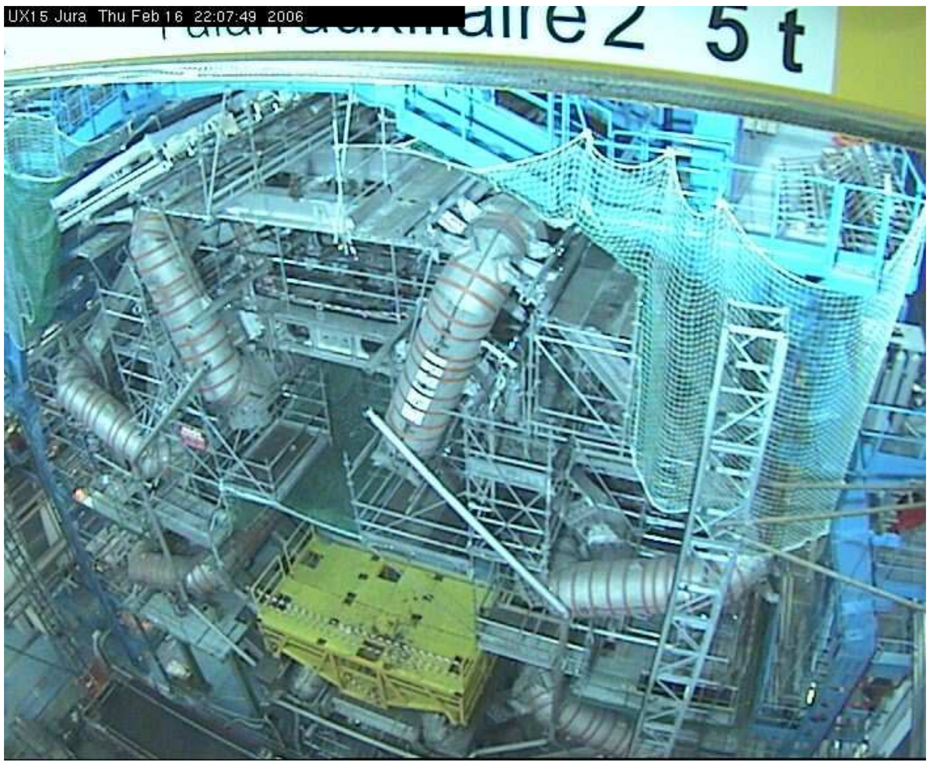
If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P^T_\pm < 1$, then there will be one additional term in the limit $m_e \to 0$,

$$\frac{1}{4}\, 2\ P^T_- P^T_+\ \mathrm{Re}(\mathcal{M}_{-+}\mathcal{M}^*_{+-}).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.

III. Hadron Collider Physics

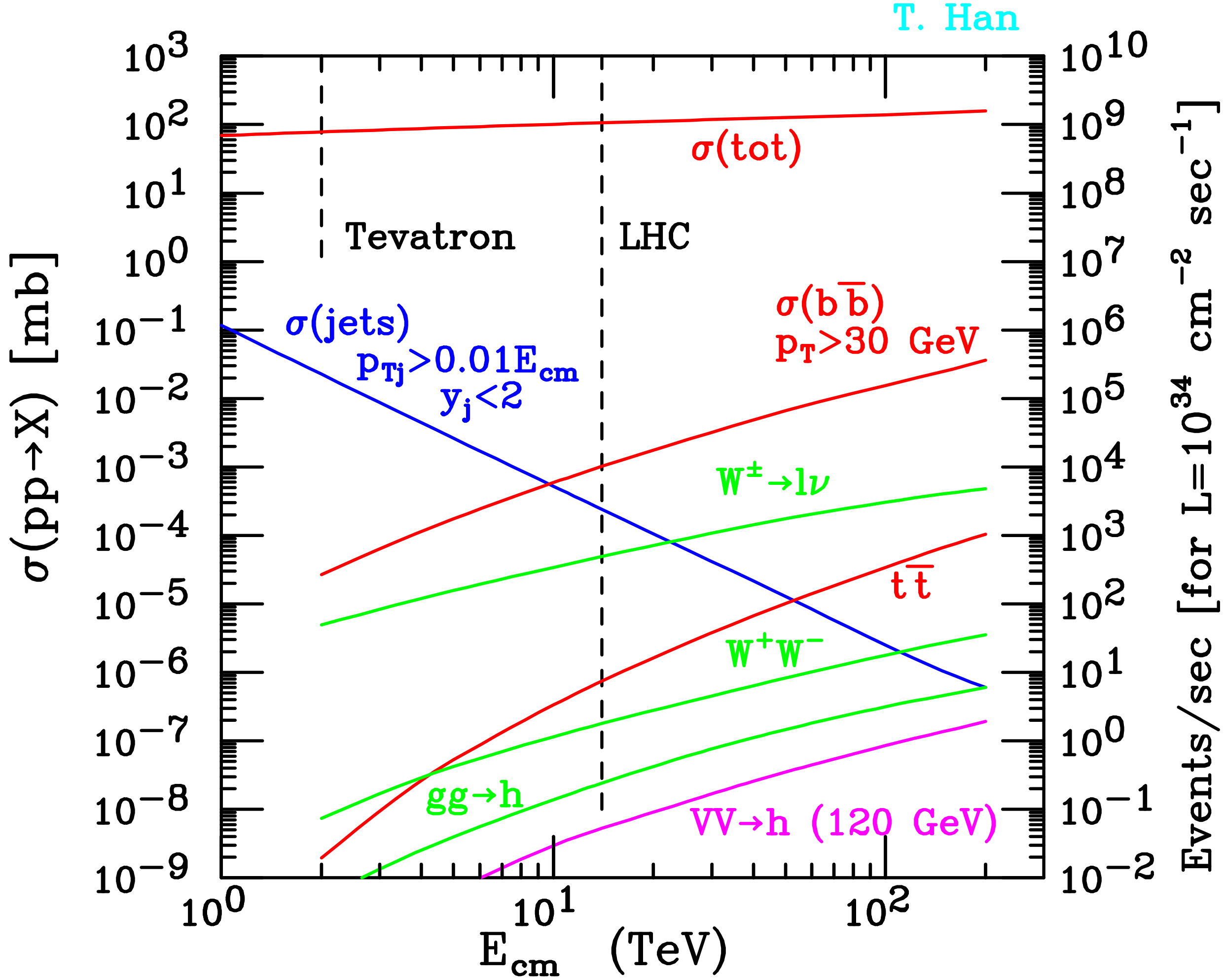
(A). New HEP frontier: the LHC
The Higgs discovery and more excitements ahead ...



