Numerical resummation in SCET

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Work in collaboration with
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1803.0704x

SCET has allowed for some of the most precise resummation results available today, but each observable takes lots of time

Big advantage of SCET is separation at Lagrangian level

- To resum, need following:
  1. Factorization for observable
  2. Fixed order computations of factorization ingredients
  3. Solving RG equations

Each of the three steps depends on the observable and needs to repeated

Can this be done in a way where the observable dependence can be computed numerically?

For a simple observable we know the factorization theorem and can easily obtain an analytical solution

Consider the factorization the thrust cumulant

$$\Sigma(\tau) = H(\mu) \int \mathrm{d}\tau_n \, \Sigma'_{J_n}(\tau_n, \mu) \int \mathrm{d}\tau_{\bar{n}} \, \Sigma'_{J_{\bar{n}}}(\tau_{\bar{n}}, \mu) \int \mathrm{d}\tau_s \, \Sigma'_S(\tau_s, \mu) \; \Theta[\tau > \tau_n + \tau_{\bar{n}} + \tau_s]$$

$$F(\tau_F, \mu) \equiv \Sigma'_F(\tau_F, \mu) = \frac{\mathrm{d}\Sigma_F(\tau_F)}{\mathrm{d}\tau_F}$$

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One can define a "max version" of thrust (taking max of thrust for each emission), which has a multiplicative factorization theorem

$$\Sigma_{\max}(\tau) = H(\mu) \, \Sigma^{\max}_{J_n}(\tau_n, \mu) \, \Sigma^{\max}_{J_{\bar{n}}}(\tau_{\bar{n}}, \mu) \, \Sigma^{\max}_S(\tau_s, \mu)$$

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This multiplicative version knows nothing about details of observable, only about the singular behavior of a single emission

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Resum logs by evolving jet and soft functions from their characteristic scales $\mu_S = Q\tau$, $\mu_J = Q\sqrt{\tau}$ to $\mu_H = Q$

$$\Sigma_{\mathrm{NLL}}(\tau) = \exp\left\{ \int_{\sqrt{\tau}Q}^{Q} \frac{\mathrm{d}\mu}{\mu} \left(4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln \frac{\mu^2}{\tau Q^2} - 4\gamma_J[\alpha_s(\mu)] \right) \right\}$$
$$\times \exp\left\{ \int_{\tau Q}^{Q} \frac{\mathrm{d}\mu}{\mu} 4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln \frac{\tau Q}{\mu} \right\} \frac{e^{-\gamma_E(2\eta_J + \eta_S)}}{\Gamma(1 + (2\eta_J + \eta_S))}$$

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This uses the anomalous dimensions for the soft and jet functions

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} J_{n_i}(\tau;\mu) = \left\{ -2\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln \frac{\tau Q^2}{\mu^2} - 2\gamma_J[\alpha_s(\mu)] \right\} J_n(\tau;\mu)$$
$$+ 2\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \int_0^\tau \mathrm{d}\tau' \, \frac{J_{n_i}(\tau;\mu) - J_{n_i}(\tau';\mu)}{\tau - \tau'} \, ,$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S(\tau;\mu) = \left\{ 2\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln \frac{\tau^2 Q^2}{\mu^2} - 2\gamma_S[\alpha_s(\mu)] \right\} S(\tau;\mu)$$
$$- 4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \int_0^\tau \mathrm{d}\tau' \, \frac{S(\tau;\mu) - S(\tau';\mu)}{\tau - \tau'} \, .$$

The resummation of large logarithms for the "max" observable is much simpler

Factorization for $\Sigma_{\max}$ multiplicative

$$\Sigma_{\max}(\tau) = H(\mu)\, \Sigma^{\max}_{J_n}(\tau_n, \mu)\, \Sigma^{\max}_{J_{\bar{n}}}(\tau_{\bar{n}}, \mu)\, \Sigma^{\max}_{S}(\tau_s, \mu)$$

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RG equations are multiplicative

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \Sigma_{J_n}^{\rm max}(\tau;\mu) = \left\{ -2\Gamma_{\rm cusp}[\alpha_s(\mu)] \ln \frac{\tau Q^2}{\mu^2} - 2\gamma_J[\alpha_s(\mu)] \right\} \Sigma_{J_n}(\tau;\mu)$$

