Ignazio Scimemi (UCM)

2D-evolution and ζ-prescription

Most recent results in collaboration with
Alexey Vladimirov

arXiv:1706.01473 and arXiv:1803….

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....TMD factorization ....

.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A\zeta_B} = Q^2$$

…and similar formulas are valid for SIDIS (EIC) and hadron production in e+e- colliders

The pathological behavior is associated to a particular kind of divergences: rapidity divergences

The renormalization of the rapidity divergences is responsible for the a new resummation scale

We have new non-perturbative effects which cannot be included in PDFs.

The case of unpolarized TMDs:
the perturbative calculable part of unpolarized TMDs is known at NNLO!
How can we use this information?
Which scale prescription allows an optimal extraction of TMD's?
What is the range of validity of the TMD factorization theorem?
Do LHC data have an impact on TMD extraction?

Status of unpolarized TMDs in perturbation theory

Perturbative Calculations

- Evolution to N3LO Y. Li, H.X. Zhu, arXiv:1604.01404 A. Vladimirov, arXiv:1610.05791
- Soft function at NNLO M.G. Echevarría, I.S., A. Vladimirov, arXiv:1511.05590.
- NNLO coefficients for TMDPDFs M.G. Echevarría, I.S., A. Vladimirov, arXiv1604.07869, T. Lübbert, J. Oredsson, M. Stahlhofen, arXiv:1602.01829, T. Gehrmann, T. Lübbert, Li Lin Yang arXiv:1403.6451
- NNLO coefficients for TMD Fragmentation Functions M.G. Echevarría, I.S., A. Vladimirov, arXiv:1509.06392, arXiv:1604.07869

Phenomenology

- Global Fits (SIDIS+DY) A. Bacchetta et al. arxiv:1703.10157,
- DY and Z-boson fits (ResBos, D'Alesio et al. arXiv:1410.4522 up to NNLL)
- Implementation of standard CSS (DYres/DyqT, Cute)
- LHC data
- TMD extraction using higher order corrections (ARTEMIDE) arXiv:1706.01473

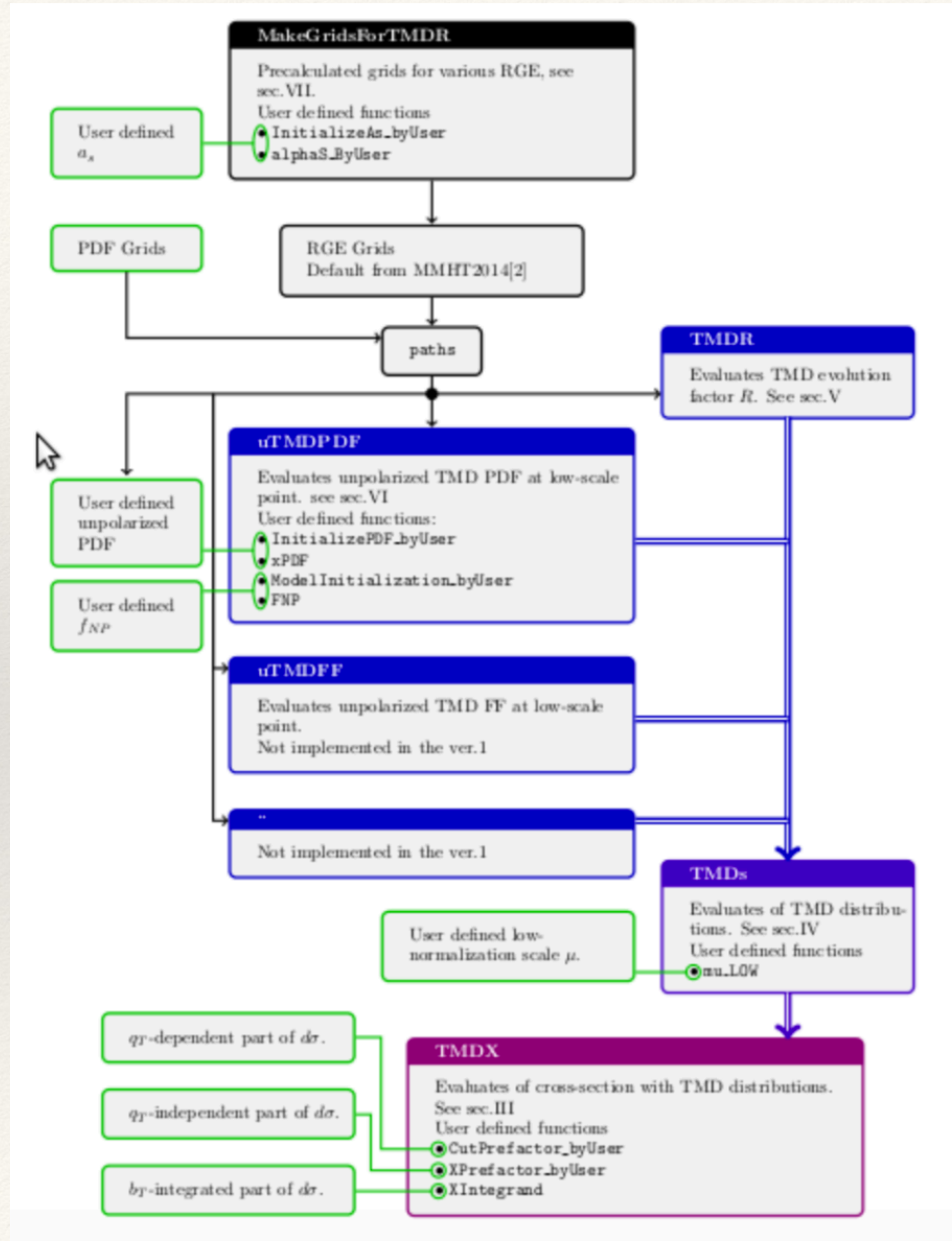
ζ-prescription

IT IS POSSIBLE TO MAKE A COMPLETE ANALYSIS OF UNPOLARIZED TMD IN DRELL-YAN AND SIDIS USING **NNLO** RESULTS

The study of polarized TMDs at the same precision is just started (see Daniel Gutierrez talk):

D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558

arTeMiDe



- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6. min at NNLO)

Currently ver 1.1

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs

Cross section and TMD structure

Lepton tensor cuts

E.w. charges

Hard Coefficient

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}|C_V(q,\mu)|^2 \int \frac{d^2\vec{b}}{4\pi} e^{i(\vec{b}\vec{q})}$$

$$\times F_{f\leftarrow h_1}(x_1,\vec{b};\mu,\zeta) F_{f'\leftarrow h_2}(x_2,\vec{b};\mu,\zeta) + Y,$$

Y-term 1/Q^2 corrections

Evolution factor

Low energy TMD

$$F_{f\leftarrow h}(x,\mathbf{b};\mu_f,\zeta_f) = R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)] F_{f\leftarrow h}(x,\mathbf{b};\mu_i,\zeta_i)$$

$$R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)] = \exp\left[\int_P \left(\gamma_F^f(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^f(\mu,\mathbf{b})\frac{d\zeta}{\zeta}\right)\right]$$

$$F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = \sum_q \int_x^1 \frac{dz}{z} C_{f\leftarrow q}(z,\mathbf{L}_\mu;\mu,\zeta) f_{q\leftarrow h}\left(\frac{x}{z},\mu\right) f_{NP}(z,\mathbf{b})$$

Matching (Wilson) coefficient

PDF

Non-perturbative input

Cross section and TMD structure

Lepton tensor cuts

E.w. factors

Hard Coefficient

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}|C_V(q,\mu)|^2\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}$$

$$\times F_{f\leftarrow h_1}(x_1,\vec{b};\mu,\zeta)F_{f'\leftarrow h_2}(x_2,\vec{b};\mu,\zeta) + Y,$$

Only small qT data

Evolution factor

Low energy TMD

$$F_{f\leftarrow h}(x,\mathbf{b};\mu_f,\zeta_f) = R^f[\mathbf{b};(\mu_f,\zeta_f)\leftarrow(\mu_i,\zeta_i)]F_{f\leftarrow h}(x,\mathbf{b};\mu_i,\zeta_i)$$

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$$F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = \sum_q\int_x^1\frac{dz}{z}C_{f\leftarrow q}(z,\mathbf{L}_\mu;\mu,\zeta)f_{q\leftarrow h}\left(\frac{x}{z},\mu\right)f_{NP}(z,\mathbf{b})$$

