Joint resummation of two angularities at NNLL

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Motivation and overview

Event generators

- Fully differential
- No strict order counting
- No resummation uncertainties

Precise theory calculations

- Resummation of large logarithms
- Matched to fixed-oder predictions
- Uncertainty estimates
- Typically limited to one observable

- **Ideally: Precise and differential predictions**

- Precise prediction for the $e^+e^- \to 2$ jets cross section differential in two angularities e_α and e_β

↪ e_α and e_β are jointly resummed to NNLL

↪ Matched to NLO fixed order

↪ Numerical study:
Profile scales, uncertainties, comparison to Monte Carlo

Angularities

- Event shape angularities Berger, Kucs, Sterman (2003)

$$e_\alpha = \frac{1}{Q}\sum_i E_i (\sin\theta_i)^{2-\alpha}(1-|\cos\theta_i|)^{\alpha-1} \approx \frac{2^{\alpha-1}}{Q}\sum_i E_i \theta_i^\alpha$$

Sum over colored particles

Energy of particle i

Angle of particle i with respect to appropriately chosen axis

- Axis that is insensitive to recoil: winner-takes-all axis
- Special cases are $\alpha = 2$: Thrust, $\alpha = 1$: Broadening
- Parameter α determines the weight of the angle

NLO

$E_1 > E_2 > E_3$

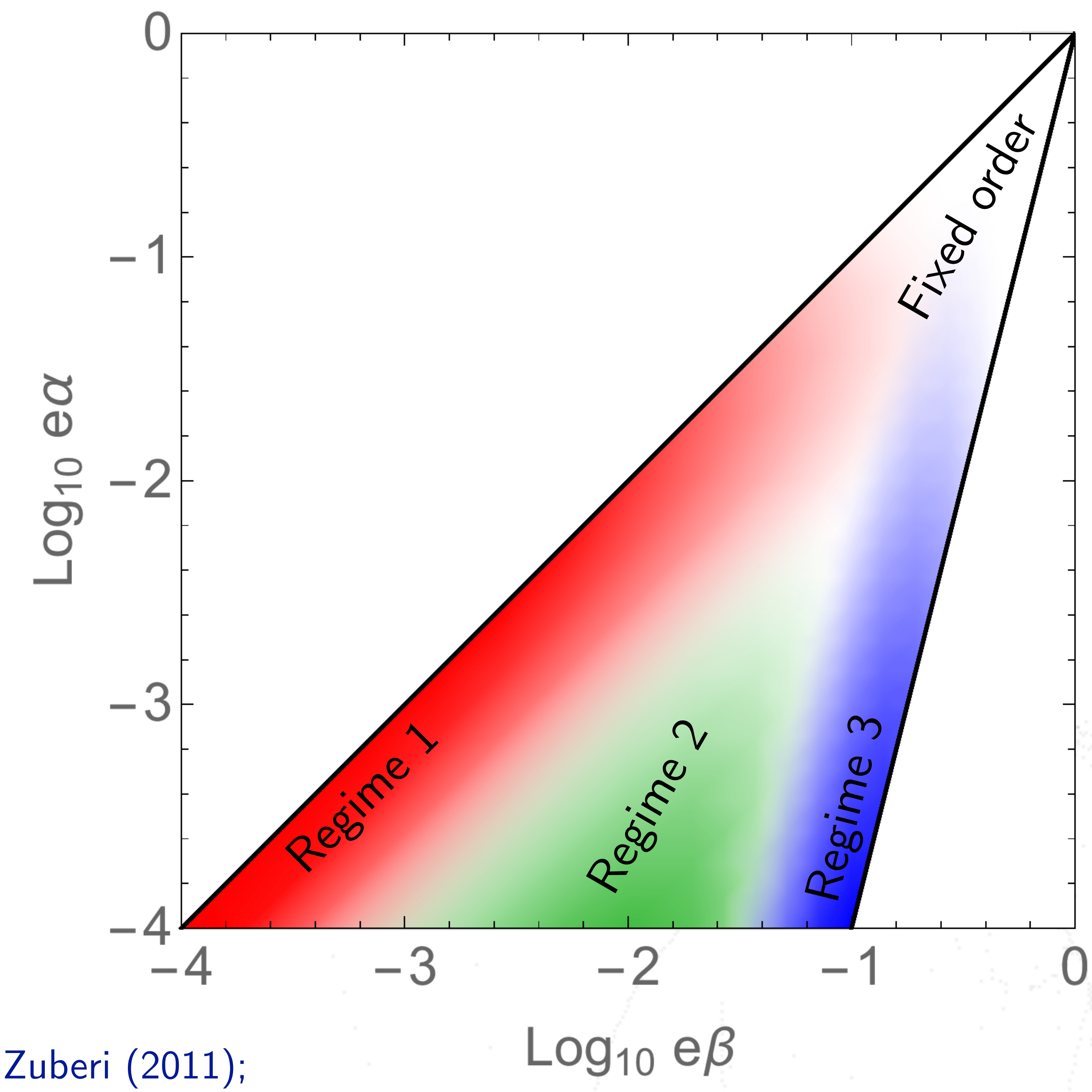
SCET+ framework

- Three different regions:

$$\text{Regime 1}: \quad e_\beta \sim e_\alpha$$

$$\text{Regime 2}: \quad e_\beta \gg e_\alpha \gg e_\beta^{\alpha/\beta}$$

$$\text{Regime 3}: \quad e_\alpha \sim e_\beta^{\alpha/\beta}$$



- Regime 1 and 3: **SCET I**

  Regime 1 governed by e_β measurement

  Regime 3 governed by e_α measurement

- Regime 2: **SCET +** Bauer, Tackmann, Walsh, Zuberi (2011); Procura, Waalewijn, LZ (2014); Larkoski, Moult, Neill (2015)

Mode	Scaling $(-,+,\perp)$	Measurement
n-collinear	$Q(1,\lambda^{2r/\beta},\lambda^{r/\beta})$	e_β
n-collinear-soft	$Q\left(\lambda^{\frac{\alpha r-\beta}{\alpha-\beta}},\lambda^{\frac{(\alpha-2)r-(\beta-2)}{\alpha-\beta}},\lambda^{\frac{(\alpha-1)r-(\beta-1)}{\alpha-\beta}}\right)$	e_β, e_α
soft	$Q(\lambda,\lambda,\lambda)$	e_α

$\beta/\alpha < r < 1$

and $\lambda \sim e_\alpha \sim e_\beta^{1/r}$

SCET+ framework

- Three different regions:

$$\text{Regime 1}: \quad e_\beta \sim e_\alpha$$

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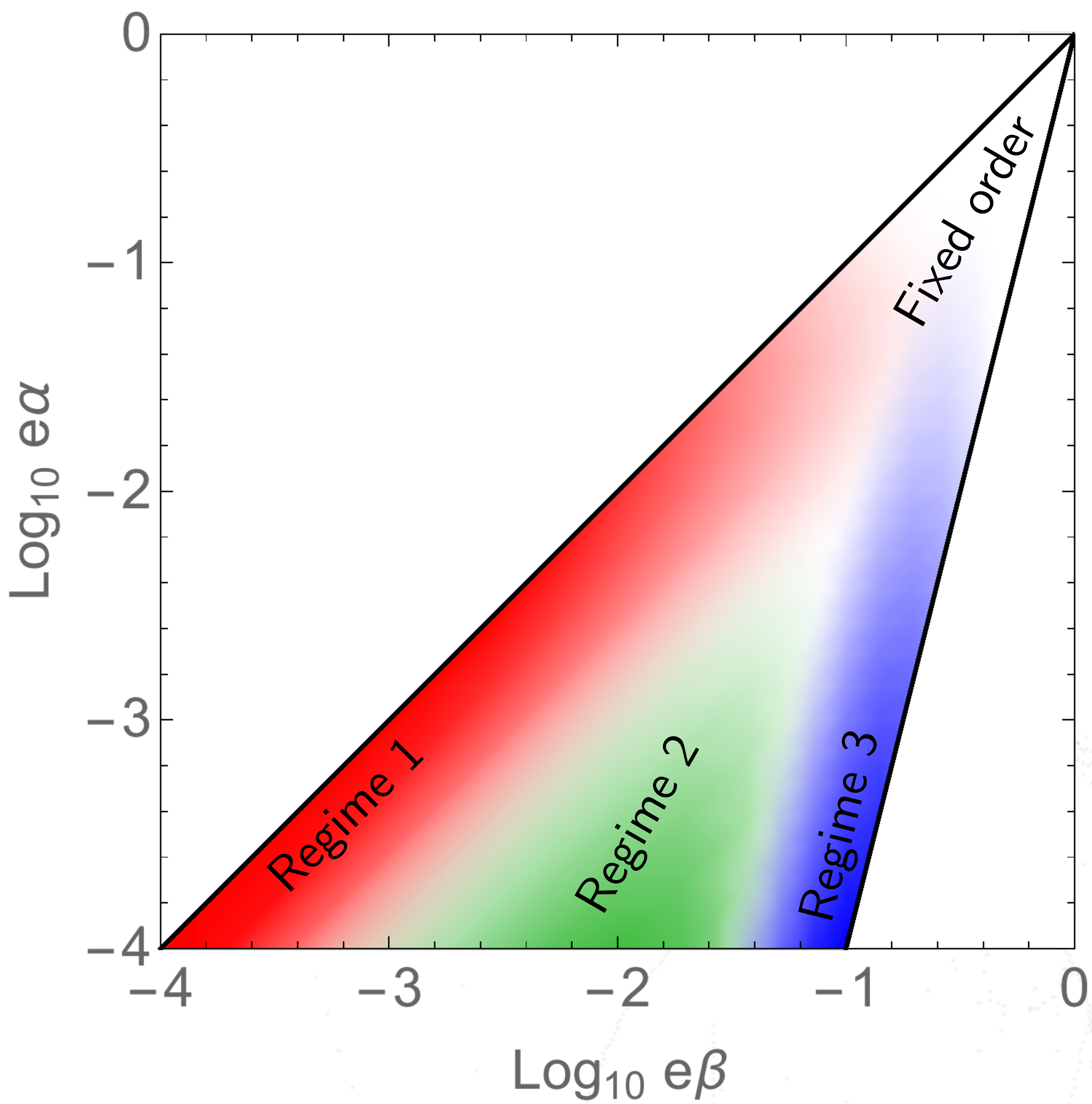