ATLAS (90m underground)

CMS

LHC Event rates for various SM processes:



$10^{34}/\mathrm{cm}^2/\mathrm{s} \Rightarrow 100\ \mathrm{fb}^{-1}/\mathrm{yr}.$

Annual yield # of events $= \sigma \times L_{int}$:

10B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...

Discovery of the Higgs boson opened a new chapter of HEP!

Theoretical challenges:

Unprecedented energy frontier

(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi/m_\pi^2 \sim 120 \text{ mb}.$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7 \ (\frac{s}{\text{GeV}^2})^{0.0808} \ \text{mb}, & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2} \ \ln^2 \frac{s}{s_0}, & \text{Froissart bound.} \end{cases}$$

(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2) \ \hat{\sigma}_{parton}(s).$$

- Accurate (higher orders) partonic cross sections $\hat{\sigma}_{parton}(s)$.
- Parton distribution functions to the extreme (density):

$$Q^2 \sim (a \ few \ TeV)^2, \quad x \sim 10^{-3} - 10^{-6}.$$

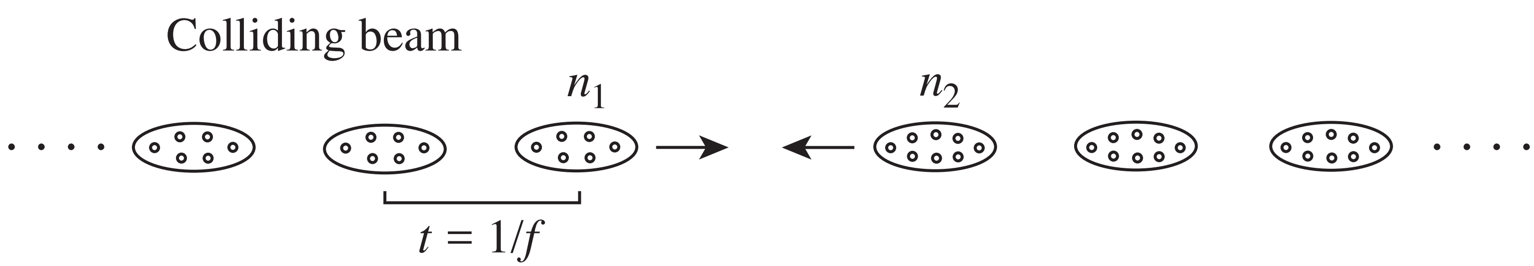
Experimental challenges:

- The large rate turns to a hostile environment:

  $\approx$ 1 billion event/sec: impossible read-off !

  $\approx$ 1 interesting event per $1,000,000$: selection (triggering).

  $\approx$ 25 overlapping events/bunch crossing:



$\Rightarrow$ Severe backgrounds!

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

$(\eta = 2.5 \Rightarrow 10^\circ; \qquad \eta = 5 \Rightarrow 0.8^\circ.)$

With optimal triggering and kinematical selections:

$p_T \geq 30 - 100$ GeV, $\quad |\eta| \leq 3 - 5; \quad \not{E}_T \geq 100$ GeV.

(B). Special kinematics for hadron colliders

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

Then the parton c.m. frame moves randomly, even by event:

$$\begin{aligned} \beta_{cm} &= \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :} \\ y_{cm} &= \frac{1}{2}\ln\frac{1+\beta_{cm}}{1-\beta_{cm}} = \frac{1}{2}\ln\frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty). \end{aligned}$$

The four-momentum vector transforms as

$$\begin{aligned} \begin{pmatrix} E' \\ p'_z \end{pmatrix} &= \begin{pmatrix} \gamma & -\gamma\ \beta_{cm} \\ -\gamma\ \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} \\ &= \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}. \end{aligned}$$

This is often called the “boost”.

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$\begin{aligned} E_T &= \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \\ p^\mu &= (E_T \cosh y,\ p_T \sin\phi,\ p_T \cos\phi,\ E_T \sinh y), \\ \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy. \end{aligned}$$

Due to random boost between Lab-frame/c.m. frame event-by-event,

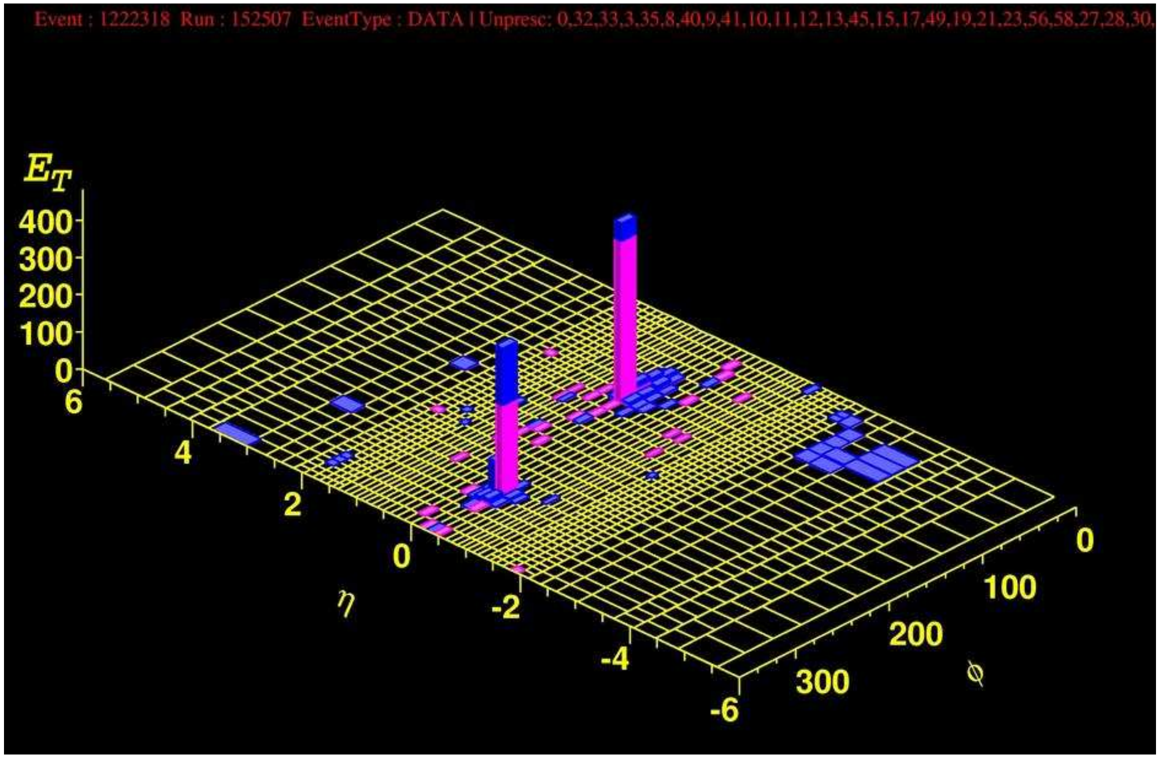
$$y' = \frac{1}{2} \ln \frac{E' + p'_z}{E' - p'_z} = \frac{1}{2} \ln \frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm}.$$

