Electroweak Corrections to Inclusive Processes

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Outline

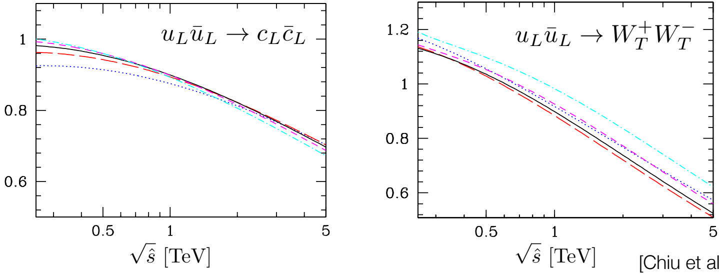
- Factorization
- PDFs and running
- Soft functions and running
- EW Gauge boson PDFs

AM, Waalewijn: 1802.08687, Fornal, AM, Waalewijn: 1803.today

Electroweak Double Logs

Ciafaloni, Ciafaloni, Comelli   Fadin, Lipatov, Martin, Melles   Kühn, Penin, Smirnov
Denner and Pozzorini

- $\alpha_W \ln^2 Q^2/M^2$ corrections which grow with energy and decrease cross section.
- $\sim$ 10% at the LHC, become $\sim$ 100% at the FCC.
- A problem in searches for new physics in tails of distributions.



[Chiu et al.]

Inclusive Processes

- In QCD, inclusive rates such as DIS do not have double logarithms.
- Ciafaloni et al. pointed out that EW inclusive processes can have double logs because particles are not EW singlets.
- Chiu et al. summed EW double logs using RG evolution in SCET. Considering EW exclusive proceses
- Here we consider EW inclusive processes such as DIS or DY where the final state has invariant mass $\gg M_Z$.

PDFs are QCD singlets

Quark distributions are color invariant. PDFs

$$O_q(r^-) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \mathrm{d}\xi \, e^{-i\xi r^-} \left[\bar{q}(\bar{n}\xi) \, \mathcal{W}(\bar{n}\xi)\right] \bar{\not{n}} \left[\mathcal{W}^\dagger(0) \, q(0)\right],$$

$$\langle p | O_q(xp^-) | p \rangle \equiv f_q(x, \mu)$$

$$\langle p | \bar{q} \ldots q | p \rangle = f_q(x, \mu), \qquad \langle p | \bar{q} \ldots T^a \ldots q | p \rangle = 0$$

so equal probabilities to find the different colors q_1, q_2, q_3

EW Non-singlet PDFs

The action of $SU(2) \times U(1)$ on proton states $|p\rangle$ is *not defined*, because the symmetry is broken. Only definite property is electric charge,

$$Q\,|p\rangle = |p\rangle$$

For weak interactions,

$$\langle p|\bar{q}\ldots t^a \ldots q|p\rangle \neq 0$$

In particular

$$\langle p|\bar{q}\ldots t^3 \ldots q|p\rangle = \frac{1}{2}\left[f_{u_L}(x,\mu) - f_{d_L}(x,\mu)\right] \neq 0, \quad f_{u_L}(x,\mu) \neq f_{d_L}(x,\mu)$$

Bauer, Ferland, Webber: 1703.08562, 1712.07147

AM, Waalewijn: 1802.08687

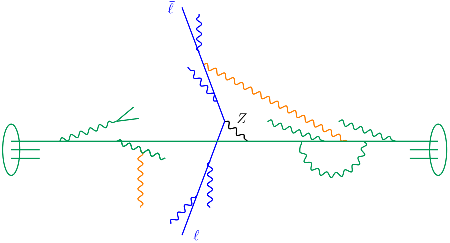
Factorization

1. Integrate out hard scale and match onto operators at Q in unbroken theory
2. Factorize into collinear and soft sectors Chiu et al.
3. Square the amplitude, and factor into PDFs, Fragmentation functions and soft functions.
4. Run using the RGE to M_Z
5. Match onto broken phase