- All regimes describe cross-section up to power corrections

$$\frac{d^2\sigma}{de_\alpha\, de_\beta} = \frac{d^2\sigma_1}{de_\alpha\, de_\beta}\left[1 + \mathcal{O}\left(e_\alpha^{\min(2/\alpha,1)}\right)\right]$$

$$\frac{d^2\sigma}{de_\alpha\, de_\beta} = \frac{d^2\sigma_2}{de_\alpha\, de_\beta}\left\{1 + \mathcal{O}\left[\left(\frac{e_\beta}{e_\alpha^{\beta/\alpha}}\right)^{\frac{\alpha \min(2/\alpha,1)}{\alpha-\beta}}, \left(\frac{e_\alpha}{e_\beta}\right)^{\frac{\beta \min(2/\beta,1)}{\alpha-\beta}}\right]\right\}$$

$$\frac{d^2\sigma}{de_\alpha\, de_\beta} = \frac{d^2\sigma_3}{de_\alpha\, de_\beta}\left[1 + \mathcal{O}\left(e_\beta^{\min(2/\beta,1)}\right)\right]$$

Get large when approaching Regime 3

Get large when approaching Regime 1

- Regime 2 resums most logs, but involves also two expansions

Factorization theorems

Regime 1:

- Single differential jet functions
- Double differential soft function

$$\frac{d^2\sigma_1}{de_\alpha\, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\beta\, J(e^n_\beta) \int de^{\bar{n}}_\beta\, J(e^{\bar{n}}_\beta) \times \int de^s_\alpha\, de^s_\beta\, S(e^s_\alpha, e^s_\beta)\, \delta(e_\alpha - e^s_\alpha)\, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta - e^s_\beta)$$

$$\frac{d^2\sigma_2}{de_\alpha\, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\beta\, J(e^n_\beta) \int de^{ns}_\alpha\, de^{ns}_\beta\, \mathscr{S}(e^{ns}_\alpha, e^{ns}_\beta) \times \int de^{\bar{n}}_\beta\, J(e^{\bar{n}}_\beta) \int de^{\bar{n}s}_\alpha\, de^{\bar{n}s}_\beta\, \mathscr{S}(e^{\bar{n}s}_\alpha, e^{\bar{n}s}_\beta) \times \int de^s_\alpha\, S(e^s_\alpha)\, \delta(e_\alpha - e^{ns}_\alpha - e^{\bar{n}s}_\alpha - e^s_\alpha)\, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta - e^{ns}_\beta - e^{\bar{n}s}_\beta)$$

$$\frac{d^2\sigma_3}{de_\alpha\, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\alpha\, de^n_\beta\, J(e^n_\alpha, e^n_\beta) \int de^{\bar{n}}_\alpha\, de^{\bar{n}}_\beta\, J(e^{\bar{n}}_\alpha, e^{\bar{n}}_\beta) \times \int de^s_\alpha\, S(e^s_\alpha)\, \delta(e_\alpha - e^n_\alpha - e^{\bar{n}}_\alpha - e^s_\alpha)\, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta)$$

Factorization theorems

Regime 3:

- Double differential jet functions
- Single differential soft function

$$\frac{d^2\sigma_1}{de_\alpha \, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\beta \, J(e^n_\beta) \int de^{\bar{n}}_\beta \, J(e^{\bar{n}}_\beta) \times \int de^s_\alpha \, de^s_\beta \, S(e^s_\alpha, e^s_\beta) \, \delta(e_\alpha - e^s_\alpha) \, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta - e^s_\beta)$$

$$\frac{d^2\sigma_2}{de_\alpha \, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\beta \, J(e^n_\beta) \int de^{ns}_\alpha \, de^{ns}_\beta \, \mathscr{S}(e^{ns}_\alpha, e^{ns}_\beta) \times \int de^{\bar{n}}_\beta \, J(e^{\bar{n}}_\beta) \int de^{\bar{n}s}_\alpha \, de^{\bar{n}s}_\beta \, \mathscr{S}(e^{\bar{n}s}_\alpha, e^{\bar{n}s}_\beta) \times \int de^s_\alpha \, S(e^s_\alpha) \, \delta(e_\alpha - e^{ns}_\alpha - e^{\bar{n}s}_\alpha - e^s_\alpha) \, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta - e^{ns}_\beta - e^{\bar{n}s}_\beta)$$

$$\frac{d^2\sigma_3}{de_\alpha \, de_\beta} = \hat{\sigma}_0 H(Q^2) \int de^n_\alpha \, de^n_\beta \, J(e^n_\alpha, e^n_\beta) \int de^{\bar{n}}_\alpha \, de^{\bar{n}}_\beta \, J(e^{\bar{n}}_\alpha, e^{\bar{n}}_\beta) \times \int de^s_\alpha \, S(e^s_\alpha) \, \delta(e_\alpha - e^n_\alpha - e^{\bar{n}}_\alpha - e^s_\alpha) \, \delta(e_\beta - e^n_\beta - e^{\bar{n}}_\beta)$$

Factorization theorems

Regime 2:

- Single differential jet functions
- Single differential soft function
- Double differential collinear-soft function

$$\frac{\mathrm{d}^2\sigma_1}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} = \hat{\sigma}_0 H(Q^2) \int \mathrm{d}e_\beta^n\, J(e_\beta^n) \int \mathrm{d}e_\beta^{\bar{n}}\, J(e_\beta^{\bar{n}}) \times \int \mathrm{d}e_\alpha^s\, \mathrm{d}e_\beta^s\, S(e_\alpha^s, e_\beta^s)\, \delta(e_\alpha - e_\alpha^s)\, \delta(e_\beta - e_\beta^n - e_\beta^{\bar{n}} - e_\beta^s)$$

$$\frac{\mathrm{d}^2\sigma_2}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} = \hat{\sigma}_0 H(Q^2) \int \mathrm{d}e_\beta^n\, J(e_\beta^n) \int \mathrm{d}e_\alpha^{ns}\, \mathrm{d}e_\beta^{ns}\, \mathscr{S}(e_\alpha^{ns}, e_\beta^{ns}) \times \int \mathrm{d}e_\beta^{\bar{n}}\, J(e_\beta^{\bar{n}}) \int \mathrm{d}e_\alpha^{\bar{n}s}\, \mathrm{d}e_\beta^{\bar{n}s}\, \mathscr{S}(e_\alpha^{\bar{n}s}, e_\beta^{\bar{n}s}) \times \int \mathrm{d}e_\alpha^s\, S(e_\alpha^s)\, \delta(e_\alpha - e_\alpha^{ns} - e_\alpha^{\bar{n}s} - e_\alpha^s)\, \delta(e_\beta - e_\beta^n - e_\beta^{\bar{n}} - e_\beta^{ns} - e_\beta^{\bar{n}s})$$

$$\frac{\mathrm{d}^2\sigma_3}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} = \hat{\sigma}_0 H(Q^2) \int \mathrm{d}e_\alpha^n\, \mathrm{d}e_\beta^n\, J(e_\alpha^n, e_\beta^n) \int \mathrm{d}e_\alpha^{\bar{n}}\, \mathrm{d}e_\beta^{\bar{n}}\, J(e_\alpha^{\bar{n}}, e_\beta^{\bar{n}}) \times \int \mathrm{d}e_\alpha^s\, S(e_\alpha^s)\, \delta(e_\alpha - e_\alpha^n - e_\alpha^{\bar{n}} - e_\alpha^s)\, \delta(e_\beta - e_\beta^n - e_\beta^{\bar{n}})$$

NNLL ingredients

	Fixed-order	Non-cusp	Cusp and Beta
LL	tree	-	1-loop
NLL	tree	1-loop	2-loop
NNLL	1-loop	2-loop	3-loop

- Fixed order ingredients @ 1-loop:

  $J(e_\beta)$ Hornig, Lee, Ovanesyan (2009)

  $S(e_\alpha)$ Hornig, Lee, Ovanesyan (2009)

  $J(e_\alpha, e_\beta)$ Larkoski, Moult, Neill (2014)

  $S(e_\alpha, e_\beta)$ Larkoski, Moult, Neill (2014)

  } Recalculated

  $\mathscr{S}(e_\alpha, e_\beta)$ Kasemets, Waalewijn, LZ (2016)

- Non-cusp anomalous dimensions @ 2-loop:

  $\gamma_{H,1}$ 3-loop: Moch, Vermaseren, Vogt (2005)

  $\gamma_{S,1}$ Bell, Rahn, Talbert (2016)

  remaining ones by consistency

$$\gamma_H(\alpha_s) + 2\gamma_J(\alpha_s, \alpha) + \gamma_S(\alpha_s, \alpha) = 0$$

$$\gamma_H(\alpha_s) + 2\gamma_J(\alpha_s, \beta) + 2\gamma_{\mathscr{S}}(\alpha_s, \alpha, \beta) + \gamma_S(\alpha_s, \alpha) = 0$$

Consistency relations

- Integrating the double-differential jet and soft function yields single differential ones

$$\int \mathrm{d}e_\beta \, J(e_\beta, e_\alpha) = J(e_\alpha) \,, \qquad \int \mathrm{d}e_\alpha \, S(e_\beta, e_\alpha) = S(e_\beta)$$

This does not hold when integrating over the other angularity

- Consistency between the factorization theorems

$$J(e_\alpha, e_\beta) = \int \mathrm{d}e_\beta^n \, J(e_\beta^n) \mathscr{J}(e_\alpha, e_\beta - e_\beta^n) + \text{power corrections}$$

and a similar relation between $S(e_\alpha, e_\beta)$, $S(e_\alpha)$ and $\mathscr{S}(e_\alpha^{ns}, e_\beta^{ns})$