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Resummation is just product of exponentials

$$\Sigma^{\text{NLL}}_{\max}(\tau) = \exp\left\{ \int_{\sqrt{\tau}Q}^{Q} \frac{\mathrm{d}\mu}{\mu} \left(4\Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu^2}{\tau Q^2} - 4\gamma_J[\alpha_s(\mu)] \right) \right\} \times \exp\left\{ \int_{\tau Q}^{Q} \frac{\mathrm{d}\mu}{\mu} 4\Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\tau Q}{\mu} \right\} .$$

One can express the more complicated observable Σ(v) in term of simpler observable Σ_{max}(v)

$$\Sigma(\tau) = H(\mu) \int \mathrm{d}\tau_n \, \Sigma'_{J_n}(\tau_n, \mu) \int \mathrm{d}\tau_{\bar{n}} \, \Sigma'_{J_{\bar{n}}}(\tau_{\bar{n}}, \mu) \int \mathrm{d}\tau_s \, \Sigma'_S(\tau_s, \mu) \; \Theta[\tau > \tau_n + \tau_{\bar{n}} + \tau_s]$$

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Combining the two, we can write

$$\Sigma(\tau) = \Sigma_{\max}(\tau) \int \mathrm{d}\tau_n\, \mathcal{F}'_{J_n}(\tau_n, \mu) \int \mathrm{d}\tau_{\bar{n}}\, \mathcal{F}'_{J_{\bar{n}}}(\tau_{\bar{n}}, \mu) \int \mathrm{d}\tau_s\, \mathcal{F}'_S(\tau_s, \mu)\; \Theta[\tau - \tau_n - \tau_{\bar{n}} - \tau_s]$$

with

$$\mathcal{F}_F(\tau_F, \tau, \mu) = \frac{\Sigma_F(\tau_F, \mu)}{\Sigma_F^{\max}(\tau, \mu)} = \frac{\Sigma_F^{\max}(\delta\tau, \mu)}{\Sigma_F^{\max}(\tau, \mu)}\, \frac{\Sigma_F(\tau_F, \mu)}{\Sigma_F^{\max}(\delta\tau, \mu)}$$

"Transfer function"

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Observable dependence isolated in transfer function

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \Sigma_F^{\max}(\tau; \mu) = \left\{ 2 a_F \Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln \frac{\mu_F^2}{\mu^2} - 2\gamma_F[\alpha_s(\mu)] \right\} \Sigma_F^{\max}(\tau; \mu)$$

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Can transfer function be computed numerically?

While transfer function is IR finite, there are still UV divergences in SCET we need to deal with

Individual transfer functions contains UV divergences

$$\text{QCD}: \quad \int \mathrm{d}y_{qg}\,\mathrm{d}y_{\bar{q}g}\;\Theta[\min(y_{qg}, y_{\bar{q}g}, 1-y_{qg}-y_{\bar{q}g}) < \tau]\,\Theta[0 < y_{ij} < 1]$$

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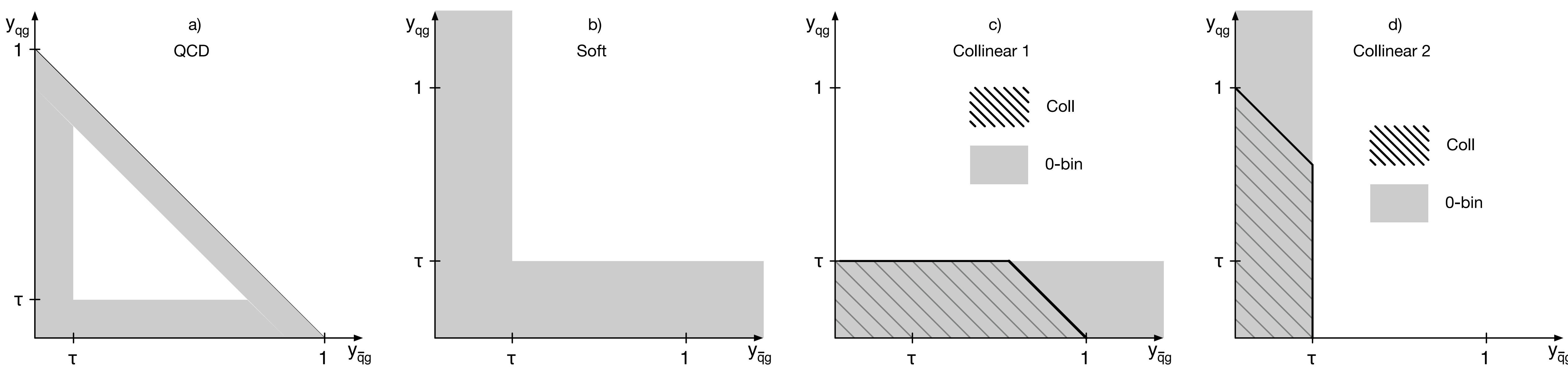