Integrating out at the Hard Scale

Consider lepton-hadron interactions such as DIS or Drell-Yan

$$\mathcal{L} = \sum \mathcal{H}_i O_i$$



$$O_{\ell q}^{(3)} = (\bar{\ell}_1 \gamma^\mu t^a \ell_2)\,(\bar{q}_3 \gamma_\mu t^a q_4)\,, \qquad O_{\ell q} = (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{q}_3 \gamma_\mu q_4)\,,$$

$$O_{\ell u} = (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{u}_3 \gamma_\mu u_4)\,, \qquad O_{\ell d} = (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{d}_3 \gamma_\mu d_4)\,,$$

$$O_{eq} = (\bar{e}_1 \gamma^\mu e_2)\,(\bar{q}_3 \gamma_\mu q_4)\,,$$

$$O_{eu} = (\bar{e}_1 \gamma^\mu e_2)\,(\bar{u}_3 \gamma_\mu u_4)\,, \qquad O_{ed} = (\bar{e}_1 \gamma^\mu e_2)\,(\bar{d}_3 \gamma_\mu d_4)\,,$$

Factor into Collinear and Soft

BPS

$$q \to \mathcal{S} \underbrace{[\mathcal{W}^\dagger \psi]}_{\text{still call it } q}$$

Collinear Wilson line:

$$\mathcal{W}(x) = \mathrm{P}\exp\left\{ i \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot \left[g_3 A_n(x + s\bar{n}) + g_2 W_n(x + s\bar{n}) + g_1 \mathsf{y}_q B_n(x + s\bar{n}) \right] \right\},$$

Soft Wilson Line:

$$\mathcal{S} = \mathrm{P}\exp\left\{ i \int_{-\infty}^{0} \mathrm{d}s\, n \cdot \left[g_3 A_s(s\, n) + g_2 W_s(s\, n) + g_1 \mathsf{y}_q B_s(s\, n) \right] \right\}.$$

Operators:

$$O_{\ell q}^{(3)} \to (\bar{\ell}_1 \mathcal{S}_1^\dagger \gamma^\mu t^a \mathcal{S}_2 \ell_2)\, (\bar{q}_3 \mathcal{S}_3^\dagger \gamma_\mu t^a \mathcal{S}_4 q_4)$$

Squaring (as needed in the cross section)

$$O_{\ell q}^{(3)} O_{\ell q}^{(3)\dagger} \to \left(\bar{\ell}_1 \mathcal{S}_1^\dagger \gamma^\mu t^a \mathcal{S}_2 \ell_2\right) \left(\bar{q}_3 \mathcal{S}_3^\dagger \gamma_\mu t^a \mathcal{S}_4 q_4\right)$$
$$\left(\bar{\ell}_2 \mathcal{S}_2^\dagger \gamma^\nu t^b \mathcal{S}_1 \ell_1\right) \left(\bar{q}_4 \mathcal{S}_4^\dagger \gamma_\nu t^b \mathcal{S}_3 q_3\right)$$

$\ldots = 1,\ t^a$. Also PDFs have additional factors such as γ matrices

$$\sigma \sim \sum_X \langle p_1 p_2 | \mathcal{L}_{\text{hard}} | \mu^+ \mu^- X \rangle \langle \mu^+ \mu^- X | \mathcal{L}_{\text{hard}} | p_1 p_2 \rangle$$

$$\sim |\mathcal{H}|^2 \underbrace{\langle p_1 | \bar{q}_3 \ldots q_3 | p_1 \rangle}_{\text{PDF}} \quad \underbrace{\langle p_3 | \bar{q}_4 \ldots q_4 | p_2 \rangle}_{\text{PDF}}$$

$$\underbrace{\langle 0 | \mathcal{S}_1^\dagger \cdots \mathcal{S}_1 \mathcal{S}_2^\dagger \cdots \mathcal{S}_2 \mathcal{S}_3^\dagger \cdots \mathcal{S}_3 \mathcal{S}_4^\dagger \cdots \mathcal{S}_4 | 0 \rangle}_{\text{soft}}$$

$$\underbrace{\sum_{X_1} \langle 0 | \ell_1 | \mu^- X_1 \rangle \langle \mu^- X_1 | \bar{\ell}_1 | 0 \rangle}_{\text{FF}} \quad \underbrace{\sum_{X_2} \langle 0 | \bar{\ell}_2 | \mu^+ X_2 \rangle \langle \mu^+ X_2 | \ell_2 | 0 \rangle}_{\text{FF}}$$

