QCD resummation with the ARES method

Andrea Banfi

University of Sussex

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Outline

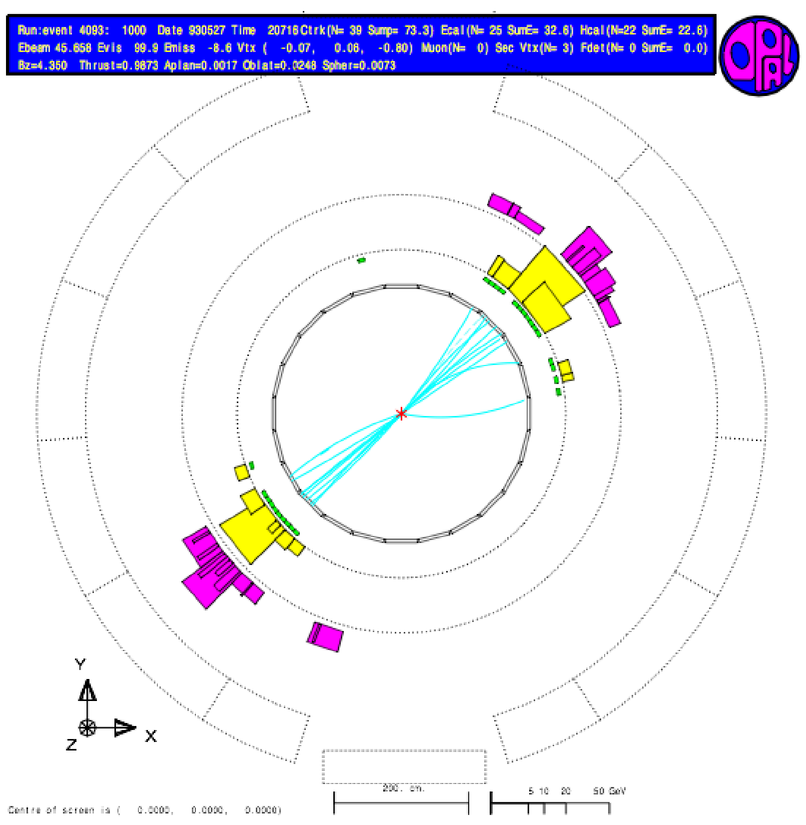
- Resummation of final-state observables
- Observable properties (rIRC safety)
- The ARES method for QCD resummation
- Jet rates in e^+e^- annihilation
- Current work in progress and outlook

Final-state observables

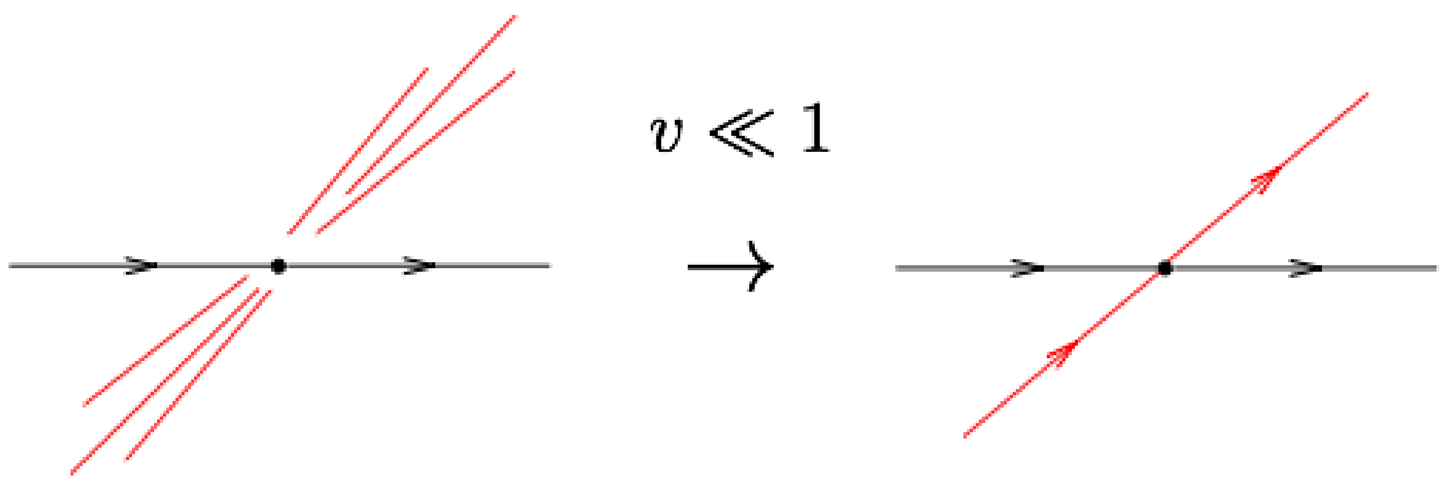
- We consider a generic final-state observable, a function $V(p_1, \ldots, p_n)$ of all possible final-state momenta $p_1, \ldots, p_n$

- Examples: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$



THE NARROW-JET LIMIT

- Selecting events close to the Born limit, i.e. $v \ll 1$, produces large logarithms of the resolution variable v due to incomplete real-virtual cancellations

$$\Sigma(v) \simeq 1 - C\,\frac{\alpha_s}{\pi}\ln^2\frac{1}{v} + \dots$$

LO NLO

breakdown of perturbation theory!