In the massless limit, rapidity $\to$ pseudo-rapidity:

$$y \to \eta = \frac{1}{2} \ln \frac{1 + \cos\theta}{1 - \cos\theta} = \ln \cot \frac{\theta}{2}.$$

`Exercise 4.1:  Verify all the above equations.`

The "Lego" plot:



A CDF di-jet event on a lego plot in the $\eta - \phi$ plane.

$\phi, \Delta y = y_2 - y_1$ is boost-invariant.
Thus the "separation" between two particles in an event

$$\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$$ is boost-invariant,

and lead to the "cone definition" of a jet.

(C). Characteristic observables: Crucial for uncovering new dynamics.

Selective experimental events

$\Longrightarrow$ Characteristic kinematical observables

(spatial, time, momentaum phase space)

$\Longrightarrow$ Dynamical parameters

(masses, couplings)

Energy momentum observables $\Longrightarrow$ mass parameters

Angular observables $\Longrightarrow$ nature of couplings;

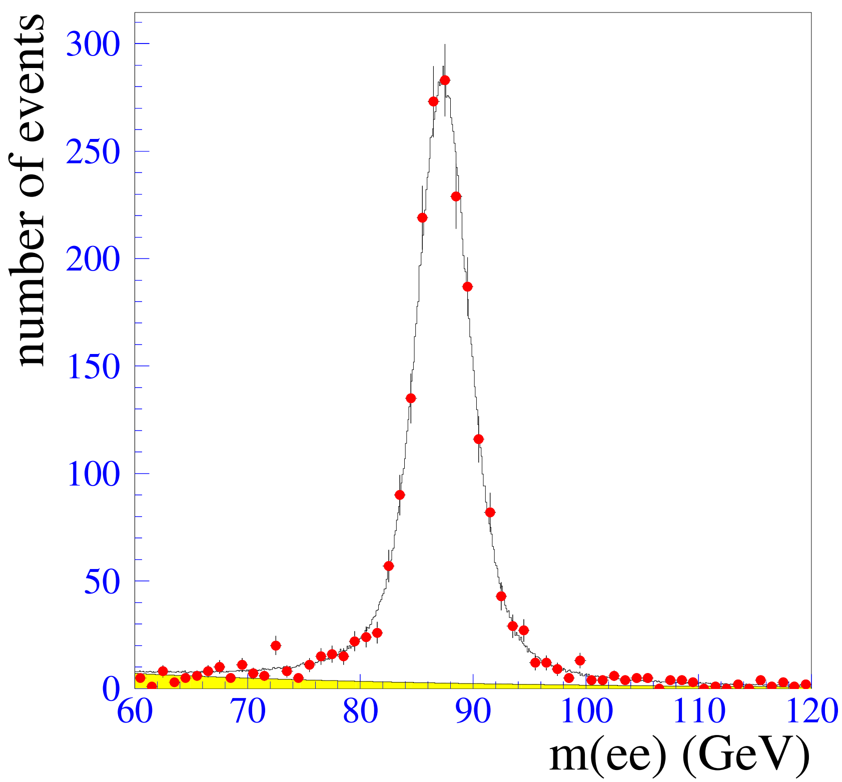
Production rates, decay branchings/lifetimes $\Longrightarrow$ interaction strengths.

(D). Kinematical features:
(a). s-channel singularity: bump search we do best.

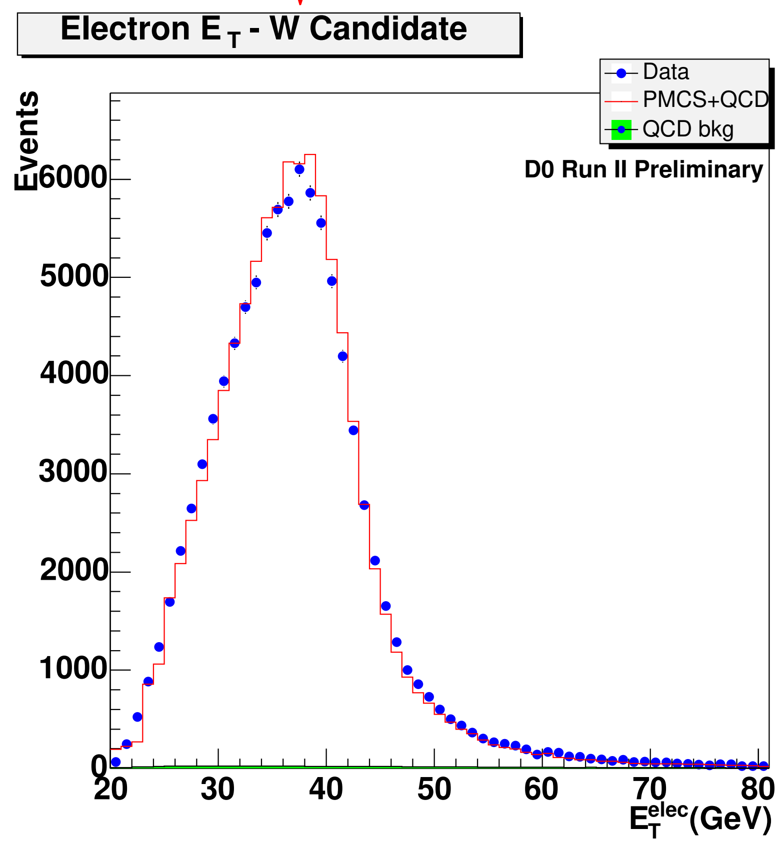
- invariant mass of two-body $R \to ab:\ m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\frac{d\hat{\sigma}}{dm_{ee}^2\ dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{1}{m_{ee}^2 \sqrt{1 - 4p_{eT}^2/m_{ee}^2}}$$