Cross section is an overall gauge singlet, but can have

$$\langle p_1 | \bar{q}_3 \ldots t^a \ldots q_3 | p_1 \rangle$$

contracted with

$$\langle 0 | \mathcal{S}_1^\dagger t^a \mathcal{S}_1 \mathcal{S}_2^\dagger t^b \mathcal{S}_2 \ldots | 0 \rangle$$

For QCD, $t^a \to T^A$ and PDFs vanish unless $T^A \to 1$. $\mathcal{S}^\dagger \mathcal{S} = 1$, so soft function drops out.

PDFs

PDFs with $Q = 0$ in unbroken phase:

$q,\ \bar{q}$	$f_q^{(I=0)}(x,\mu)$	$f_q^{(I=1,I_3=0)}(x,\mu)$	
$u,\ \bar{u}$	$f_u^{(I=0)}(x,\mu)$		
$d,\ \bar{d}$	$f_d^{(I=0)}(x,\mu)$		
W	$f_W^{(I=0)}(x,\mu)$	$f_W^{(I=1,I_3=0)}(x,\mu)$	$f_W^{(I=2,I_3=0)}(x,\mu)$
B	$f_B^{(I=0)}(x,\mu)$		
$WB,\ BW$		$f_{WB}^{(I=1,I_3=0)}(x,\mu)$	
$H,\ \bar{H}$	$f_H^{(I=0)}(x,\mu)$	$f_H^{(I=1,I_3=0)}(x,\mu)$	
$\widetilde{H}H,\ \bar{H}\widetilde{\bar{H}}$		$f_{\widetilde{H}H}^{(I=1,I_3=1)}(x,\mu)$	

PDFs in broken phase:

$u_L,\ u_R,\ d_L,\ d_R$
$W_T^+,\ W_T^-,\ Z_T,\ Z_T\gamma,\ \gamma Z_T,\ \gamma$
$W_L^+,\ W_L^-,\ Z_L,\ Z_L h,\ hZ_L,\ h$

Anomalous Dimensions

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_i(x,\mu,\nu) = \sum_j \int_0^1 \mathrm{d}z\ \gamma_{\mu,ij}(z,\mu,\nu)\ f_j\left(\frac{x}{z},\mu,\nu\right)$$

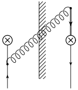
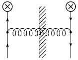
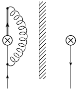
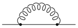
$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} f_i(x,\mu,\nu) = \gamma_{\nu,i}(z,\mu,\nu)\ f_i\,(x,\mu,\nu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \mathcal{S}(\mu,\nu) = \gamma_{\mu,\mathcal{S}}(\mu,\nu)\ \mathcal{S}(\mu,\nu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} \mathcal{S}(\mu,\nu) = \gamma_{\nu,\mathcal{S}}(\mu,\nu)\ \mathcal{S}(\mu,\nu)$$

$\hat{\gamma}$: drop overall α/π

Collinear Anomalous Dimensions

Graph	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\frac{2}{(1-z)_+} - 2 - 2\,\delta(1-z)\ln\frac{\nu}{\bar{n}\cdot p}$	$-\ln\frac{\mu^2}{M^2}$
	$1-z$	0
Total$_1$	$\frac{2}{(1-z)_+} - z - 1 - 2\ln\frac{\nu}{\bar{n}\cdot p}\,\delta(1-z)$	$-\ln\frac{\mu^2}{M^2}$
	$2\left(\ln\frac{\nu}{\bar{n}\cdot p} + 1\right)\delta(1-z)$	$\ln\frac{\mu^2}{M^2}$
	$-\frac{1}{2}\delta(1-z)$	0
Total$_2$	$\left(2\ln\frac{\nu}{\bar{n}\cdot p} + \frac{3}{2}\right)\delta(1-z)$	$\ln\frac{\mu^2}{M^2}$

double logs and rapidity divergences for non-singlet PDFs.
Color factors $C_F - C_A/2$ and C_F for adjoint PDFs.