- All relations checked at 1-loop

Matching of cross sections

- Combine cross sections to get a expression which is valid everywhere in phase space → See also talk tomorrow from Gillian Lusterman

$$\sigma = \sigma_2(\mu_J, \mu_S, \mu_{\mathscr{S}}) + \left[\sigma_1(\mu_J, \mu_S^{\mathrm{R1}}) - \sigma_2(\mu_J, \mu_S^{\mathrm{R1}}, \mu_S^{\mathrm{R1}})\right] + \left[\sigma_3(\mu_J^{\mathrm{R3}}, \mu_S) - \sigma_2(\mu_J^{\mathrm{R3}}, \mu_S, \mu_J^{\mathrm{R3}})\right] + \left[\sigma_{\mathrm{FO}}(\mu_{\mathrm{FO}}) - \sigma_1(\mu_{\mathrm{FO}}, \mu_{\mathrm{FO}}) - \sigma_3(\mu_{\mathrm{FO}}, \mu_{\mathrm{FO}}) + \sigma_2(\mu_{\mathrm{FO}}, \mu_{\mathrm{FO}}, \mu_{\mathrm{FO}})\right]$$

- E.g. in fixed-order region $\mu_J = \mu_S = \mu_{\mathscr{S}} = \mu_J^{\mathrm{R3}} = \mu_S^{\mathrm{R1}} = \mu_{\mathrm{FO}}$ so $\sigma = \sigma_{\mathrm{FO}}(\mu_{\mathrm{FO}})$
- For a smooth transition: Profile scales

1D results

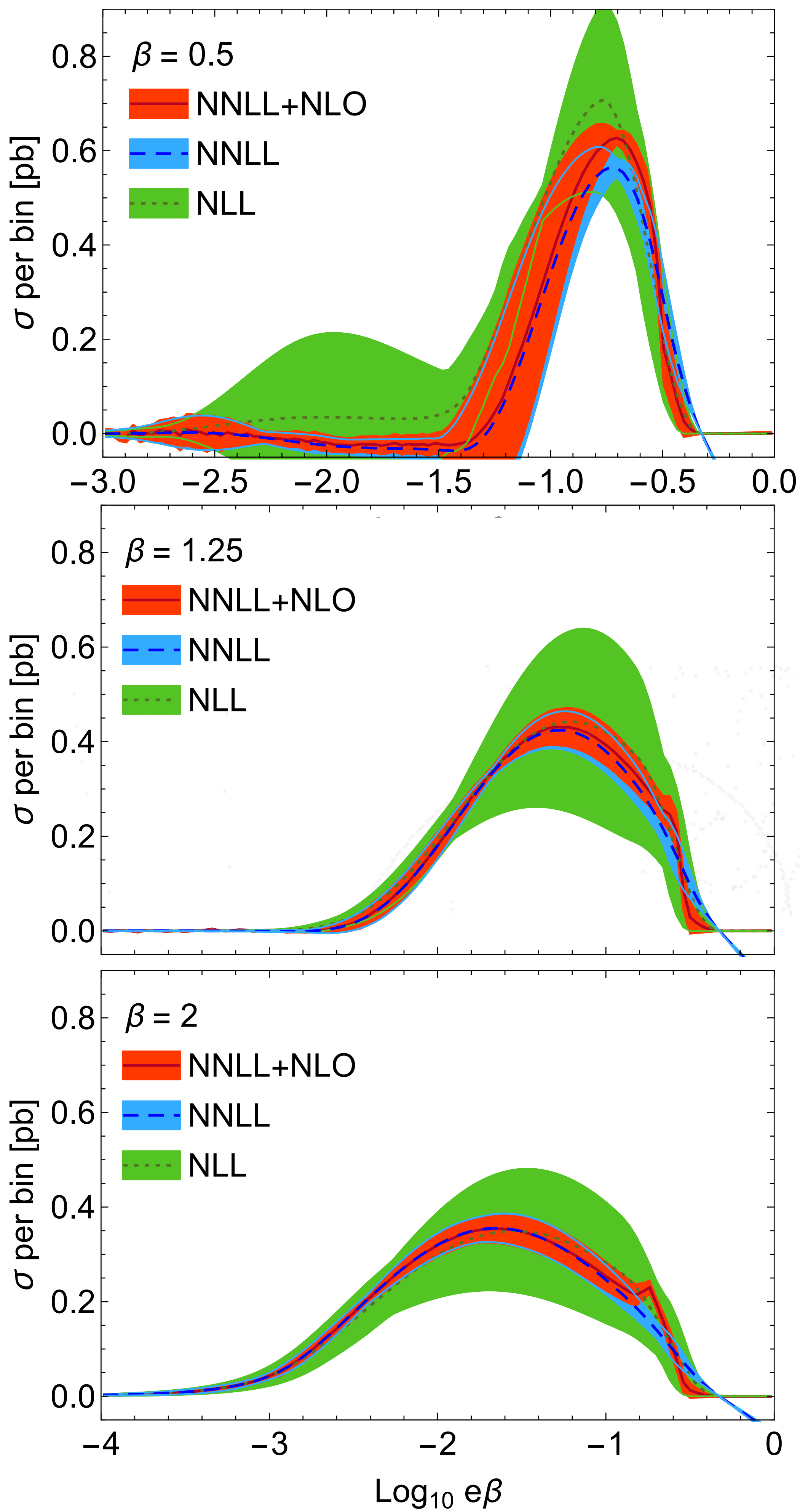
- Profiles used for thrust generalised to angularities Gangal, Stahlhofen, Tackmann (2014); Mo, Tackmann, Waalewijn (2017);

$$\mu_J = Q[f_{\text{run}}(e_\beta, t_1, t_3)]^{1/\beta}$$
$$\mu_S = Q f_{\text{run}}(e_\beta, t_1, t_3)$$

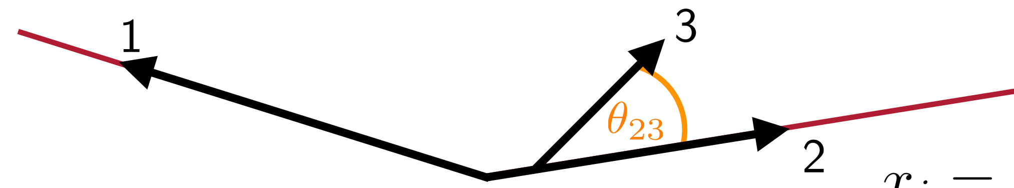
interpolates between the canonical region ($e_\beta < t_1$) and the fixed-order region ($e_\beta > t_3$)

- Profile scale variations
  - Fixed-order scale variations
  - Variations of the transition parameters
  - Resummation variations

$$\mu_J^{\text{vary}} = Q\left[f_{\text{run}} f_{\text{vary}}(e_\beta, t_3)^a\right]^{1/\beta - b}$$
$$\mu_S^{\text{vary}} = Q\, f_{\text{run}} f_{\text{vary}}(e_\beta, t_3)^a$$



2D results: NLO



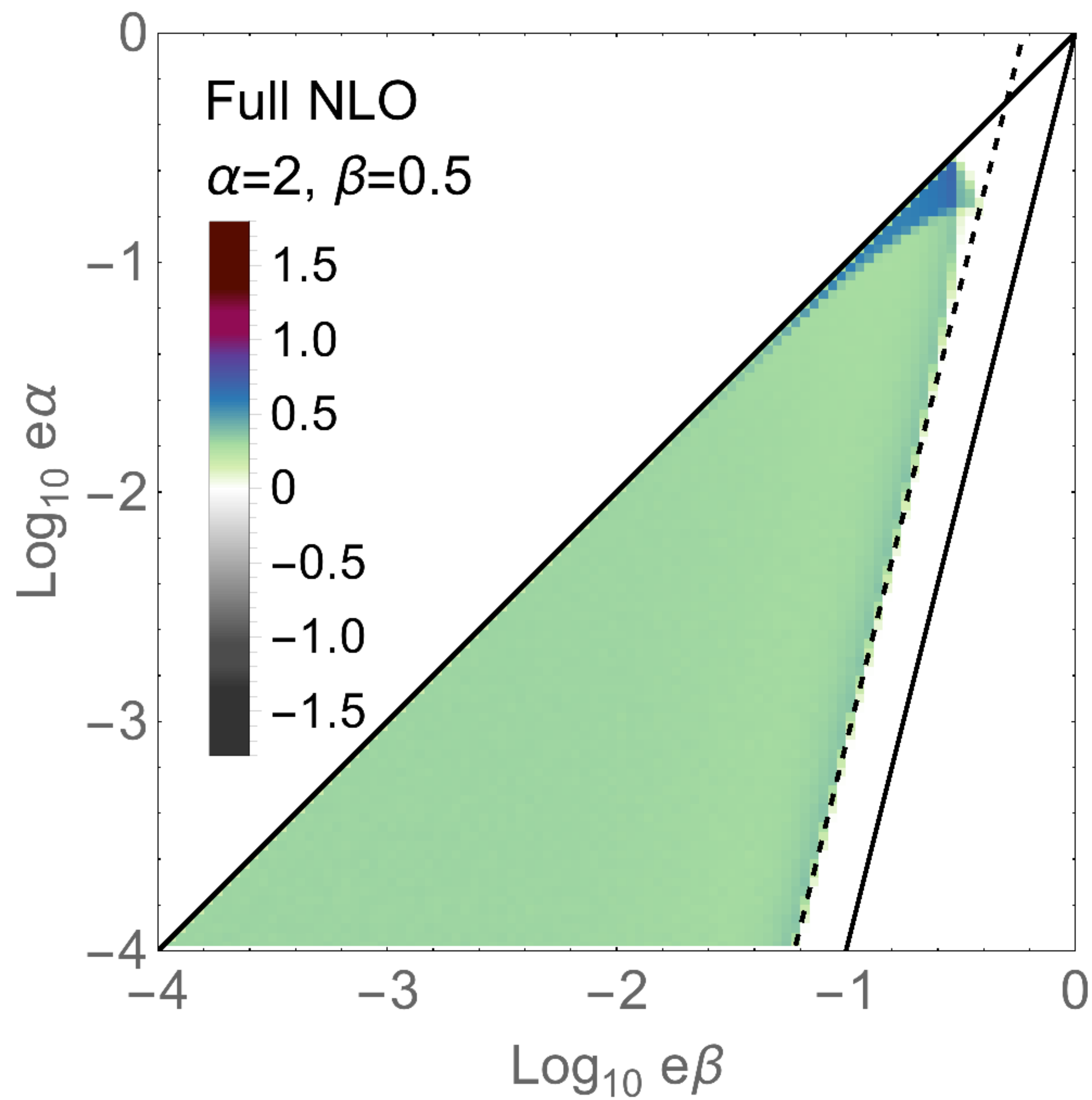
- NLO, assume

$$e_\alpha = \frac{1}{2} x_3 \, (1 - \cos^2\theta_{23})^{1-\alpha/2} (1 - |\cos\theta_{23}|)^{\alpha-1}$$