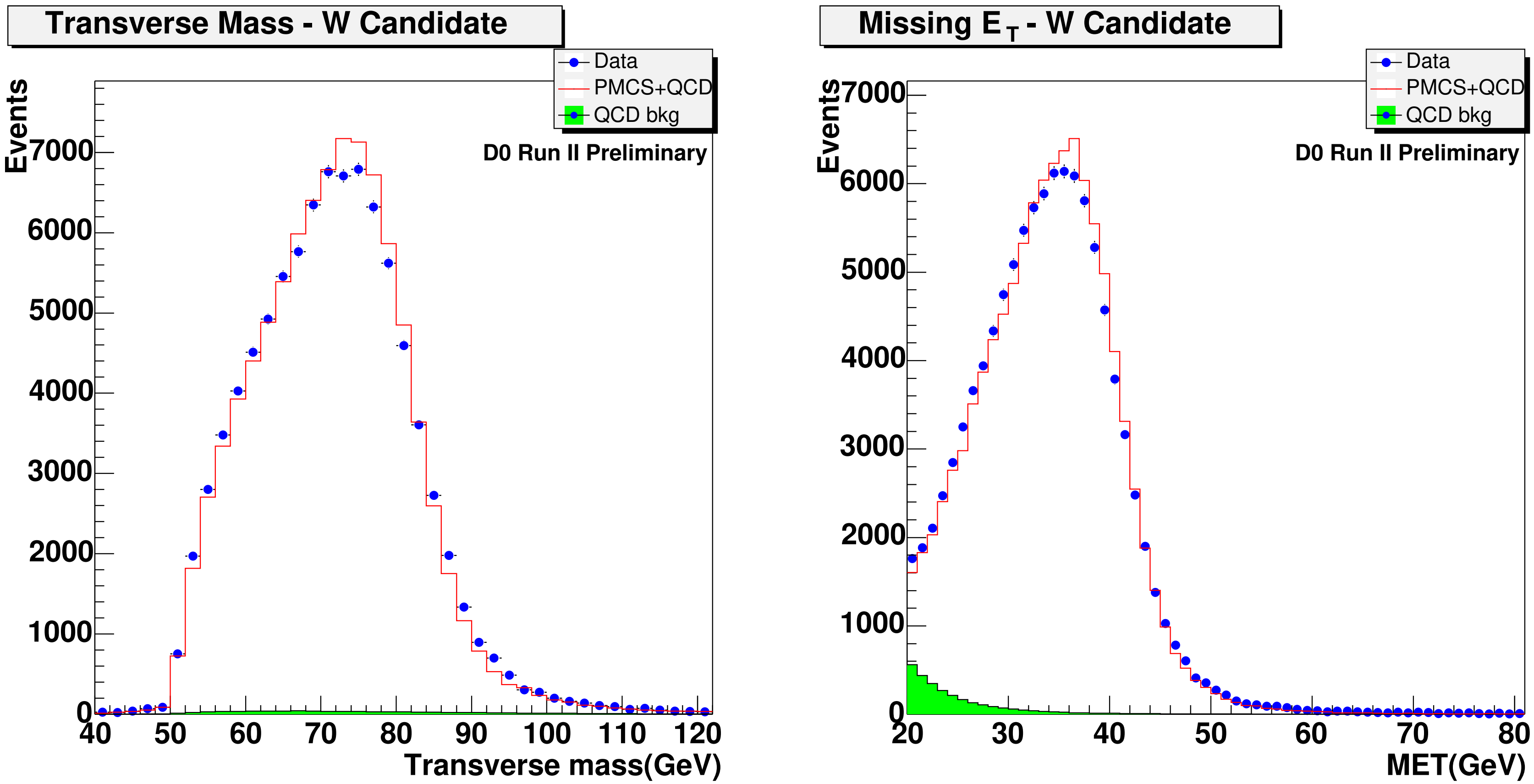
$Z \to e^+e^-$



$W \to e\nu$

- "transverse" mass of two-body $W^- \to e^- \bar{\nu}_e$:

$$\begin{aligned} m^2_{e\nu\ T} &= (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \\ &= 2 E_{eT} E_T^{miss} (1 - \cos\phi) \le m^2_{e\nu}. \end{aligned}$$



If $p_T(W) = 0$, then $m_{e\nu\ T} = 2E_{eT} = 2E_T^{miss}$.

Exercise 5.1: For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}}\ \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

where $p_{eT} = p_e \sin\theta^*$ is the transverse momentum and θ^* is the polar angle in the c.m. frame. Comment on the apparent singularity at $p_{eT}^2 = s/4$.

