Factorization, Renormalization and Resummation at Subleading Power

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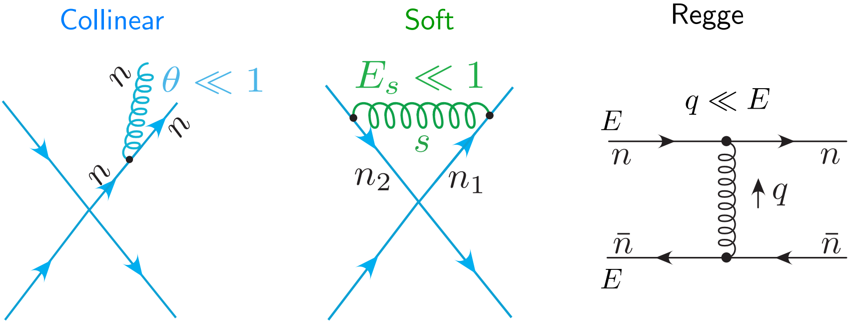
Limits of QCD

- Significant progress in understanding QCD made by considering limits where we have a power expansion in some small kinematic quantity.



- All orders behavior described by factorization theorems:

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau} = H^{(0)} J_\tau^{(0)} \otimes J_\tau^{(0)} \otimes S_\tau^{(0)} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q\tau}, \tau\right)$$

Power Corrections for Event Shapes

- "Standard" factorization theorems describe only leading term.
- More generally, can consider expanding an observable in τ

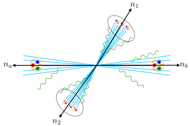
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} &= \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ \quad \text{Leading Power (LP)} \\
&+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau \quad \text{Next to Leading Power (NLP)} \\
&+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \tau \log^m \tau \\
&+ \cdots \\
&= \frac{d\sigma^{(0)}}{d\tau} + \frac{d\sigma^{(2)}}{d\tau} + \frac{d\sigma^{(4)}}{d\tau} + \cdots
\end{aligned}
$$

- Why do we want to understand power corrections?

Application: Fixed Order Subtractions

- IR divergences in fixed order calculations can be regulated using event shape observables. [Boughezal, Focke, Petriello, Liu], [Gaunt, Stahlhofen, Tackmann, Walsh]

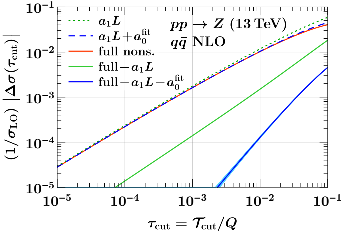
$$\boxed{\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}$$



$$\int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

Compute using factorization in soft/collinear limits:

Power Correction



$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N)$$

Application: Bootstrap

- Bootstrap approaches aim to completely reconstruct amplitudes or cross sections from limits.
- Most success in planar $\mathcal{N} = 4$.
- Some recent applications in QCD. [Li, Zhu][Duhr et al.]

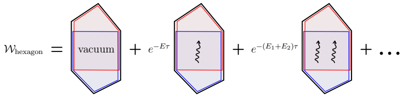
Remaining Parameters in Symbol of 6-Point MHV Remainder Function

Constraint	$L=2$	$L=3$	$L=4$
1. Integrability	75	643	5897
2. Total S_3 symmetry	20	151	1224
3. Parity invariance	18	120	874
4. Collinear vanishing (T^0)	4	59	622
5. OPE leading discontinuity	0	26	482
6. Final entry	0	2	113
7. Multi-Regge limit	0	2	80
8. Near-collinear OPE (T^1)	0	0	4
9. Near-collinear OPE (T^2)	0	0	0

[Dixon et al.]

LL All Powers

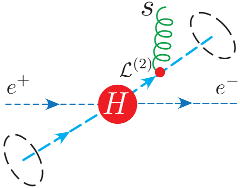
NLP, NNLP



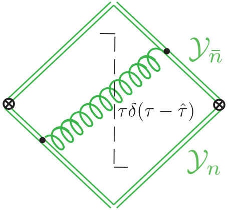
[Basso, Sever, Vieira]

Outline

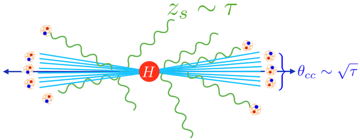
- Factorization at Subleading Power in SCET