ζ-prescription

Matching (Wilson) coefficient

PDF

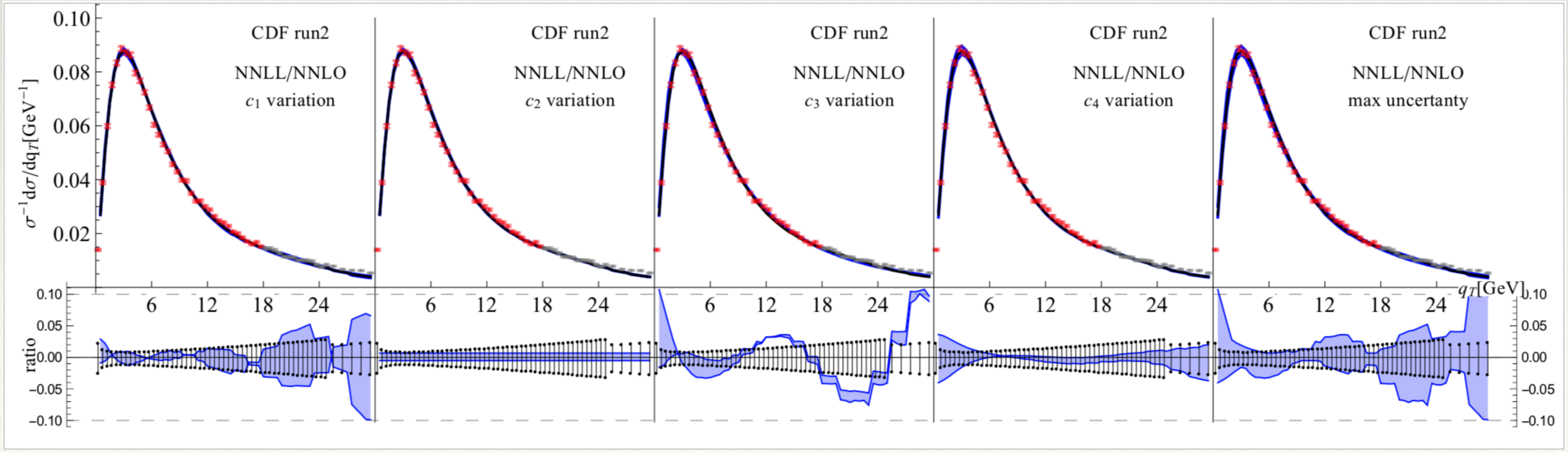
Gaussian? Exponential?

How can we compare different extractions of TMDs?

- Can we compare different extractions of TMDs?
- What kind of non-perturbative information can we really extract when we use a particular TMD implementation?

The problem goes beyond the usual variation of scales

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}|C_V(Q,c_2Q)|^2\Big\{R^f[\vec{b};(c_2Q,Q^2)\to(c_3\mu_i,\zeta_{c_3\mu_i});c_1\mu_i]\Big\}$$
$$\times F_{f\leftarrow h_1}(x,\vec{b};c_4\mu_{\rm OPE},\zeta_{c_4\mu_{\rm OPE}})F_{f'\leftarrow h_2}(x,\vec{b};c_4\mu_{\rm OPE},\zeta_{c_4\mu_{\rm OPE}})$$



Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

TMD (standard) anomalous dimension

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}(\mu,b) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

TMD rapidity anomalous dimension

Collinear overlap

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu,\zeta) = -\Gamma(\mu)$$

$$\mu \frac{d}{d\mu} \mathcal{D}(\mu,b) = \Gamma(\mu)$$

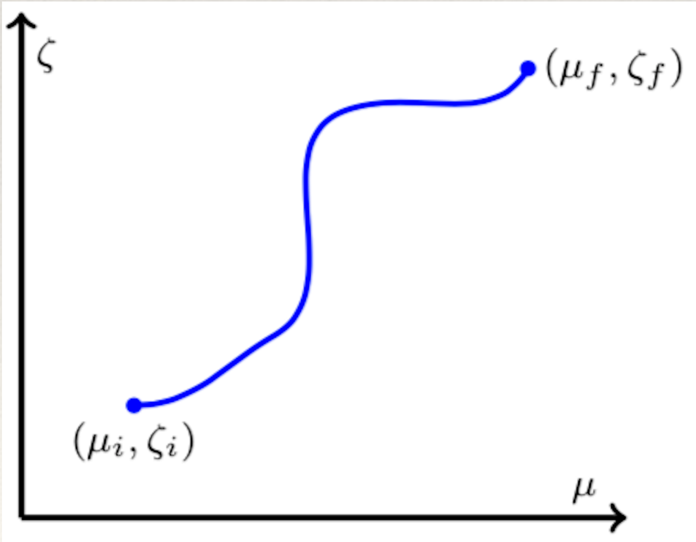
Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

Integrability Condition…

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\mu \frac{d}{d\mu}\mathcal{D}(\mu,b)$$

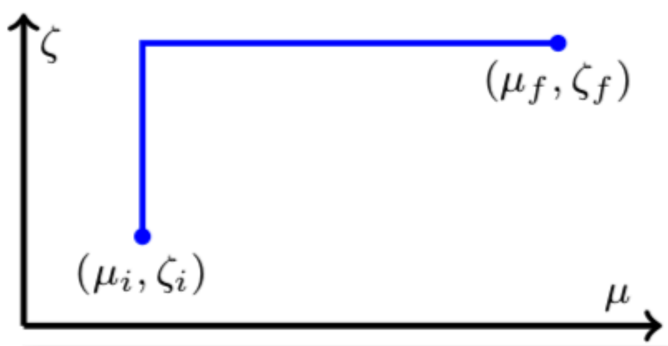
…ensures the path independence of the evolution factor…



$$R[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)] = \exp\left[\int_P \left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right]$$

Ambiguity in the TMD evolution

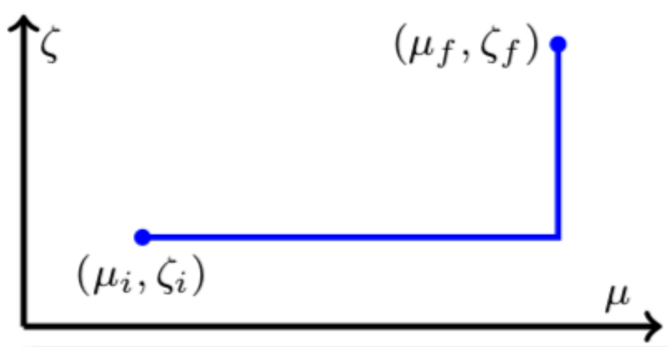
Coupled evolution of TMD and truncation of the perturbative series



Solution 1

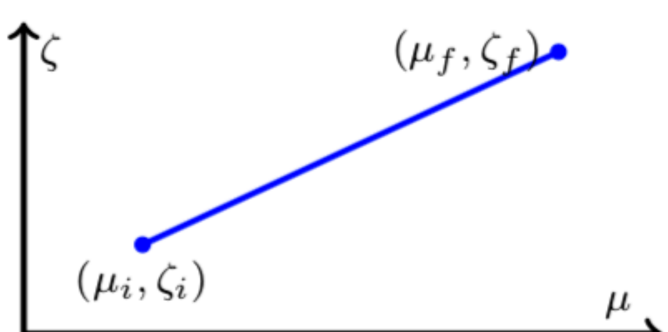
$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

given in [Collins' textbook]



Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_i) - \mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$



Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t), \zeta(t)) \frac{\mu_f - \mu_i}{(\mu_f - \mu_i)t + \mu_i} - \mathcal{D}(\mu(t), b) \frac{\zeta_f - \zeta_i}{(\zeta_f - \zeta_i)t + \zeta_i}\Big) dt$$

Ambiguity in the TMD evolution

Coupled evolution of TMD and truncation of the perturbative series

In practice due to the truncation of the perturbative series:
Transitivity and reversibility of evolution is lost

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) \neq -\mu \frac{d}{d\mu}\mathcal{D}(\mu,b)$$

For Q=Mz the solution path dependence enormous:
at b=0.5 an error of about 18% at N3LO on the evolution factor R

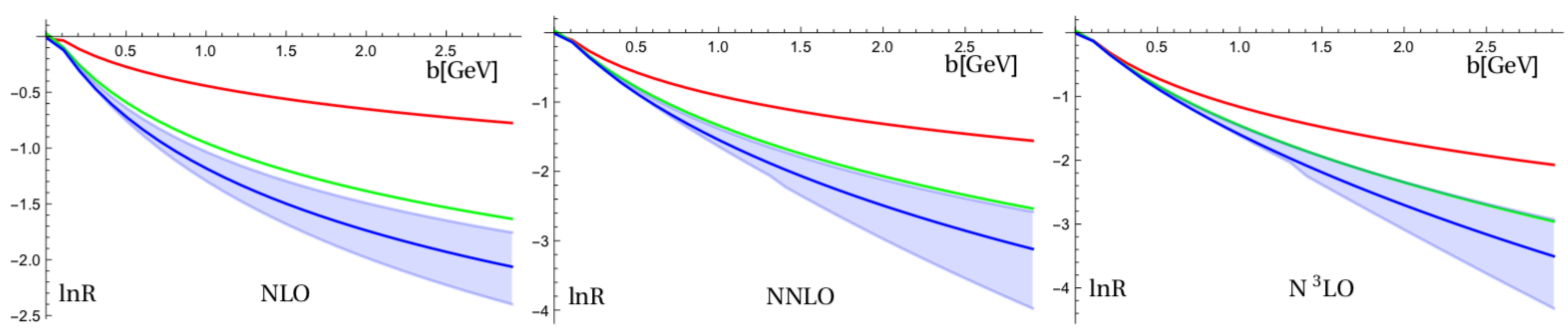


FIG. 3: Comparisons of different solution for $\ln R((M_Z, M_Z^2) \to (\mu_b, \mu_b^2))$ where $\mu_b = C_0/b + 2$. The blue line is the solution 1. The red Line is the solution 2. The green line is the solution 3. The error band is obtained from the improved $\mathcal{D}$ solution at $\mu_0 = \mu_i$ by variation of $\mu_0 \in (0.5, 2)\mu_i$. The blue line with error-band corresponds to the solution used in [18].

