Heavy quark jet substructure

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In collaboration with
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Probing substructure of QCD jets

- Jet substructure observables allow us a deeper understanding of QCD dynamics in a jet and are designed to uncover specific features of a jet 1609.07483, I. Moult et. al.
- Grooming of jets reduces jet contamination from initial state radiation, underlying event and pile up .
- First analytic calculation for groomed massless parton jets , 1603.09338, Larkoski et. al.
- For massive quarks, see recent work on top quark jet mass 1608.01318 I.Stewart et. al.
- We wish to develop the factorization theorem for analytical calculation of groomed jet substructure observables for heavy quark jets, with a specific interest in b quark jets .
- First step of the theoretical effort towards utilizing b quark jets as probes of QGP.

Energy correlators for groomed jets

Simplest jet substructure observables are the class of energy correlator functions

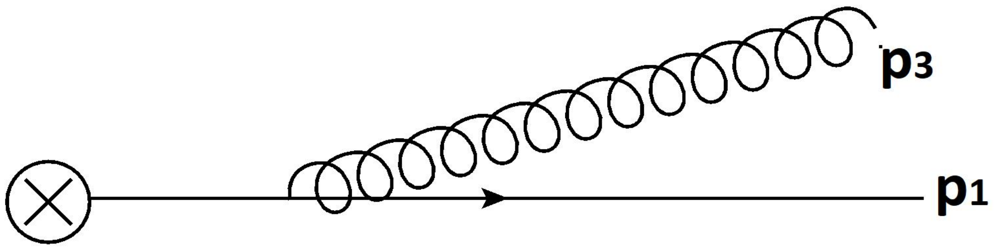
$$e_2^{(\alpha)}\Big|_{e^+e^-} = \frac{1}{E_J^2}\sum_{i<j\in J} E_i E_j \left(\frac{2p_i\cdot p_j}{E_iE_j}\right)^{\alpha/2} \qquad z_i \equiv \frac{E_i}{E_J}, \qquad \theta_{ij}^2 \equiv \frac{2p_i\cdot p_j}{E_iE_j},$$

Soft drop grooming algorithm

$$\frac{\min[E_i, E_j]}{E_i+E_j} > z_{\text{cut}}\left(\sqrt{2}\frac{\sin\frac{\theta_{ij}}{2}}{\sin\frac{R}{2}}\right)^{\beta}$$

We will set $\beta = 0$. This wil remove any radiation with energy fraction less than z_{cut} .

