Massive N-Jettiness Soft Function at NNLO

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N-jettiness

The N-jettiness event shape variable [Stewart et al, 2010]

$$\mathcal{T}_N = \sum_k \min_i \{n_i \cdot q_k\}$$

$n_i\ (i = a, b, 1, ..., N)$: light-like reference vectors representing the directions of initial and final jets
q_k: momenta of final-state partons
$\mathcal{T}_N \to 0$: q_k is a soft or collinear radiation

In this limit, the cross section is factorized as [Stewart et al, 2009,2010]

$$\frac{d\sigma}{d\mathcal{T}_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^{N} J_n\right)$$

B_i: beam functions, known up to NNLO [Gaunt et al, 2014]
J_n: jet function, known up to NNLO [Becher et al 2006, 2010]
S: soft function, known up to NNLO for two or three massless partons [Boughezal et al 2015, Compell et al 2017]

N-jettiness with massive particles

Application of N-jettiness

- Resummed distributions in DY, DIS, Higgs processes [Stewart et al, 2009-2010; Kang et al, 2012-2015; Berger, 2010; Jouttenus, 2013; Alioli, 2015, $\cdots$] .

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- NNLO calculations in top quark decay, $e^+e^- \to t\bar{t}$, $H/V + j$ [J.Gao et al, 2013, Gaunt et al, 2015;Boughezal et al, 2015; Gaunt et al, 2015; Berger et al, 2016; Abelof et al, 2016] .

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- No new collinear singularities generated by the heavy particle, $\mathcal{T}_N$ is still able to control all the infrared (both soft and collinear) singularities.
- Soft currents become complicated.

Kinematics

Consider the process

$$P_1 + P_2 \to Q + X$$

P_1 and P_2: incoming hadrons
Q: the massive colored particle
X: inclusive hadronic final state.

Two light-like vectors

$$n^\mu = (1, 0, 0, 1), \qquad \bar{n}^\mu = (1, 0, 0, -1)$$

The momenta are

$$p_1^\mu = \frac{m}{2} n^\mu \,, \quad p_2^\mu = \frac{m}{2} \bar{n}^\mu, \quad p_3^\mu = m v^\mu = \frac{m}{2}(n^\mu + \bar{n}^\mu)$$

m the mass of particle Q.
0-jettiness event shape variable:

$$\tau \equiv \mathcal{T}_0 = \sum_k \min\{n \cdot q_k, \bar{n} \cdot q_k\}$$

Soft function

The soft function is defined by the vacuum matrix element

$$S(\tau,\mu) = \sum_{X_s} \left\langle 0 \middle| \bar{\mathbf{T}}\, Y_n^\dagger Y_{\bar n} Y_v \middle| X_s \right\rangle \left\langle X_s \middle| \mathbf{T}\, Y_n Y_{\bar n}^\dagger Y_v^\dagger \middle| 0 \right\rangle$$

$$\underbrace{\delta\left(\tau - \sum_k \min\left(n\cdot \hat{P}_k, \bar{n}\cdot \hat{P}_k\right)\right)}_{\text{measurement function}}$$

where the soft Wilson lines are defined as

$$Y_n(x) = \mathbf{P}\exp\left(ig_s \int_{-\infty}^{0} ds\, n\cdot A_s^a(x+sn)t^a\right)$$

$$Y_{\bar n}^\dagger(x) = \bar{\mathbf{P}}\exp\left(-ig_s \int_{-\infty}^{0} ds\, \bar{n}\cdot A_s^a(x+s\bar{n})t^a\right)$$

$$Y_v^\dagger(x) = \mathbf{P}\exp\left(ig_s \int_{0}^{\infty} ds\, v\cdot A_s^a(x+sv)t^a\right)$$

Structure of the soft function

The cross section in the limit of $\tau \to 0$ can be written as

$$\frac{d\sigma}{dYd\tau} = \sigma_0 H(\mu^2) \int dt_a dt_b d\tau_s B_1(t_a, x_a, \mu) B_2(t_b, x_b, \mu)$$
$$\times S(\tau_s, \mu) \delta\left(\tau - \tau_s - \frac{t_a + t_b}{m}\right)\left(1 + \mathcal{O}\left(\frac{\Lambda^2}{m^2}, \frac{\tau}{m}\right)\right)$$

Since the all-order cross section does not depend on the renormalization scale μ, the dependence on the scale μ of the hard, beam and soft functions will cancel against each other order by order in α_s.