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\right]$$

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the PT series in the region $\alpha_s L \sim 1$, with $L = \ln(1/v)$

$$\Sigma(v) \simeq e^{\underbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}}} \times \left(\overset{1}{\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}}} \overset{+}{+} \overset{\alpha_s}{\underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}}} \overset{+\ldots}{+\ldots} \right)$$

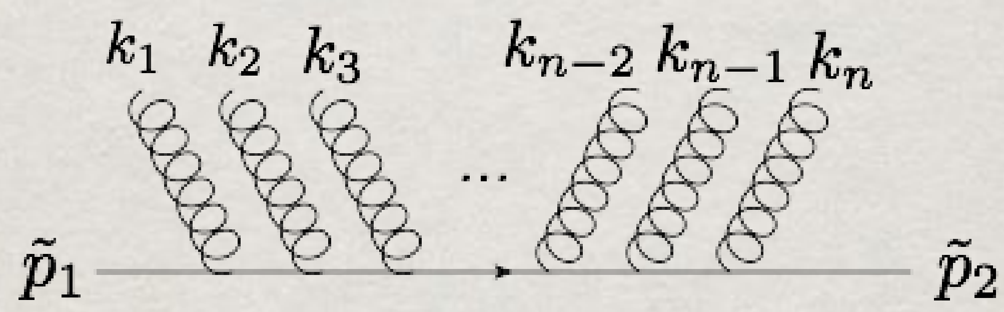
The ARES method

- NNLL corrections are often sizeable and important for precision physics
- The most important limitation is the analytical treatment of the observable in some (smartly defined) conjugate space
- The Automated Resummer for Event Shapes (ARES) is a semi-numerical approach that:
  - does not rely on analytical properties of the observable
  - is NNLL accurate and extendable to higher orders
  - is fully general for a very broad category of observables (~ all that can be possibly resummed at NNLL accuracy)
  - is flexible and automated (only input: observable’s routine in suitable limits)

Basic observable properties

- We consider an infrared and collinear (IRC) safe observable normalised as

$$v = V(\{\tilde{p}\}, k_1, \ldots, k_n) \leq 1$$



- In the Born limit, $V(\{\tilde{p}\}) = 0$
- In the limit $v \to 0$, quasi-Born kinematics, all secondary emissions $k_1, \ldots, k_n$ are soft and/or collinear