Fixed order cross section



$$x_i = E_i/E_J$$

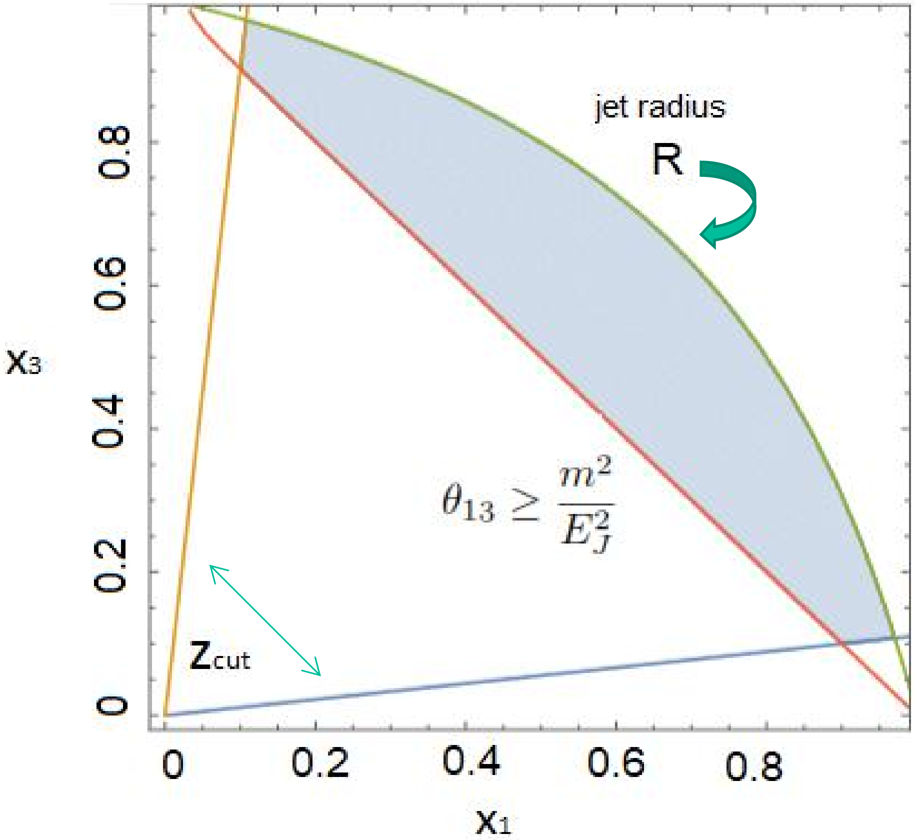
phase space

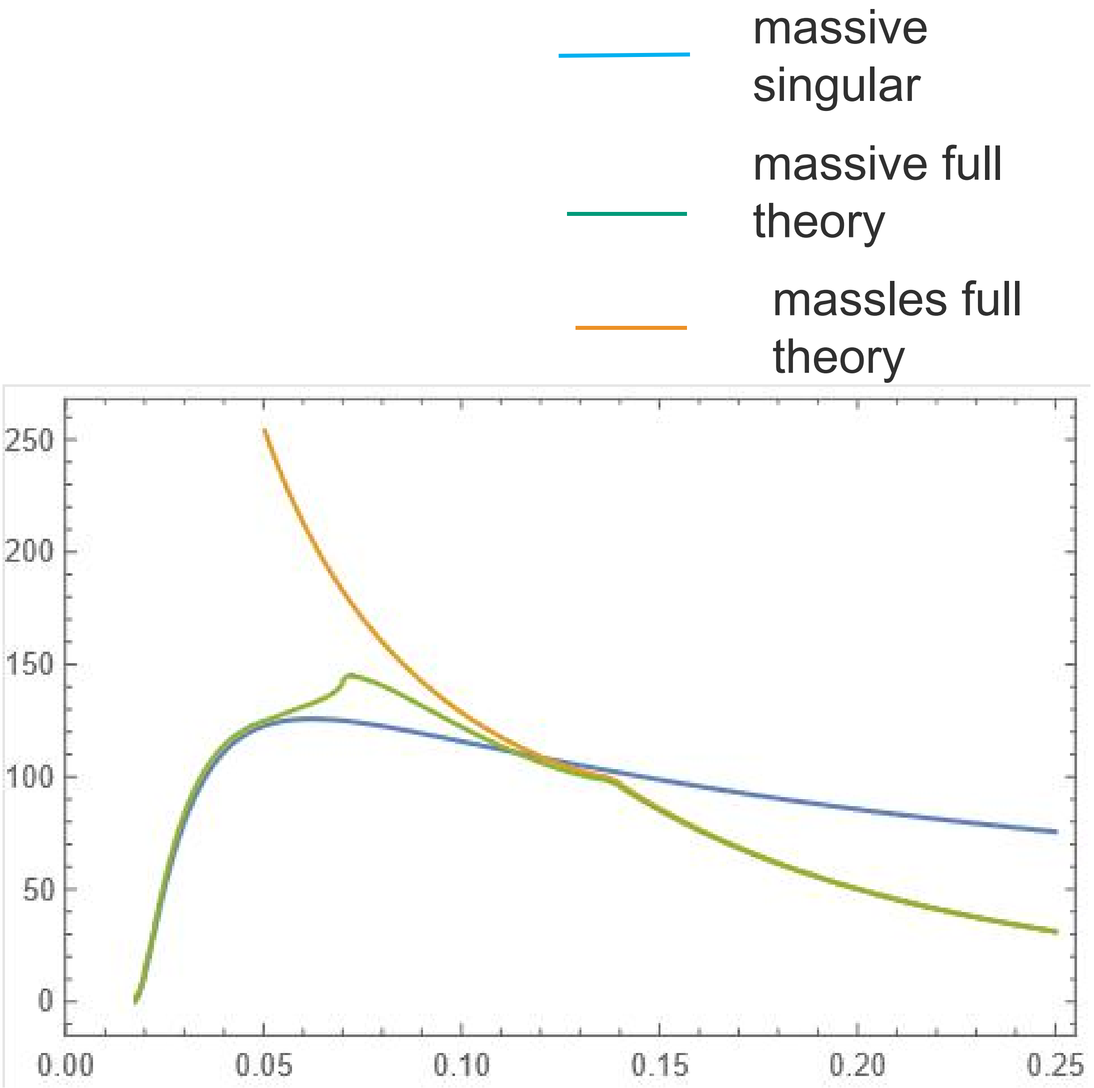
$$\frac{1}{\sigma_0}\frac{d\sigma}{de_2^\alpha} = \frac{\alpha_s}{2\pi}C_F\int dx_1 dx_3\left\{\frac{x_1^2+(2-x_1-x_3)^2}{(1-x_1)(x_1+x_3-1)} - \frac{m^4}{E_J^4}\frac{x_3^2}{4(x_1-1)^2(x_1+x_3-1)^2}\right.$$

$$\left.+\frac{m^2}{E_J^2}\frac{(1-2x_1-2x_3)x_1^2+2(x_1+x_3)^2x_1-3(2-x_1-x_3)^2+8(2-x_1-x_3)-6}{2(x_1-1)^2(x_3+x_1-1)^2}\right\}$$

$$\delta\left(e_2^\alpha - \frac{x_1x_3}{(x_1+x_3)^2}\left(4\frac{(x_1+x_3-1)}{x_1x_3}\right)^{\alpha/2}\right)$$

Phase Space


jet radius
R
$\theta_{13} \geq \frac{m^2}{E_J^2}$
Zcut
x3
x1
0
0.2
0.4
0.6
0.8


massive singular
massive full theory
massles full theory
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200
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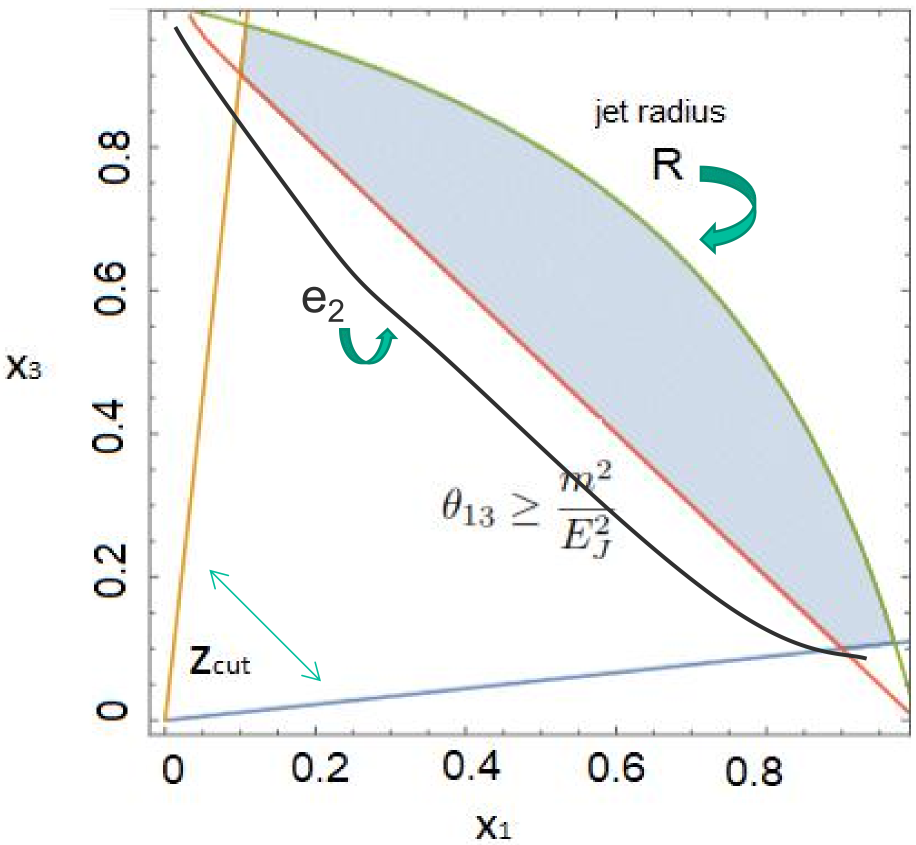
Phase Space