$\Rightarrow$We can derive the logarithmic terms of the scale μ in the soft function from the knowledge of the (N)NLO hard and beam functions.

Structure of the soft function

According to dimensional analysis, the bare soft function can be written in QCD perturbation theory as

$$S(\tau) = \delta(\tau) + \frac{1}{\tau}\sum_{n=1}^{\infty}\left(\frac{Z_{\alpha_s}\alpha_s}{4\pi}\right)^n \left(\frac{\tau}{\mu}\right)^{-2n\epsilon} s^{(n)}(\mu).$$

It is convenient to discuss the renormalization group equation by using Laplace transformed soft function

$$\begin{aligned}\tilde{S}(L,\mu) &= \int_0^\infty d\tau \exp\left(-\frac{\tau}{e^{\gamma_E}\mu e^{L/2}}\right) S(\tau) \\ &= 1 + \sum_{n=1}^{\infty}\left(\frac{Z_{\alpha_s}\alpha_s}{4\pi}\right)^n e^{n(L-2\gamma_E)\epsilon}\Gamma(-2n\epsilon)s^{(n)}\ .\end{aligned}$$

Then the corresponding renormalized soft function $\tilde{s}$ is defined as

$$\tilde{s} = Z_s^{-1}\tilde{S}\ , \quad \frac{d\ln Z_s}{d\ln\mu} = -\gamma_s$$

LO one-gluon soft current, or the eikonal current:

$$J_a^{\mu(0)}(q) = \sum_{i=1}^{3} \mathbf{T}_i^a \frac{p_i^\mu}{p_i \cdot q}$$

NLO soft function:

$$S^{(1)}(\tau) = \frac{2e^{\gamma_E \epsilon}\mu^{2\epsilon}}{\pi^{1-\epsilon}} \int d^d q \delta(q^2) J_a^{\mu(0)\dagger} d_{\mu\nu}(q) J_a^{\nu(0)}(q) F(n, \bar{n}, q)$$

The factor $F(n, \bar{n}, q)$ is the measurement function, defined as

$$F(n, \bar{n}, q) = \delta(q^+ - \tau)\Theta(q^- - q^+) + \delta(q^- - \tau)\Theta(q^+ - q^-)$$

NLO bare soft function:

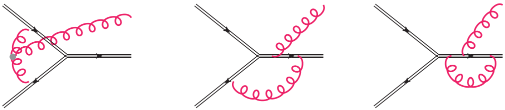
$$s^{(1)} = \frac{4e^{\gamma_E \epsilon}}{\epsilon \Gamma(1-\epsilon)} \Bigg(\mathbf{T}_1 \cdot \mathbf{T}_1 + \mathbf{T}_2 \cdot \mathbf{T}_2 +$$

$$\mathbf{T}_3 \cdot \mathbf{T}_3 \epsilon \left(\epsilon \psi^0 \left(1 + \frac{\epsilon}{2}\right) - \epsilon \psi^0 \left(\frac{1+\epsilon}{2}\right) - 1 \right) \Bigg)$$

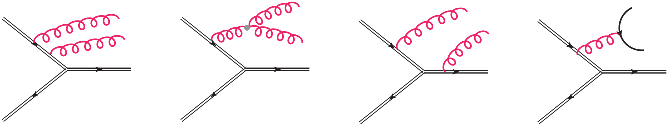
NNLO soft function

The NNLO contribution consists of two parts

$$S^{(2)}(\tau) = S^{(2)}_{\mathrm{VR}}(\tau) + S^{(2)}_{\mathrm{DR}}(\tau)$$



Virtual real corrections



Double real corrections

NNLO soft function: virtual real corrections

The virtual-real contribution to the soft function is given by

$$S^{(2)}_{\mathrm{VR}}(\tau) = \frac{e^{2\gamma_E\epsilon}\mu^{4\epsilon}}{\pi^{2-\epsilon}} 2\mathrm{Re}\left[\int d^D q \delta(q^2) J_a^{\mu(0)\dagger} d_{\mu\nu}(q) J_a^{\nu(1)}(q) F(n,\bar{n},q)\right]$$