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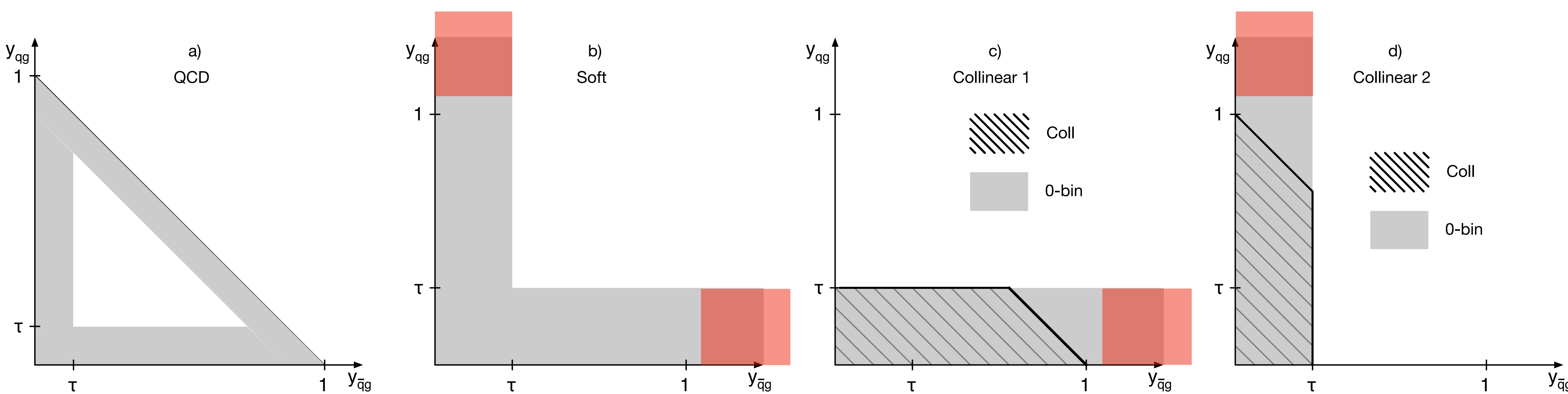