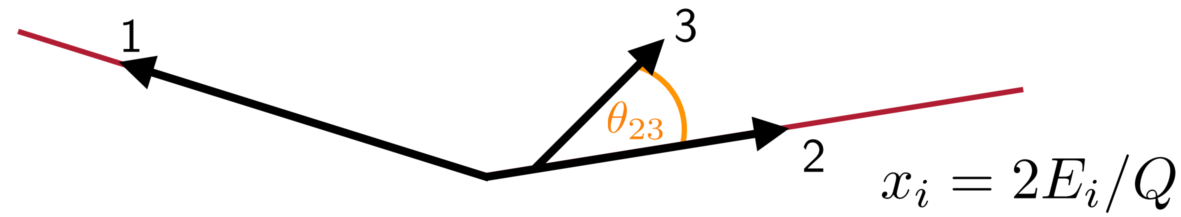
  and use

$$\frac{d^2\sigma}{dx_q \, dx_{\bar{q}}} = \sigma_0 \, \frac{\alpha_s C_F}{2\pi} \, \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

- Checked against `EVENT2`



2D results: NLO



- NLO, assume

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and use

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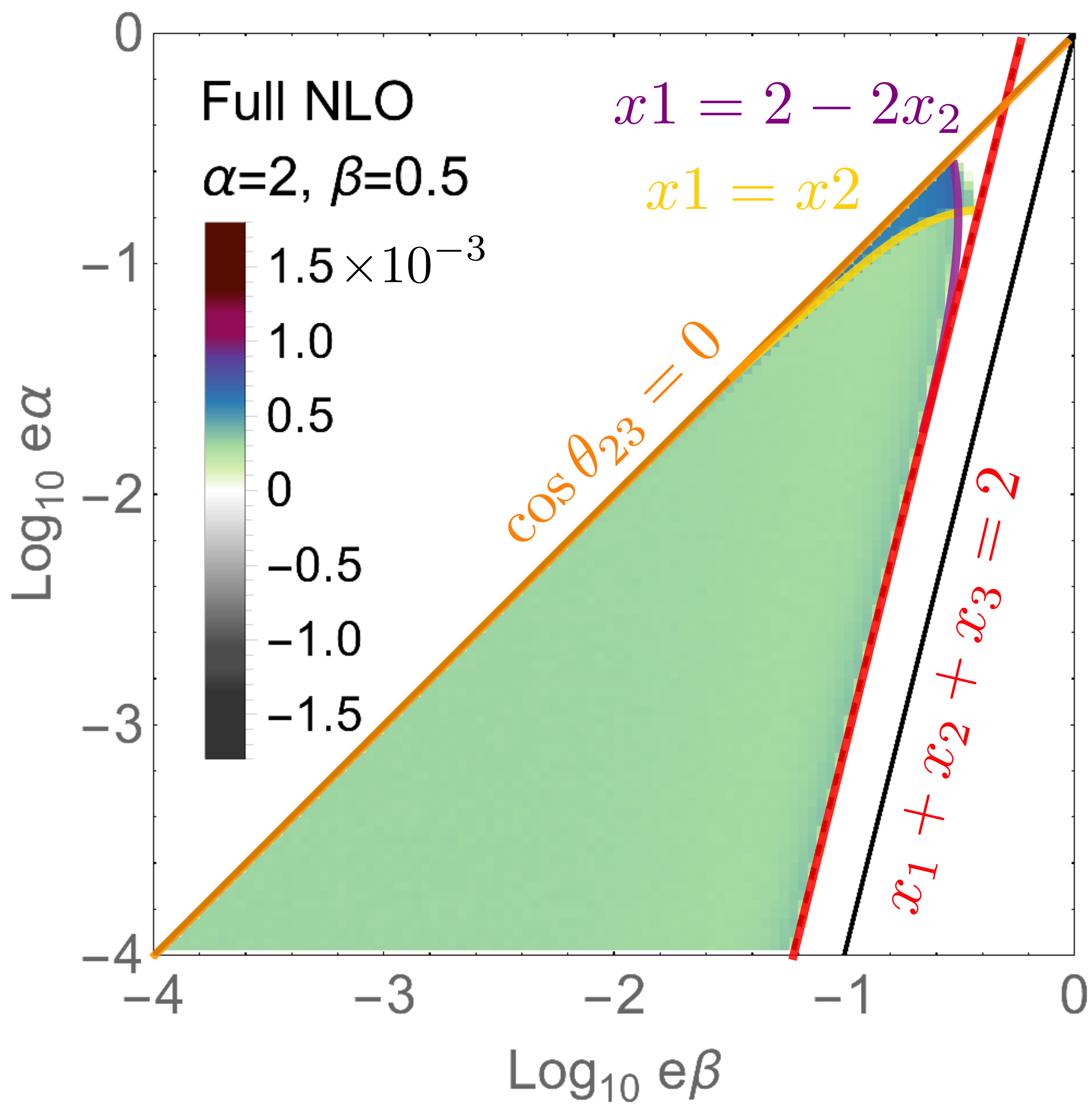
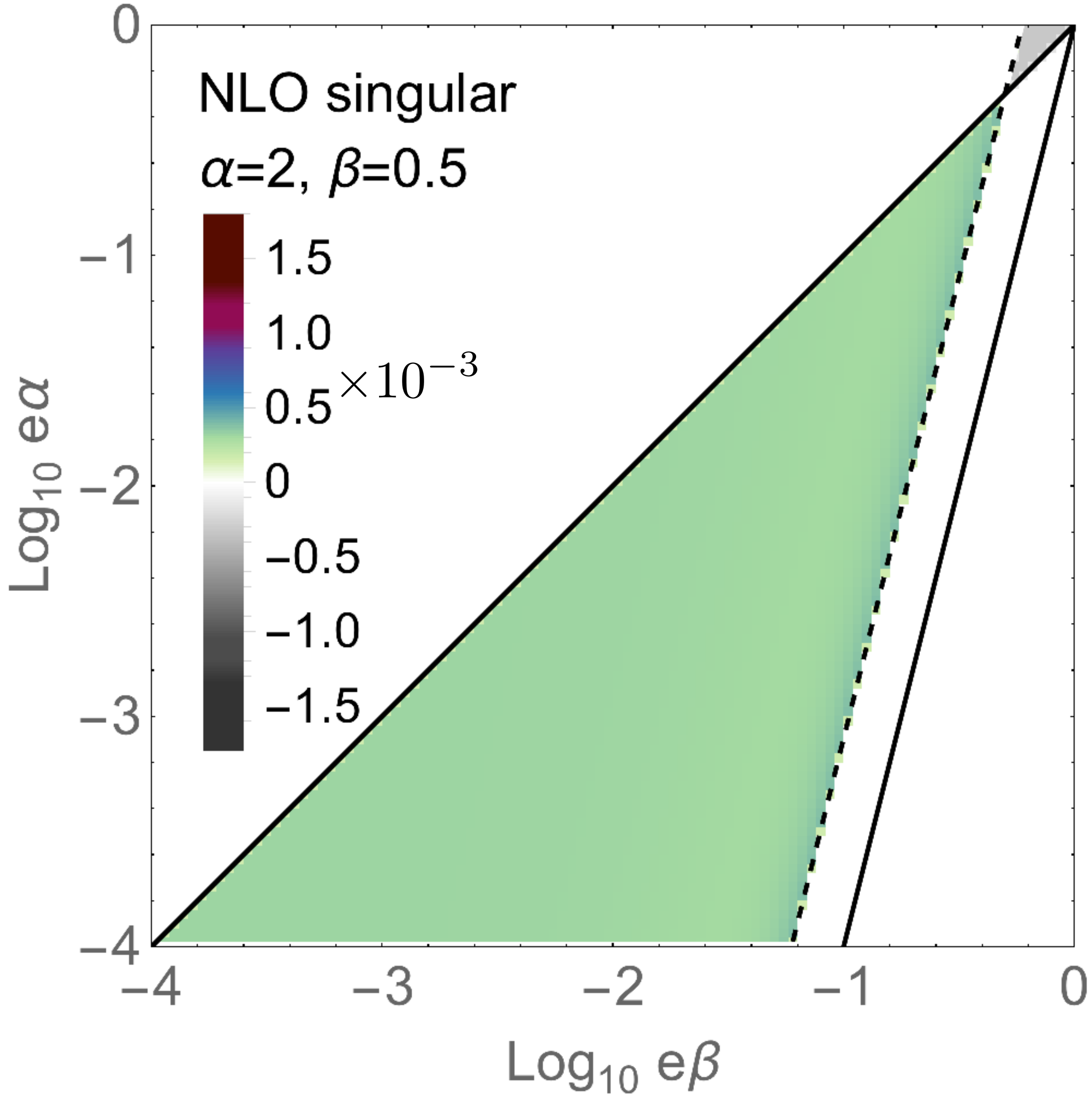
- Checked against `EVENT2`