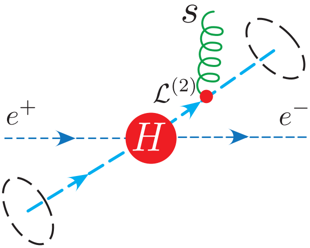
- Renormalization at Subleading Power



- Leading Log Resummation at Next-to-Leading Power for Thrust



Factorization at Subleading Power in SCET

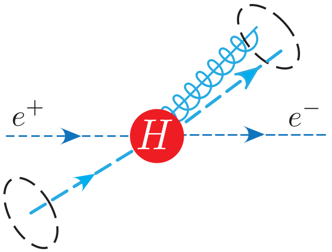


Subleading Power SCET

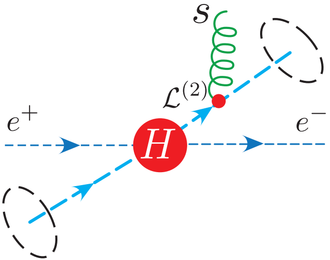
- SCET naturally organizes power expansion

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \sum_{i\geq 0} \mathcal{L}_{\rm hard}^{(i)} + \sum_{i\geq 0} \mathcal{L}^{(i)}$$

Subleading Hard Scattering Operators



Subleading Lagrangians



Soft-Collinear Factorization at Subleading Power

- BPS field redefinition decouples LP soft and collinear interactions.
- Working in an expansion in τ (not α_s), subleading power Lagrangians enter as T-products:

$$\langle 0| T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\ \mathcal{L}_{\rm dyn}]\}|X\rangle$$

$$= \langle 0| T\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x\ (\mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots)]\}|X\rangle$$

$$= \langle 0| T\left\{\tilde{O}_j^{(k)}(0)\exp[i\int d^4x \mathcal{L}^{(0)}]\left(1 + i\int d^4y \mathcal{L}^{(1)} + \frac{1}{2}\left(i\int d^4y \mathcal{L}^{(1)}\right)\left(i\int d^4z \mathcal{L}^{(1)}\right) + i\int d^4z \mathcal{L}^{(2)} + \cdots\right)\right\}|X\rangle$$

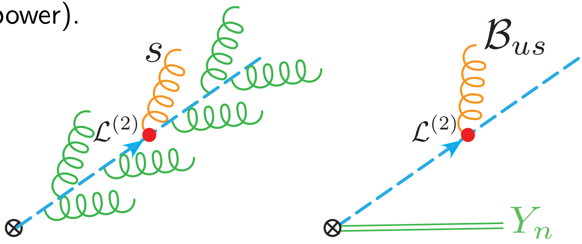
$$= \langle 0| T\left\{\tilde{O}_j^{(k)}(0)\left(1 + i\int d^4y \mathcal{L}^{(1)} + \frac{1}{2}\left(i\int d^4y \mathcal{L}^{(1)}\right)\left(i\int d^4z \mathcal{L}^{(1)}\right) + i\int d^4z \mathcal{L}^{(2)}\right)\right\}|X\rangle_{\mathcal{L}^{(0)}} + \cdots .$$

- Only need to consider a finite number of insertions.
- Decoupling of leading power dynamics $\implies$ states still factorize.

$$|X\rangle = |X_n\rangle |X_s\rangle$$

Gauge Invariant Ultrasoft Fields

- At subleading power, explicit ultrasoft fields appear.
- Wilson lines from field redefinition can be arranged into gauge invariant "gluon" operators plus Wilson lines (analogous to $\mathcal{B}_{\perp n}$ at leading power).



$$Y_{n_i}^{(r)\dagger} iD_{us}^{(r)\mu} Y_{n_i}^{(r)} = i\partial_{us}^{\mu} + [Y_{n_i}^{(r)\dagger} iD_{us}^{(r)\mu} Y_{n_i}^{(r)}] = i\partial_{us}^{\mu} + T_{(r)}^{a} g \mathcal{B}_{us(i)}^{a\mu}$$

- Provides gauge invariant description of soft sector at subleading power.