Soft Anomalous Dimensions

$$\mathcal{S}^{ab}_{12\ldots n} = \text{tr}\left[(\mathcal{S}_1 t^{a_1} \mathcal{S}_1^\dagger)(\mathcal{S}_2 t^{a_2} \mathcal{S}_2^\dagger)\ldots(\mathcal{S}_{n_I} t^{a_{n_I}} \mathcal{S}_{n_I}^\dagger)\right],$$

n_I = number of indices

$$\hat{\gamma}_{\nu,\mathcal{S}} = -\frac{1}{2} n_I\, c_A \ln\frac{\mu^2}{M^2}$$

$$\hat{\gamma}_{\mu,\mathcal{S}_{12}} = c_A\left[\ln\frac{\mu^2}{\nu^2} - L_{12}\right],$$

$$\hat{\gamma}_{\mu,\mathcal{S}_{123}} = c_A\left[\frac{3}{2}\ln\frac{\mu^2}{\nu^2} - \frac{1}{2}(L_{12}+L_{13}+L_{23})\right],$$

where

$$L_{ij} \equiv \ln\left|\frac{n_i\cdot n_j}{2}\right| .$$

imaginary parts cancel.

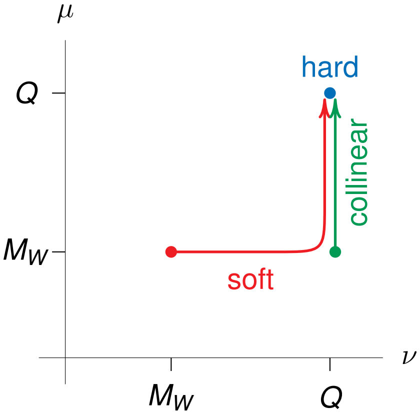
Angular Dependence of Soft Anomalous Dimension

$$\hat{\gamma}_{\mu,\mathcal{S}_{1234}} = c_A \left[2 \ln \frac{\mu^2}{\nu^2} - \begin{pmatrix} L_{12}+L_{34} & 0 & 0 \\ 0 & L_{13}+L_{24} & 0 \\ 0 & 0 & L_{14}+L_{23} \end{pmatrix} \right] - \begin{pmatrix} 0 & -w & w \\ -v & 0 & v \\ -u & u & 0 \end{pmatrix},$$

using the conformal cross ratios (which depend on angles)

$$u = \ln \frac{(n_1 \cdot n_2)(n_3 \cdot n_4)}{(n_1 \cdot n_3)(n_2 \cdot n_4)},$$
$$v = \ln \frac{(n_1 \cdot n_2)(n_3 \cdot n_4)}{(n_1 \cdot n_4)(n_2 \cdot n_3)},$$
$$w = \ln \frac{(n_1 \cdot n_3)(n_2 \cdot n_4)}{(n_1 \cdot n_4)(n_2 \cdot n_3)} = v - u.$$

Evolution



μ anom dim of soft:

$$\gamma^{\rm DL}_{\mu,\mathcal{S}} = n_l \frac{\alpha_2}{\pi} \ln \frac{\mu^2}{\nu^2} \,.$$

μ anom dim of PDF:

$$U^{\rm DL}_{\mu} \approx \exp \left[-\frac{\alpha_2}{\pi} \ln^2 \frac{Q}{M_W} \right] ,$$

At single-log, get angular dependence from soft running.