NLO one-loop soft current: [Bierenbaum et al,2011]

$$J_a^{\mu(1)}(q) = i f_{abc} \sum_{i\neq j=1}^{3} \mathbf{T}_i^b \mathbf{T}_j^c \left(\frac{p_i^\mu}{p_i\cdot q} - \frac{p_j^\mu}{p_j\cdot q}\right) g_{ij}(\epsilon,q,p_i,p_j)$$

The factor $g_{ij}(\epsilon,q,p_i,p_j)$ is given by

$$\Big\{ \Big[p_i\cdot q(p_j\cdot q)^2 M_1 + \frac{1}{2} p_j\cdot q(p_i\cdot p_j p_i\cdot q - m_i^2 p_j\cdot q)M_2 + i\leftrightarrow j\Big]$$

$$+ [p_i\cdot p_j p_i\cdot q p_j\cdot q - m_i^2(p_j\cdot q)^2 - m_j^2(p_i\cdot q)^2] \frac{p_i\cdot q p_j\cdot q}{p_i\cdot p_j} M_3 \Big\}$$

$$\times \frac{2p_i\cdot p_j}{m_i^2(p_j\cdot q)^2 - 2p_i\cdot p_j p_i\cdot q p_j\cdot q + m_j^2(p_j\cdot q)^2}$$

NNLO soft function: virtual real corrections

We have used the HypExp package to manipulate the hypergeometric functions appearing in the intermediate steps. The result of virtual real contribution is

$$
\begin{aligned}
s_{\mathrm{VR}}^{(2)} =& -\frac{8\mathrm{C_A}\mathrm{C_F}}{\epsilon^3} + \frac{8\mathrm{C_A^2}}{\epsilon^2} + \frac{4\mathrm{C_A}}{3\epsilon}\left((\pi^2 - 6 - 24\ln 2)\mathrm{C_A} + 3\pi^2\mathrm{C_F}\right) \\
&+ \frac{4\mathrm{C_A}}{3}\left(\mathrm{C_A}(\pi^2 - 33\zeta_3 + 12(\ln^2 2 + 2\ln 2)) + 16\zeta_3\mathrm{C_F}\right) \\
&- \epsilon\frac{\mathrm{C_A}}{15}\Bigg[2\mathrm{C_A}\Big(\pi^4 + 30\pi^2(3 - 4\ln 2 + \ln^2 2) \\
&- 10(24 + 3\ln^4 2 + 72\mathrm{Li}_4(1/2) - 124\zeta_3 + 63\zeta_3\ln 2)\Big) + \pi^4\mathrm{C_F}\Bigg] \\
&+ \mathcal{O}(\epsilon^2)
\end{aligned}
$$

NNLO soft function: double real corrections

Two types of contributions

$$S_{\mathrm{DR}}^{(2)}(\tau) = S_{\mathrm{gg}}^{(2)}(\tau) + S_{\mathrm{q\bar{q}}}^{(2)}(\tau)$$

A soft quark-antiquark pair contribution [Catani et al, 2001]

$$|\mathcal{M}(q_1, q_2, p_1, p_2, p_3)|^2 \simeq (g_s^4 \mu^{4\epsilon}) \sum_{i,j=1}^{3} \mathcal{T}_{ij}(q_1, q_2) |\mathcal{M}(p_1, p_2, p_3)|^2$$

with

$$\mathcal{T}_{ij}(q_1, q_2) = -T_R \mathbf{T}_i \cdot \mathbf{T}_j \frac{2p_i \cdot p_j q_1 \cdot q_2 + p_i \cdot (q_1 - q_2) p_j \cdot (q_1 - q_2)}{2(q_1 \cdot q_2)^2 p_i \cdot (q_1 + q_2) p_j \cdot (q_1 + q_2)}$$