Exercise 5.2: Show that for an on-shell decay $W^- \to e^-\bar{\nu}_e$:

$$m_{e\nu\ T}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \leq m_{e\nu}^2.$$

Exercise 5.3: Show that if W/Z has some transverse motion, δP_V, then:

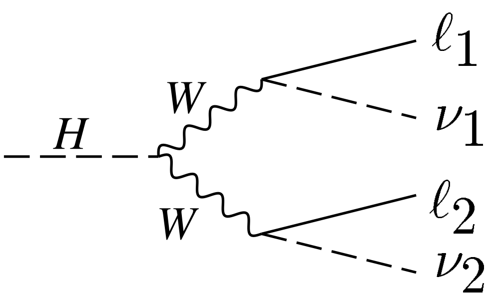
$$p'_{eT} \sim p_{eT}\ [1 + \delta P_V/M_V],$$
$$m_{e\nu\ T}'^2 \sim m_{e\nu\ T}^2\ [1 - (\delta P_V/M_V)^2],$$
$$m_{ee}'^2 = m_{ee}^2.$$

- $H^0 \to W^+W^- \to j_1 j_2\ e^-\bar{\nu}_e$:

cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1 T} + E_{W_2 T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2$$
$$= (\sqrt{p^2_{jjT} + M_W^2} + \sqrt{p^2_{e\nu T} + M_W^2})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \le M_H^2.$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not p}_T = -\sum_{obs}\ \vec{p}_T^{\,obs}$.



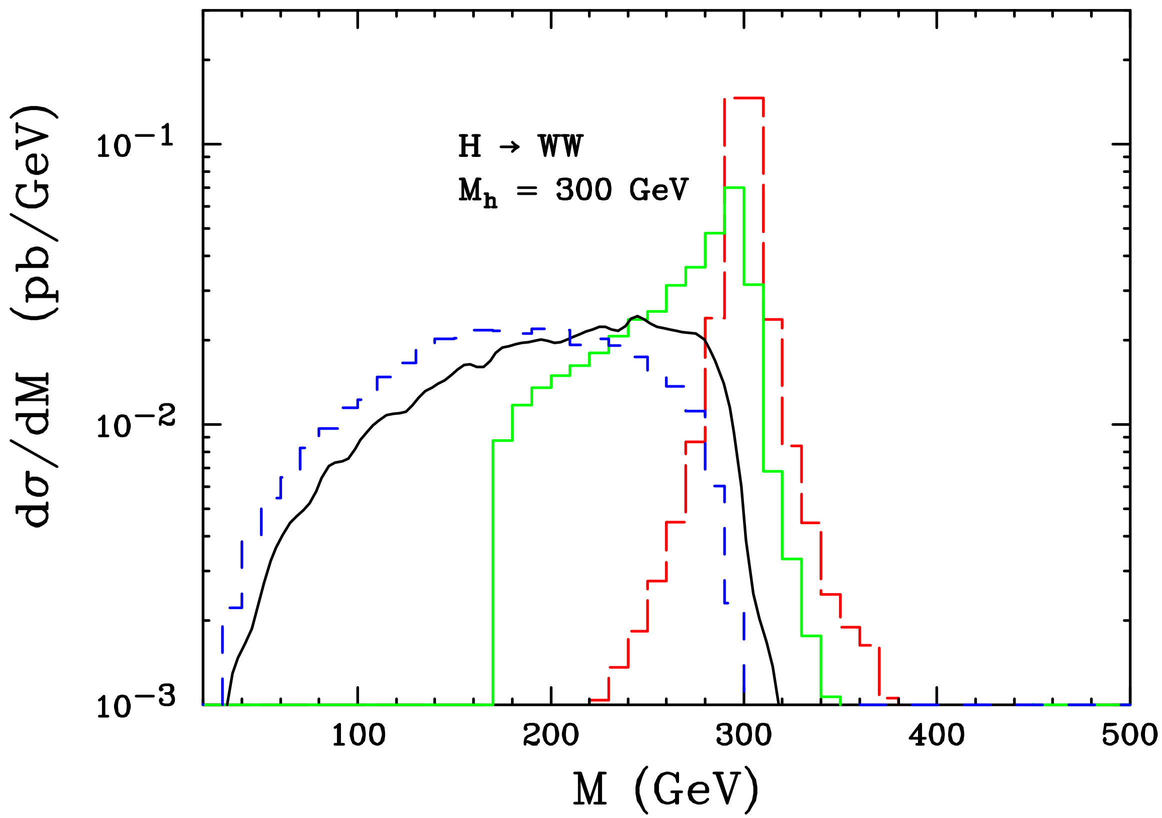
- $H^0 \to W^+W^- \to e^+\nu_e\ e^-\bar{\nu}_e$:

"effecive" transverse mass:

$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2$$
$$m_{eff\ T} \approx E_{e1T} + E_{e2T} + E_T^{miss}$$

cluster transverse mass (II):

$$m^2_{WW\ C} = \left(\sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not p_T\right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not p}_T)^2$$
$$m_{WW\ C} \approx \sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not p_T$$



M_{WW} invariant mass (WW fully reconstructable): - - - - - - - - -

$M_{WW,\ T}$ transverse mass (one missing particle ν): ————————

$M_{eff,\ T}$ effetive trans. mass (two missing particles): - - - - - - - -

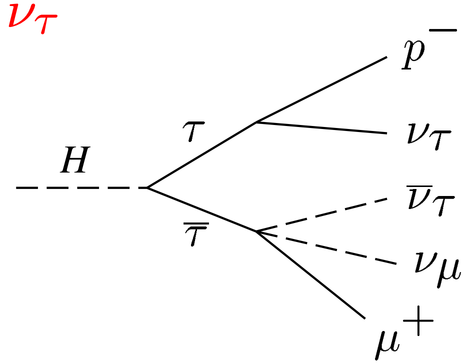
$M_{WW,\ C}$ cluster trans. mass (two missing particles): ————————

YOU design an optimal variable/observable for the search.

• cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+\ \bar{\nu}_\tau\ \nu_\mu, \quad \rho^-\ \nu_\tau$$

A lot more complicated with (many) more $\nu' s$?



Not really!

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau/E_\tau = 2m_\tau/m_H \approx 1.5^\circ \quad (m_H = 120\ \text{GeV}).$$

We can thus take

$$\vec{p}_{\tau^+} = \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu' s}, \quad \vec{p}_+^{\,\nu' s} \approx c_+ \vec{p}_{\mu^+}.$$
$$\vec{p}_{\tau^-} = \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu' s}, \quad \vec{p}_-^{\,\nu' s} \approx c_- \vec{p}_{\rho^-}.$$

where $c_\pm$ are proportionality constants, to be determined.