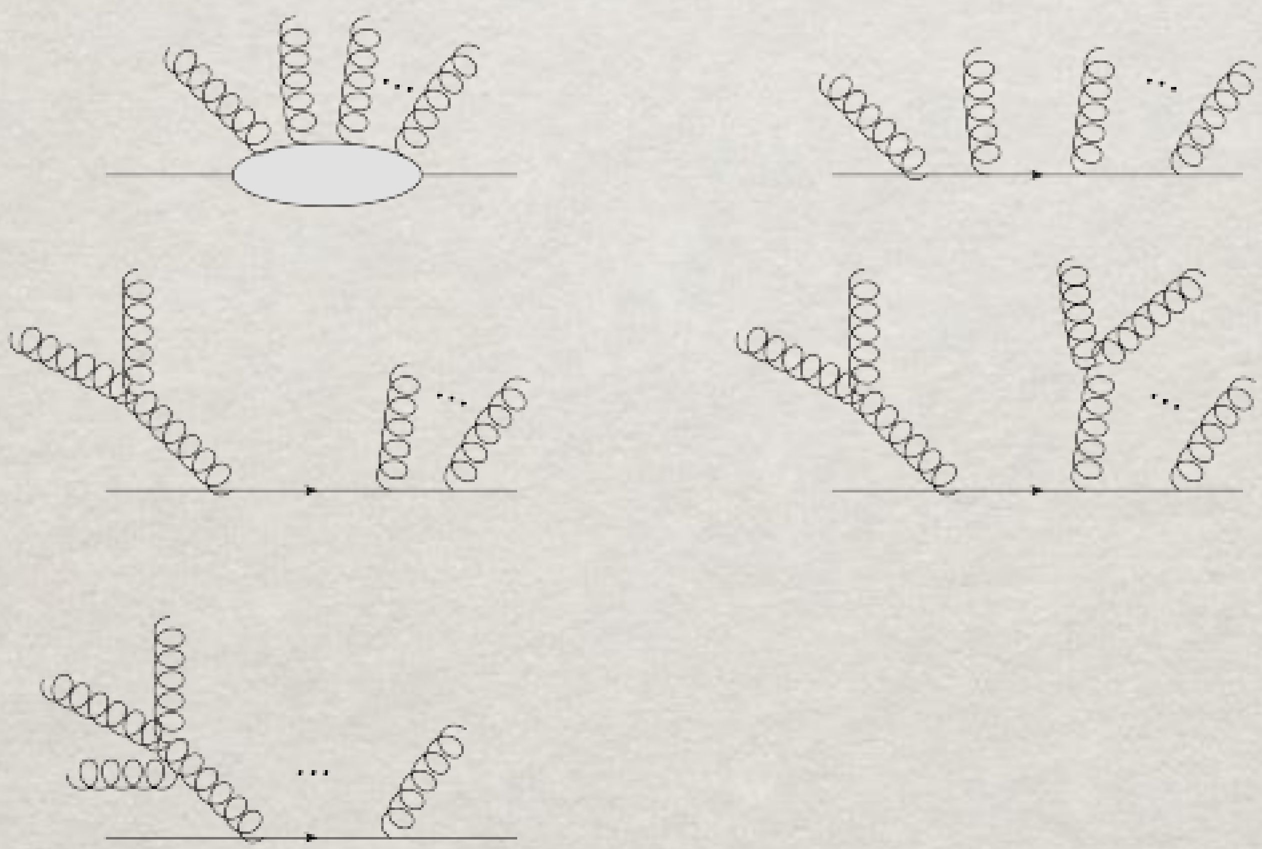
Recursive IRC safety

- We restrict ourselves to recursive IRC (rIRC) safe observables, for which
  - the observable scaling properties when we make all emissions simultaneously soft-collinear are the same with any number of secondary emissions
  - such scaling properties are unchanged after an infinitely soft emission or a perfect collinear splitting
- Examples of rIRC observables:
  - most global event shapes
  - Durham and Cambridge jet resolution parameters (no JADE and Geneva)
  - transverse momentum of the leading jet in Higgs or vector boson production

Implications of rIRC safety

- The only emissions that contribute to $\Sigma(v) = \mathrm{Prob}[V(\{\tilde{p}\}, k_1, \ldots, k_n)] < v$ in the limit $v \to 0$, up to powers of v, are those for which

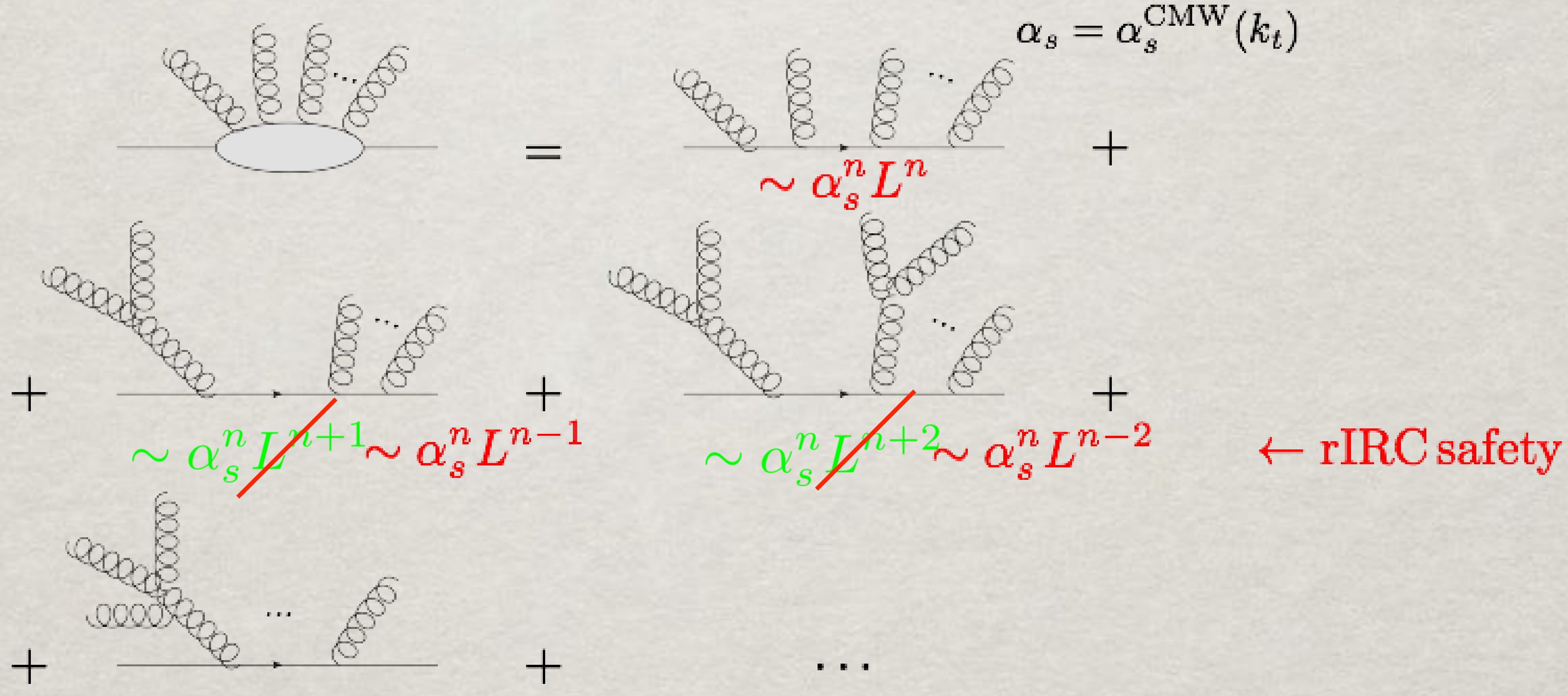
$$V(\{\tilde{p}\}, k_1) \sim V(\{\tilde{p}\}, k_2) \sim \cdots \sim V(\{\tilde{p}\}, k_n) \sim V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$$