$$S_{\mathrm{q\bar{q}}}^{(2)}(\tau) = \frac{4 e^{2\gamma_E \epsilon} \mu^{4\epsilon}}{\pi^{2-2\epsilon}} \int d^d q_1 d^d q_2 \delta(q_1^2) \delta(q_2^2) \sum_{i,j=1}^{3} \mathcal{T}_{ij}(q_1, q_2) F(n, \bar{n}, q_1, q_2)$$

NNLO soft function: double real corrections

The two-gluon soft current [Catani et al, 2001; Czakon, 20011]

$$J^{\mu\nu(0)}_{a_1a_2}(q_1,q_2)=\frac{1}{2}\left\{J^{\mu(0)}_{a_1},J^{\nu(0)}_{a_2}\right\}+if_{a_1a_2a_3}\sum_{i=1}^{3}\mathbf{T}_i^{a_3}\left\{\frac{p_i^\mu q_1^\nu-p_i^\nu q_2^\mu}{q_1\cdot q_2 p_i\cdot(q_1+q_2)}\right.$$
$$\left.-\frac{p_i\cdot(q_1-q_2)}{2p_i\cdot(q_1+q_2)}\left[\frac{p_i^\mu p_i^\nu}{p_i\cdot q_1 p_i\cdot q_2}+\frac{g^{\mu\nu}}{q_1\cdot q_2}\right]\right\}$$

$$S^{(2)}_{\mathrm{gg}}(\tau)=\frac{2e^{2\gamma_E\epsilon}}{\pi^{2-2\epsilon}}\int d^dq_1d^dq_2\delta(q_1^2)\delta(q_2^2)$$
$$\times J^{\mu_1\nu_1(0)\dagger}_{a_1a_2}(q_1,q_2)d_{\mu_1\mu_2}(q_1)d_{\nu_1\nu_2}(q_2)J^{\mu_2\nu_2(0)}_{a_1a_2}(q_1,q_2)F(n,\bar{n},q_1,q_2)$$

The measurement function

$$\begin{aligned}F(n,\bar{n},q_1,q_2)&=\delta(q_1^++q_2^+-\tau)\Theta(q_1^--q_1^+)\Theta(q_2^--q_2^+)\\&+\delta(q_1^++q_2^--\tau)\Theta(q_1^--q_1^+)\Theta(q_2^+-q_2^-)\\&+\delta(q_1^-+q_2^+-\tau)\Theta(q_1^+-q_1^-)\Theta(q_2^--q_2^+)\\&+\delta(q_1^-+q_2^--\tau)\Theta(q_1^+-q_1^-)\Theta(q_2^+-q_2^-)\end{aligned}$$

NNLO soft function: double real corrections

It is convenient to perform the phase space integration in the light-cone coordinates.

$$\int d^d q = \frac{1}{2}\int d^{d-2}q_T dq^+ dq^- ,$$

Then we insert two identities

$$1 = \int d\tau_1 \delta(\tau_1 - q_1^{\pm}), \quad 1 = \int d\tau_2 \delta(\tau_2 - q_2^{\pm})$$

to extract the contributions from the two hemispheres.
Finally, we need to calculate four-fold integrals over a unit hypercube.

$$I = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dt \frac{x^{1+2\epsilon} y^{-1+2\epsilon} (1-z)^{-2\epsilon} z^{-1-2\epsilon} (1-t)^{-\frac{1}{2}-\epsilon} t^{-\frac{1}{2}-\epsilon}}{[x^2 + z(1-x^2)]\,(1 - 2xy + x^2y^2 + 4txy)}.$$

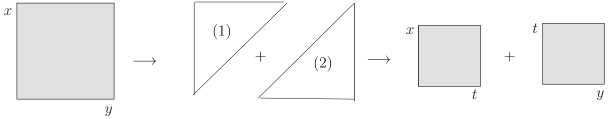
Two different methods

- Mellin-Barnes representation

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dz \frac{Y^z}{X^{\lambda+z}}\Gamma(\lambda+z)\Gamma(-z)$$