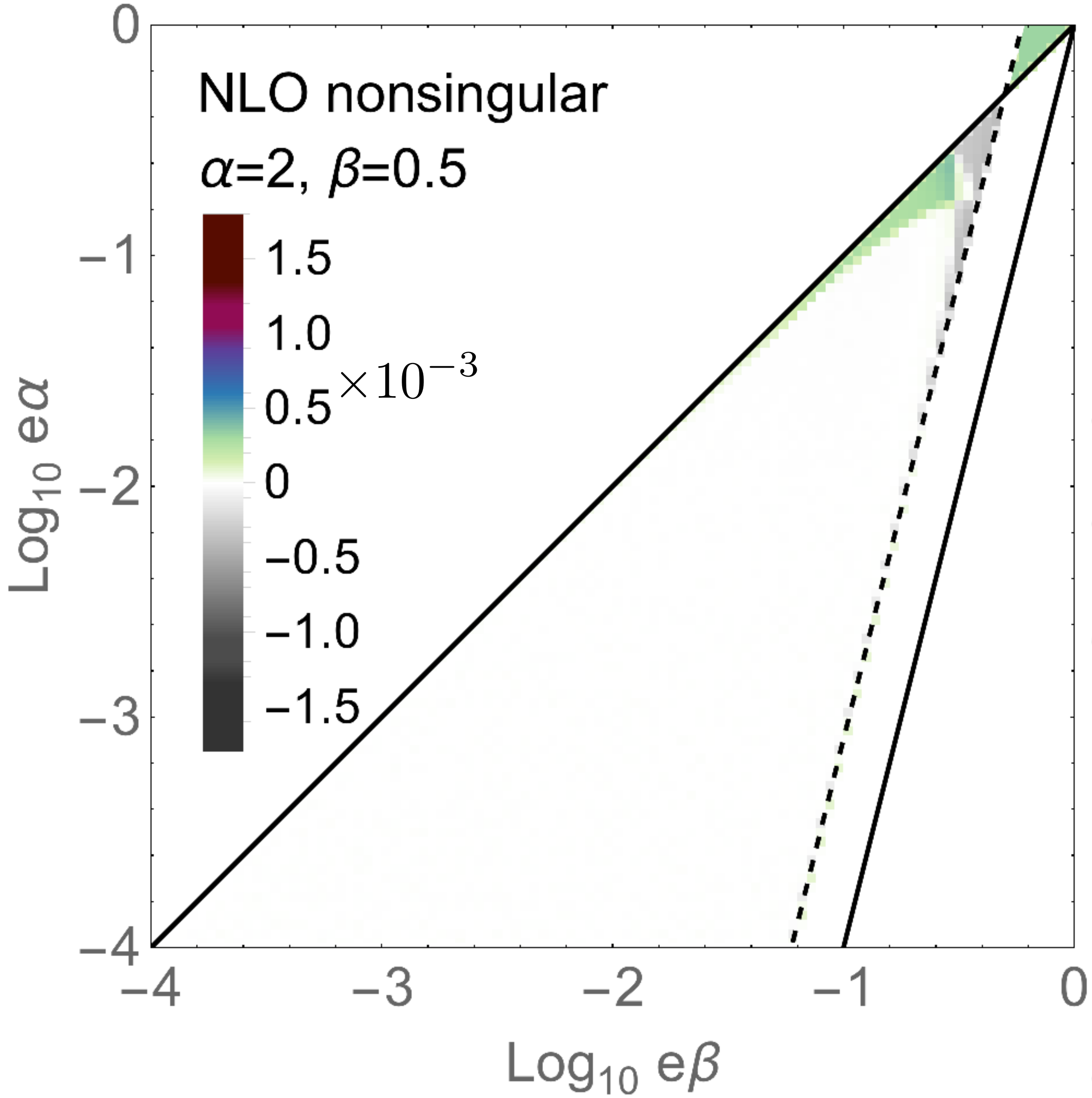
- Phase-space boundaries at NLO

$$e_\beta \geq e_\alpha \geq 2^{\frac{\alpha-\beta}{\beta}} e_\beta^{\frac{\alpha}{\beta}}$$



2D results: nonsingular contribution





The plots are normalized to the full NLO cross section

2D profile scales

- Canonical scales in each region

	$\log(\mu_J^{\text{can}}/Q)$	$\log(\mu_{\mathscr{S}}^{\text{can}}/Q)$	$\log(\mu_S^{\text{can}}/Q)$
1	$1/\beta \log e_\beta$		$\log e_\beta$
2	$1/\beta \log e_\beta$	$(1-\beta)/(\alpha-\beta) \log e_\alpha + (\alpha-1)/(\alpha-\beta) \log e_\beta$	$\log e_\alpha$
3	$1/\alpha \log e_\alpha$		$\log e_\alpha$

- Construct scales in terms of logarithms of angularities
- Step 1: Polynomial that interpolates between the canonical scales between regions (one free parameter from transition point)
- Step 2: Transition to the nonperturbative regime: Freeze α_s below 2 GeV
- Step 3: Transition to fixed order
  - Choose a square shape for FO region: $t = \min(\log e_a, \log e_b)$
  - Polynomial in t that interpolates between canonical scale across regions and μ_{FO} (parameters t_1 and t_3 as in 1D case)

Differential vs cumulative scale setting

- Differential scale setting: $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta}$ with $\mu_i(e_\alpha, e_\beta)$
- Cumulative scale setting: $\Sigma(e_\alpha^c, e_\beta^c)$ with $\mu_i(e_\alpha^c, e_\beta^c)$

$$\Sigma(e_\alpha^c, e_\beta^c) = \int_0^{e_\alpha^c} \mathrm{d}e_\alpha \int_0^{e_\beta^c} \mathrm{d}e_\beta \, \frac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta}$$

- Differentiating the latter gives

$$\frac{\mathrm{d}^2\Sigma}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} = \frac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} + \sum_i \frac{\mathrm{d}^2\Sigma}{\mathrm{d}\ln\mu_i\,\mathrm{d}e_\beta}\frac{\mathrm{d}\ln\mu_i}{\mathrm{d}e_\alpha} + \sum_j \frac{\mathrm{d}^2\Sigma}{\mathrm{d}\ln e_\alpha\,\mathrm{d}\ln\mu_j}\frac{\mathrm{d}\ln\mu_j}{\mathrm{d}e_\beta} + \sum_{i,j} \frac{\mathrm{d}^2\Sigma}{\mathrm{d}\ln\mu_i\,\mathrm{d}\ln\mu_j}\frac{\mathrm{d}\ln\mu_i}{\mathrm{d}e_\alpha}\frac{\mathrm{d}\ln\mu_j}{\mathrm{d}e_\beta}$$

- At NNLL differential scale setting does not capture all logarithms

See e.g. Almeida, Ellis, Lee, Sterman, Sung (2014)

- Our scales undergo rapid changes in transition regions, leading to artifacts when using cumulant scale setting → We use differential scale setting

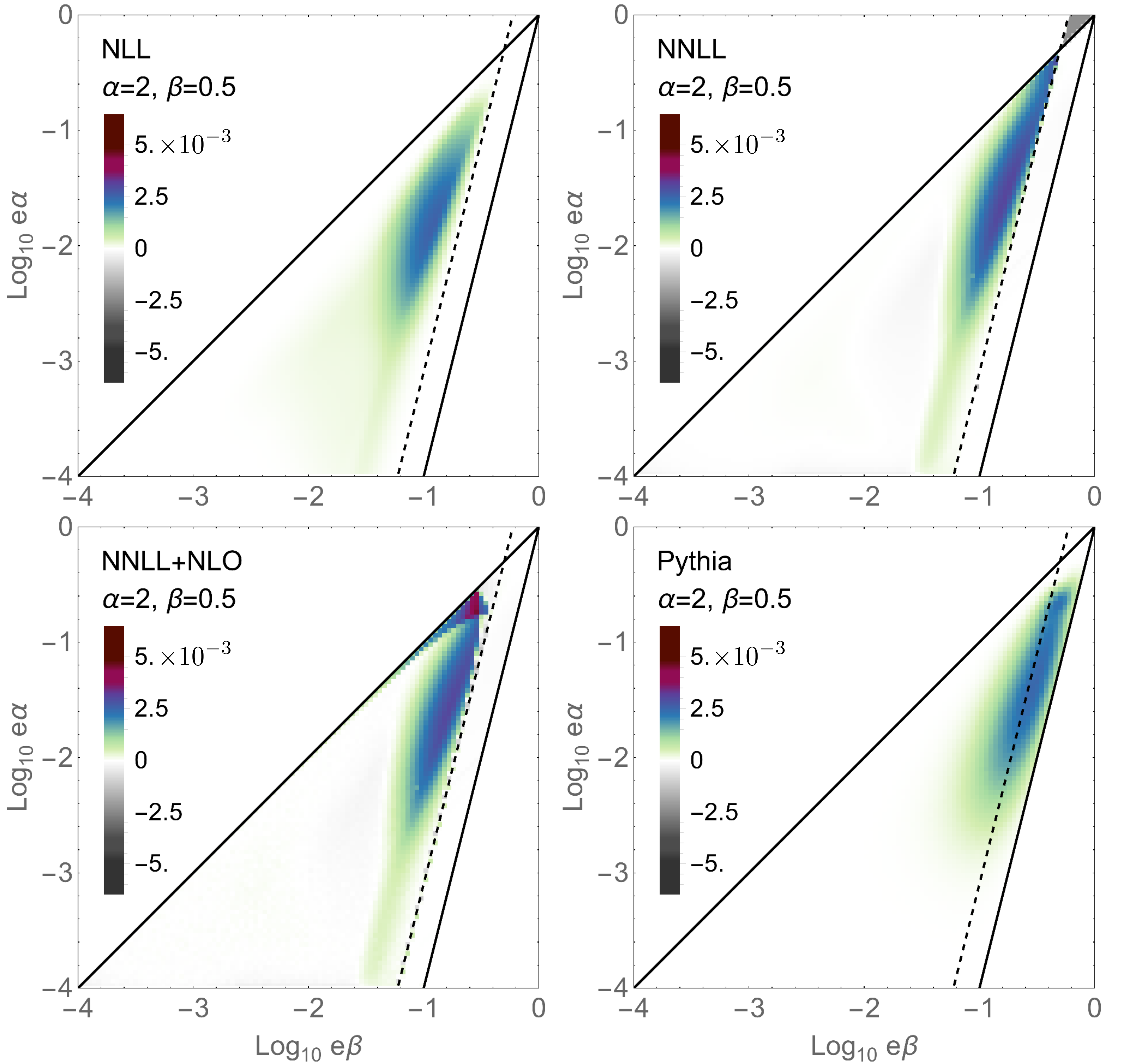
- Work in progress: Include additional terms

2D results

- NLL switches of at NLO boundary due to profiles
- Comparison to Pythia:

  Peak region outside the NLO phase-space

Normalized such that the cross section in each plot integrates to 1

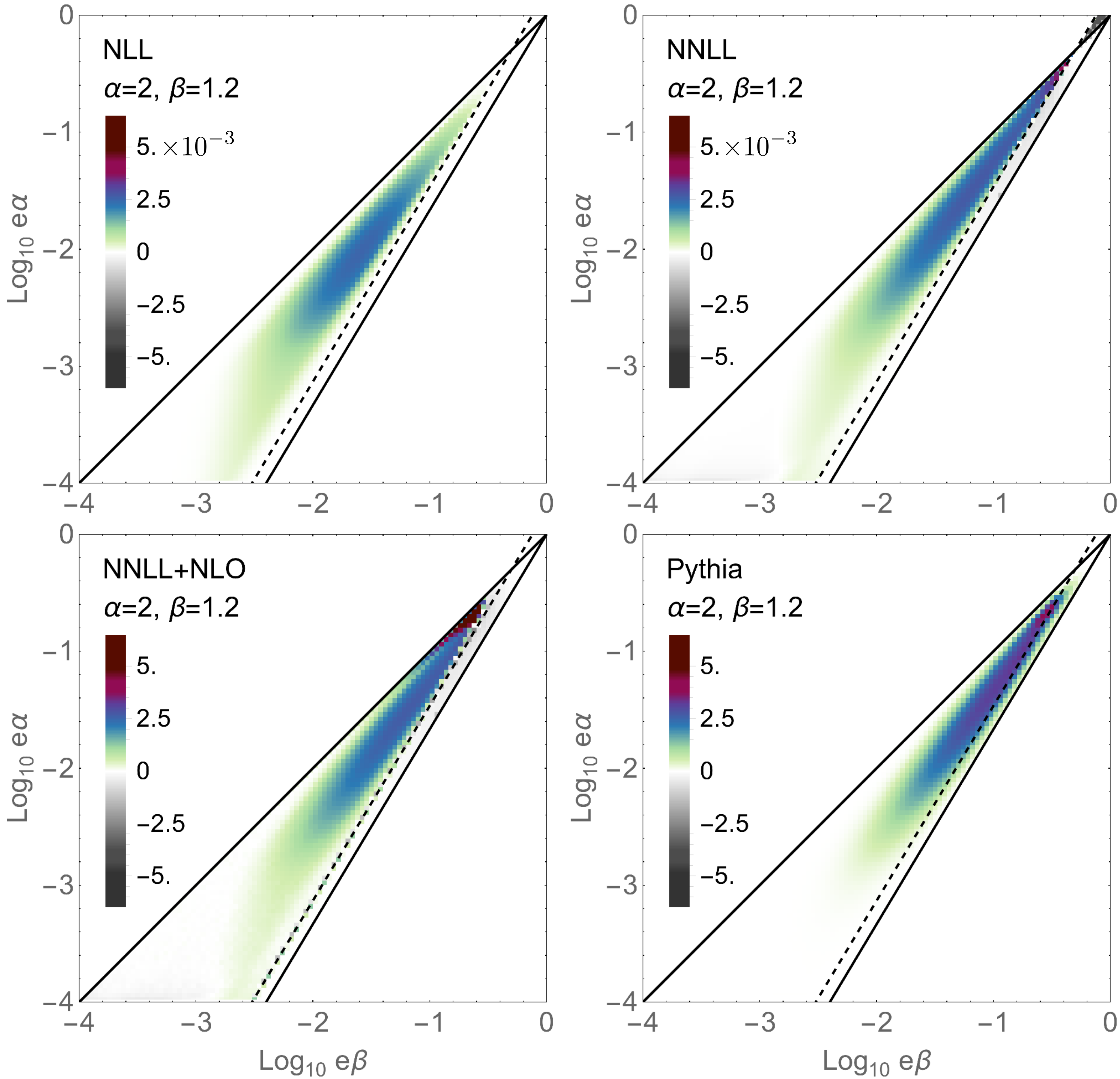


2D results

- Comparison to Pythia:

  Peak region inside the NLO phase-space

  Pythia more similar to NNLL than NLL



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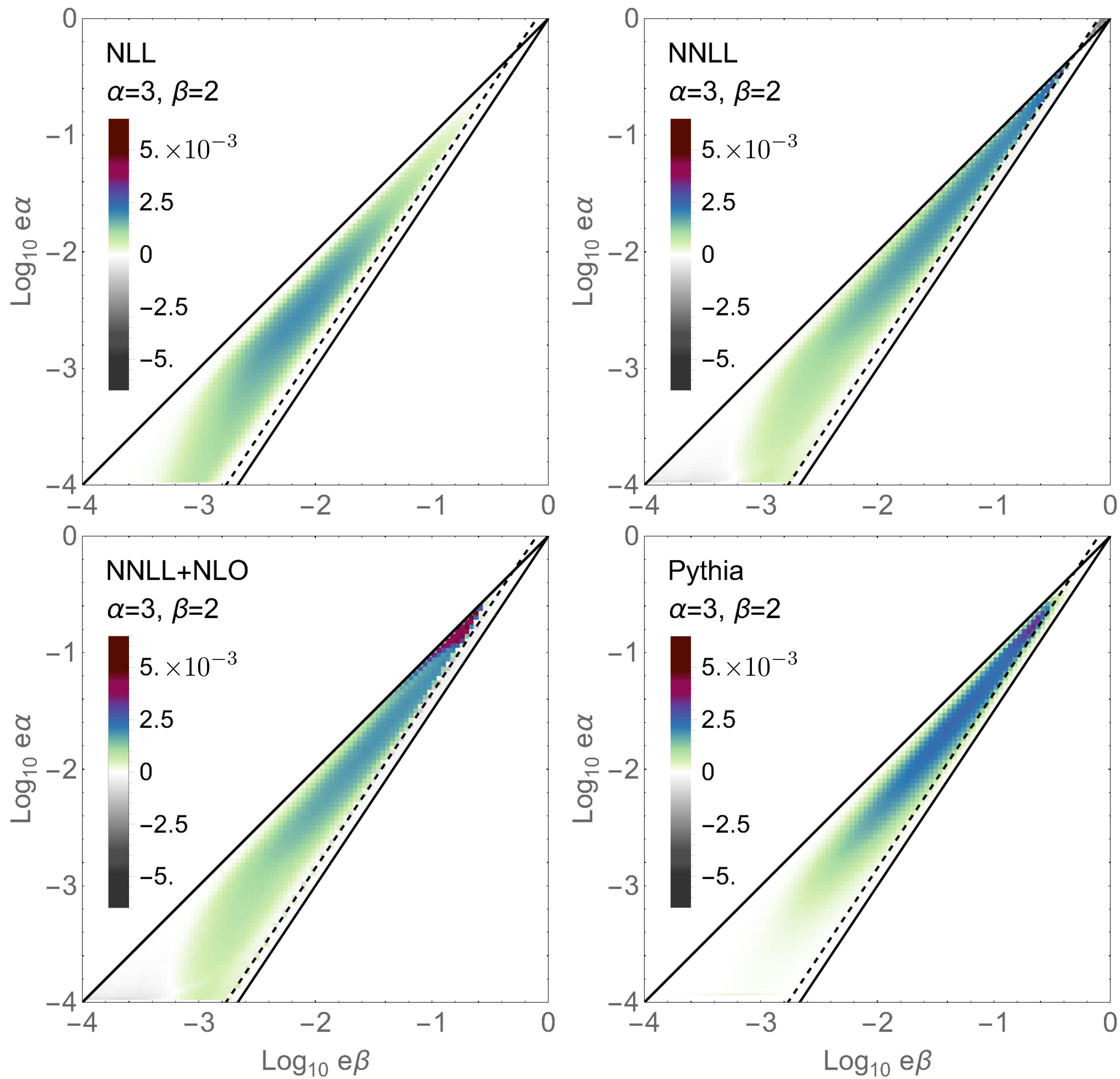
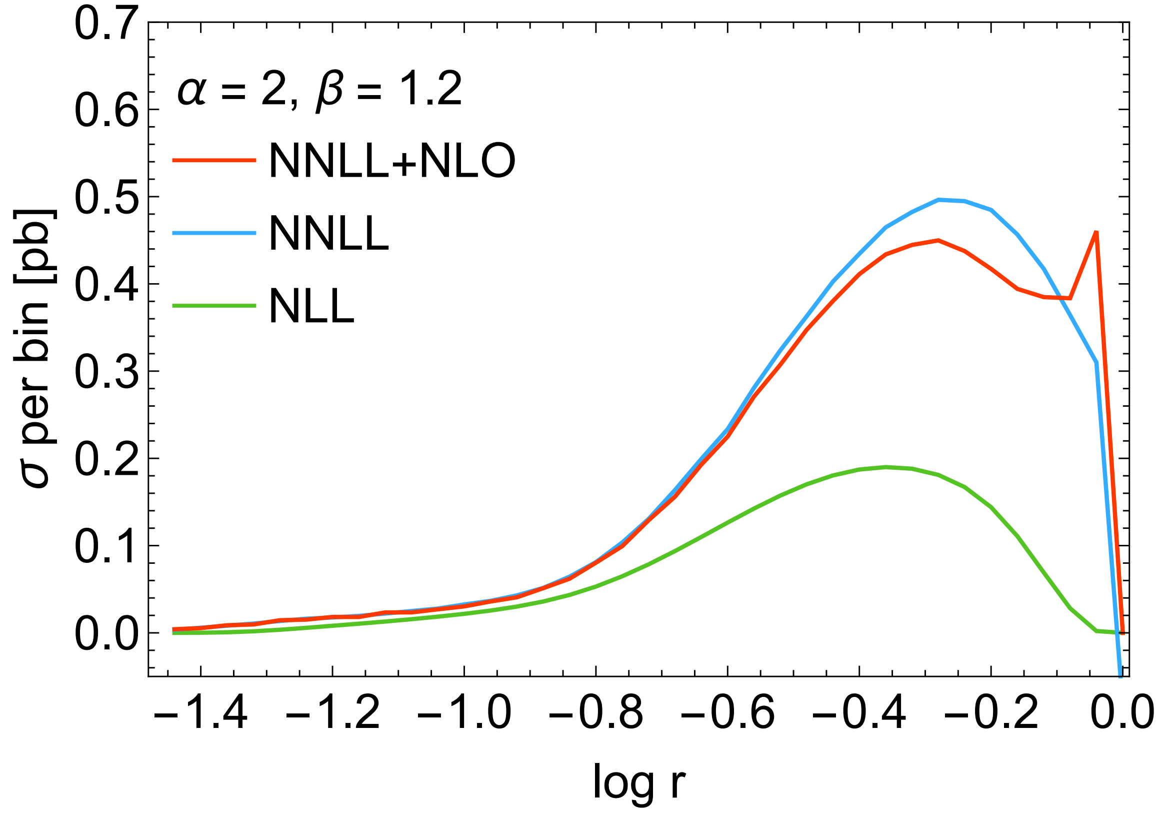