- This, together with QCD coherence, is enough to establish the relative importance of soft-collinear contributions at all logarithmic orders

Relevant emissions

- At NLL the only relevant emissions are soft and collinear gluons widely separated in angle (rapidity separation $\sim \ln(1/v)$)



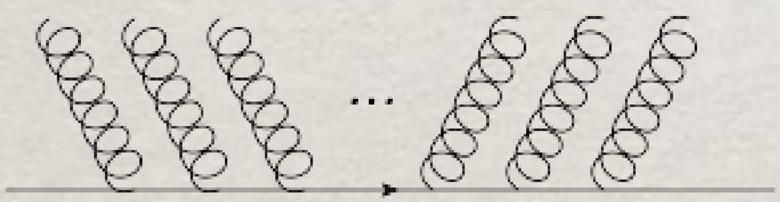
- Any other emission gives a contribution of relative order α_s

NLL RESUMMATION

- Unresolved emissions and virtual corrections result in a double-logarithmic Sudakov exponent, the radiator [Banfi Salam Zanderighi '05]

$$\Sigma(v) = e^{-R_{\rm NLL}(v)} \mathcal{F}_{\rm NLL}(v)$$

- The effect of multiple soft and collinear gluons widely separated in angle is encoded in the single-logarithmic function $\mathcal{F}_{\rm NLL}(v)$



Measure defined by the soft-collinear ensemble

$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}] \qquad R' \equiv -v\frac{dR}{dv} = \sum_{\ell_i} R'_{\ell_i}$$

$$\mathcal{F}_{\rm NLL}(v) = \left\langle \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}^{\rm NLL}(\{\tilde{p}\},\{k_i\})}{v}\right)\right\rangle$$

- The function $V_{\rm sc}^{\rm NLL}(\{\tilde{p}\},\{k_i\})$ is the observable for soft and collinear emissions widely separated in angle. For jet rates, this is different from $V_{\rm sc}(\{\tilde{p}\},\{k_i\})$, the observable in the soft-collinear limit

NNLL RESUMMATION

- The NNLL radiator, encoding the cancellation of unresolved real emissions and virtual corrections, can be computed for an arbitrary observable

[see B. el-Menoufi's talk]

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)} \left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$

Clustering
(jet algorithms only)

Correlated emission

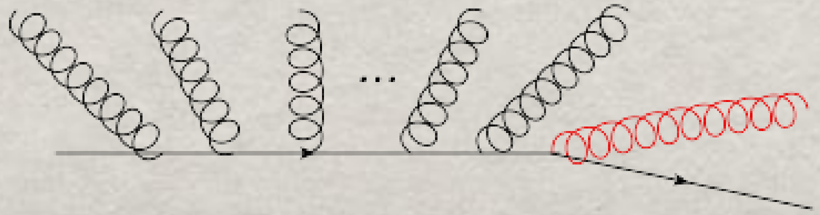
$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