- Sector decomposition: to isolate the overlapping singularities

$$\int_0^1 dx \int_0^1 dy\, x^{-1-a\epsilon}\, y^{-b\epsilon}\, \Big(x+(1-x)\,y\Big)^{-1}$$



$$\int_0^1 dx\, x^{-1-(a+b)\epsilon} \int_0^1 dt\, t^{-b\epsilon}\, \Big(1+(1-x)\,t\Big)^{-1}$$

$$+\int_0^1 dy\, y^{-1-(a+b)\epsilon} \int_0^1 dt\, t^{-1-a\epsilon}\, \Big(1+(1-y)\,t\Big)^{-1}$$

NNLO soft function: double real corrections

The above integral is evaluated to be

$$I|_{\mathrm{MB}} = \frac{0.392699}{\epsilon^3} + \frac{1.08879}{\epsilon^2} + \frac{4.09324}{\epsilon} + 15.1676 + 50.7918\epsilon$$

$$I|_{\mathrm{SecDec}} = \frac{0.392695}{\epsilon^3} + \frac{1.08876}{\epsilon^2} + \frac{4.09301}{\epsilon} + 15.1668 + 50.7878\epsilon$$

The final result of the double-real contribution is

$$\begin{aligned} s_{\mathrm{DR}}^{(2)} &= \frac{8\mathrm{C_A}\mathrm{C_F} - 32\mathrm{C_F^2}}{\epsilon^3} \\ &+ \frac{1}{\epsilon^2}(46.667\mathrm{C_A}\mathrm{C_F} - 8\mathrm{C_A^2} - 2.667 n_f \mathrm{C_F}) \\ &- \frac{1}{\epsilon}(-67.226\mathrm{C_A}\mathrm{C_F} + 2.667 n_f \mathrm{C_A} - 5.6423\mathrm{C_A^2} - 4.444 n_f \mathrm{C_F} + 263.189\mathrm{C_F^2}) \\ &+ (-316.07\mathrm{C_A}\mathrm{C_F} - 2.957 n_f \mathrm{C_A} + 54.485\mathrm{C_A^2} + 4.853 n_f \mathrm{C_F} + 641.097\mathrm{C_F^2}) \\ &+ \epsilon(-531.488\mathrm{C_A}\mathrm{C_F} - 2.905 n_f \mathrm{C_A} + 92.248\mathrm{C_A^2} + 10.171 n_f \mathrm{C_F} + 874.517\mathrm{C_F^2}) \\ &+ \mathcal{O}(\epsilon^2)\ . \end{aligned}$$

Cross-check with RG equations

Because of the independence of the cross section on the renormalisation scale μ, in Laplace space, RG equation for soft function is

$$\frac{d \ln \tilde{s}}{d \ln \mu} = \gamma_s = -\frac{d \ln H}{d \ln \mu} - \frac{d \ln \tilde{B}_1}{d \ln \mu} - \frac{d \ln \tilde{B}_2}{d \ln \mu}$$

The expression for renormalisation factor Z_s is

$$\ln Z_s = \frac{\alpha_s}{4\pi}\left(\frac{\gamma_s^{(0)\prime}}{4\epsilon^2} + \frac{\gamma_s^{(0)}}{2\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\beta_0\gamma_s^{(0)\prime}}{16\epsilon^2} + \frac{\gamma_s^{(1)\prime} - 4\beta_0\gamma_s^{(0)}}{16\epsilon^2} + \frac{\gamma_s^{(1)}}{4\epsilon}\right)$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
SCET prediction	-24.8889	96.8889	158.568	354.032
real calculation	-24.8889	96.8888	158.577	353.820
difference	0	-1×10^{-4}	9×10^{-3}	-0.212

Comparison of the coefficients of $\epsilon^{-i}, i = 1, 2, 3, 4$ in double real contribution in two different methods.