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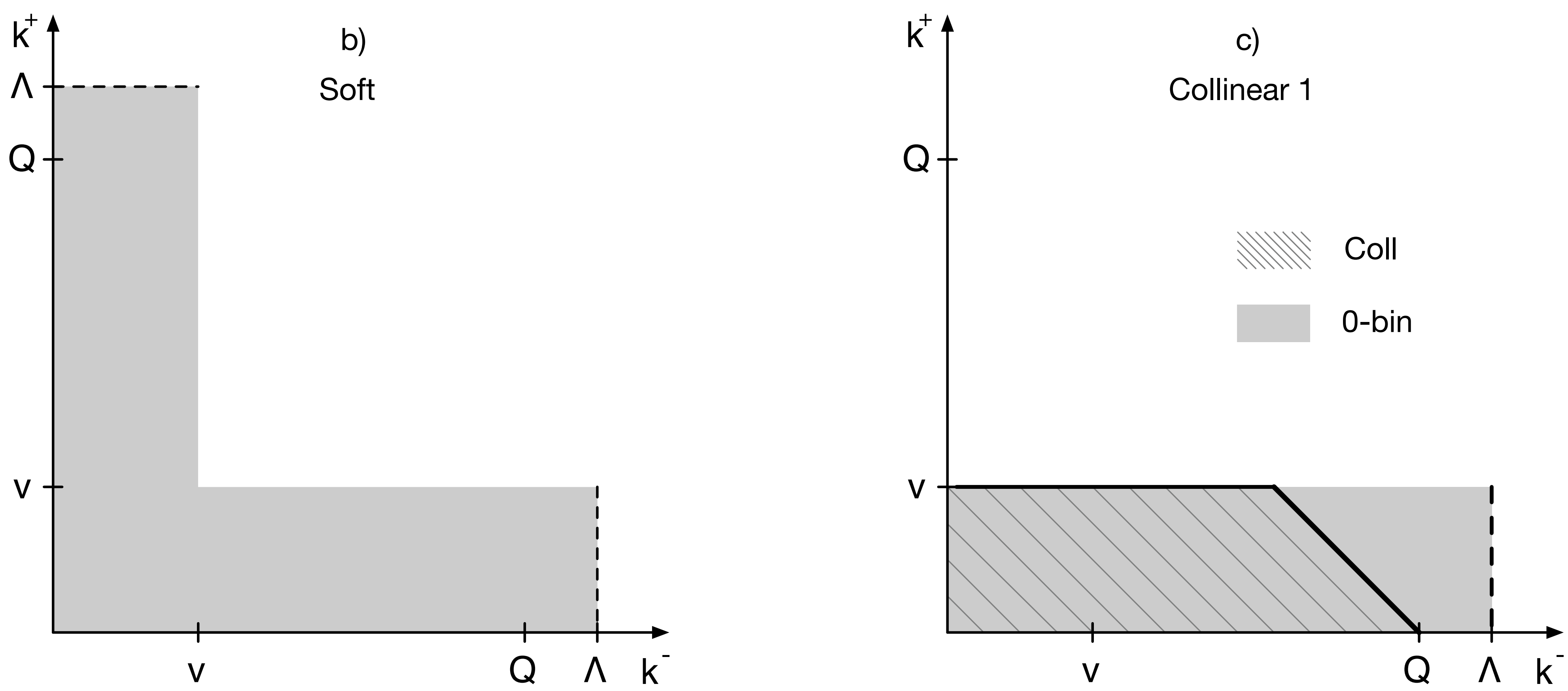

This gives SCET with two UV regulators (just like SCET_{II} with rapidity regulator).

With new regulator, soft and jet functions become (in Laplace)

$$\tilde{S}_{\text{bare}}(u;\mu,\Lambda) = 1 + C_F \frac{\alpha_s}{\pi}\left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{\mu}{\Lambda} - \ln^2\frac{Q\,u_0}{\Lambda\,u} + 2\ln^2\frac{\mu}{\Lambda} - \frac{\pi^2}{4}\right]$$

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Characteristic scales

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Note that jet function is single logarithmic

The transfer function can be computed numerically by literally summing all possible diagrams

Banfi, Salam, Zanderighi ('04)
Banfi, McAslan, Monni, Zanderighi ('14)

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1) To compute transfer function at N^kLL, need ingredients at $N^{k-1}LL$

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$$\mathcal{F}_F(\tau_F, \tau, \mu) = \frac{\Sigma_F^{\max}(\delta\tau, \mu)}{\Sigma_F^{\max}(\tau, \mu)} \frac{\Sigma_F(\tau_F, \mu)}{\Sigma_F^{\max}(\delta\tau, \mu)}$$

1) To compute transfer function at N^kLL, need ingredients at $N^{k-1}LL$

Double logarithmic structure same for Σ and Σ_{max}, one power of log cancels

$$\begin{aligned}\frac{\Sigma_{\max}(\delta v)}{\Sigma_{\max}(v)} &= \exp\left\{L_{\delta v} g_1(\alpha_s L_{\delta v}) - L_v g_1(\alpha_s L_v) + g_2(\alpha_s L_{\delta v}) - g_2(\alpha_s L_v)\right\} \\ &= \exp\left\{L_\delta \left[g_1(\alpha_s L_v) + \alpha_s L_v g_1'(\alpha_s L_v)\right] + \ldots\right\},\end{aligned}$$

The transfer function can be computed numerically by literally summing all possible diagrams

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$$\mathcal{F}_{J_n}^{\mathrm{NLL}}(\tau_n, \tau, Q) = \Theta[\tau_n > 0]$$