Comparison

Bauer, Ferland, Webber: 1703.08562, 1712.07147

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_q^{(I=1,I_3=0)}(x,\mu) = \frac{\alpha_2}{\pi} \int_0^{1-M/\mu} \mathrm{d}z \left[-\frac{1}{4}\widetilde{P}_{QQ}(z)\, f_q^{(I=1,I_3=0)}\left(\frac{x}{z},\mu\right) + \frac{1}{4}N_c\widetilde{P}_{QG}(z)\, f_W^{(I=1,I_3=0)}\left(\frac{x}{z},\mu\right) + \ldots \right].$$

and add a correction that follows from sum rules

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_q^{(I=1,I_3=0)}(x,\mu) = \frac{\alpha_2}{\pi} f_q^{(I=1,I_3=0)}(x,\mu) \int_0^{1-M/\mu} \mathrm{d}z\, z \left[-\frac{3}{4}\widetilde{P}_{QQ}(z) - \frac{3}{4}\widetilde{P}_{GQ}(z)\right] + \ldots$$

$$= \frac{\alpha_2}{\pi}\left(\frac{3}{2}\ln\frac{M}{\mu} + \frac{9}{8}\right) f_q^{(I=1,I_3=0)}(x,\mu) + \ldots ,$$

$z < 1$ are the same, and the Sudakov double logs agree.

Matching to Broken Phase

At $\mu = M_Z$, match onto unbroken theory:

$$
\begin{aligned}
\langle p|O_u^{(I=0)}|p\rangle &= f_{u_R} \\
\langle p|O_q^{(I=0)}|p\rangle &= f_{u_L} + f_{d_L} \\
\langle p|O_q^{(I=1,a=3)}|p\rangle &= \frac{1}{2} f_{u_L} - \frac{1}{2} f_{d_L} \\
\langle p|O_W^{(I=0)}|p\rangle &= f_{W^+} + f_{W^-} + \cos^2\theta_W f_Z + \sin^2\theta_W f_\gamma \\
&\quad + \sin\theta_W \cos\theta_W \left(f_{Z\gamma} + f_{\gamma Z}\right) \\
\langle p|O_W^{(I=1,I_3=0)}|p\rangle &= f_{W^+} - f_{W^-} \\
&\vdots
\end{aligned}
$$

Tree-level matching enough for NLL order.

Higgs Sector

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\,\varphi^+ \\ v + h - i\varphi^3 \end{pmatrix} ,$$

Equivalence theorem: $\varphi^\pm \to W_L^\pm$, $\varphi^3 \to Z_L$.

$$\langle p | O_H^{(I=0)} | p \rangle = f_{W_L^+} + \frac{1}{2}(f_h + f_{Z_L} + f_{Z_L h} + f_{h Z_L})$$

Longitudinal gauge boson emission given by scalars and equivalence theorem. Used already in previous results Chiu et al.

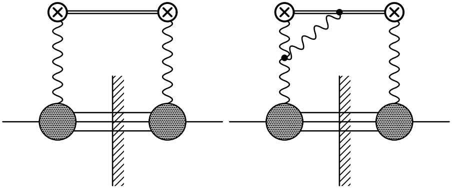
W,Z PDFs

In the broken phase:

$$O_{W_T^+}(r^-) = -\frac{1}{2\pi r^-} \int_{-\infty}^{\infty} \mathrm{d}\xi \, e^{-i\xi r^-} \, \bar{n}_\mu [W^{-\mu\lambda}(\bar{n}\xi) \, \mathcal{W}(\bar{n}\xi)] \, \bar{n}_\nu [\mathcal{W}^\dagger(0) \, W^{+\nu}{}_\lambda(0)] \,,$$

$$O_\gamma(r^-) = -\frac{1}{2\pi r^-} \int_{-\infty}^{\infty} \mathrm{d}\xi \, e^{-i\xi r^-} \, \bar{n}_\mu F^{\mu\lambda}(\bar{n}\xi) \, \bar{n}_\nu F^\nu{}_\lambda(0) \,,$$

where $\mathcal{W}$ is a QED Wilson line. Similarly for W^-, Z and $Z\gamma$.