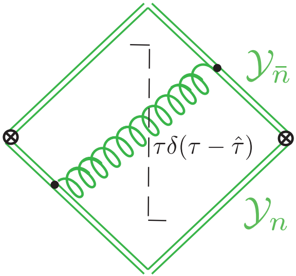
Factorization

- EFT makes subleading power factorization (at least formally) straightforward.
- Cross section expressed as matrix elements of gauge invariant fields:

$$\chi_n, \mathcal{B}_{\perp n}, \mathcal{P}_\perp, \mathcal{B}^{a\mu}_{us(n)}, \psi_{us(n)}, \partial^\mu_{us}, Y_n$$

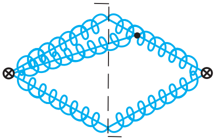
- With interactions decoupled, just as at leading power, factorization amounts to manipulation into matrix elements of soft and collinear fields (with additional convolutions).
- Renormalization of these operators is significantly more complicated than at LP. It is required to sum subleading power logarithms.

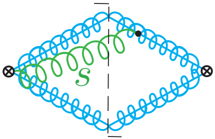
Renormalization at Subleading Power for Thrust



Fixed Order Calculation

- Compute power corrections for thrust at lowest order





$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau} = 8C_A\left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{Q^2\tau}\right)-\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{Q^2\tau^2}\right)\right]\theta(\tau)+\mathcal{O}(\alpha_s^2)$$

$$= 8C_A\left(\frac{\alpha_s}{4\pi}\right)\log\tau\ \theta(\tau)+\mathcal{O}(\alpha_s^2)$$

- No virtual corrections at lowest order ($\delta(\tau)\sim 1/\tau$).
- Divergences cancel between soft and collinear.
- Log appears at first non-vanishing order:
  - At LP, $\log(\tau)/\tau$ arises from RG evolution of $\delta(\tau)$
  - At NLP $\log(\tau)$ arises from RG evolution of "nothing"?

An Important Illustrative Example

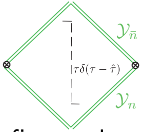
[Moult, Stewart, Vita, Zhu]

- Consider the power suppressed soft function:

$$S^{(2)}_{g,\tau\delta}(\tau,\mu) = \frac{1}{(N_c^2-1)}\mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\,\tau\ \delta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- This soft function vanishes at lowest order

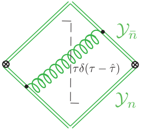
$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s^0)} =$$



$$= \tau\delta(\tau) = 0$$

- It has a UV divergence at the first order

$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s)} = 2$$



$$= g^2\theta(\tau)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{(Q\tau)^2}\right) + \mathcal{O}(\epsilon)\right)$$

- What renormalizes this function?

An Important Illustrative Example

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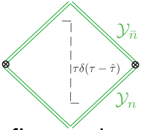

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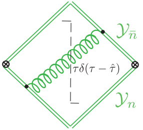


$$= g^2\theta(\tau)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{(Q\tau)^2}\right) + \mathcal{O}(\epsilon)\right)$$

- What renormalizes this function?

$\Longrightarrow$ Mixing with another operator!

An Important Illustrative Example [Moult, Stewart, Vita, Zhu]

- We can use a simple trick to find the missing operator.
- The RG for the leading power soft function is known:

$$\mu \frac{dS^{(0)}_{g,\delta}(\tau,\mu)}{d\mu} = \int d\tau' \, 2\Gamma^g_{\rm cusp} \left(2\left[\frac{\theta(\tau-\tau')}{\tau-\tau'}\right]_+ - \log\left(\frac{\mu^2}{Q^2}\right) \delta(\tau-\tau') \right) S^{(0)}_{g,\delta}(\tau',\mu)$$

- Multiplying by τ, we find

$$\mu \frac{d}{d\mu} \tau S^{(0)}_{g,\delta}(\tau,\mu) = \int d\tau' ((\tau-\tau')+\tau') \, 2\Gamma^g_{\rm cusp} \left(2\left[\frac{\theta(\tau-\tau')}{\tau-\tau'}\right]_+ - \log\left(\frac{\mu^2}{Q^2}\right) \delta(\tau-\tau') \right) S^{(0)}_{g,\delta}(\tau',\mu)$$

- Simplifying, we have

$$\mu \frac{d}{d\mu} \tau S^{(0)}_{g,\delta}(\tau,\mu) = \int d\tau' \, 4\Gamma^g_{\rm cusp} \theta(\tau-\tau') S^{(0)}_{g,\delta}(\tau',\mu) + \int d\tau' \gamma^S_g(\tau-\tau')\tau' S^{(0)}_{g,\delta}(\tau',\mu)$$