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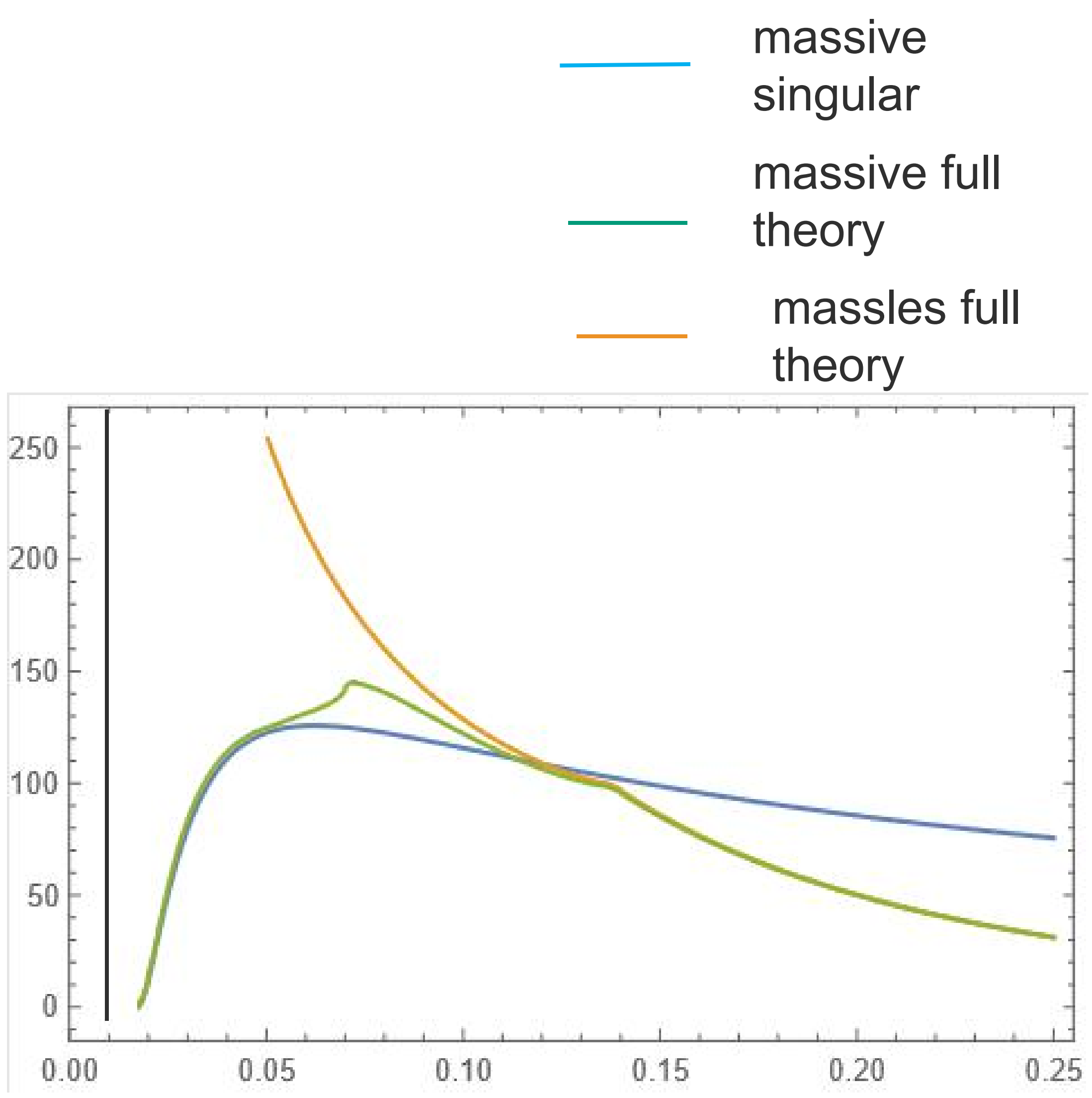

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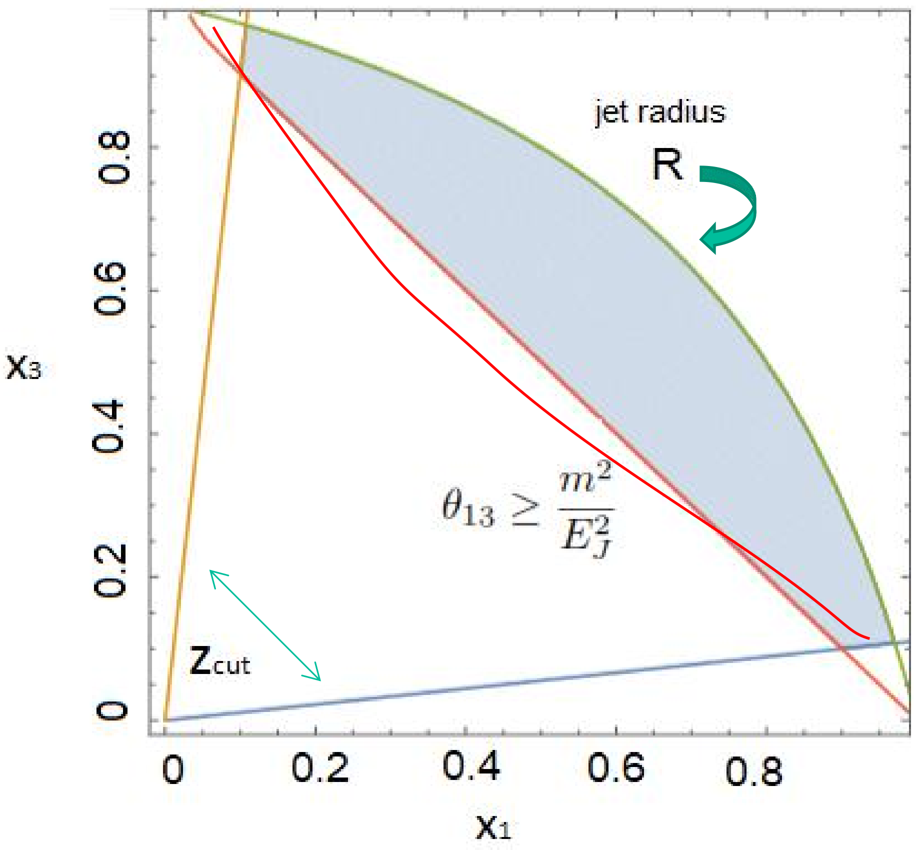

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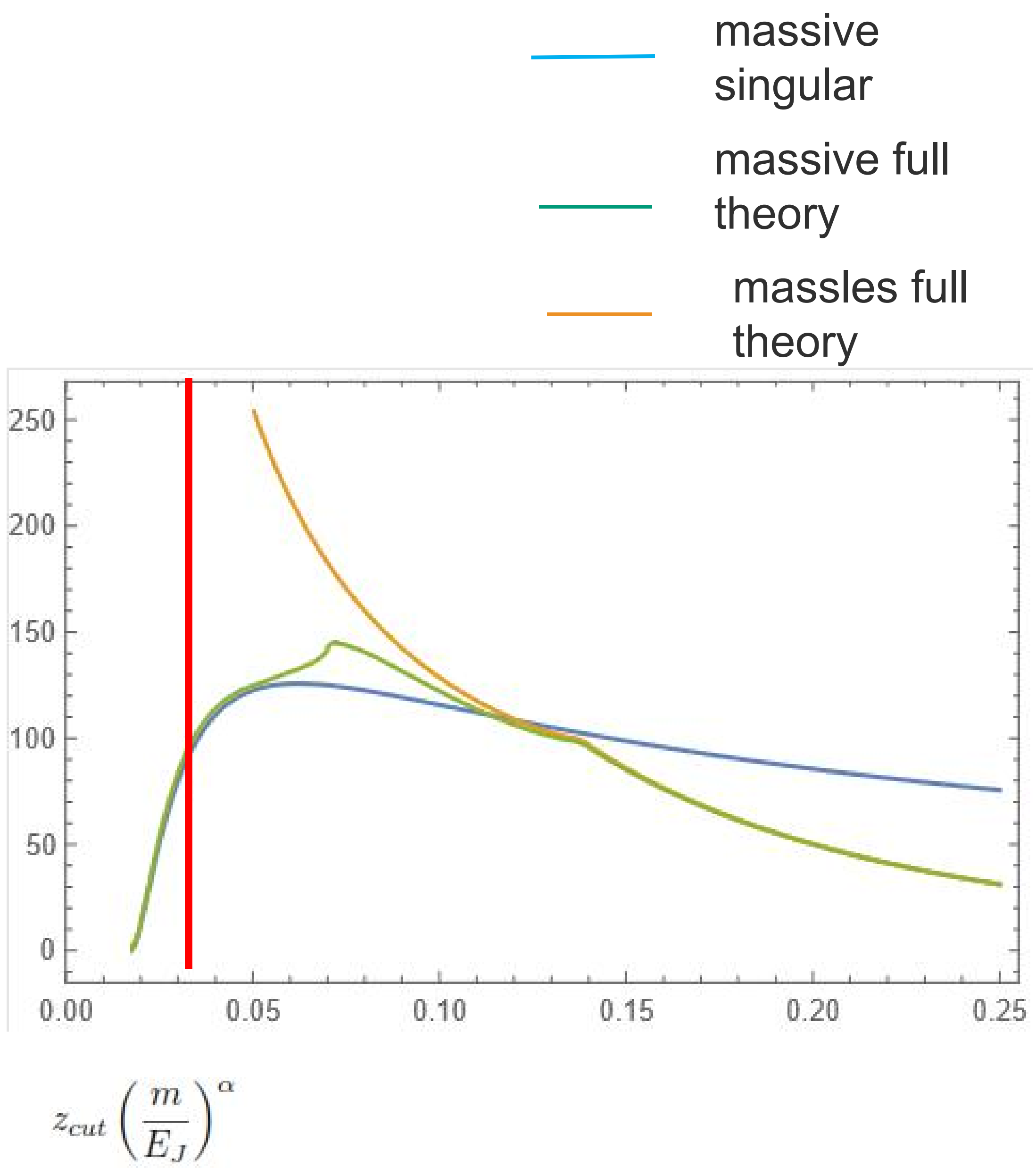