This is applicable to any decays of fast-moving particles, like

$$T \to Wb \to \ell\nu,\ b.$$

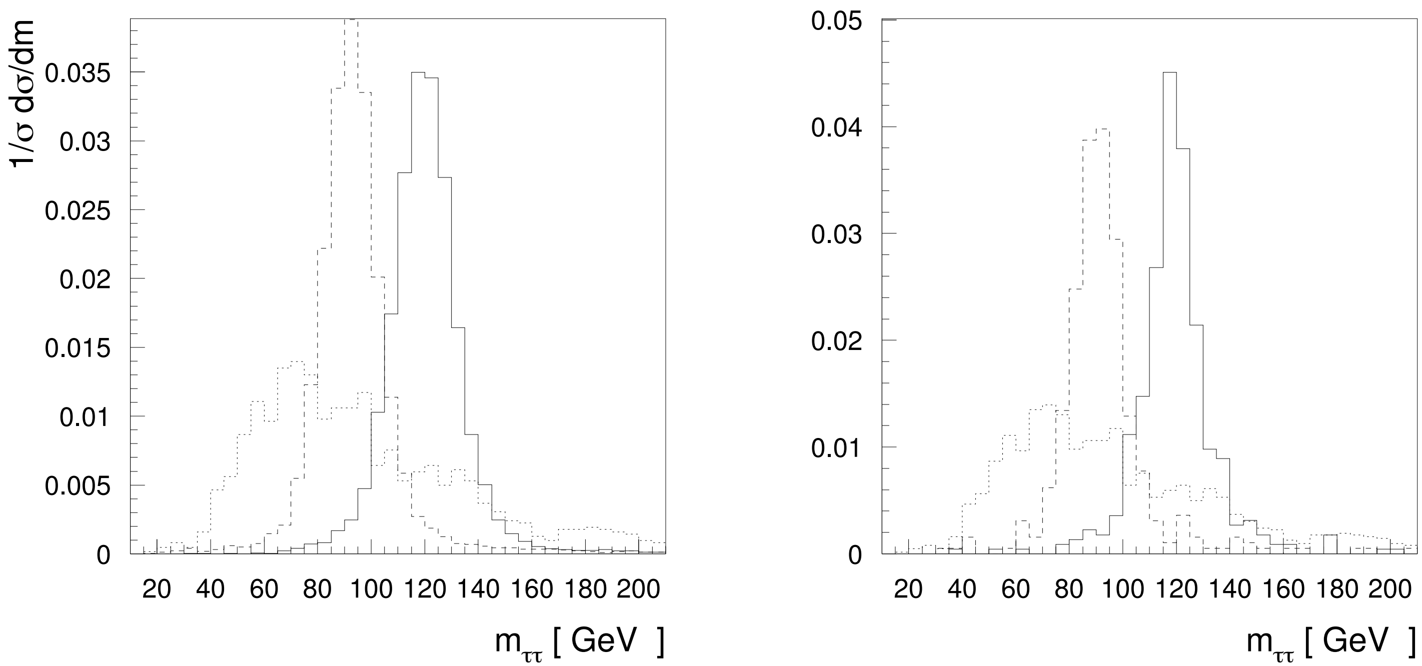
Experimental measurements: p_{ρ^-}, p_{μ^+}, $\not{p}_T$:

$$c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x = (\not{p}_T)_x,$$
$$c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y = (\not{p}_T)_y.$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle, or the Higgs should have a non-zero transverse momentum.



(b). Two-body versus three-body kinematics

- Energy end-point and mass edges:

utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \ \tilde{\mu}_R^-$$

$$\text{with two} - \text{body decays}: \ \ \tilde{\mu}_R^+ \to \mu^+ \tilde{\chi}_0, \ \ \ \tilde{\mu}_R^- \to \mu^- \tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$

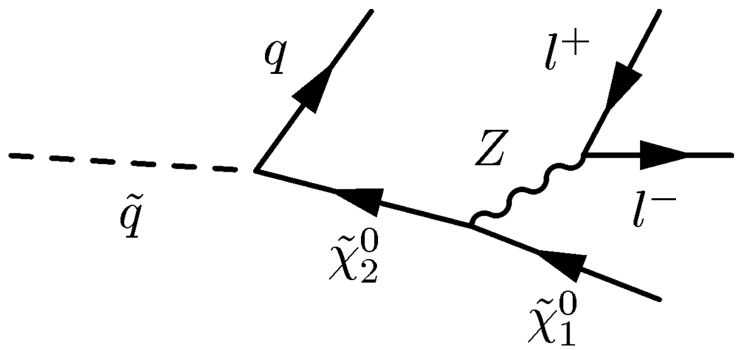
$$\text{with} \ \ \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \ \ \gamma = (1-\beta)^{-1/2}.$$

Energy end-point: $E_\mu^{lab} \Rightarrow M_{\tilde{\mu}_R}^2 - m_\chi^2$.

Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

Same idea can be applied to hadron colliders ...