- Performing the integral, we have

$$\mu \frac{d}{d\mu} \tau S^{(0)}_{g,\delta}(\tau,\mu) = 4\Gamma^g_{\rm cusp} \, S^{(2)}_{g,\theta}(\tau,\mu) + \int d\tau' \gamma^S_g(\tau-\tau',\mu)\tau' S^{(0)}_{g,\delta}(\tau',\mu)$$

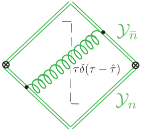
- Here we have defined a new power suppressed soft function

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} {\rm tr}\langle 0|\mathcal{Y}^T_{\bar n}(0)\mathcal{Y}_n(0)\theta(\tau-\hat\tau)\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar n}(0)|0\rangle$$

Perturbative View

- Returning to our perturbative calculation of the subleading power soft function

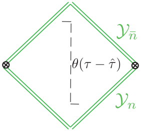
$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s)} = 2$$



$$= g^2\theta(\tau)\left(\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{(Q\tau)^2}\right)+\mathcal{O}(\epsilon)\right)$$

- UV divergence now easily understood as mixing with θ function operator, which is non-vanishing at lowest order

$$S^{(2)}_{g,\theta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s^0)} =$$



$$= \theta(\tau)$$

- Similar θ function counterterm observed by Paz in subleading power jet function at one-loop. Our example enables us to prove their all orders structure.

θ-Function Operators

- At subleading power we require θ-jet and θ-soft functions

$$J^{(2)}_{\mathcal{B}_n,\theta}(\tau,\mu) = \frac{(2\pi)^3}{(N_c^2-1)}\mathrm{tr}\Big\langle 0\Big|\, \mathcal{B}^{\mu a}_{n\perp}(0)\,\delta(Q+\bar{\mathcal{P}})\delta^2(\mathcal{P}_\perp)\,\theta(\tau-\hat{\tau})\,\mathcal{B}^{\mu a}_{n\perp,\omega}(0)\,\Big|0\Big\rangle$$

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)}\mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar n}(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar n}(0)|0\rangle$$

- They are power suppressed due to $\theta(\tau)\sim 1$ instead of $\delta(\tau)\sim 1/\tau$.
- Arise only through mixing at cross section level.
- We find this type of mixing is a generic behavior at subleading power.
- Extension to higher power straightforward

$$S^{(n)}_{g,(n,m),\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)}\mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar n}(0)\mathcal{Y}_n(0)(\tau-\hat{\tau})^m\hat{\tau}^{n-m}\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar n}(0)|0\rangle$$

Renormalization

- These subleading jet and soft functions satisfy a 2×2 mixing RG

$$\mu \frac{d}{d\mu} \begin{pmatrix} J^{(2)}_{\mathcal{B}_n,\tau\delta}(\tau,\mu) \\ J^{(2)}_{\mathcal{B}_n,\theta}(\tau,\mu) \end{pmatrix} = \int d\tau' \begin{pmatrix} \gamma^J_{\mathcal{B}_n,\tau\delta\to\tau\delta}(\tau-\tau') & \gamma^J_{\mathcal{B}_n,\tau\delta\to\theta}\delta(\tau-\tau') \\ 0 & \gamma^J_{\mathcal{B}_n,\theta\to\theta}(\tau-\tau') \end{pmatrix} \begin{pmatrix} J^{(2)}_{\mathcal{B}_n,\tau\delta}(\tau',\mu) \\ J^{(2)}_{\mathcal{B}_n,\theta}(\tau',\mu) \end{pmatrix}$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau,\mu) \\ S^{(2)}_{g,\theta}(\tau,\mu) \end{pmatrix} = \int d\tau' \begin{pmatrix} \gamma^S_{g,\tau\delta\to\tau\delta}(\tau-\tau',\mu) & \gamma^S_{g,\tau\delta\to\theta}\delta(\tau-\tau') \\ 0 & \gamma^S_{g,\theta\to\theta}(\tau-\tau',\mu) \end{pmatrix} \begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau',\mu) \\ S^{(2)}_{g,\theta}(\tau',\mu) \end{pmatrix}$$

- We can now solve this equation to renormalize the operators, and resum subleading power logarithms.
- Consider for concreteness the soft function. Fourier transforming

$$\tilde{F}(y) = Q \int d\tau \; e^{-iQ\tau y/2} \; F(Q\tau)$$

we have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\tau\delta}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\tau\delta}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