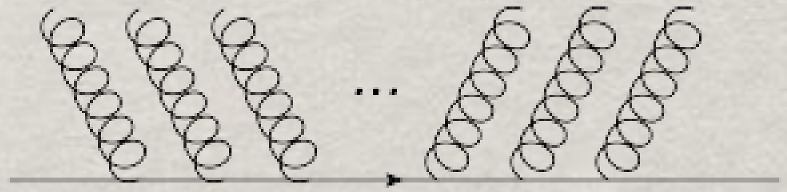
Hard collinear
Recoil

Running coupling
Rapidity
(jet algorithms)

Soft wide-angle

Soft-collinear
(event shapes)





NNLL RESUMMATION

- All NNLL corrections can be written in terms of finite integrals in four dimensions

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$

Clustering
(jet algorithms only)

Correlated emission

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

Hard collinear
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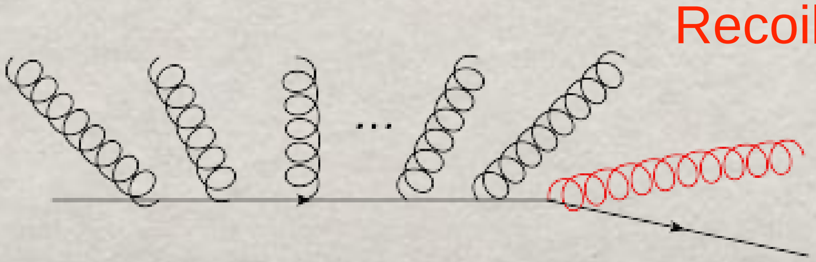
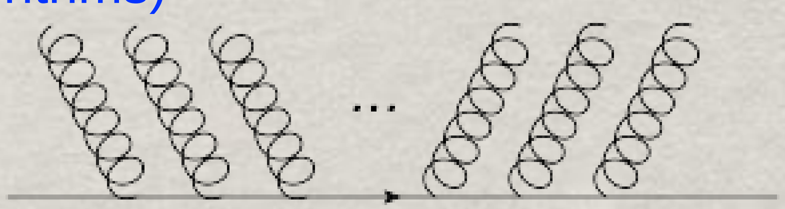




NNLL RESUMMATION

- Every NNLL correction requires to determine an approximate expression for the observable in the relevant kinematic limit

$$\Sigma(v) = e^{-R_{\mathrm{NNLL}}(v)}\left[\mathcal{F}_{\mathrm{NLL}}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(v)\right]$$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\}) \neq V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Clustering
(jet algorithms only)

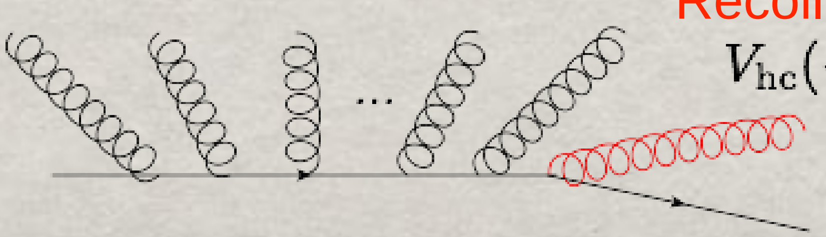
Correlated emission

$$\delta\mathcal{F}_{\mathrm{NNLL}} = \delta\mathcal{F}_{\mathrm{clust}} + \delta\mathcal{F}_{\mathrm{correl}} + \delta\mathcal{F}_{\mathrm{hc}} + \delta\mathcal{F}_{\mathrm{rec}} + \delta\mathcal{F}_{\mathrm{wa}} + \delta\mathcal{F}_{\mathrm{sc}}$$

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Hard collinear
Recoil

$V_{\mathrm{hc}}(\{\tilde{p}\},\{k_i\})$



$V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})$

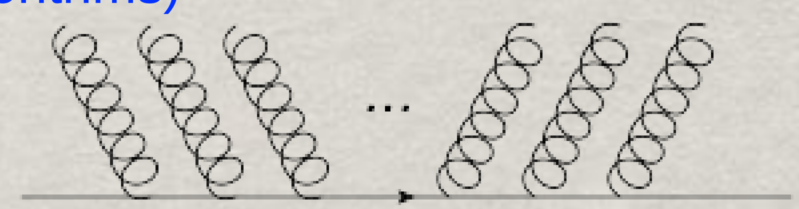
Running coupling
Rapidity
(jet algorithms)

$V_{\mathrm{wa}}(\{\tilde{p}\},\{k_i\})$

Soft wide-angle

$V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})$

Soft-collinear
(event shapes)



CAESAR VS ARES

[Banfi Salam Zanderighi ‘05]

- Establishes the range in which actual emissions can be considered soft and collinear
- Uses the actual observable subroutine and computes its soft-collinear limit numerically
- Requires careful extrapolations to be extended to NNLL