2D Evolution field: Notation and ideal case

The evolution scales are treated on equally

$$\vec{\nu} = \left(\ln \frac{\mu^2}{1\ \mathrm{GeV}^2}, \ln \frac{\zeta}{1\ \mathrm{GeV}^2} \right)$$

Differentiation

$$\vec{\nabla} = \frac{d}{d\vec{\nu}} = (\mu^2 \frac{d}{d\mu^2}, \zeta \frac{d}{d\zeta}), \qquad \mathbf{curl} = (-\zeta \frac{d}{d\zeta}, \mu^2 \frac{d}{d\mu^2})$$

Evolution field

$$\mathbf{E}(\vec{\nu}, b) = (\frac{\gamma_F(\vec{\nu})}{2}, -\mathcal{D}(\vec{\nu}, b))$$

TMD Evolution

$$\vec{\nabla} F(x, b; \vec{\nu}) = \mathbf{E}(\vec{\nu}, b) F(x, b; \vec{\nu})$$

Integrability Condition and Scalar Potential

$$\vec{\nabla} \times \mathbf{E} = 0 \Rightarrow \mathbf{E}(\vec{\nu}, b) = \vec{\nabla} U(\vec{\nu}, b)$$

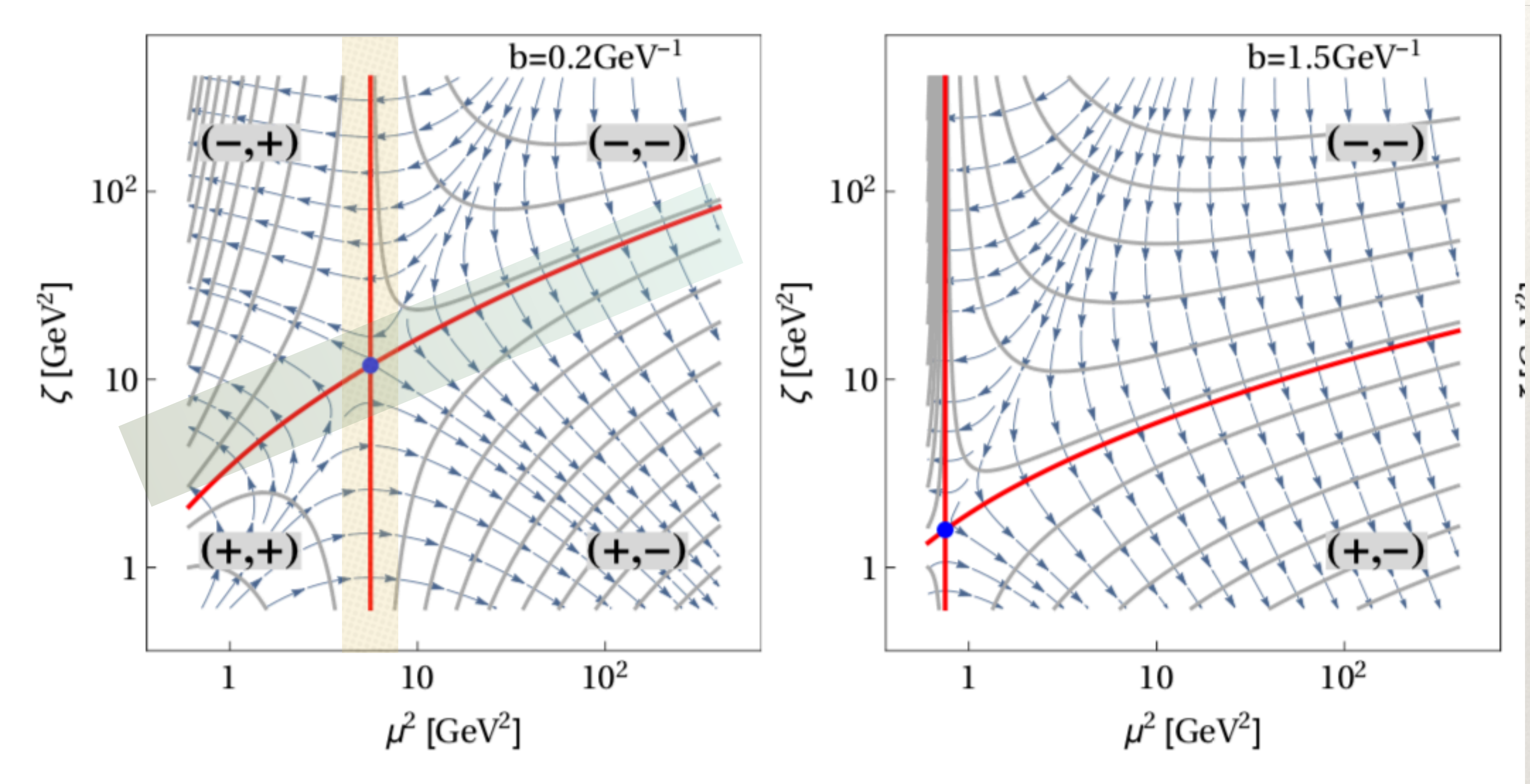
Evolution kernel

$$\ln R[b; \vec{\nu}_f \to \vec{\nu}_i] = U(\vec{\nu}_f, b) - U(\vec{\nu}_i, b)$$

with

$$U(\vec{\nu}, b) = \int^{\nu_1} \frac{\Gamma(s)s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu}, b)\nu_2 + \mathrm{const}(b)$$

2D Evolution field: Notation and ideal case



● =saddle point

Singularities: Landau pole (on the left, not shown) and saddle point $\mathbf{E}(\vec{\nu}_{\text{saddle}}, b) = \vec{0}$

Equipotential/null-evolution curves: $\vec{\omega}(t, \vec{\nu}_B, b) = (t, \omega(t, \vec{\nu}_B, b)) \quad \rightarrow \frac{d\vec{\omega}}{dt} \cdot \vec{\nabla} U(\vec{\omega}, b) = 0$

Special null-evolution curves: $\mu = \mu_{\text{saddle}}$ and $\vec{\nu}_B = \vec{\nu}_{\text{saddle}}$

Truncation of the perturbative series

The truncation induces a difference

$$\delta\Gamma(\mu,b)=\Gamma(\mu)-\mu\frac{d\mathcal{D}(\mu,b)}{d\mu},$$

$$\delta\Gamma^{(N)}=2\sum_{n=1}^{N}\sum_{k=0}^{n}n\bar{\beta}_{n-1}(a_s)a_s^{n-1}d^{(n,k)}\mathbf{L}_\mu^k$$

$$\text{with }\bar{\beta}_n(a_s)=\beta(a_s)-\sum_{k=0}^{n-1}\beta_k a_s^{k+2}$$

$$\mathbf{L}_\mu=\ln\left(\frac{X^2b^2}{4e^{-2\gamma_E}}\right)$$

$$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu^N)\text{ with perturbative }D$$

$$\delta\Gamma^{(N)}\sim\mathcal{O}(a_s^{N+1}\mathbf{L}_\mu)\text{ with resummed }D$$

$$\ln\frac{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_1}\{\mu_2,\zeta_2\}]}{R[b;\{\mu_1,\zeta_1\}\xrightarrow{P_2}\{\mu_2,\zeta_2\}]}=\frac{1}{2}\int_{\Omega(P_1\cup P_2)}d^2\nu\,\delta\Gamma(\vec{\nu},b)=\int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\delta\Gamma(\mu,b)\ln\left(\frac{\zeta_1(\mu)}{\zeta_2(\mu)}\right)$$

The path dependence is enhanced by the difference in rapidity scale

At large value of impact parameter the breaking of integrability condition becomes crucial

Recovering path independence

Helmholtz decomposition of evolution fields

$$\mathbf{E}(\vec{\nu},b) = \tilde{\mathbf{E}}(\vec{\nu},b) + \mathbf{\Theta}(\vec{\nu},b)$$

Basic properties of evolution fields

$$\text{curl}\tilde{\mathbf{E}} = 0, \qquad \vec{\nabla}\cdot\vec{\Theta} = 0, \qquad \tilde{\mathbf{E}}\cdot\mathbf{\Theta} = 0.$$