Solution of the RGE

- The general solution to this RG can be written as

$$\tilde{S}^{(2)}_{g,\tau\delta}(y,\mu) = e^{\int\limits_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_{11} \log(iy\mu' e^{\gamma_E})} \left[\tilde{S}^{(2)}_{g,\tau\delta}(y,\mu_0) + X(\mu,\mu_0)\tilde{S}^{(2)}_{g,\theta}(y,\mu_0)\right]$$

- If $\gamma_{11} = \gamma_{22}$, as will occur in our case

$$X(\mu,\mu_S)|_{\gamma_{11}=\gamma_{22}} = \gamma_{12} \log\left(\frac{\mu}{\mu_S}\right)$$

- For LL, the boundary conditions are

$$S^{(2)}_{g,\tau\delta}(\tau,\mu_S) = 0 + \mathcal{O}(\alpha_s)\,,$$
$$S^{(2)}_{g,\theta}(\tau,\mu_S) = \theta(\tau) + \mathcal{O}(\alpha_s)$$

Resumed Soft Function

- We find the final result for the renormalized subleading power soft function:

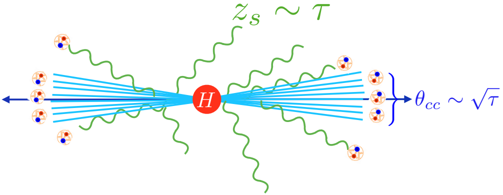
$$S^{(2)}_{g,\tau\delta}(Q\tau,\mu) = \theta(\tau)\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right) e^{\frac{1}{2}\gamma_{11}\log^2\left(\frac{\mu}{Q\tau}\right)}$$

- Expanded perturbatively, we see a simple series:

$$S^{(2)}_{g,\tau\delta}(Q\tau,\mu) = \theta(\tau)\left[\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right) + \frac{1}{2}\gamma_{12}\gamma_{11}\log^3\left(\frac{\mu}{Q\tau}\right) + \cdots\right]$$

- In particular, we find
  - First log generated by mixing with the θ function operators.
  - The single log is then dressed by Sudakov double logs from the diagonal anomalous dimensions.
- Example also useful for understanding power suppressed RG consistency.

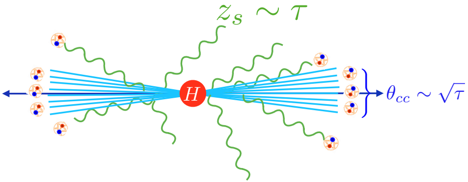
Leading Log Resummation at Next-to-Leading Power for Thrust in $H \to gg$



LL Resummation for Thrust at NLP

- Simple playground is pure glue QCD for Thrust in $H \to gg$

$$\tau = 1 - \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



- Represents simplest possible example to highlight features of subleading power resummation.
- Extension to NLL, inclusion of quark operators, etc. interesting but won't be covered here.

LL Resummation for Thrust at NLP

- Power corrections arise from two distinct sources:
  - Power corrections to scattering amplitudes.
  - Power corrections to kinematics.
- Each represent RG independent classes of power corrections:

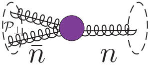
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{kin,LL}}}{\mathrm{d}\tau} + \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{hard,LL}}}{\mathrm{d}\tau}$$

- We will see that to LL, each class reduces to a mixing with θ function operators, equivalent to the 'illustrative' example shown above.
- This immediately implies exponentiation into a LL Sudakov for thrust at subleading power.

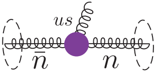
Matrix Element Corrections

[Moult, Stewart, Vita]

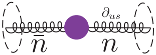
- Matrix element corrections arise from operators involving an additional $\mathcal{B}_{n\perp}$, $\mathcal{B}_{us}$ or ∂_{us}.
- We have performed an explicit matching to the required operators



$$\mathcal{O}^{(2)}_{\mathcal{P}\mathcal{B}1} = C^{(2)}_{\mathcal{P}\mathcal{B}1} if^{abc}\mathcal{B}^a_{n\perp,\omega_1}\cdot\left[\mathcal{P}_\perp\mathcal{B}^b_{\bar{n}\perp,\omega_2}\cdot\right]\mathcal{B}^c_{\bar{n}\perp,\omega_3}H\,,$$
$$\mathcal{O}^{(2)}_{\mathcal{P}\mathcal{B}2} = C^{(2)}_{\mathcal{P}\mathcal{B}2} if^{abc}\left[\mathcal{P}_\perp\cdot\mathcal{B}^a_{\bar{n}\perp,\omega_3}\right]\mathcal{B}^b_{n\perp,\omega_1}\cdot\mathcal{B}^c_{\perp\bar{n},\omega_2}H$$