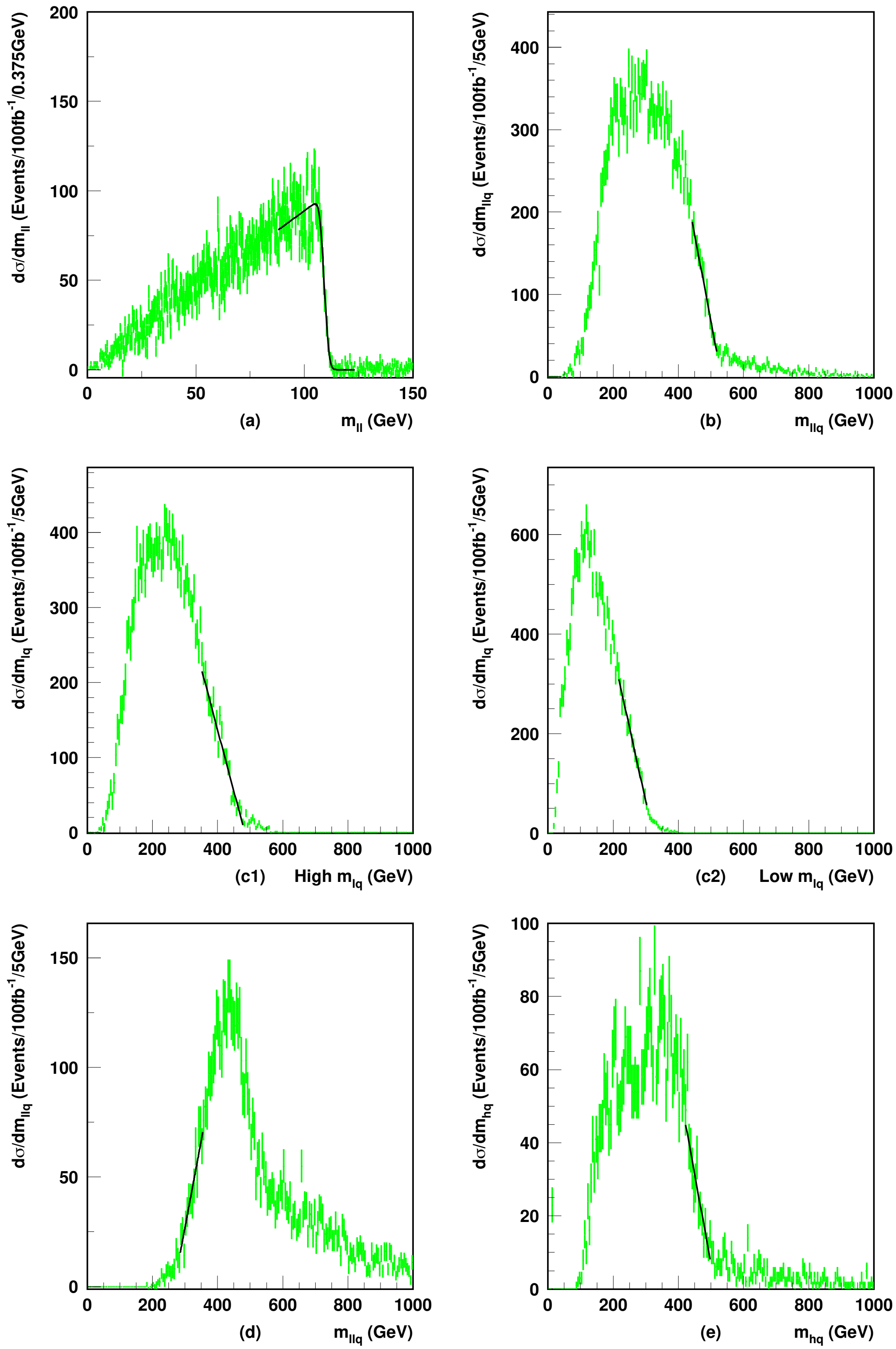
Consider a squark cascade decay:



1[st] edge : $M^{max}(\ell\ell) = M_{\chi_2^0} - M_{\chi_1^0}$;

2[nd] edge : $M^{max}(\ell\ell j) = M_{\tilde{q}} - M_{\chi_1^0}$.

`Exercise 5.4: Verify these relations.`

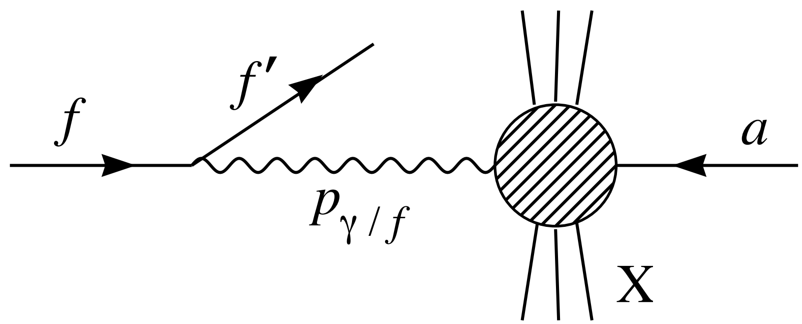


dσ/dm_ll (Events/100fb^-1/0.375GeV)
(a) m_ll (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(b) m_llq (GeV)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c1) High m_lq (GeV)
dσ/dm_lq (Events/100fb^-1/5GeV)
(c2) Low m_lq (GeV)
dσ/dm_llq (Events/100fb^-1/5GeV)
(d) m_llq (GeV)
dσ/dm_hq (Events/100fb^-1/5GeV)
(e) m_hq (GeV)

(c). t-channel singularity: splitting.

- Gauge boson radiation off a fermion:

The familiar Weizsäcker-Williams approximation



$$\sigma(fa \to f'X) \approx \int dx\ dp_T^2\ P_{\gamma/f}(x,p_T^2)\ \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x,p_T^2) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\left(\frac{1}{p_T^2}\right)\Big|_{m_e}^{E}.$$

† The kernel is the same as $q \to qg^*$ $\Rightarrow$ generic for parton splitting;

† The form $dp_T^2/p_T^2 \to \ln(E^2/m_e^2)$ reflects the collinear behavior.

- Generalize to massive gauge bosons:

$$P^T_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{8\pi^2}\,\frac{1+(1-x)^2}{x}\,\frac{p_T^2}{(p_T^2+(1-x)M_V^2)^2},$$
$$P^L_{V/f}(x,p_T^2) = \frac{g_V^2+g_A^2}{4\pi^2}\,\frac{1-x}{x}\,\frac{(1-x)M_V^2}{(p_T^2+(1-x)M_V^2)^2}.$$

Special kinematics for massive gauge boson fusion processes:

For the accompanying jets,

At low-p_{jT},

$$\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}$$

At high-p_{jT},

$$\left.\begin{array}{l} \frac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \frac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}$$

has become important tools for Higgs searches, single-top signal etc.

(E). Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

$$i \sum_{\tau}^{L,R} g_\tau^f \, \gamma^\mu \; P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$\begin{aligned} A_{FB}^{i,f} &\equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i \mathcal{A}_f, \\ \mathcal{A}_f &= \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}. \end{aligned}$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.