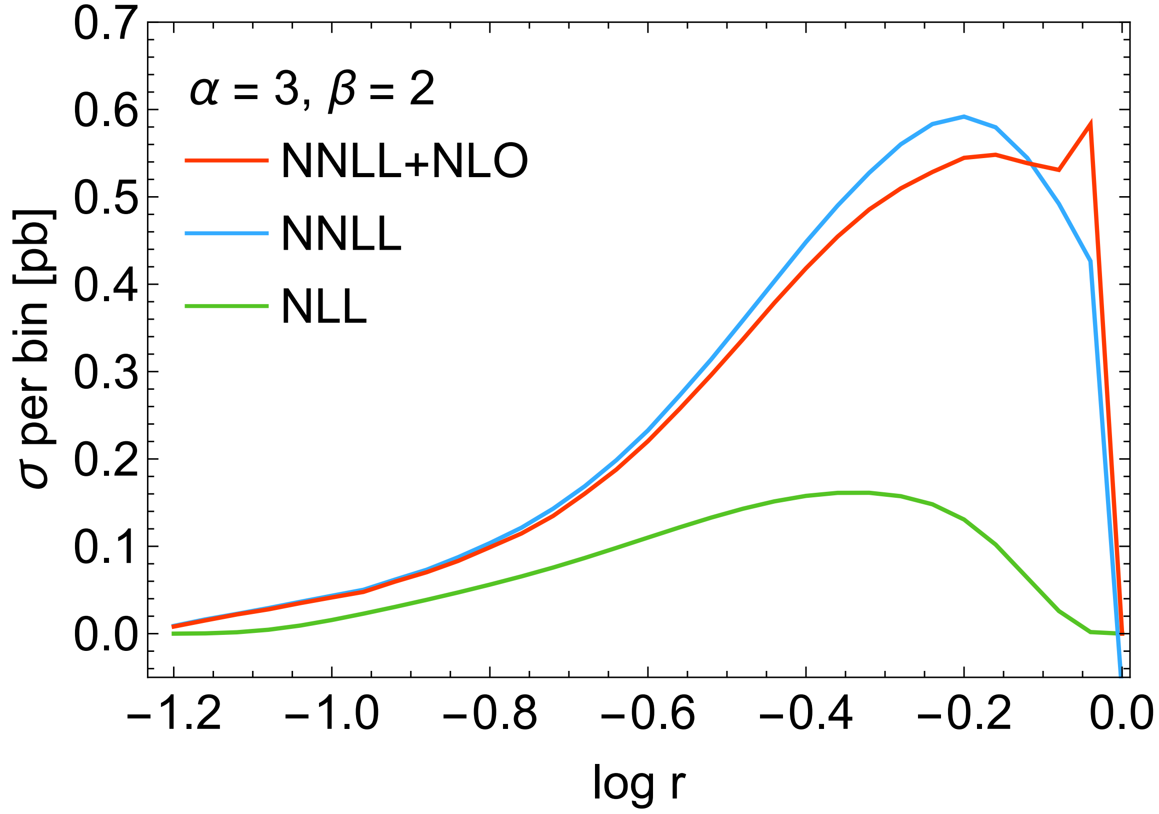
Normalized such that the cross section in each plot integrates to 1

Ratio plots

- Ratio observable $r = e_\alpha / e_\beta$ is not IRC safe
- Differential cross section can be calculated by marginalizing the resummed double differential cross section ("Sudakov safety") Larkoski, Thaler (2013)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \int \mathrm{d}e_\alpha \mathrm{d}e_\beta \frac{\mathrm{d}\sigma}{\mathrm{d}e_\alpha \mathrm{d}e_\beta} \delta\Big(r - \frac{e_\alpha}{e_\beta}\Big)$$





Conclusions

- We calculated the $e^+e^- \to 2$ jets cross section differential in two angularities e_α and e_β in SCET+
- We matched the cross section predictions from the different phase-space regions and constructed 2D profile scales for a smooth transition
- Work in progress:
  - ⤳ Plan: NNLL+NLO
    At the moment only NLL'+NLO due to differential scale setting
  - ⤳ 2D scale variations to estimate uncertainties
  - ⤳ Validation at $\mathcal{O}(\alpha_s^2)$ by comparison to EVENT2

Thank you!

Back up

Technical detail

- Double-differential soft and jet functions implemented as cumulant to avoid complicated plus distributions

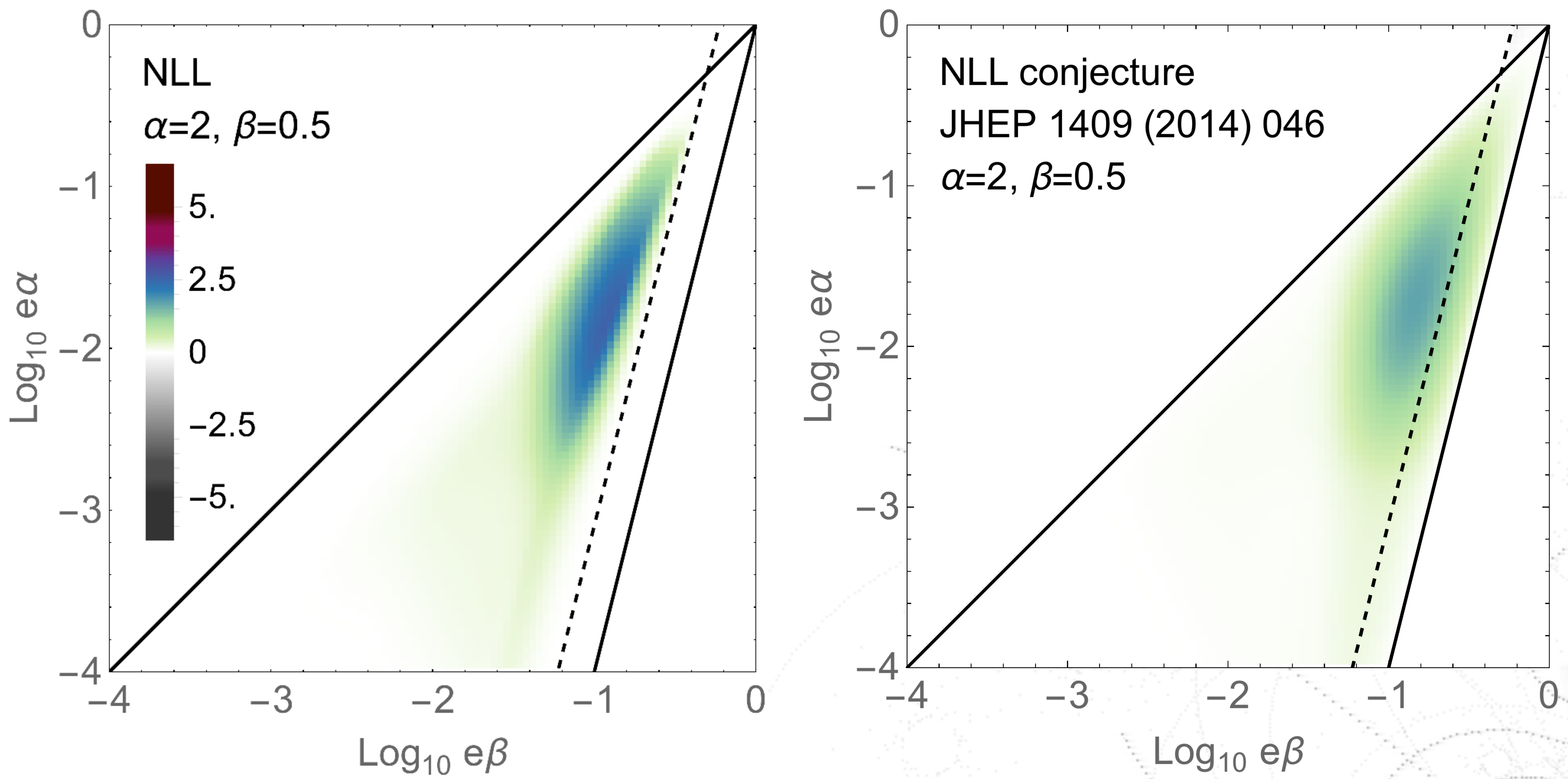
$$
\begin{aligned}
S^{(1)}(Qe_\alpha, Q^\beta e_\beta, \mu) = C_F \Bigg\{ \Bigg[&- \frac{16}{(\beta-1)} \frac{1}{\mu^\beta} \mathcal{L}_1\Big(\frac{Q^\beta e_\beta}{\mu^\beta}\Big) + 8 \ln \frac{Q^2}{\mu^2} \frac{1}{\mu^\beta} \mathcal{L}_0\Big(\frac{Q^\beta e_\beta}{\mu^\beta}\Big) \\
&+ \Big(- 2(\beta-1) \ln^2 \frac{Q^2}{\mu^2} + \frac{\pi^2}{3(\beta-1)} \Big) \delta(Q^\beta e_\beta) \Bigg] \delta(Qe_\alpha) \\
&- \frac{8}{\alpha-\beta} \frac{\mathrm{d}}{\mathrm{d}(Qe_\alpha)} \frac{\mathrm{d}}{\mathrm{d}(Q^\beta e_\beta)} \theta(e_\alpha)\theta(e_\beta - e_\alpha) \Bigg[\ln \frac{Qe_\alpha}{\mu} - \ln \frac{Q^\beta e_\beta}{\mu^\beta} \\
&+ \frac{1}{2}(\beta-1) \ln \frac{Q^2}{\mu^2} \Bigg]^2 \Bigg\}
\end{aligned}
$$