$$\mathcal{O}^{(2)}_{\mathcal{B}(us(n))} = C^{(2)}_{\mathcal{B}(us(n))}\left(if^{abd}\left(\mathcal{Y}_n^T\mathcal{Y}_{\bar{n}}\right)^{dc}\right)\left(\mathcal{B}^a_{n\perp,\omega_1}\cdot\mathcal{B}^b_{\bar{n}\perp,\omega_2}\bar{n}\cdot g\mathcal{B}^c_{us(n)}\right)\,,$$
$$\mathcal{O}^{(2)}_{\mathcal{B}(us(\bar{n}))} = C^{(2)}_{\mathcal{B}(us(\bar{n}))}\left(if^{abd}\left(\mathcal{Y}_{\bar{n}}^T\mathcal{Y}_{n}\right)^{dc}\right)\left(\mathcal{B}^a_{n\perp,\omega_1}\cdot\mathcal{B}^b_{\bar{n}\perp,\omega_2}n\cdot g\mathcal{B}^c_{us(\bar{n})}\right)$$



$$\mathcal{O}^{(2)}_{\partial\mathcal{B}(us)(0)} = C^{(2)}_{n\cdot\partial}\mathcal{B}^{\mu a}_{\perp n,\omega_1} in\cdot\partial\mathcal{B}^{\mu b}_{\perp\bar{n},\omega_2}\left(\mathcal{Y}_{\bar{n}}^T\mathcal{Y}_n\right)^{ab}H\,,$$
$$\mathcal{O}^{(2)}_{\partial\mathcal{B}(us)(\bar{0})} = C^{(2)}_{\bar{n}\cdot\partial}\mathcal{B}^{\mu a}_{\perp\bar{n},\omega_2} i\bar{n}\cdot\partial\mathcal{B}^{\mu b}_{\perp n,\omega_1}\left(\mathcal{Y}_{\bar{n}}^T\mathcal{Y}_n\right)^{ab}H$$

- Wilson coefficients of soft operators are fixed to all orders using RPI:

$$C^{(2)}_{\mathcal{B}(us(n))} = -\frac{\partial C^{(0)}}{\partial\omega_1}$$

Factorization for Matrix Element Corrections

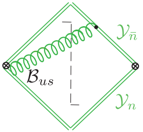
- By RG consistency, it is sufficient to consider the power suppressed soft function, involving a ∂_{us} or $\mathcal{B}_{us}$

$$\frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{Y}_{\bar n}^T(x)\mathcal{Y}_n(x)\bar n\cdot\mathcal{B}_{us(n)}(x)\delta(\tau_{us}-\hat\tau_{us})\mathcal{Y}_n^T(0)\mathcal{Y}_{\bar n}(0)|0\rangle=\int\frac{d^4r}{(2\pi)^4}e^{-ir\cdot x}S^{(2)}_{n\mathcal{B}_{us}}(\tau_{us},r)$$

which appears in the factorization as

$$\frac{d\sigma^{(2)}_{\mathcal{B}_{us},n}}{d\tau}=H_{\bar n\cdot\mathcal{B}}(Q^2)\int d\tau_n d\tau_{\bar n}d\tau_{us}\delta(\tau-\tau_n-\tau_{\bar n}-\tau_{us})$$
$$\cdot\left[\int\frac{d^4r}{(2\pi)^4}S^{(2)}_{n\mathcal{B}_{us}}(\tau_{us},r)\right]\cdot\left[\int\frac{dk^-}{2\pi}\mathcal{J}_{\bar n}(\tau_{\bar n},k^-)\right]\cdot\left[\int\frac{dl^+}{2\pi}\mathcal{J}_n(\tau_n,l^+)\right]$$

- These operators mix with a θ function soft function just as with the 'illustrative' example considered above. Resummation is identical.