Scalar potentials

$$\tilde{\mathbf{E}}(\vec{\nu},b) = \vec{\nabla}\tilde{U}(\vec{\nu},b) \qquad \mathbf{\Theta}(\vec{\nu},b) = \mathbf{curl}\, V(\vec{\nu},b)$$

Ideally one could repair the truncation using decomposition of the evolution field

$$\text{curl}\mathbf{E} = \text{curl}\mathbf{\Theta} = \frac{\delta\Gamma(\vec{\nu},b)}{2} \neq 0$$

The integrability condition is re-established defining the evolution kernel as

$$\ln R[b;\vec{\nu}_f \to \vec{\nu}_i] = \tilde{U}(\vec{\nu}_f,b) - \tilde{U}(\vec{\nu}_i,b)$$

$$\nabla^2\tilde{U}(\vec{\nu},b) = \frac{1}{2}\frac{d\gamma_F(\vec{\nu})}{d\nu_1}$$

However in order to fix completely the evolution potential one needs boundary condition for the evolution field:
at the moment no theoretically solid non-perturbative input is known

Recovering path independence

We modify anomalous dimensions such that integrability restored

$$\mu \frac{d\mathcal{D}(\mu,b)}{d\mu} = -\zeta \frac{d\gamma_F(\mu,\zeta)}{d\zeta}$$

It can be done from both sides of the equation.

Improved $\mathcal{D}$

Facilitate

$$\mu \frac{d\mathcal{D}}{d\mu} = \Gamma.$$

by

$$\mathcal{D}(\mu,b) = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu}\Gamma(\mu) + \mathcal{D}(\mu_0,b)$$

- In the spirit of [Collins' text book].
- Already used in many studies
- However, it is not the best way

Improved γ

We set

$$\zeta \frac{d\gamma_F}{d\zeta} \equiv -\mu \frac{d\mathcal{D}}{d\mu} = \delta\Gamma - \Gamma$$

Or

$$\gamma_F(\mu,\zeta) \to \gamma_M(\mu,\zeta,b)$$

$$\gamma_M = (\Gamma - \delta\Gamma)\ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V$$

- Completely self consistent
- Very natural

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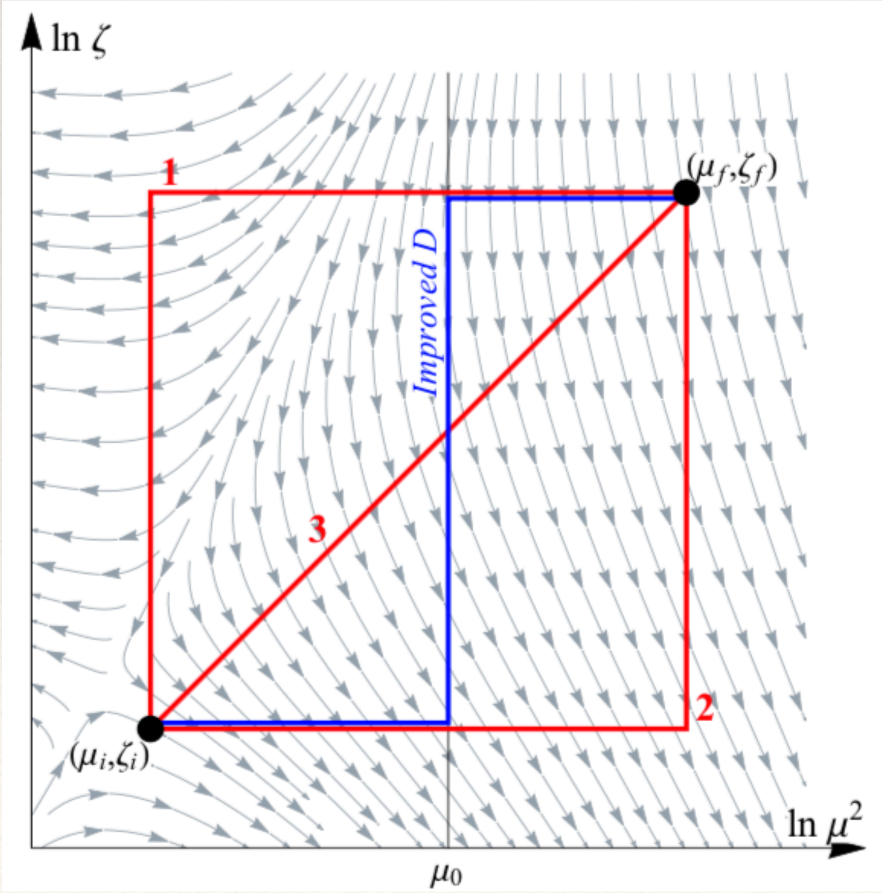
Improved D scenario

$$\mathcal{D}(\mu,b)=\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma(\mu')+\mathcal{D}(\mu_0,b) \longrightarrow \tilde{U}(\vec{\nu},b;\mu_0)=\int_{\ln\mu_0^2}^{\nu_1}\frac{\Gamma(s)(s-\nu_2)-\gamma_V(s)}{2}ds-\mathcal{D}(\mu_0,b)\nu_2+const(b)$$

The truncation effects should be minimized by the choice of μ_0

$$\ln R[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i);\mu_0]=\int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\left(\Gamma(\mu)\ln\left(\frac{\mu^2}{\zeta_f}\right)-\gamma_V(\mu)\right)-\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\Gamma(\mu)\ln\left(\frac{\zeta_f}{\zeta_i}\right)-\mathcal{D}(\mu_0,b)\ln\left(\frac{\zeta_f}{\zeta_i}\right).$$

This is a mixture of solution 1 and 2. The solution dependence is parameterized by μ_0



The minimization occurs only when one finds a μ_0 such that

$$\delta\Gamma(\mu_0,b)=0$$

In order to compare fits one should agree on a conventional μ_0 scale

Improved γ scenario

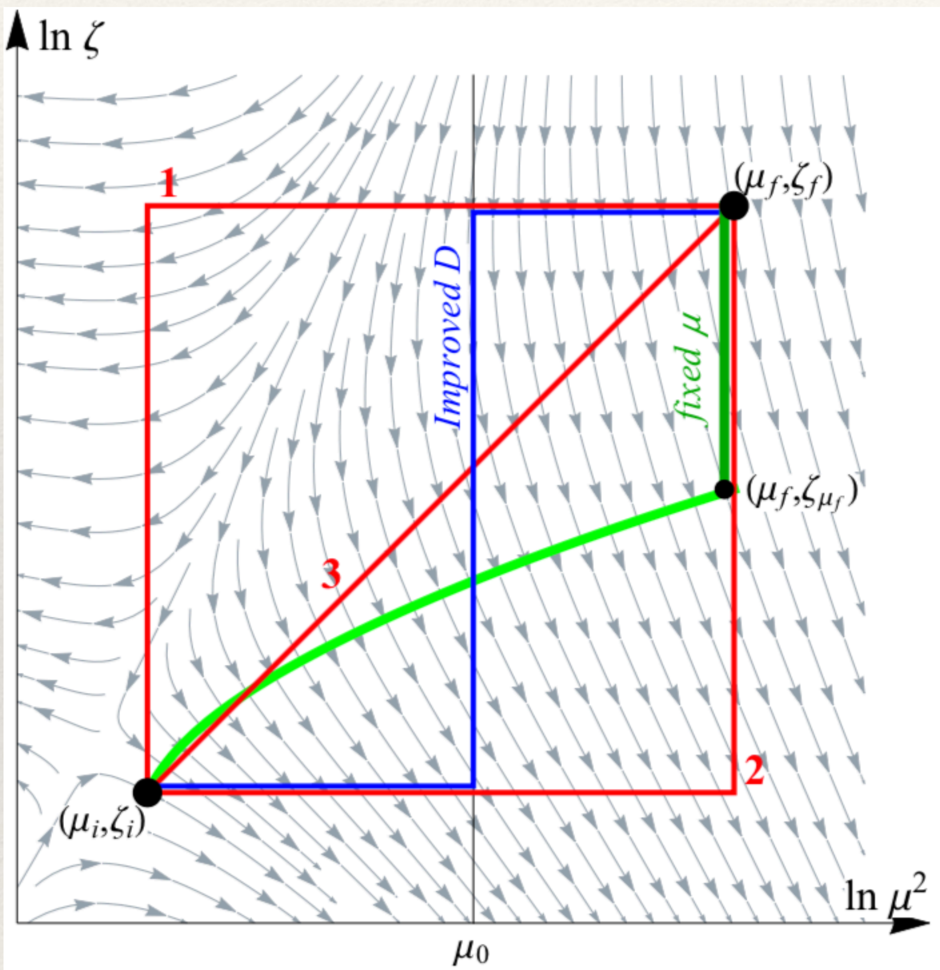
$$\gamma_M(\mu,\zeta,b) = (\Gamma(\mu) - \delta\Gamma(\mu,b))\mathbf{l}_\zeta - \gamma_V(\mu) \longrightarrow \tilde{U}(\vec{\nu},b) = \int^{\nu_1} \frac{(\Gamma(s) - \delta\Gamma(s,b))s - \gamma_V(s)}{2} ds - \mathcal{D}(\vec{\nu},b)\nu_2 + const(b)$$

$$\ln R[b;(\mu_f,\zeta_f) \to (\mu_i,\zeta_i)] = -\int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu}\left(2\mathcal{D}(\mu,b) + \gamma_V(\mu)\right) + \mathcal{D}(\mu_f,b)\ln\left(\frac{\mu_f^2}{\zeta_f}\right) - \mathcal{D}(\mu_i,b)\ln\left(\frac{\mu_i^2}{\zeta_i}\right)$$