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$z_{cut}\left(\frac{m}{E_J}\right)^{\alpha}$

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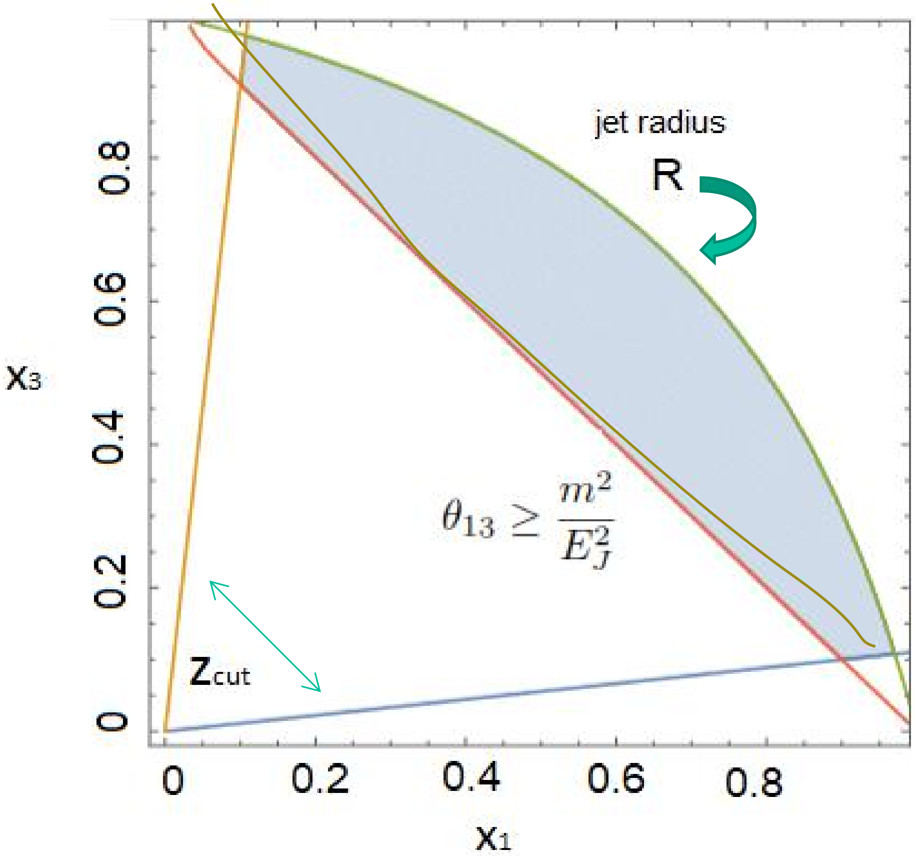

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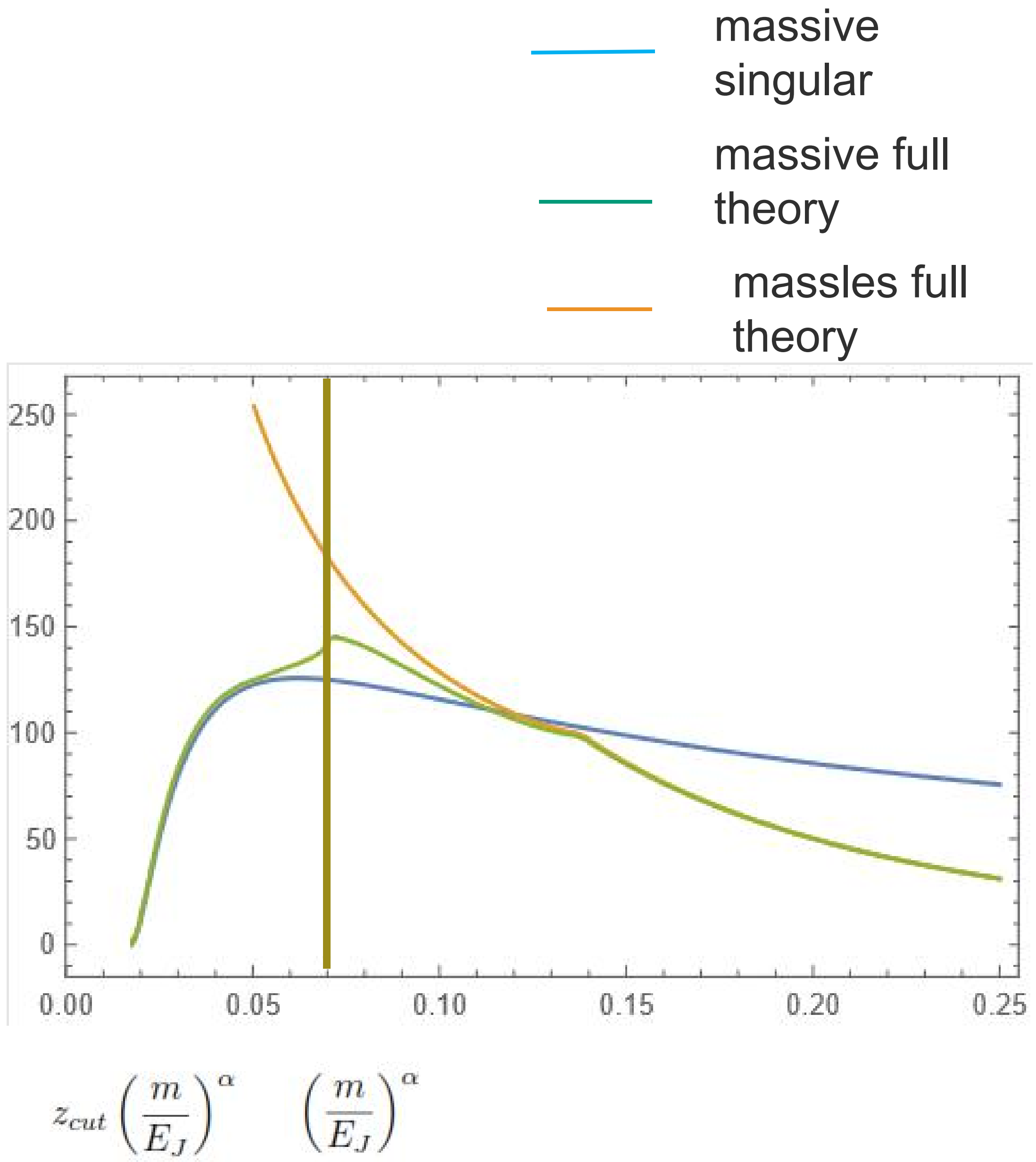