At hadronic level:

$$A_{FB}^{\mathsf{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \Big(P_q(x_1) P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1) P_q(x_2) \Big) \, \mathsf{sign}(x_1 - x_2)}{\int dx_1 \sum_q \Big(P_q(x_1) P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1) P_q(x_2) \Big)}.$$

Perfectly fine for Z/Z'-type:

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

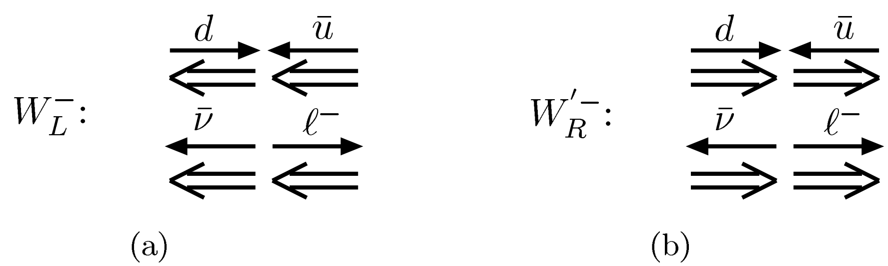
In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

It is the boost-direction of $\ell^+\ell^-$.

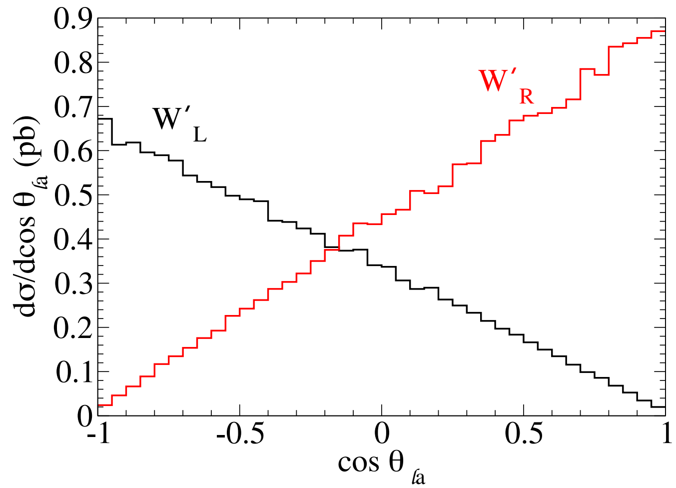
How about $W^{\pm}/W'^{\pm}(\ell^{\pm}\nu)$-type?

In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$,
AND ℓ^{+} (ℓ^{-}) along the direction with $\bar{q}$ (q) $\Rightarrow$ OK at the Tevatron,

But: (1). cann't get the boost-direction of $\ell^{\pm}\nu$ system;
(2). Looking at $\ell^{\pm}$ alone, no insight for W_L or W_R!



In $p\bar{p}$ collisions: (1). a reconstructable system
(2). with spin correlation → only tops $W' \to t\bar{b} \to \ell^{\pm}\nu\ \bar{b}$:



(F). CP asymmetries A_{CP}:

To non-ambiguously identify CP-violation effects,
one must rely on CP-odd variables.

Definition: A_{CP} vanishes if CP-violation interactions do not exist (for the relevant particles involved).

This is meant to be in contrast to an observable:
that'd be *modified* by the presence of CP-violation,
but is *not zero* when CP-violation is absent.

$$\text{e.g. } M_{(\chi^{\pm}\ \chi^0)}, \quad \sigma(H^0,\ A^0), \ldots$$

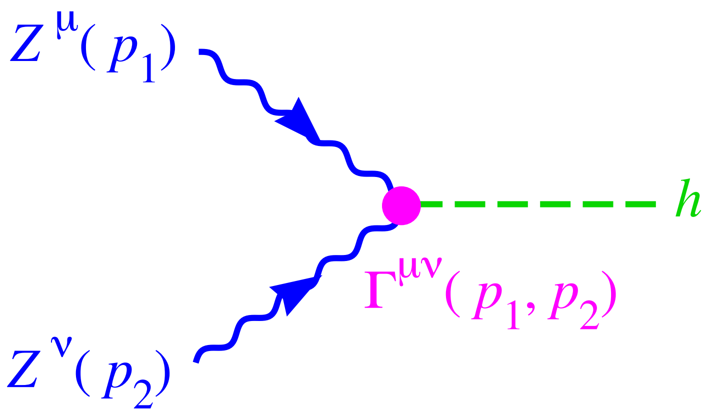
Two ways:
a). Compare the rates between a process and its CP-conjugate process:

$$\frac{R(i \to f) - R(\bar{i} \to \bar{f})}{R(i \to f) + R(\bar{i} \to \bar{f})}, \quad \text{e.g.} \quad \frac{\Gamma(t \to W^+ q) - \Gamma(\bar{t} \to W^- \bar{q})}{\Gamma(t \to W^+ q) + \Gamma(\bar{t} \to W^- \bar{q})}.$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M} \sim M_1 + M_2 \sin\theta,$$
$$A_{CP} = \sigma^F - \sigma^B = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

E.g. 1: $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$



$$\Gamma^{\mu\nu}(p_1, p_2) = i\frac{2}{v}\, h[a\ M_Z^2 g^{\mu\nu} + b\ (p_1^\mu p_2^\nu - p_1 \cdot p_2 g^{\mu\nu}) + \tilde{b}\ \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}]$$

$a = 1,\ b = \tilde{b} = 0$ for SM.

In general, $a,\ b,\ \tilde{b}$ complex form factors,

describing new physics at a higher scale.

For $H \to Z(p_1)Z^*(p_2) \to e^+(q_1)e^-(q_2),\ \mu^+\mu^-$, define:

$$O_{CP} \sim (\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2),$$
$$\text{or}\ \ \cos\theta = \frac{(\vec{p}_1 - \vec{p}_2) \cdot (\vec{q}_1 \times \vec{q}_2)}{|\vec{p}_1 - \vec{p}_2||\vec{q}_1 \times \vec{q}_2)|}.$$

E.g. 2: $H \to t(p_t)\bar{t}(p_{\bar{t}}) \to e^+(q_1)\nu_1 b_1,\ e^-(q_2)\nu_2 b_2.$

$$-\frac{m_t}{v}\bar{t}(a + b\gamma^5)t\ H$$
$$O_{CP} \sim (\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{p}_{e^+} \times \vec{p}_{e^-}).$$

thus define an asymmetry angle.