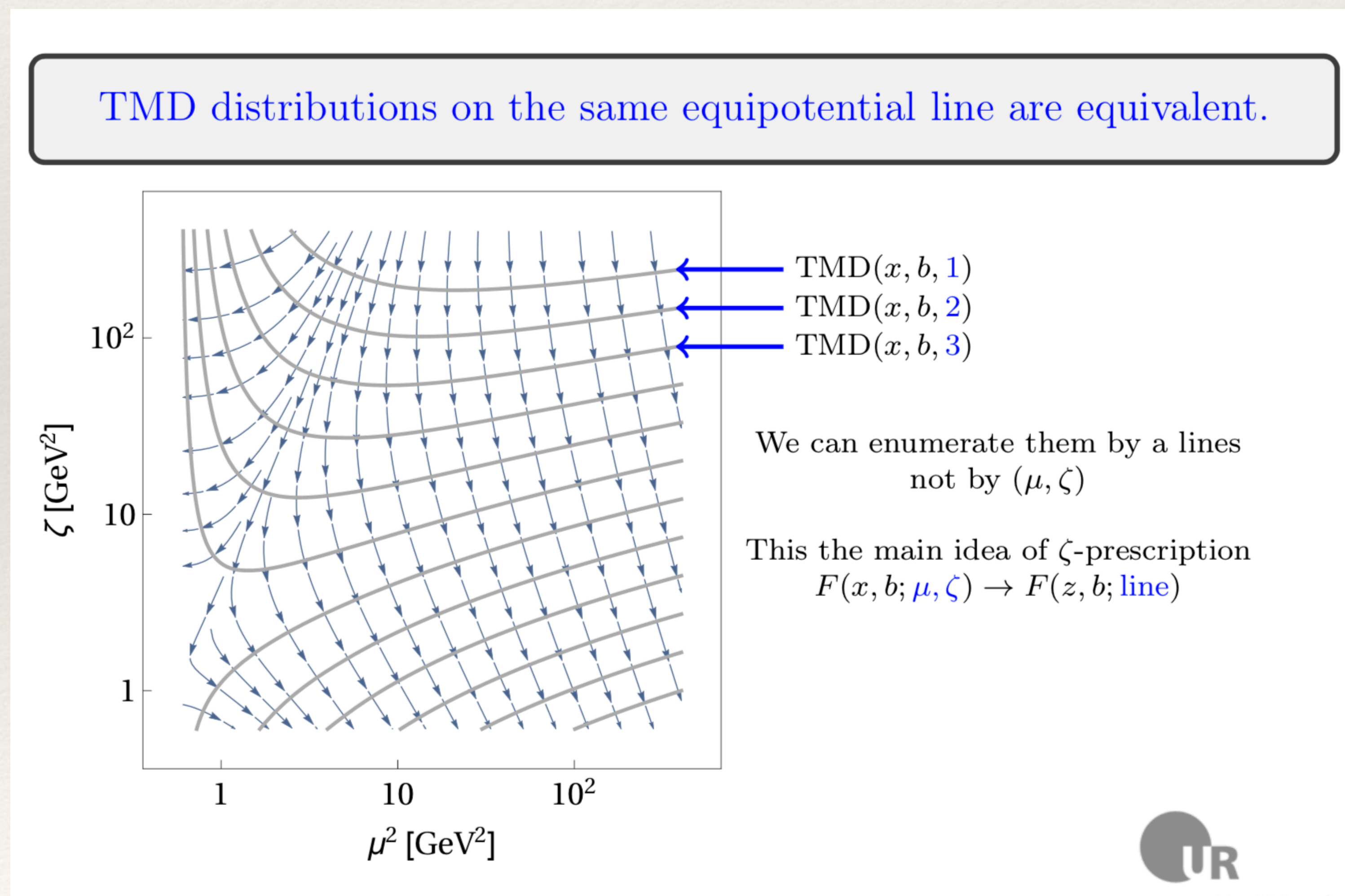
Clear advantages:

- No more the intermediate scale μ_0
- Path independence
- Simplicity
- It can be combined with non-perturbative models for D (at large b)

ζ-prescription



We can provide evolution first on an *equi-potential line* and then on a vertical line.



ζ-prescription

We have just to evolve from an equi-potential/null-evolution line to the final point

$$F(x,b;\mu_f,\zeta_f) = R[b;(\mu_f,\zeta_f)\to(\mu_f,\zeta_{\mu_f}(\vec{\nu}_B,b))]F(x,b;\vec{\nu}_B)$$

This is realized choosing $\zeta_\mu(b)$ such that

$$\frac{\gamma_F(\mu,\zeta_\mu(b))}{2\mathcal{D}(\mu,b)} = \frac{\mu^2}{\zeta_\mu(b)}$$

$$\mu^2\frac{dF(x,\mathbf{b};\mu,\zeta_\mu)}{d\mu^2} = 0.$$

$$F_{f\leftarrow h}(x,\vec{b};\mu,\zeta)=\sum_q\int_x^1\frac{dz}{z}C_{f\leftarrow q}(z,\mathbf{L}_\mu;\mu,\zeta)f_{q\leftarrow h}\left(\frac{x}{z},\mu\right)f_{NP}(z,\mathbf{b})$$

ζ-prescription

In this prescription the structure of coefficient is much simpler

$$C_{q\leftarrow q}(x,\mathbf{L}_\mu;\mu,\zeta_\mu)=\delta(\bar{x})+a_s(\mu)C_F\Big[-2\mathbf{L}_\mu\Big(\frac{2}{(1-x)_+}-1-x\Big)+2\bar{x}+\delta(\bar{x})\Big(-3\mathbf{L}_\mu-\frac{\pi^2}{6}\Big)\Big]+\cdots$$

We do not introduce undesired power corrections

We have several proof of scale stability: TMD area, ...

$$\int_0^1 dx C_{q\leftarrow q}(x,\mathbf{L}_\mu;\mu,\zeta_\mu)=1+a_s(\mu)C_F\left(1-\frac{\pi^2}{6}\right)+\cdots$$

Cancellation of logs

$$\mu^2\frac{d}{d\mu^2}C_{f\leftarrow f'}(x,\mathbf{b};\mu,\zeta_\mu)\otimes f_{f'\leftarrow h}(x,\mu)=0\ .$$

TMD on equi-potential lines

The TMDs one equi-potential lines are not evolved so one can define a TMD by a single parameter line

$$F(x,b;\vec{\nu}_B) = F(x,b;\vec{\nu}'_B), \qquad \vec{\nu}'_B \in \vec{\omega}(\vec{\nu}_B, b).$$

ONE CAN HAVE AN EVOLUTION ONLY WHEN MOVING BETWEEN DIFFERENT LINES

$$F(x,b;\vec{\nu}_B) = R[b;\vec{\nu}_B \to \vec{\nu}'_B]F(x,b;\vec{\nu}'_B)$$

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ONE CAN HAVE AN EVOLUTION ONLY WHEN MOVING BETWEEN DIFFERENT LINES

$$F(x,b;\vec{\nu}_B) = R[b;\vec{\nu}_B \to \vec{\nu}'_B]F(x,b;\vec{\nu}'_B)$$

Outcome: the modeling of the non-perturbative part of the TMD does not depend anymore on the relation between renormalization scale and impact parameter.

Question: Is there a preferred line?