- Convolutions with cumulative distributions

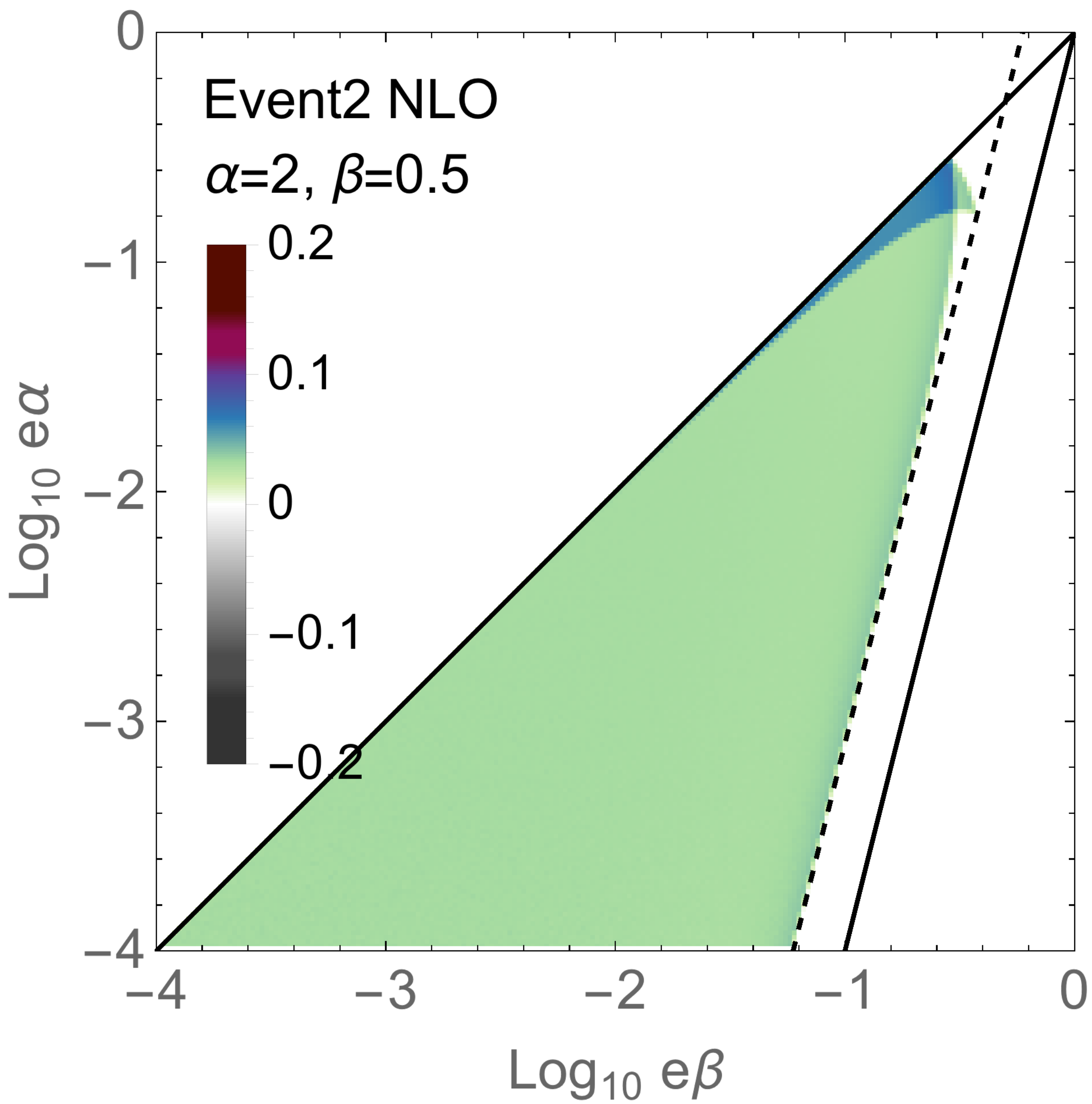
$$
\begin{aligned}
\int_0^{y_c} \mathrm{d}y \int_0^{y} \mathrm{d}x\, F'(x) G'(y-x) &= \int_0^{y_c} \mathrm{d}x \int_0^{y_c - x} \mathrm{d}y\, F'(x) G'(y) = \int_0^{y_c} \mathrm{d}x\, F'(x) G(y_c - x) \\
&= F(y_c) G(y_c) + \int_0^{y_c} \mathrm{d}x\, F'(x) [G(y_c - x) - G(y_c)]
\end{aligned}
$$

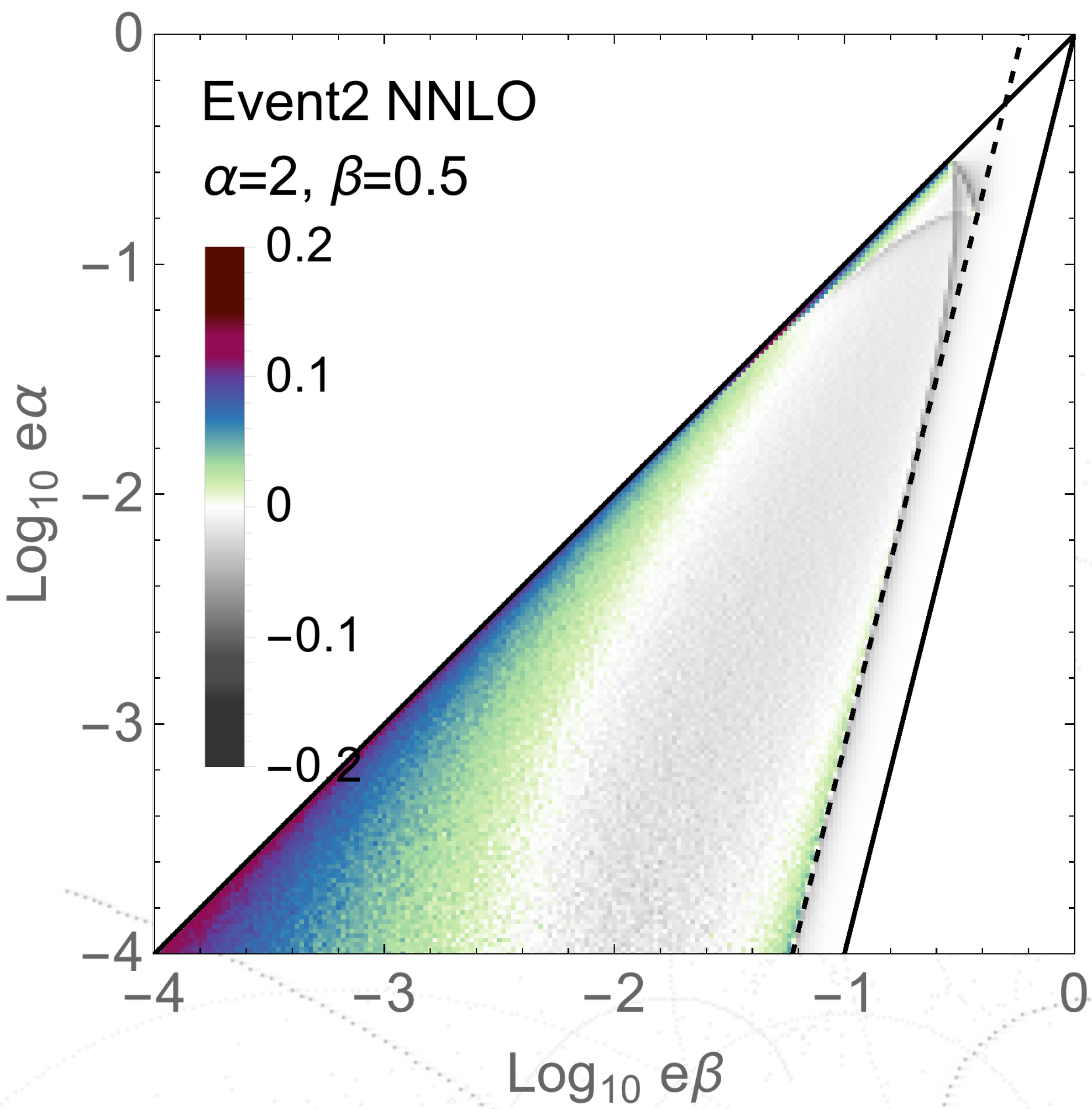
Larkoski, Moult, Neill: NLL conjecture

- Boundary theories for the measurement of two angularities on a single jet were identified and factorization theorems derived
- Interpolating function across the bulk region



EVENT2 results





- Preliminary results!

2D profile scales

- Canonical scales in each region

	$\log(\mu_J^{\mathrm{can}}/Q)$	$\log(\mu_{\mathscr{S}}^{\mathrm{can}}/Q)$	$\log(\mu_S^{\mathrm{can}}/Q)$
1	$1/\beta \log e_\beta$		$\log e_\beta$
2	$1/\beta \log e_\beta$	$(1-\beta)/(\alpha-\beta) \log e_\alpha + (\alpha-1)/(\alpha-\beta) \log e_\beta$	$\log e_\alpha$
3	$1/\alpha \log e_\alpha$		$\log e_\alpha$

- Construct scales in terms of logarithms of angularities

- Step 1: Polynomial that interpolates between the canonical scales between regions (one free parameter from transition point)

$$\log_{10} \frac{\mu_J^{\mathrm{region}}}{Q} = g\Big[\log_{10} e_\beta, \frac{1}{\beta}, \frac{1}{\alpha} \log_{10} e_\alpha, \log_{10} e_\alpha + \tilde{t}_r\Big(\log_{10}\left(2^{(\beta-\alpha)/\alpha} e_\alpha^{\beta/\alpha}\right) - \log_{10} e_\alpha\Big), \log_{10}\left(2^{(\beta-\alpha)/\alpha} e_\alpha^{\beta/\alpha}\right)\Big]$$

$g(x, a, b, x_1, x_2)$ interpolates between ax for $x < x_1$ and b for $x > x_2$

2D profile scales

- Canonical scales in each region

	$\log(\mu_J^{\text{can}}/Q)$	$\log(\mu_{\mathscr{S}}^{\text{can}}/Q)$	$\log(\mu_S^{\text{can}}/Q)$
1	$1/\beta \log e_\beta$		$\log e_\beta$
2	$1/\beta \log e_\beta$	$(1-\beta)/(\alpha-\beta) \log e_\alpha + (\alpha-1)/(\alpha-\beta) \log e_\beta$	$\log e_\alpha$
3	$1/\alpha \log e_\alpha$		$\log e_\alpha$

- Step 2: Transition to the nonperturbative regime: Freeze α_s below 2 GeV
- Step 3: Transition to fixed order
  - Choose a square shape for FO region: $t = \min(\log e_a, \log e_b)$
  - Polynomial $h(t, t_1, t_3)$ that interpolates between 1 for $t < t_1$ and 0 for $t > t_3$
  - E.g. $(\mu_j^{\text{region}})^h (\mu_{\text{FO}})^{(1-h)}$