Can compute it in terms of proton structure functions, similar to the analysis for the photon PDF AM, Nason, Salam, Zanderighi

$$
\begin{aligned}
x f_{W_T^+}(x,\mu) = \frac{\alpha_2(\mu)}{16\pi} \int_x^1 \frac{\mathrm{d}z}{z} \Bigg\{ \Bigg[\int_{\frac{m_p^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2} \frac{Q^4}{(Q^2+M_W^2)^2} \\
\times \left(-z^2 F_L^{(\bar\nu)}(x/z,Q^2) + \left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2^{(\bar\nu)}(x/z,Q^2) \right) \Bigg] \\
+ z p_{\gamma q}(z) \left(\ln \frac{\mu^2}{M_W^2(1-z)+\mu^2} - \frac{M_W^2(1-z)}{M_W^2(1-z)+\mu^2} \right) F_2^{(\bar\nu)}(x/z,\mu^2) \\
- z^2 F_2^{(\bar\nu)}(x/z,\mu^2) \Bigg\} + \mathcal{O}(\alpha_2^2)\,.
\end{aligned}
$$

W in proton: $h = 1, 0, -1$ states.

$$\begin{aligned} f_{W_T} &= (h=1) + (h=-1) \\ f_{\Delta W_T} &= (h=1) - (h=-1) \\ f_{W_L} &= (h=0) \end{aligned}$$

$f_{\Delta W_T} \neq 0$ even in unpolarized proton because of parity violation in the weak interactions.

Can compute ΔW in terms of the F_3 structure function,

$$W_{\mu\nu}(p,q) = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right) - \frac{iF_3}{2p\cdot q}\,\epsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma,$$

Longitudinal PDFs

$$f_{W_L^+}(x,\mu) = \frac{M_W^2}{2\pi r^-}\int_{-\infty}^{\infty} \mathrm{d}\xi\, e^{-i\xi r^-}\, \langle p | [\bar n \cdot W^-(\bar n \xi)\, \mathcal{W}(\bar n \xi)]\, [\mathcal{W}^\dagger(0)\, \bar n \cdot W^+(0)] | p \rangle\,,$$

and similarly for W^-, Z, Zh [Not $Z\gamma$] and h. Cannot be writen in terms of field-strength tensors, but in terms of Higgs using equivalence theorem.

$$O_{H^+}(r^-) = \frac{r^-}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\xi\, e^{-i\xi r^-} [H^\dagger(\bar n \xi)\, \mathcal{W}(\bar n \xi)]_1\, [\mathcal{W}^\dagger(0)\, H(0)]^1\,,$$

$$\begin{aligned}\mathcal{W}^\dagger(x) H(x) &= P\left\{ -i \int_0^\infty \mathrm{d}s\, \bar n \cdot [g_2 W(x+\bar n s) + g_1 B(x + \bar n s)] \right\} H(x) \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\,\varphi^+(x) \\ v + h(x) - i\varphi^3(x) \end{pmatrix} - i\frac{v}{\sqrt{2}} \int_0^\infty \mathrm{d}s \begin{pmatrix} \frac{g_2}{\sqrt{2}}\, \bar n \cdot W^+(x+\bar n s) \\ -\frac{1}{2} g_Z\, \bar n \cdot Z(x + \bar n s) \end{pmatrix} + \ldots\end{aligned}$$