Renormalized soft function

For a color octet production

$$\tilde{s}^{(1)} = 2\mathrm{C_A}(L - 2\ln 2) - \mathrm{C_F}(2L^2 + \pi^2),$$

$$\tilde{s}^{(2)} = \mathrm{C_F^2} K_{FF} + \mathrm{C_A^2} K_{AA} + \mathrm{C_A C_F} K_{AF} + \mathrm{C_A} n_f K_{Af} + \mathrm{C_F} n_f K_{Ff} - \frac{A_1}{4}$$

with

$$K_{FF} = 2L^4 + 2\pi^2 L^2 + \frac{247\pi^4}{90} \, ,$$

$$K_{AA} = \frac{17L^2}{3} + \left(4\zeta_3 + \frac{98}{9} - \frac{2\pi^2}{3} - \frac{68}{3}\ln 2\right)L + \frac{17\pi^2}{6} \, ,$$

$$K_{AF} = -\frac{58L^3}{9} + \left(-\frac{134}{9} + \frac{2\pi^2}{3} + 8\ln 2\right)L^2 + \left(28\zeta_3 - \frac{808}{27} - \frac{40\pi^2}{9}\right)L$$
$$- \frac{568\zeta_3}{3} + \frac{4\pi^4}{9} - \frac{268\pi^2}{27} + \frac{52\pi^2}{3}\ln 2 \, ,$$

$$K_{Af} = -\frac{2}{3}L^2 + \left(\frac{8}{3}\ln 2 - \frac{20}{9}\right)L - \frac{\pi^2}{3} \, ,$$

$$K_{Ff} = \frac{4}{9}L^3 + \frac{20}{9}L^2 + \left(\frac{112}{27} + \frac{4\pi^2}{9}\right)L + 8\zeta_3 + \frac{40}{27}\pi^2$$

A_1 is the coefficient of ϵ in $s^{(2)}_{\mathrm{DR}} + s^{(2)}_{\mathrm{VR}}$

General kinematics: tW production

The top quark velocity v is

$$v^+ = \frac{1-\beta_t\cos\theta_t}{\sqrt{1-\beta_t^2}}, \quad v^- = \frac{1+\beta_t\cos\theta_t}{\sqrt{1-\beta_t^2}}, \quad v_T = \frac{\beta_t\sin\theta_t}{\sqrt{1-\beta_t^2}},$$

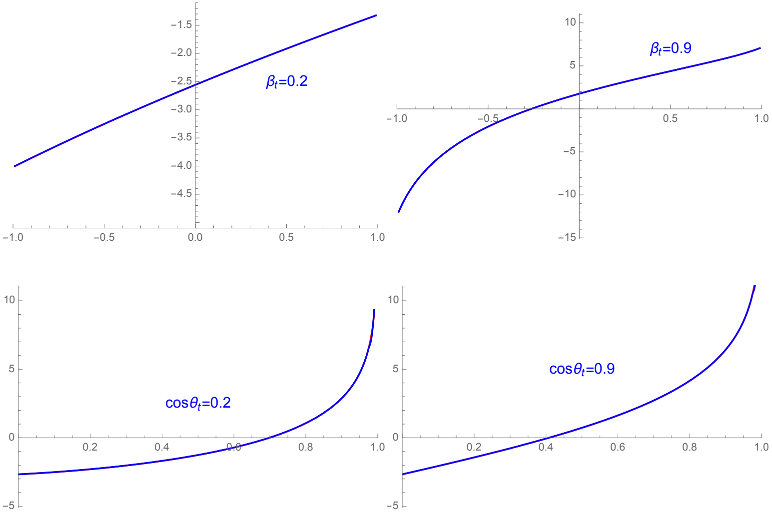
with $\beta_t \in (0,1), \quad \cos\theta_t \in (-1,1)$.
The anomalous dimension of the soft function