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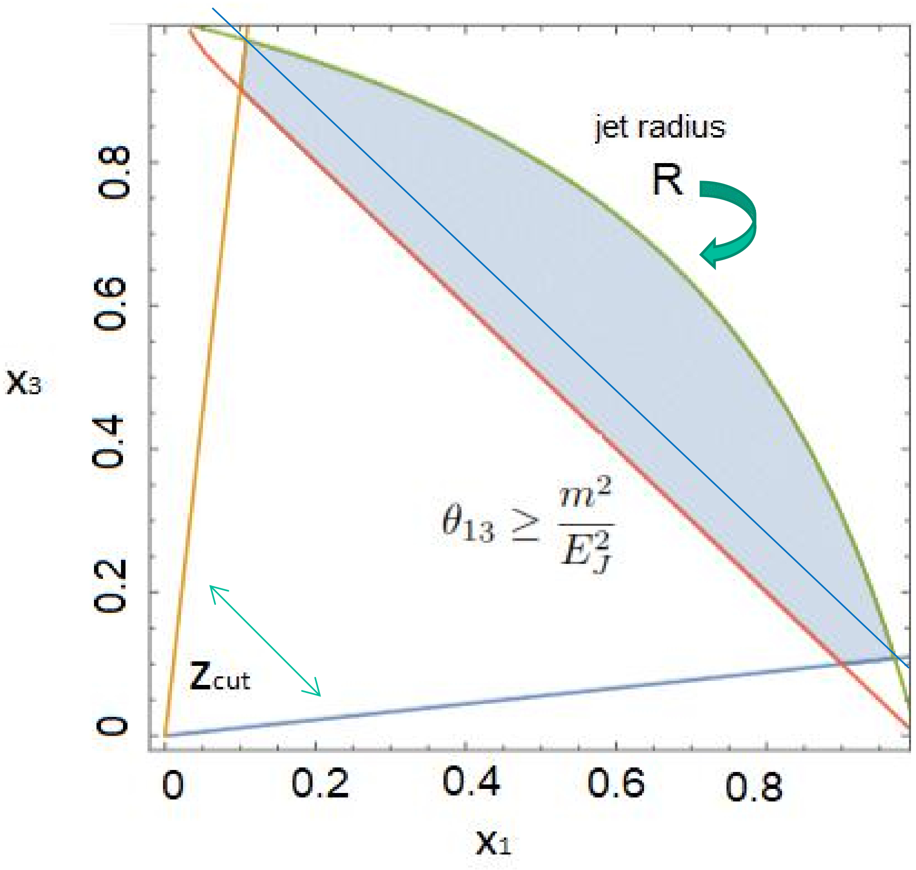

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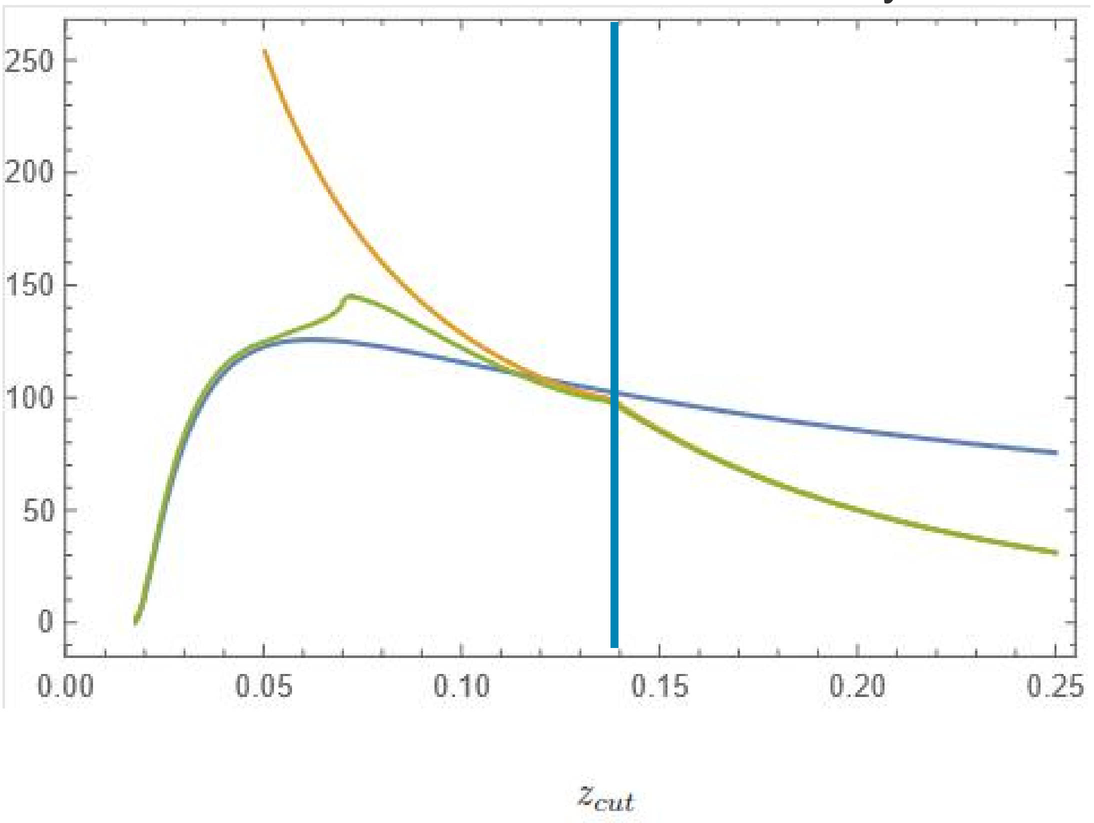