[Banfi McAslan Monni Zanderighi ‘15]

- Generates emissions that are by construction soft and collinear (no energy-momentum conservation)
- Uses analytically determined soft and collinear limits of each observable
- Can be in principle extended to any logarithmic accuracy

Soft-collinear emissions

- Integration measure for soft and collinear emissions widely separated in angle

Measure defined by the soft-collinear ensemble

$$= \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]$$

$$\langle G(\{k_i\})\rangle = \int d\mathcal{Z}[\{R'_{\ell_i}, k_i\}]\, G(\{k_i\})$$

$$= \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(\int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \sum_{\ell_i} R'_{\ell_i} \int_0^{2\pi} \frac{d\phi_i^{(\ell_i)}}{2\pi} \int_0^1 d\xi_i^{(\ell_i)} \right) G(k_1, \dots, k_n)$$

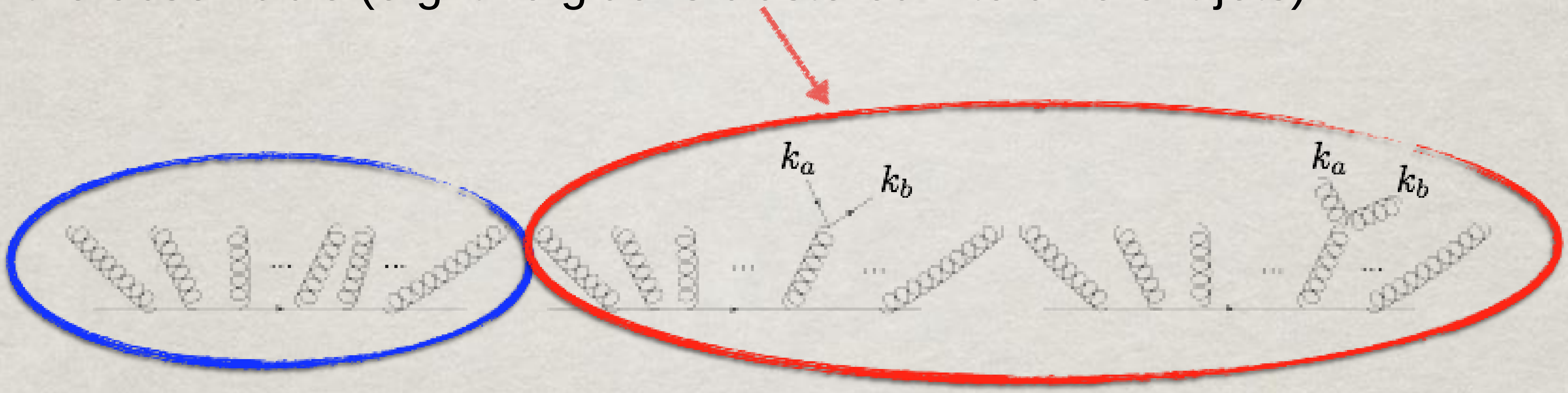
- Probability to emit a soft and collinear gluon k_i with $V(\{\tilde{p}\}, k_i) = v\,\zeta_i$ off hard leg ℓ_i with azimuthal angle $\phi_i^{(\ell_i)}$ and rapidity fraction $\xi_i^{(\ell_i)}$

$$dP(R'_{\ell_i}, k_i) \sim \frac{R'_{\ell_i}}{R'} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'} \Theta\left(\zeta_{i-1} - \zeta_i\right) \Theta\left(\zeta_i - \epsilon\right) \frac{d\phi_i^{(\ell_i)}}{2\pi} d\xi_i^{(\ell_i)}$$

- The function $G(k_1, \dots, k_n)$ can be used to integrate out the largest of the ζ_i

An example of NNLL correction

- Two soft-collinear partons close in rapidity
- Correlated emission corrections taking into account non-inclusiveness of the observable (e.g. two gluons clustered into different jets)



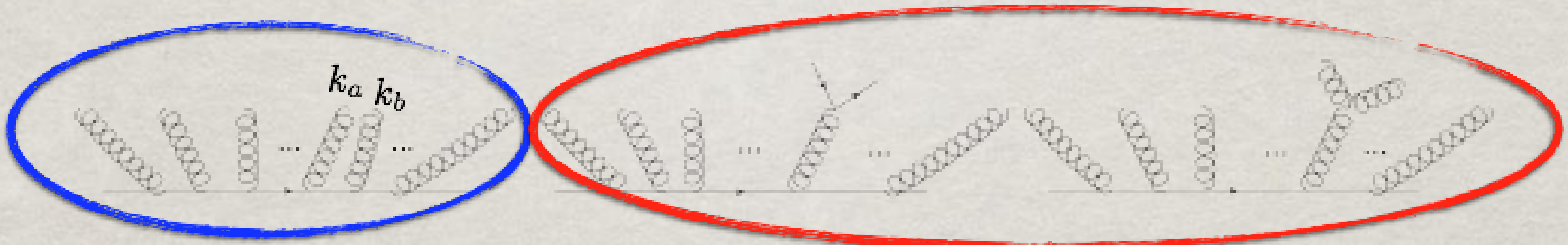
Clustering correction
(jet algorithms only)