The optimal TMD distribution

There is a consistency constraint in the TMD matching to PDFs

$$F_{f\to k}(x,b;\vec{\nu}_B)=\sum_n\sum_{f'}C^{(n)}_{f\to f'}(x,b,\vec{\nu}_B,\mu_{\rm OPE})\otimes f^{(n)}_{f'\to h}(x,\mu_{\rm OPE})$$

The values of $\mu_{\rm OPE}$ are restricted to the values of μ taken along the null-evolution curve

$$\begin{aligned}
\text{if } \nu_{B,1}<\ln\mu^2_{\rm saddle} \Rightarrow &\quad \mu_{\rm OPE}<\mu_{\rm saddle},\\
\text{if } \nu_{B,1}>\ln\mu^2_{\rm saddle} \Rightarrow &\quad \mu_{\rm OPE}>\mu_{\rm saddle},\\
\text{if } \vec{\nu}_B=(\ln\mu^2_{\rm saddle},\ln\zeta_{\rm saddle}) \Rightarrow &\quad \mu_{\rm OPE}\ \text{unrestricted}
\end{aligned}$$

Perturbative orders…

name	$\mathcal{D}$	γ_V	H	$C_{f\leftarrow f'}$	a_s(run)	PDF (evolution)
LO	a_s^1	a_s^1	a_s^0	a_s^0	lo	lo
NLO	a_s^2	a_s^2	a_s^1	a_s^1	nlo	nlo
NNLO	a_s^3	a_s^3	a_s^2	a_s^2	nnlo	nnlo

…Theoretical uncertainties…

Matching Scales

In the implementation we must choose matching prescriptions such that the perturbative series is as convergent as possible, undesired power corrections are not introduced

Hard Scale

Low Scale

$$\frac{d\sigma}{dQ^2dyd(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{ff'}^{GG'}\int\frac{d^2\vec{b}}{4\pi}e^{i(\vec{b}\vec{q})}|C_V(Q,c_2Q)|^2\Big\{R^f[\vec{b};(c_2Q,Q^2)\to(c_3\mu_i,\zeta_{c_3\mu_i});c_1\mu_i]\Big\}$$

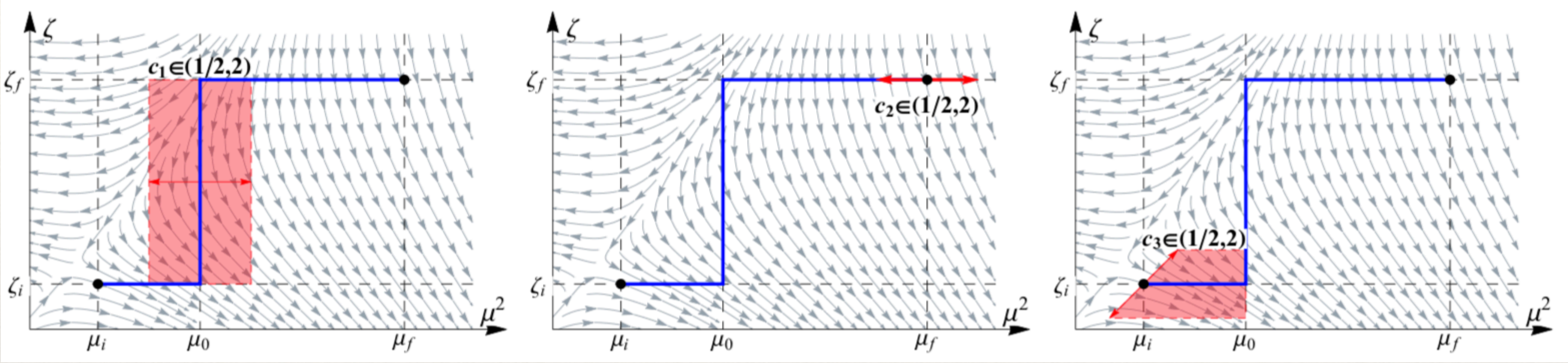
$$\times F_{f\leftarrow h_1}(x,\vec{b};c_4\mu_{\mathrm{OPE}},\zeta_{c_4\mu_{\mathrm{OPE}}})F_{f'\leftarrow h_2}(x,\vec{b};c_4\mu_{\mathrm{OPE}},\zeta_{c_4\mu_{\mathrm{OPE}}})$$

Rapidity Evolution

Small b Scale

Parameters and quality of the fits depend strongly on the choices made for the implementation

Details of scale variations



- ~~c_1 measure only solution dependence~~
- c_2 measure mismatch between H and R + solution dependence
- ~~c_3 measure mismatch between F and R + solution dependence~~
- c_4 measure mismatch between C and f

Eliminated by gamma-scenario

Eliminated by optimal TMD definition

A new error analysis: LHC

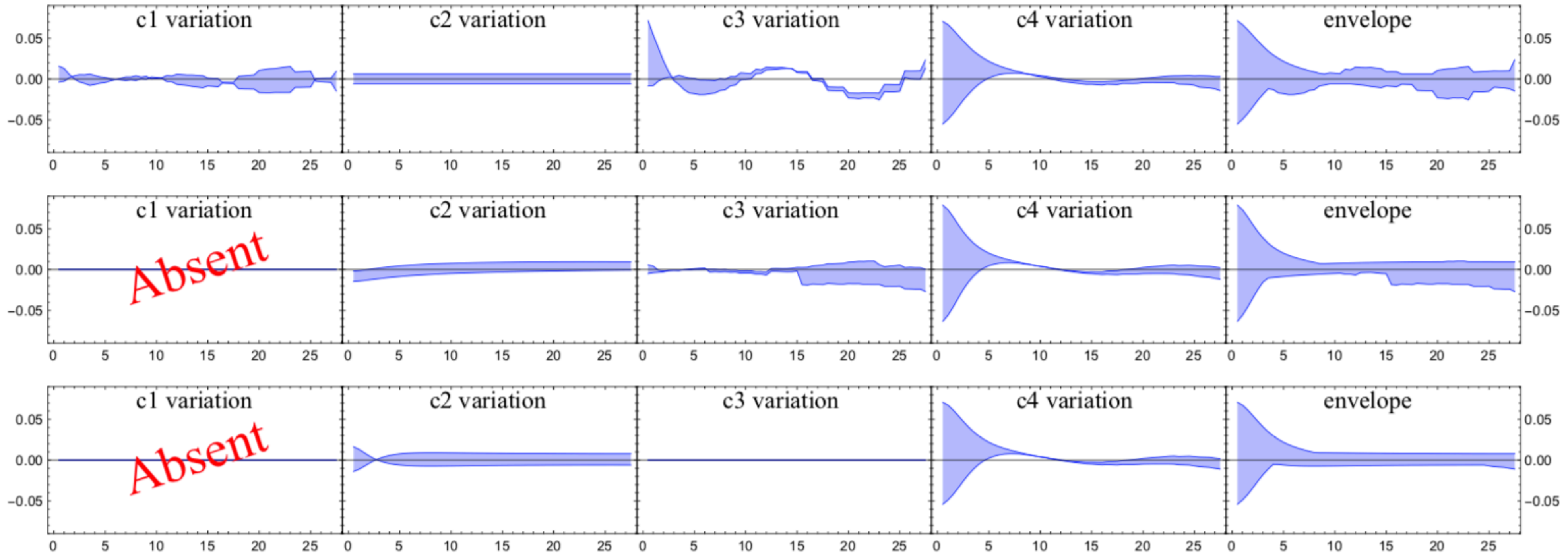


FIG. 10: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for the Z-boson production measure at ATLAS at 8 TeV [29].

A new error analysis: CDF

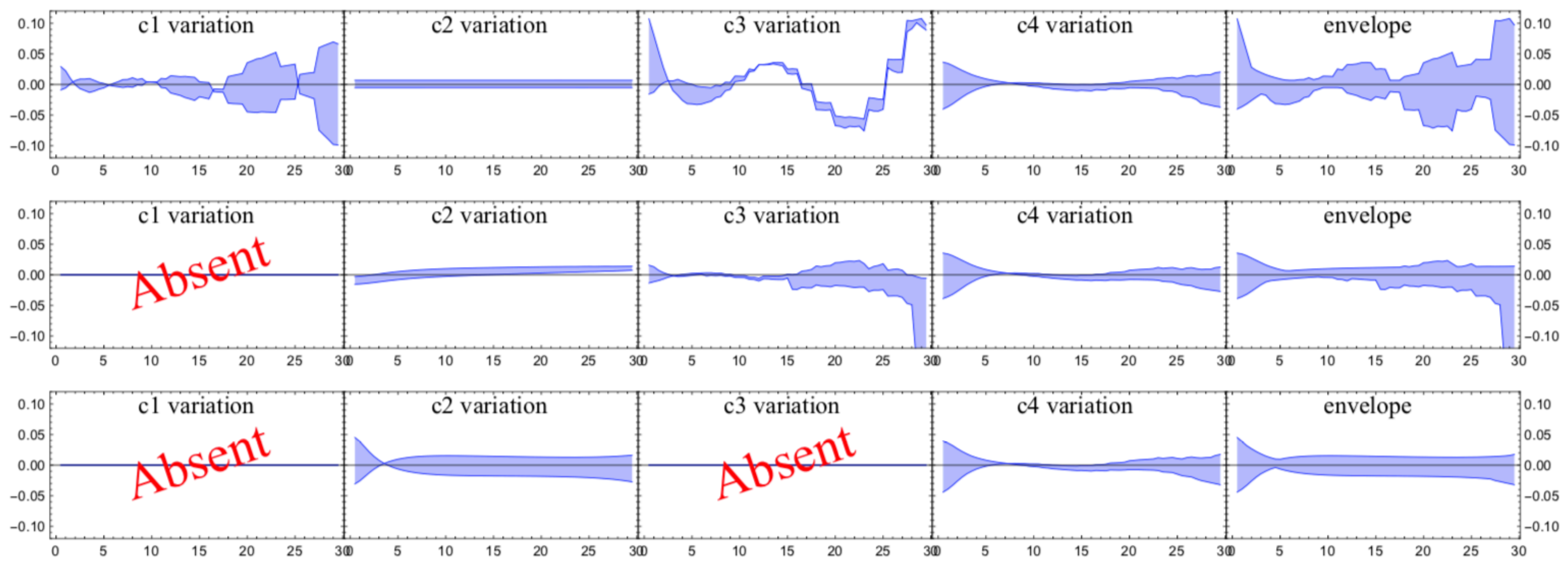


FIG. 9: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for the Z-boson production at CDF at run 2 [36].