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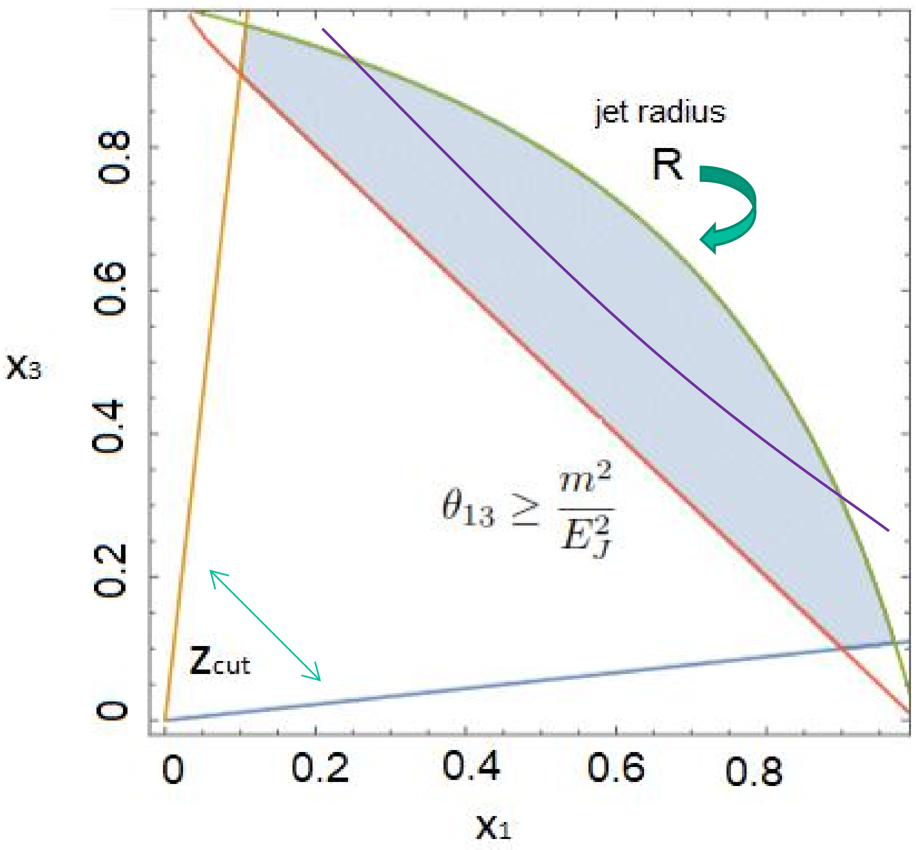
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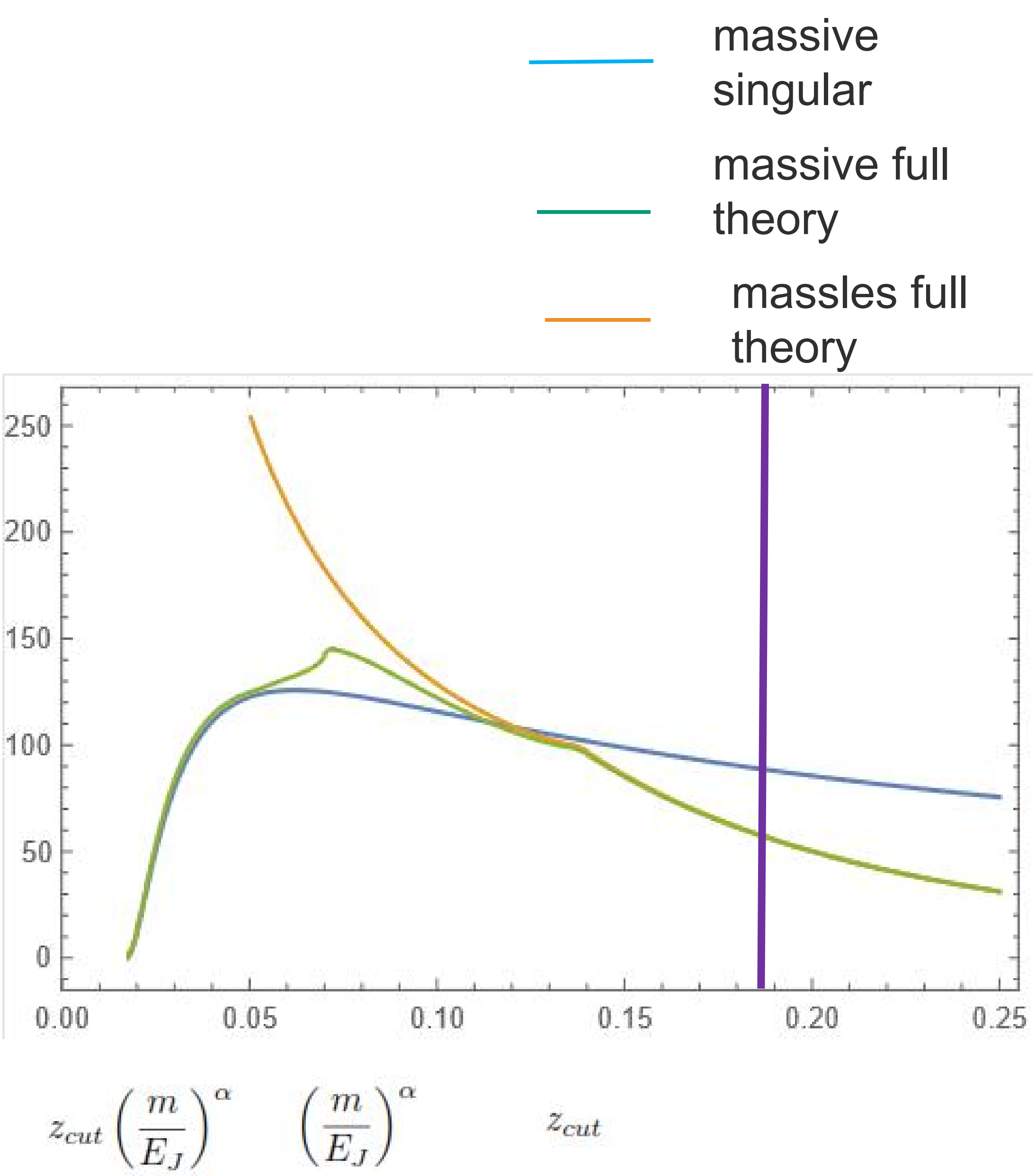
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z_{cut}

Fixed order singular cross section

Real diagram

$$\frac{\alpha_s}{2\pi} C_F \left(\frac{4}{\alpha e_2^{(\alpha)}} \ln \left(\frac{e_2^{(\alpha)}}{e_{2,min}^{(\alpha)}} \right) - \frac{2}{e_2^{(\alpha)}} \left(1 - \left(\frac{e_{2,min}^{(\alpha)}}{e_2^{(\alpha)}} \right)^{2/\alpha} \right) \right)$$

$$e_{2,min}^{(\alpha)} \sim z_{cut} \left(\frac{m}{E_J} \right)^{\alpha}$$

Virtual diagram

$$\sim \delta(e_2^{(\alpha)}) \left(\ln^2 \frac{E_J}{m}, \ln \frac{E_J}{m} \right)$$