The transfer function can be computed numerically by literally summing all possible diagrams

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$$\Sigma^{\rm NLL}(\tau) = \Sigma_{\max}(\tau)\mathcal{F}_S^{\rm NLL}(\tau, \tau, Q)$$

The transfer function can be computed numerically by literally summing all possible diagrams

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1) To compute transfer function at N^kLL, need ingredients at $N^{k-1}LL$

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3) Soft function can be written as sum over all possible diagrams

The transfer function can be computed numerically by literally summing all possible diagrams

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1) To compute transfer function at N^kLL, need ingredients at $N^{k-1}LL$

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3) Soft function can be written as sum over all possible diagrams

$$\Sigma_S(\tau_s, \mu) = \sum_{|k\rangle} |\langle k|Y_n \bar{Y}_{\bar{n}}|0\rangle|^2 \Theta(V_{\text{soft}} < \tau_s))$$

$$= \mathcal{V}_S \sum_n \left[\prod_i \int [\mathrm{d}k_i]\right] |M_S(k_1, \dots k_n)|^2 \, \theta(V_S(k_1, \dots k_n) < \tau_s)$$

The transfer function can be computed numerically by literally summing all possible diagrams

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Can we simplify this if only needed to LL?

The transfer function can be computed numerically by literally summing all possible diagrams

Banfi, Salam, Zanderighi ('04)
Banfi, McAslan, Monni, Zanderighi ('14)

To LL accuracy, only need emissions in the strongly ordered limit

$$\Sigma_S^{\mathrm{LL}}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int [\mathrm{d}k_i]\, |M_S(k_1,\ldots k_n)|^2\, \theta(V_S(k_1,\ldots k_n) < \tau_s)$$

$$= \mathcal{V}_S \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int [\mathrm{d}k_i] |M_S^{(0)}(k_i)|^2 \theta(V_S(k_1,\ldots k_n) < \tau_s)$$

The transfer function can be computed numerically by literally summing all possible diagrams

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For the max observable one can write

$$\Sigma_S^{\mathrm{max,LL}}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [\mathrm{d}k_i] |M_S(k_i)|^2 \theta\left[V_S(k_i) < \tau_s\right]$$

The transfer function can be computed numerically by literally summing all possible diagrams

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To LL accuracy, only need emissions in the strongly ordered limit

$$\Sigma_S^{\rm LL}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int [{\rm d}k_i]\, |M_S(k_1,\dots k_n)|^2\, \theta(V_S(k_1,\dots k_n) < \tau_s)$$

$$= \mathcal{V}_S \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int [{\rm d}k_i] |M_S^{(0)}(k_i)|^2 \theta(V_S(k_1,\dots k_n) < \tau_s)$$

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$$\Sigma_S^{\rm max,LL}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [{\rm d}k_i] |M_S(k_i)|^2 \theta\left[V_S(k_i) < \tau_s\right]$$

Combining these two

$$\mathcal{F}_S^{\rm NLL}(\tau_s,\tau,Q) = \frac{\Sigma_S^{\rm max,LL}(\delta\tau)}{\Sigma_S^{\rm max,LL}(\tau)} \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{\delta\tau} [{\rm d}k_i]\, |M_S^{(0)}(k_i)|^2 \Theta[V_S(k_1,\dots,k_n) < \tau_s]$$

The transfer function can be computed numerically by literally summing all possible diagrams

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regulates IR

The transfer function can be computed numerically by literally summing all possible diagrams

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To LL accuracy, only need emissions in the strongly ordered limit

$$\Sigma_S^{\rm LL}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty}\prod_{i=1}^{n}\int [{\rm d}k_i]\,|M_S(k_1,\dots k_n)|^2\,\theta(V_S(k_1,\dots k_n)<\tau_s)$$

$$= \mathcal{V}_S \sum_{n=0}^{\infty}\prod_{i=1}^{n}\int [{\rm d}k_i]|M_S^{(0)}(k_i)|^2\theta(V_S(k_1,\dots k_n)<\tau_s)$$

For the max observable one can write

$$\Sigma_S^{\rm max,LL}(\tau_s,\mu) = \mathcal{V}_S \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int [{\rm d}k_i]|M_S(k_i)|^2\theta\left[V_S(k_i)<\tau_s\right]$$