Correlated emission corrections

$$\delta\mathcal{F}_{\mathrm{correl}} = \left\langle \Theta\left(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}(\{\tilde{p}\}, k_a + k_b, \{k_i\})}{v}\right)\right\rangle$$

Event-shape vs jet rates

- Event shape variables have the property that, for fixed $V(\{\tilde{p}\}, k_i)$, their value does not depend on emissions' rapidity fractions
- This implies that some NNLL corrections are zero for event shapes
- For instance, two soft and collinear gluons close in rapidity give a NNLL clustering correction only to jet rates



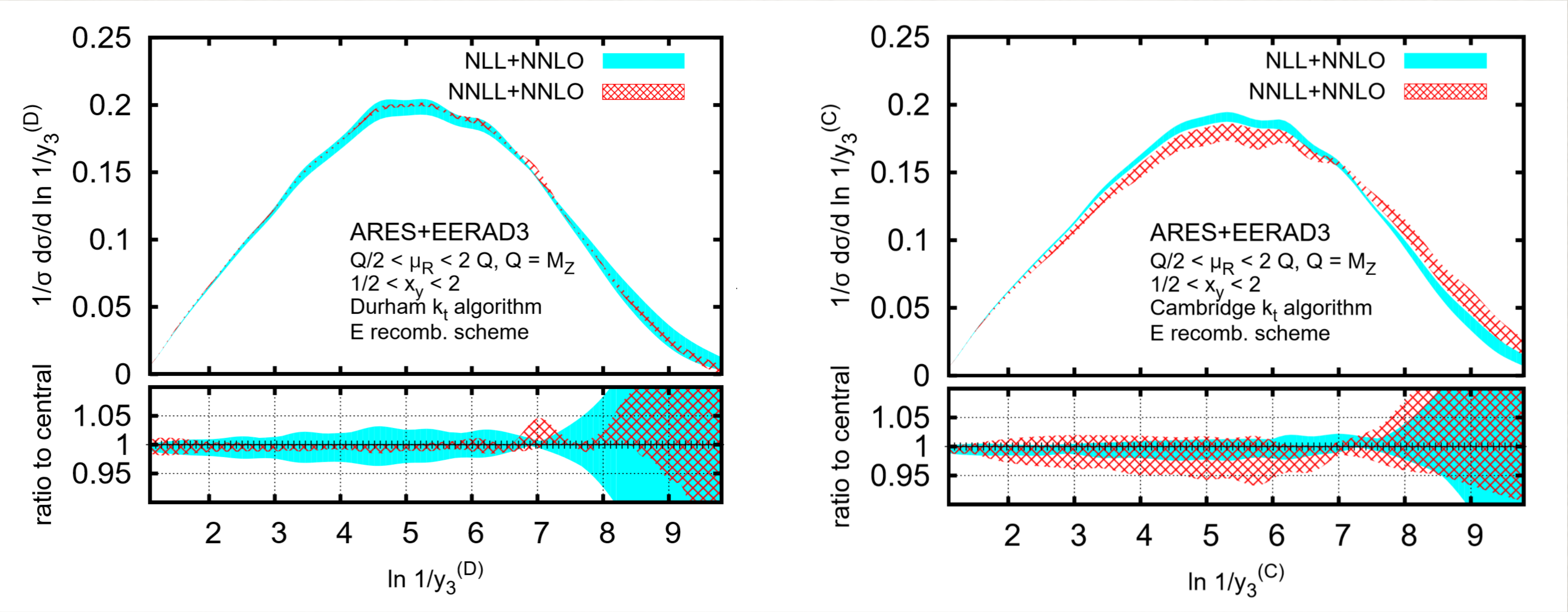
Clustering correction
(jet algorithms only)

Correlated corrections

$$\delta\mathcal{F}_{\mathrm{clust}} = \left\langle \Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\}, k_a, k_b, \{k_i\})}{v}\right)\right\rangle$$

Two-jet rate

- First-ever NNLL resummation of the two-jet rate for the Durham and Cambridge algorithms [Banfi McAslan Monni Zanderighi '16]



- NNLL resummation of the two-jet rate has been performed also for other rIRC safe jet algorithms (flavour k_t, angular-ordered Durham, inclusive k_t)