A new error analysis: E288

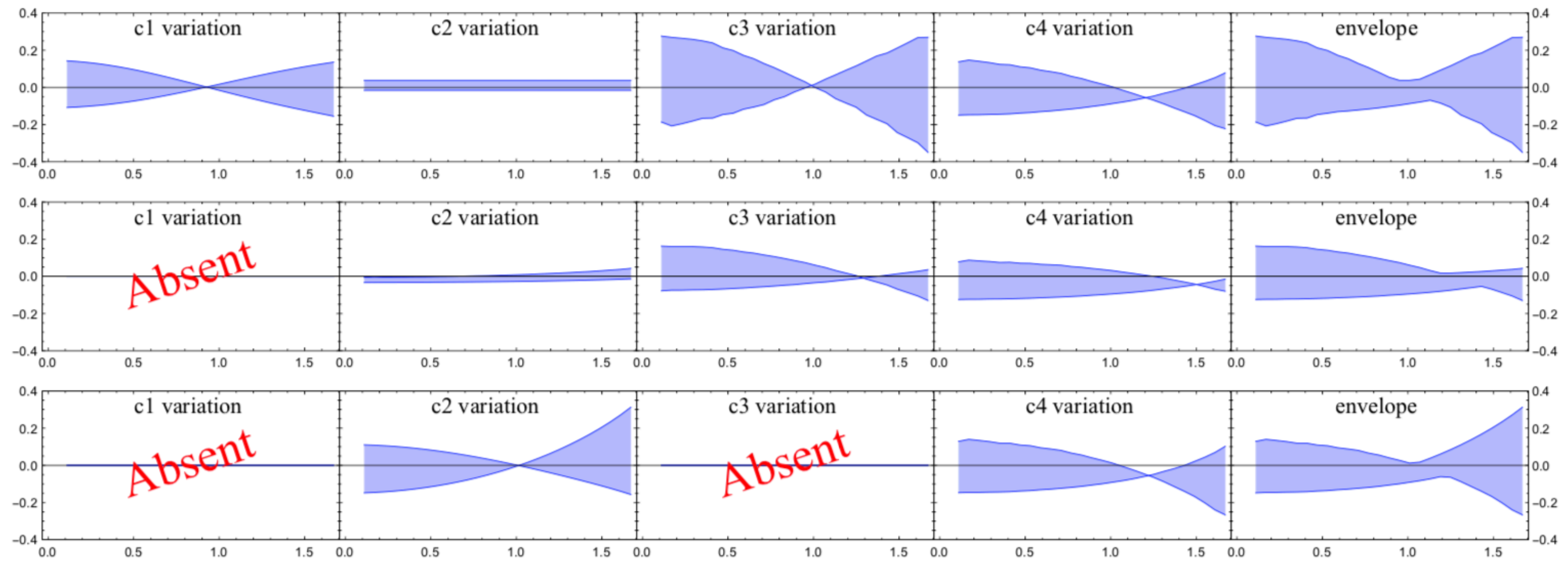
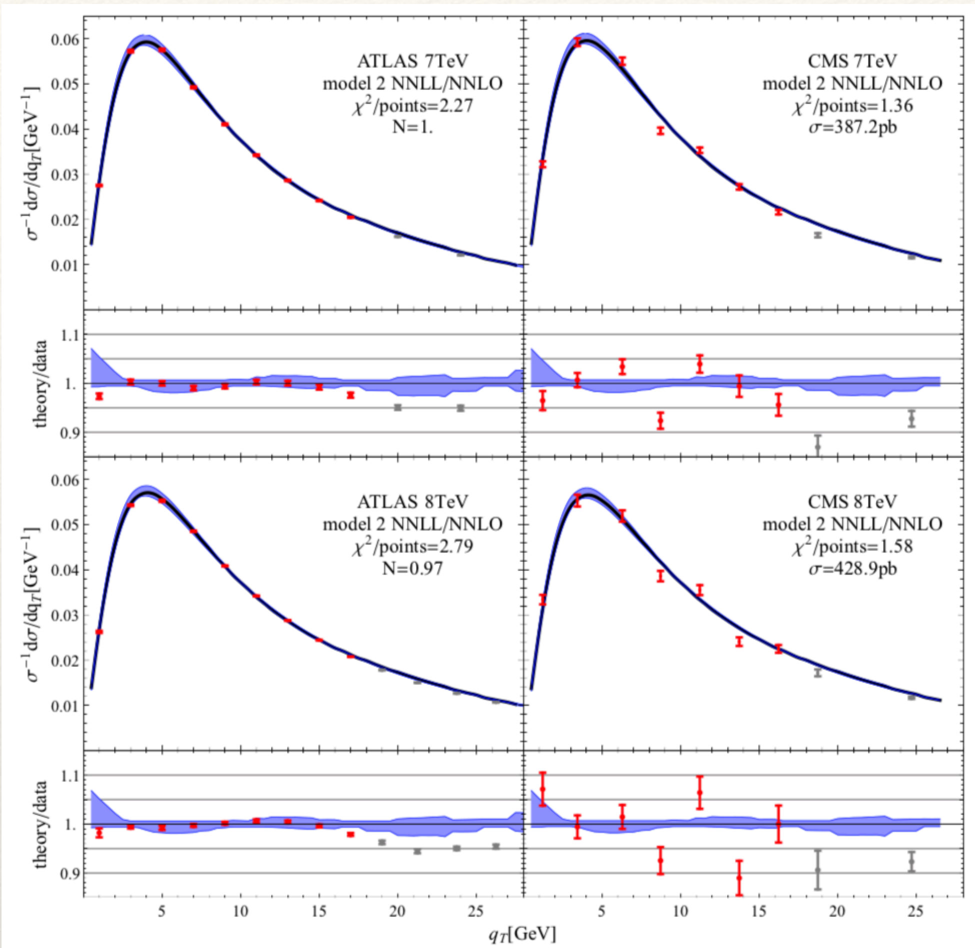
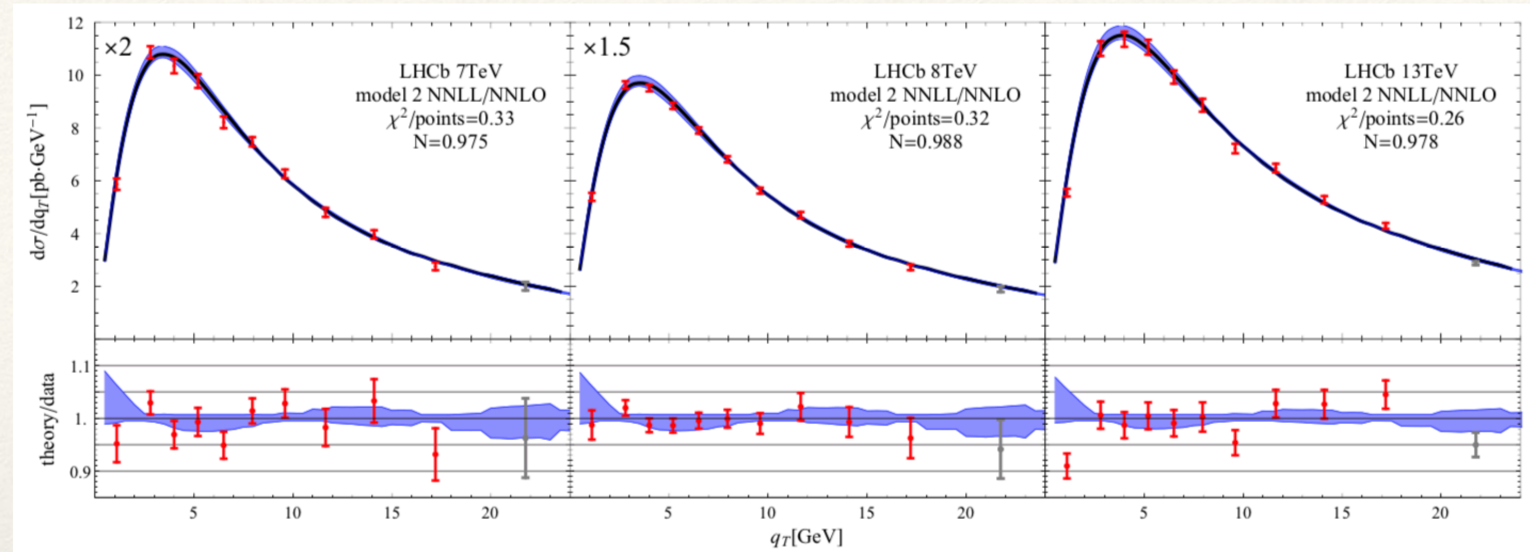


FIG. 11: Comparison of error bands obtained by the scale-variations for cross-sections given by (6.2) (top), (6.5) (middle), (6.12) (bottom). Here, the kinematics bin-integration, etc., is for Drell-Yan process measured at E288 experiment at $E_{\text{beam}} = 200\text{GeV}$ and $Q = 6 - 7\text{GeV}$ [38].