$$\begin{aligned}\gamma_s =& (\mathbf{T}_1\cdot\mathbf{T}_1+\mathbf{T}_2\cdot\mathbf{T}_2)\gamma_{\rm cusp}L - 2\mathbf{T}_1\cdot\mathbf{T}_1\gamma_{\rm cusp}\ln\frac{-s_{13}}{m^2} \\ & -2\mathbf{T}_2\cdot\mathbf{T}_2\gamma_{\rm cusp}\ln\frac{-s_{23}}{m^2} - 2\mathbf{T}_1\cdot\mathbf{T}_2\gamma_{\rm cusp}\ln\frac{s_{12}m^2}{(-s_{13})(-s_{23})} \\ & -2\gamma^Q - \gamma_B^1 - \gamma_B^2 - 2\gamma^1 - 2\gamma^2 \ni \ln(1-\beta_t^2)\end{aligned}$$

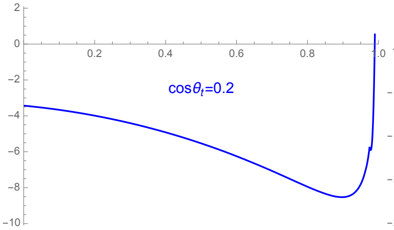
More dependence on the kinematics. Six-fold integrals. SecDec and MB are not convenient packages to deal with such kind of integrals.

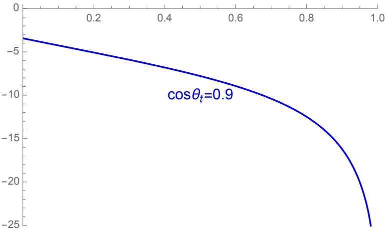
NLO coefficient of $1/\epsilon$

Blue lines: Numerical calculations; Red lines: RG predictions

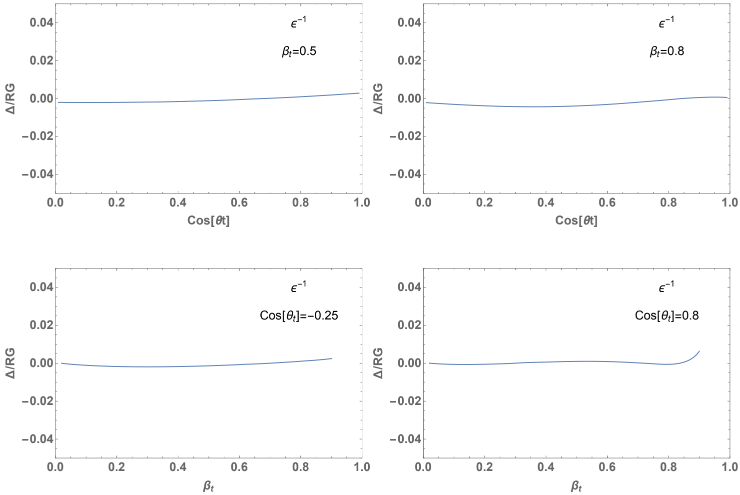


NLO coefficient of ϵ^0



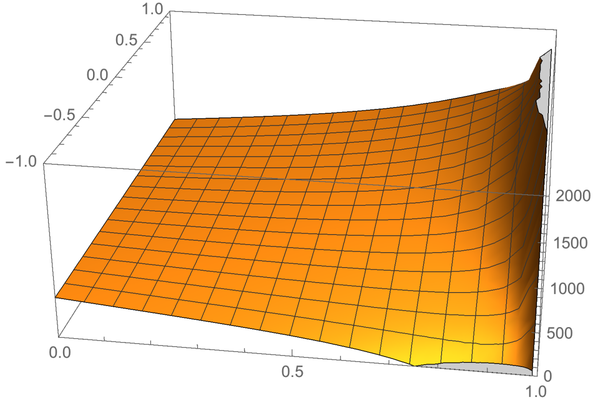


NNLO coefficient of $1/\epsilon$



NNLO coefficient of ϵ^0

Preliminary results (fitted by $\sim$ 100 phase space points):



Conclutions

- We calculate one of the indispensable ingredients in N-jettiness subtraction method, i.e., the soft function, for one massive colored particle production at hadron colliders
- Our results can be used in the differential NNLO calculation for a massive coloured particle production

Thank you !