Combining these two

$$\mathcal{F}_S^{\rm NLL}(\tau_s,\tau,Q) = \boxed{\frac{\Sigma_S^{\rm max,LL}(\delta\tau)}{\Sigma_S^{\rm max,LL}(\tau)}}\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int_{\boxed{\delta\tau}}[{\rm d}k_i]\,|M_S^{(0)}(k_i)|^2\Theta[V_S(k_1,\dots,k_n)<\tau_s]$$

regulates IR

removes dependence on δ

The transfer function can be computed numerically by literally summing all possible diagrams

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$$\mathcal{F}_S^{\mathrm{NLL}}(\tau_s, \tau, Q) = \frac{\Sigma_S^{\mathrm{max,LL}}(\delta\tau)}{\Sigma_S^{\mathrm{max,LL}}(\tau)} \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{\delta\tau} [\mathrm{d}k_i]\, |M_S^{(0)}(k_i)|^2 \Theta[V_S(k_1, \ldots, k_n) < \tau_s]$$

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Dropping terms that are beyond NLL (look at paper for details)

$$\mathcal{F}_S^{\rm NLL}(\tau_s,\tau,Q) = \delta^{R'_{\rm LL}(\tau)}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=1}^{n}\int_{\delta\tau}^{\tau}\frac{{\rm d}\tau_i}{\tau_i}R'_{\rm LL}(\tau)\right)\Theta\left[\sum_i \tau_i<\tau_s\right]$$

$$= \left[\left(\frac{\tau}{\delta\tau}\right)^{-R'_{\rm LL}(\tau)} + \int_{\delta\tau}^{\tau}\frac{{\rm d}\tau_1}{\tau_1}\left(\frac{\tau}{\tau_1}\right)^{-R'_{\rm LL}(\tau)} R'_{\rm LL}(\tau)\left(\frac{\tau_1}{\delta\tau}\right)^{-R'_{\rm LL}(\tau)} + \dots\right]\Theta\left[\sum_i \tau_i<\tau_s\right]$$

The transfer function can be computed numerically by literally summing all possible diagrams

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$$= \left[\left(\frac{\tau}{\delta\tau}\right)^{-R'_{\rm LL}(\tau)} + \int_{\delta\tau}^{\tau} \frac{{\rm d}\tau_1}{\tau_1} \boxed{\left(\frac{\tau}{\tau_1}\right)^{-R'_{\rm LL}(\tau)} R'_{\rm LL}(\tau)} \left(\frac{\tau_1}{\delta\tau}\right)^{-R'_{\rm LL}(\tau)} + \ldots \right] \Theta\left[\sum_i \tau_i < \tau_s\right]$$

$d[(\tau/\tau_1)^{-R'(\tau)}] / d\ln(\tau_1)$

The transfer function can be computed numerically by literally summing all possible diagrams

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$$\mathcal{F}_S^{\mathrm{NLL}}(\tau_s,\tau,Q) = \frac{\Sigma_S^{\mathrm{max,LL}}(\delta\tau)}{\Sigma_S^{\mathrm{max,LL}}(\tau)} \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{\delta\tau} [\mathrm{d}k_i]\, |M_S^{(0)}(k_i)|^2 \Theta[V_S(k_1,\dots,k_n) < \tau_s]$$

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$$= \left[\left(\frac{\tau}{\delta\tau}\right)^{-R'_{\mathrm{LL}}(\tau)} + \int_{\delta\tau}^{\tau} \frac{\mathrm{d}\tau_1}{\tau_1} \boxed{\left(\frac{\tau}{\tau_1}\right)^{-R'_{\mathrm{LL}}(\tau)} R'_{\mathrm{LL}}(\tau)} \left(\frac{\tau_1}{\delta\tau}\right)^{-R'_{\mathrm{LL}}(\tau)} + \dots \right] \Theta\left[\sum_i \tau_i < \tau_s\right]$$