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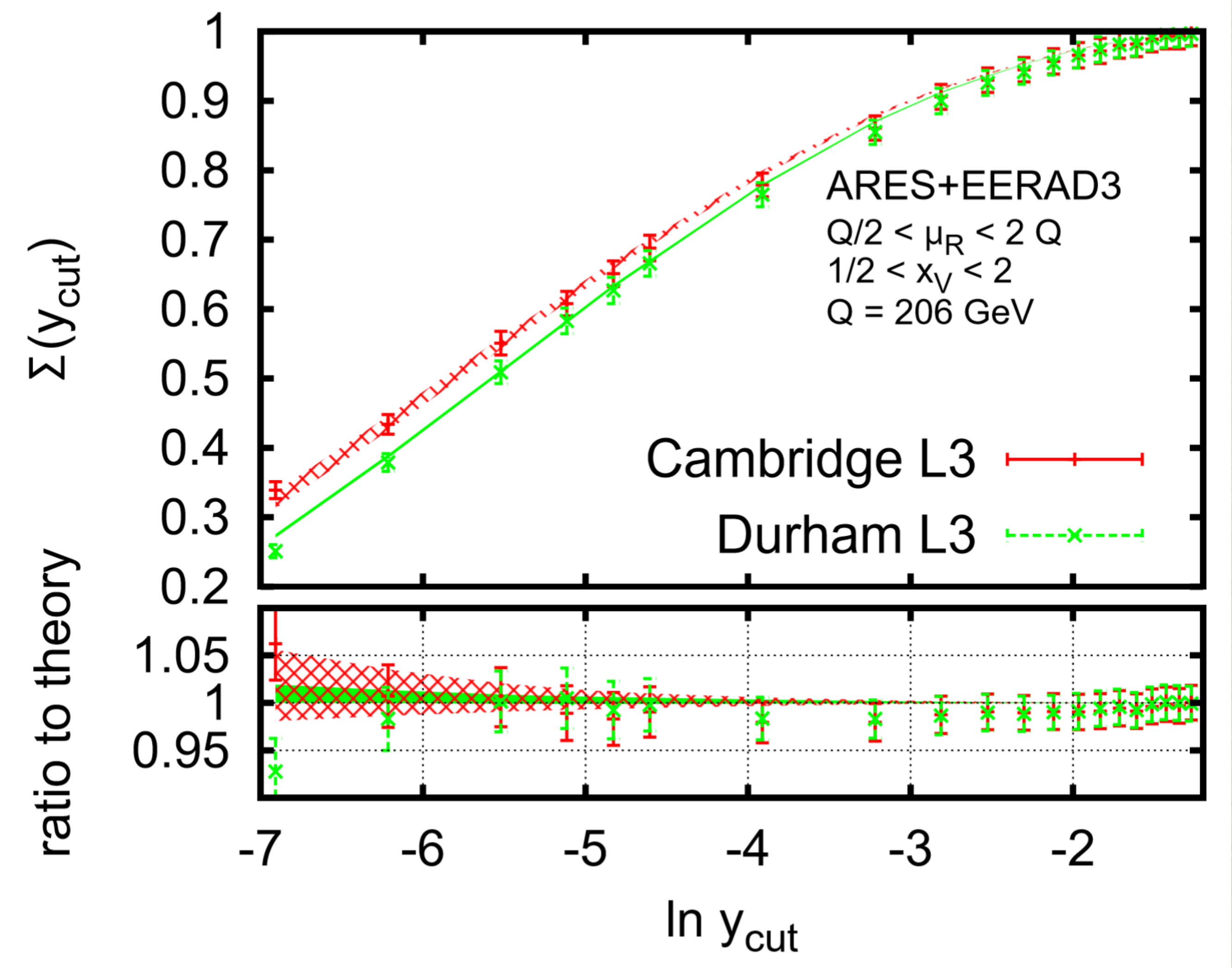

- Good agreement with LEP data $\Rightarrow$ fit of $\alpha_s(M_Z)$ in progress

Conclusions

- Novel general method for the resummation of any rIRC safe (global) observable in the two-scale regime
  - weak applicability conditions
  - cancellation of poles between real and virtual corrections performed analytically in dimensional regularisation [see B. el-Menoufi's talk]
  - contributions of resolved real emissions formulated in terms of four-dimensional integrals (suitable for Monte Carlo implementation)
- Work in progress
  - New global fit of α_s from e^+e^- event shapes
  - More NNLL resummations in multi-jet events, e.g. hadron collisions
- Ambitious goal: comparison to parton shower event generators to obtain branching algorithms that are NNLL accurate

Conclusions

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Thank you for your attention!