Develop an EFT to resum large logs in e_2 and m

EFT for massive quark jets

- Assumed hierarchy $e_2^{(\alpha)} << z_{cut} << 1$
- Assume for the heavy quark $z_q \approx 1$
- All modes have a physical constraint $\theta \geq \left(\frac{m^2}{E_J^2}\right)$
- This combined with soft drop imposes a cut-off on $\mathbf{e_2}^{(\alpha)}$ $e_2^{(\alpha)} \geq z_{cut}\left(\frac{m}{E_J}\right)^{\alpha}$

EFT modes

Wide angle radiation

- $\theta \approx 1$ requires $z \approx e_2^{(\alpha)}$ which is groomed away
  $z \approx z_{cut}$,

$$p_s \equiv E_J z_{cut}(1,1,1)$$

Global soft radiation

Collinear radiation

- $z \approx z_{cut}$ requires $\theta_{cs} \approx (e_2^{(\alpha)} / z_{cut})^{(1/\alpha)} > \theta_{min}$

$$p_{cs} \sim z_{cut} E_J \left(1, \left(\frac{e_2^{(\alpha)}}{z_{cut}}\right)^{2/\alpha}, \left(\frac{e_2^{(\alpha)}}{z_{cut}}\right)^{1/\alpha}\right)$$

Collinear Soft radiation

- $z \approx 1$ requires $\theta_c \approx (e_2^{(\alpha)})^{(1/\alpha)}$

This mode exists for $\theta_c > \theta_{min}$ or $e_2^{(\alpha)} > (m/E_J)^{\alpha}$

$$p_c \sim E_J \left(1, \left(e_2^{(\alpha)}\right)^{2/\alpha}, \left(e_2^{(\alpha)}\right)^{1/\alpha}\right)$$

Collinear radiation

EFT modes

Collinear radiation

- For $\theta \approx \theta_{min}$ we need $z \approx e_2^{(\alpha)} (\theta_{min})^\alpha$

$$p_c \sim E_J e_2^{(\alpha)}/(\theta_{\min})^\alpha \left(1, \theta^2_{min}, \theta_{\min}\right)$$

Ultra-collinear radiation

In terms of the velocity of the heavy quark $v^\mu \equiv (E_J/m\,, m/E_J, 0)$

$p_c \sim \mathbf{W}\,(v_+, v_-, 1)$ with $\mathbf{W} = m_q e_2^{(\alpha)}/(\theta_{\min})^\alpha$

is the boosted soft mode of HQET

- At $e_2^{(\alpha)} = (\theta_{\min})^\alpha = (m/E_J)^\alpha$ the collinear and ultra-collinear modes are identical

Factorization

$(m/E_J)^{\alpha} > e_2^{(\alpha)} > e_2^{(\alpha)}{}_{min}$

$$\frac{d\sigma}{de_2^{(\alpha)}} = \sigma_0(Q^2) \times S(E_J z_{cut}) \times H(m_q) \times S_c(E_J z_{cut}(e_2^{(\alpha)}/z_{cut})^{1/\alpha}) \otimes J_q(\Gamma)$$

$z_{cut} > e_2^{(\alpha)} > (m/E_J)^{\alpha}$

$$\frac{d\sigma}{de_2^{(\alpha)}} = \sigma_0(Q^2) \times S(E_J z_{cut}) S_c(E_J z_{cut}(e_2^{(\alpha)}/z_{cut})^{1/\alpha}) \otimes J_q(E_J \left(e_2^{(\alpha)}\right)^{1/\alpha})$$

E_J

↓

$E_J z_{cut}(e_2^{(\alpha)}/z_{cut})^{1/\alpha}$

↓

W

↓

Λ_{QCD}

Dominant non-perturbative corrections come from the HQET jet function

Factorization

Identical to the result in 0711.2079 , I. Stewart et. al

Massive jet function

$$J = \frac{\alpha_s C_F}{\pi}\left(\frac{1}{\alpha-1}L_C^2 + L_V^2 + L_C + \frac{1}{2}L_V\right)$$

$$L_C = \ln\left(\frac{\mu s e^{\gamma_E}}{E_J}(4\Delta)^{\frac{\alpha-1}{2}}\right)$$

$$\Delta = \frac{m^2}{E_J^2}\ ,\quad L_V = \ln\left(\frac{\mu}{m}\right)$$

Matching to HQET

Wilson coefficient

$$H_+ = \frac{\alpha_s C_F}{\pi}\left(L_V^2 + \frac{1}{2}L_V\right)$$

HQET boosted jet function

$$B_+^{(1)} = \frac{\alpha_s C_F}{\pi}\left(\frac{1}{\alpha-1}L_C^2 + L_C\right)$$