$\mathrm{d}[(\tau/\tau_1)^{-R'(\tau)}] / \mathrm{d}\ln(\tau_1)$

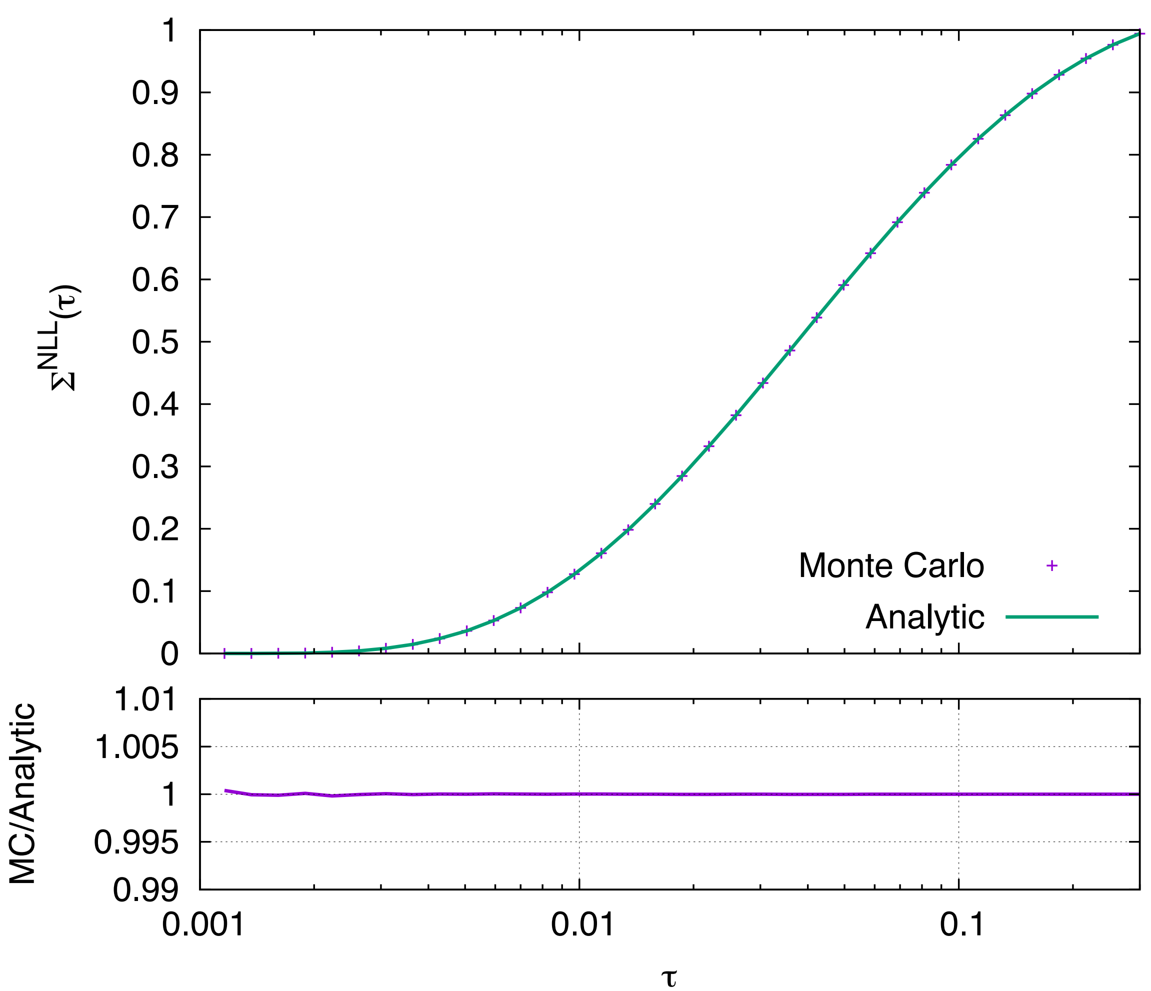
1. Start with i=0 and $\tau_0 = \tau$
2. Increase i by one
3. Generate τ_i randomly according to $(\tau_{i-1}/\tau_i)^{-R'(\tau)} = r$, with $r \in [0,1]$
4. If $\tau_i < \delta\tau$ exit algorithm, otherwise go back to step 2