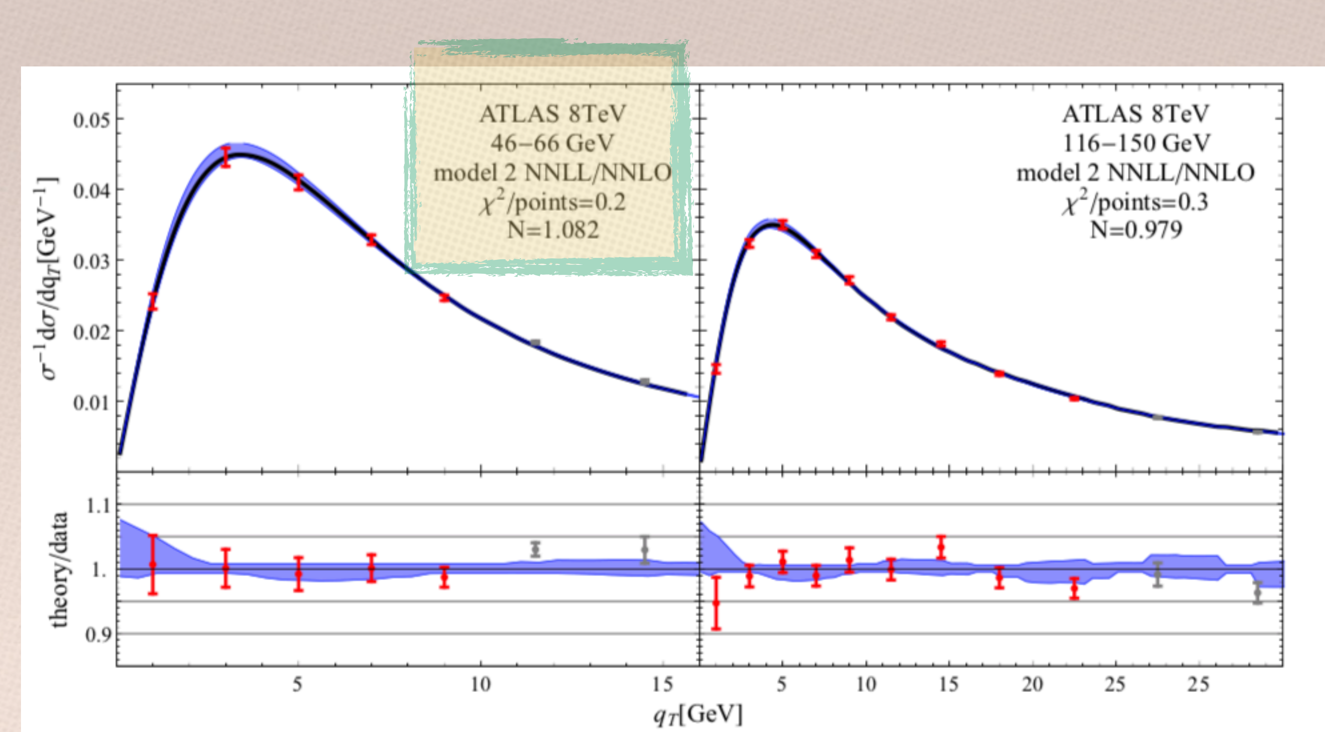
arTeMiDe version 1.1 (naive zeta-prescription)

Results for LHC in Z-production ….

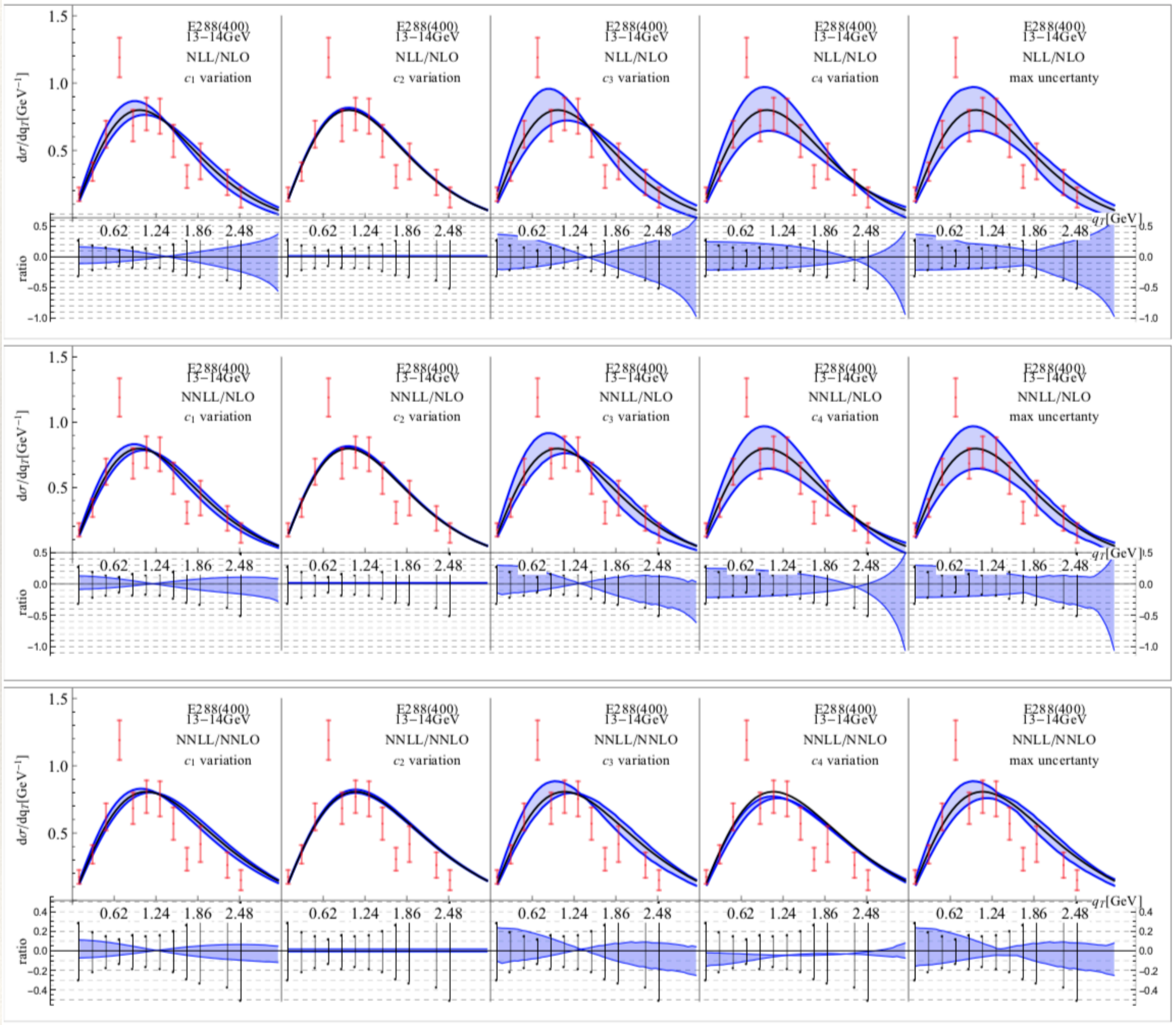




…and Drell-Yan at NNLO



E288


E288(400)
13−14GeV
NLL/NLO
c_1 variation
c_2 variation
c_3 variation
c_4 variation
max uncertanty
NNLL/NLO
NNLL/NNLO
$d\sigma/dq_T[\mathrm{GeV}^{-1}]$
ratio
q_T[GeV]
0.62
1.24
1.86
2.48

Conclusions

- A NNLO analysis is necessary for fitting data and extracting TMD (many issues solved just increasing the perturbative order).
- LHC provides very precise data that should be included in fits (especially data off the Z-boson peak). ATLAS and CMS could do better at 13 TeV!!
- Scale choices and prescription should be critically analyzed (2D-evolution and zeta-prescription, optimal TMDs)
- All this is/will be included in arTeMiDe