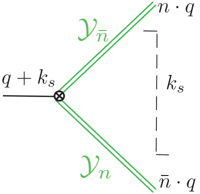


$$=\frac{\gamma_{n\mathcal{B}_{us}\to\theta}}{\epsilon}\theta(\tau)$$

$$\mu\frac{d}{d\mu}\begin{pmatrix}S_{n\mathcal{B}_{us}}(\tau,\mu)\\ S_{g,\theta}(\tau,\mu)\end{pmatrix}=\int d\tau'\begin{pmatrix}\gamma^S_{g,\delta}(\tau-\tau',\mu) & \gamma_{n\mathcal{B}_{us}\to\theta}\delta(\tau-\tau')\\ 0 & \gamma^S_{g,\delta}(\tau-\tau',\mu)\end{pmatrix}\begin{pmatrix}S_{n\mathcal{B}_{us}}(\tau',\mu)\\ S_{g,\theta}(\tau',\mu)\end{pmatrix}$$

Kinematic Corrections

- Kinematic corrections arise from
  - Phase space
  - Thrust observable definition (does not contribute at LL)
- Phase space corrections can be treated through choice of routing



$$\frac{1}{(Q+k_s)^2} = \frac{1}{Q^2} - \frac{n \cdot k_s}{Q^3} - \frac{\bar{n} \cdot k_s}{Q^3} + \mathcal{O}(\tau^2)$$

$$S^{(2)}_{g,\tau\delta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\tau\ \delta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- Are described by the 'illustrative' example considered above

$$\mu\frac{d}{d\mu}\begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau,\mu) \\ S^{(2)}_{g,\theta}(\tau,\mu) \end{pmatrix} = \int d\tau' \begin{pmatrix} \gamma^S_{g,\tau\delta\to\tau\delta}(\tau-\tau',\mu) & \gamma^S_{g,\tau\delta\to\theta}\delta(\tau-\tau') \\ 0 & \gamma^S_{g,\theta\to\theta}(\tau-\tau',\mu) \end{pmatrix} \begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau',\mu) \\ S^{(2)}_{g,\theta}(\tau',\mu) \end{pmatrix}$$

LL Resummation for Thrust at NLP [Moult, Stewart, Vita, Zhu]

- Complete result given by sum of two contributions.

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{kin,LL}}}{\mathrm{d}\tau} + \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{hard,LL}}}{\mathrm{d}\tau}$$

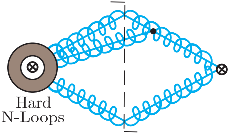
- Both have same Sudakov $\Longrightarrow$ can be directly added.
- Obtain the LL resummed result for pure glue $H \to gg$ thrust

$$\boxed{\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_A \log(\tau) e^{-\frac{\alpha_s}{4\pi}\Gamma^g_{\mathrm{cusp}}\log^2(\tau)}}$$

- Provides the first all orders resummation for an event shape at subleading power.
- Very simple result. Subleading power LL driven by cusp anomalous dimension!

Fixed Order Check

- We can explicitly check this result by fixed order calculation of the power corrections.
- RG consistency for $1/\epsilon$ poles implies that the LL power correction can be computed only from hard-collinear contributions:



[Moult, Rothen, Stewart, Tackmann, Zhu]

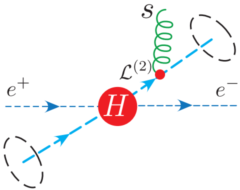
- Expanding known results for $H \to 3$ partons at NNLO, we can analytically compute the power corrections to $\mathcal{O}(\alpha_s^3)$:

$$\frac{1}{\sigma_0^H}\frac{\mathrm{d}\sigma^H}{\mathrm{d}\tau} = \frac{\alpha_s}{4\pi} 8C_A \log\tau - \left(\frac{\alpha_s}{4\pi}\right)^2 32C_A^2 \log^3\tau + \left(\frac{\alpha_s}{4\pi}\right)^3 64C_A^3 \log^5\tau + \mathcal{O}(\alpha_s^4)$$

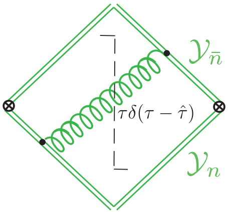
- Provides a highly non-trivial check on the correctness of our all orders resummation.

Conclusions

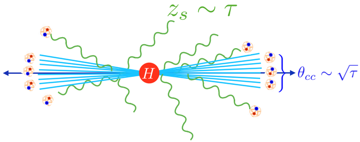
- SCET provides convenient organization of power expansion in terms of gauge invariants operators that can be separately renormalized.



- Cross section level renormalization at subleading power involves a new class of universal jet and soft functions involving θ-functions.



- Achieved first all orders resummation at subleading power for an event shape observable.



Thanks!