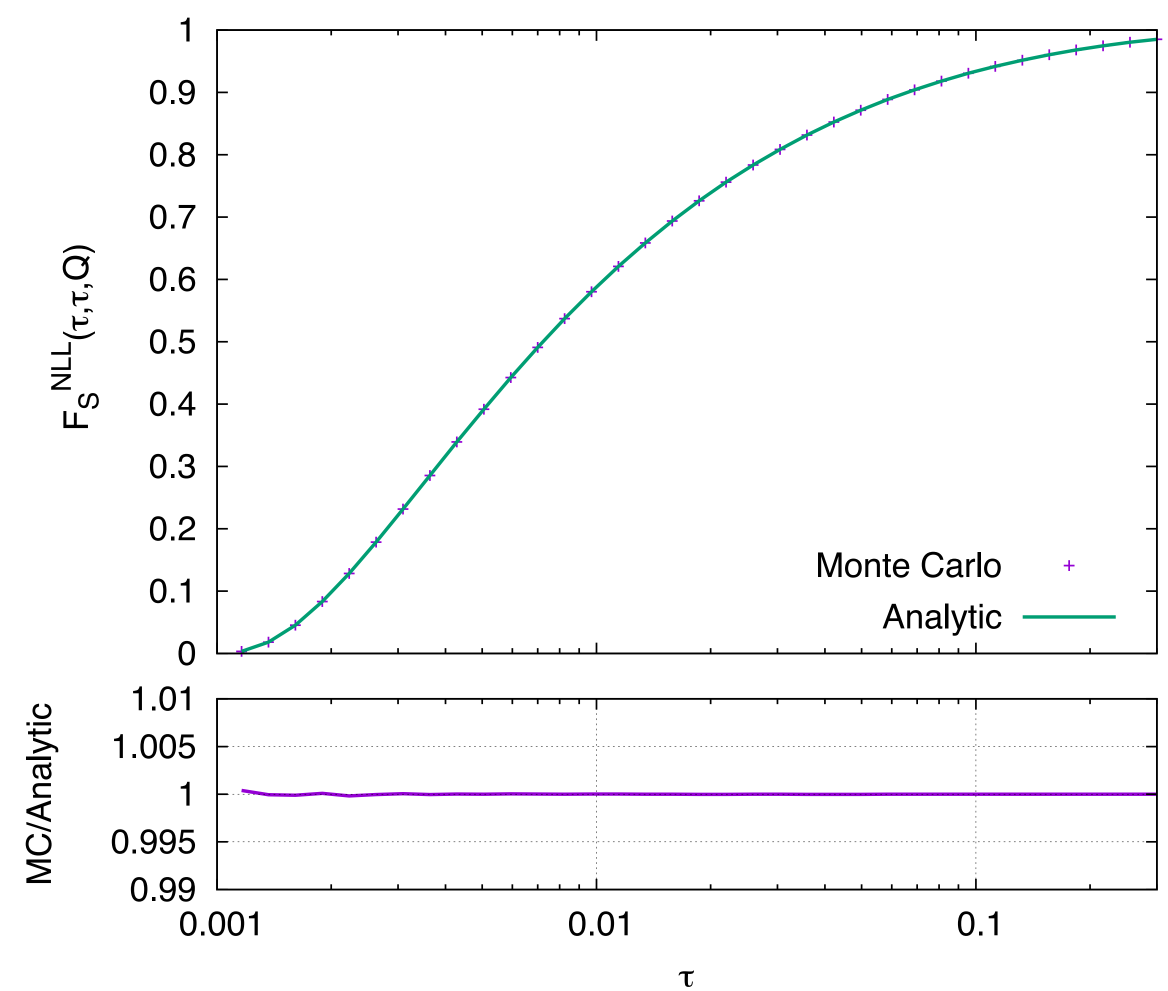
Accept event if $\Sigma_i \tau_i < \tau$

This finally allows to obtain the resummation at NLL order

Putting all information together, one finds

$$\Sigma^{\rm NLL}(\tau) = \Sigma_{\rm max}(\tau)\mathcal{F}_S^{\rm NLL}(\tau,\tau,Q)$$





This approach opens door for resummation for a large class of observables

While I have only discussed NLL, can be extended to higher logarithmic accuracy

1. Find simplified observable for class of observables
2. Compute analytical resummation to given order
3. Run generic numerical algorithm to compute resummation for any observable in given class

Questions?