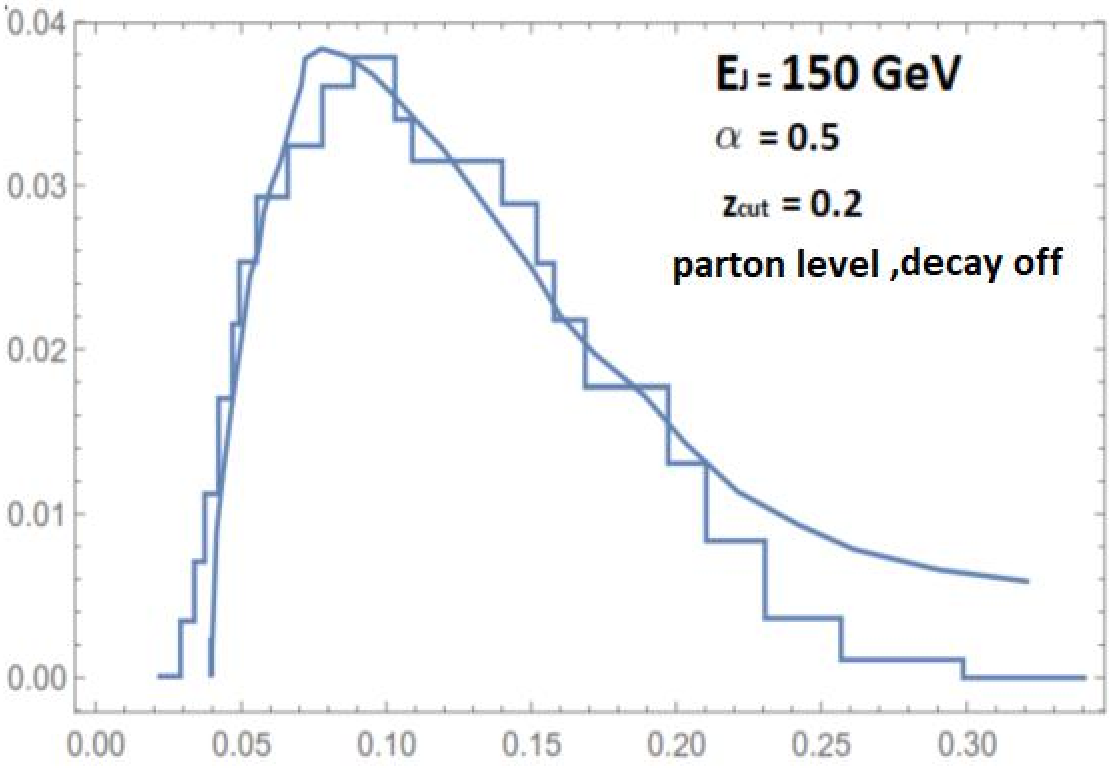
Anomalous dimension

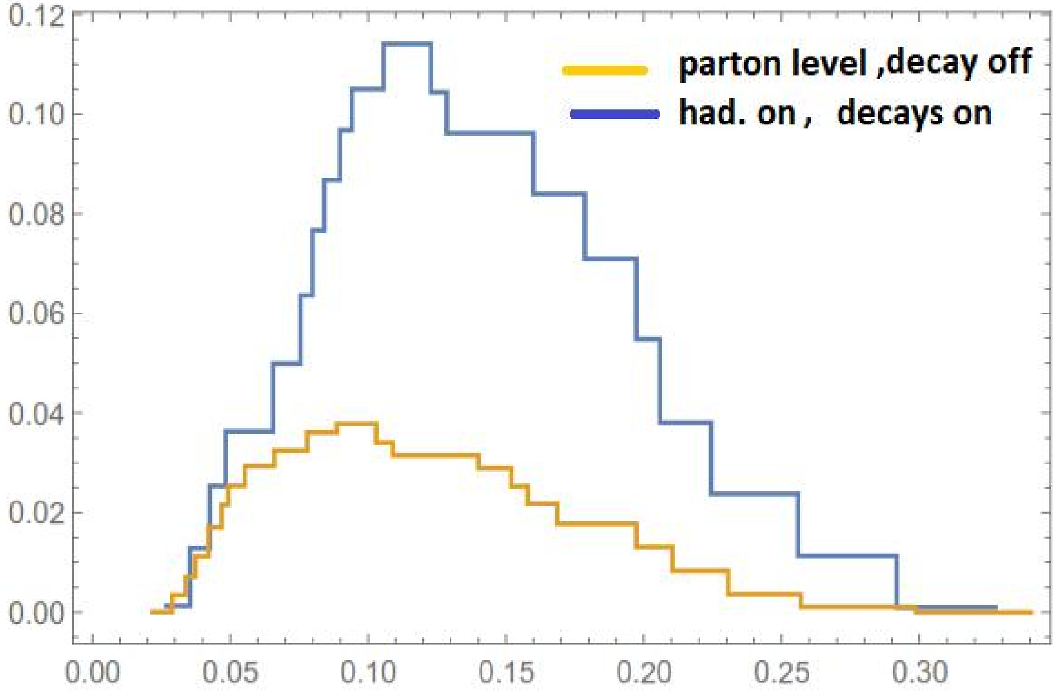
$$\gamma_\mu^J = \frac{\alpha_s C_F}{\pi}\left(\frac{\alpha}{\alpha-1}\ln\left(\frac{\mu^2(se^{\gamma_E})^{2/\alpha}}{\omega^2}\right) + \frac{3}{2}\right)$$

same as the massless jet function

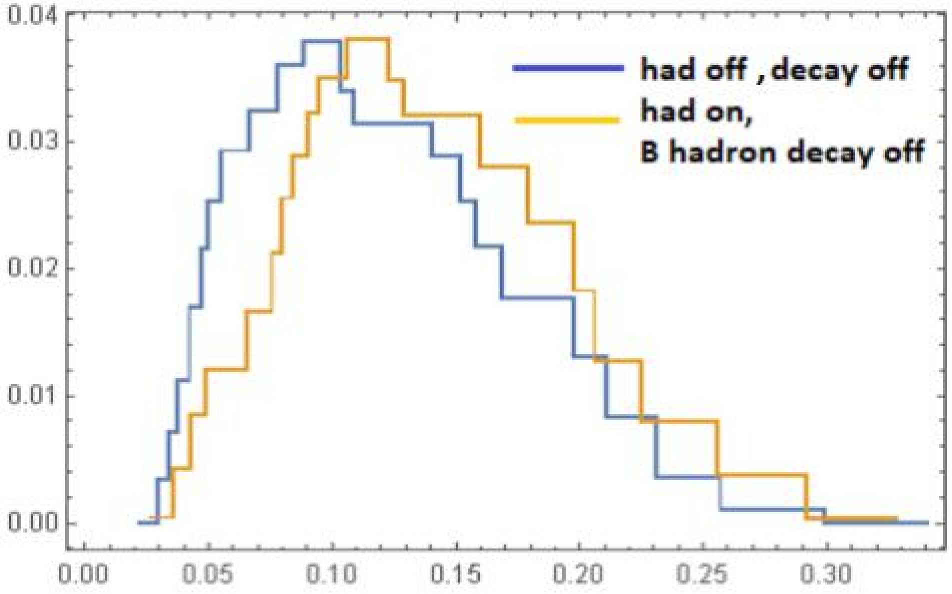
- At $1/s = (m/E_J)^\alpha$ the massive and massless jet function are identical.

Comparison of pythia with matched NLL





Hadronization corrections



- Turning off B hadron decay produces a simple shift in the partonic distribution
- A control over the prediction(with a shape function) with a reconstructed B hadron momentum
- To compare with the experimental result directly, we need to take into account b decays.
- The excess events when decay is turned on are coming from the partonic zero bin.

Hadronization corrections

- Non-perturbative corrections dominate in the HQET jet function $\longrightarrow$ $\frac{\Lambda_{QCD}}{Qe_2^{(\alpha)}}\left(\frac{Q}{m_b}\right)^{1-\alpha}$
- Using the Lorentz transformation properties of the HQET jet function, the leading non perturbative correction in Laplace space

$$P_1(s) = -\frac{s(4\Delta)^{\alpha/2}}{m} A(\alpha)$$

$$A(\alpha) = \int d\eta F(\eta,\alpha)\langle 0|\bar{h}_{v_+} W_n \mathcal{E}_T(\eta) W_n^\dagger h_{v_+}|0\rangle$$

$$F(\eta,\alpha) = e^\eta \left(1+e^{-2\eta}\right)^{\alpha/2}$$

$$\Delta = \frac{m^2}{E_J^2}$$

- **Prediction for scaling behaviour with energy and mass of the heavy quark**

$$(e_2^{(\alpha)})^{\text{avg}} = (e_2^{(\alpha)})^{\text{avg}}_{\text{perturbative}} + \frac{(4\Delta)^{\alpha/2}}{m} A(\alpha)$$

$$A(\alpha) = 0.5\text{GeV} \quad \text{for} \quad \alpha = 0.5$$

Summary and Outlook

- A first principles calculation of energy correlator observables for heavy quark jets
- An EFT to resum large logarithms in the jet observable and quark mass
- A prediction for the scaling behaviour of the leading non-perturbative corrections with the mass and energy of the jet
- An analytic understanding of jet substructure as a function of energy and mass of the heavy quark.
- Long term application for studying medium effects in QGP
- The EFT calculation can be extended by taking into account decay of the heavy quark , which can then be used to tag b jets.

Thank You