Using integration by parts, we rewrite

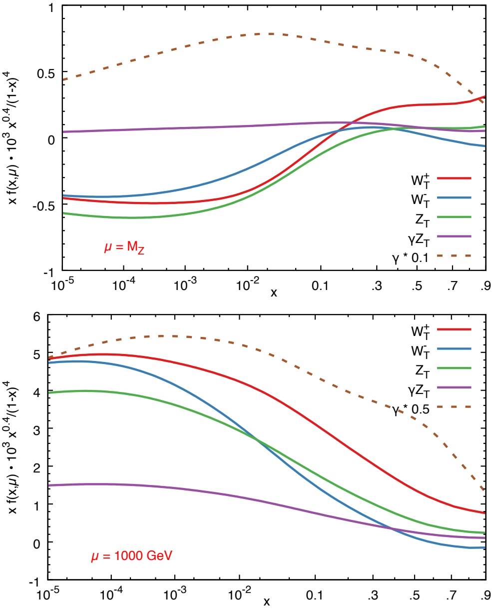
$$\int_{-\infty}^{\infty} \mathrm{d}\xi \; e^{-i\xi r^-} \int_0^{\infty} \mathrm{d}s \; \bar{n} \cdot Z(\bar{n}\xi + \bar{n}s)$$
$$= \frac{1}{i\,r^-} \int_{-\infty}^{\infty} \mathrm{d}\xi \; e^{-i\xi r^-} \, \bar{n} \cdot Z(\bar{n}\xi),$$

As a result,

$$\mathcal{W}^\dagger(\bar{n}\xi) H(\bar{n}\xi) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\left[i\varphi^+(\bar{n}\xi) - \frac{M_W}{r^-}\bar{n} \cdot W^+(\bar{n}\xi)\right] \\ v + h(\bar{n}\xi) - \left[i\varphi^3(\bar{n}\xi) - \frac{M_Z}{r^-}\bar{n} \cdot Z(\bar{n}\xi)\right] \end{pmatrix} + \ldots$$

So the φ PDFs turn into longitudinal gauge boson PDFs.

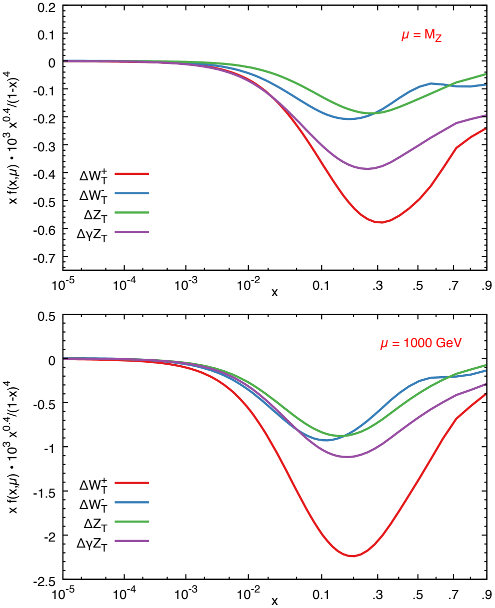
Transverse



$$W_T \propto \frac{\alpha_2}{4\pi} \ln \frac{\mu^2}{M^2}$$

Does not have to be positive because we did a $\overline{\text{MS}}$ subtraction.

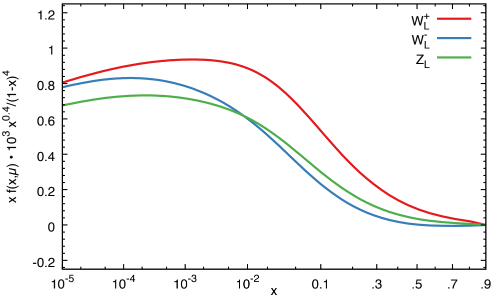
Polarized



$$\Delta W_T \propto \frac{\alpha_2}{4\pi} \ln \frac{\mu^2}{M^2}$$

More quarks than antiquarks in a proton, and left-handed quarks tend to emit $h = -1$ W bosons.

Longitudinal



$$W_L \propto \frac{\alpha_2}{4\pi}$$

Smaller by $\ln \mu^2/M^2$ relative to W_T and ΔW.
Scale independent at this order

Conclusions

- Factorizing EW cross sections leads to EW soft functions and EW non-singlet PDFs
- Computed the anomalous dimensions — rapidity divergences and Sudakov double logs
- Angular dependence in soft anomalous dimension
- Compute the matching onto the broken phase
- Calculated the EW gauge boson PDFs