Automated calculation of N-jet soft functions in SCET

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Introduction

Idea: Automation

- Find generic strategy to evaluate soft functions
- Set up a numerical method based on universal structure of divergences
  - ✓ Isolate singularities with universal phase-space parametrization
  - ✓ Compute observable dependent integrations numerically
  - ✓ SoftSERVE

Bell, Rahn, Talbert (to appear)

Aim: extend our framework for calculating N-jet soft functions

Motivations

- Soft functions are essential ingredient of factorization theorems (N-jettiness, hadronic event shapes, boosted tops and etc)
- Subtraction technique for the calculation of jet cross sections in fixed-order QCD

Catani, Grazzini (2007)
Boughezal, Focke, Liu, Petriello(2015)
Gaunt, Stahlhofen, Tackmann, Walsh (2015)

Outline

Review of dijet soft function calculation (setup and strategy)

(a) NLO: Real emission
(b) NNLO: Virtual-Real & Double-Real emissions

} **presented in previous SCET workshops**

Automating N-jet soft function calculation

(a) NLO: Real emission
Boost invariant parametrization
(b) NNLO: Virtual-Real & Double-Real emissions

N-jettiness soft function

(a) Constraints from RGE
(b) 1-jettiness ***Preliminary Results***
(c) 2-jettiness ***Preliminary Results***

Summary and outlook

Review:

Dijet soft functions

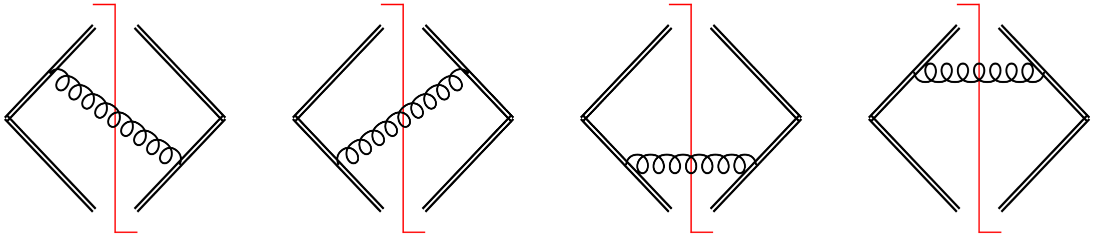
Dijet soft functions at NLO

Bell, Rahn, Talbert (2015)

✓ Soft functions with back-to-back soft Wilson lines

$$S(\tau,\mu) = \frac{1}{N_c} \sum_X \mathcal{M}(\tau;\{k_i\}) \, \mathrm{Tr} \, \langle 0|S^\dagger_{\bar{n}} S_n|X\rangle\langle X|S^\dagger_n S_{\bar{n}}|0\rangle$$

✓ One-loop: Virtual corrections scaleless, real emissions diagrams



✓ Soft function at NLO:

$$S_1 \sim \int d^d k \, \left(\frac{\nu}{k_+ + k_-}\right)^\alpha \delta(k^2)\,\theta(k^0) \;\mathcal{M}(\tau;k)\; |\mathcal{A}(k)|^2$$

$$|\mathcal{A}(k)|^2 \sim \frac{1}{k_+ k_-}$$

Setup of calculation at NLO

Bell, Rahn, Talbert (2015)

Strategy

1. Parametrization: use **transverse momentum** and **rapidity measure**

$$k_T = \sqrt{k_+ k_-}\,, \qquad y = \frac{k_+}{k_-}$$

2. Generic measurement function (inspired by **Laplace space**)

$$\boxed{\mathcal{M}(\tau;k) = \exp\Big(-\tau\, k_T\, y^{n/2}\, f(y,\theta)\Big)}$$

- k_T dependence fixed on dimensional grounds
- θ is angle between $\vec{k}_\perp$ and measurement vector $\vec{v}_\perp$
- $f(y,\theta)$ finite and non-zero in collinear limit $y \to 0$

Setup of calculation at NLO

Bell, Rahn, Talbert (2015)

3. Integrate $\mathbf{k}_T$ analytically

4. Derive a master formula

$$S_1 \sim \Gamma(-2\varepsilon-\alpha) \int_0^1 dy\, \frac{y^{-1+n\varepsilon+\alpha/2}}{(1+y)^\alpha} \int_{-1}^1 d\cos\theta\;\; \sin^{-1-2\varepsilon}\theta\;\; \left[f(y,\theta)\right]^{2\varepsilon+\alpha}$$

- singularities from $k_T \to 0$ and $y \to 0$ are factorised
- additional regulator is needed only for $n = 0$ ($\to$ SCET-2 observable)

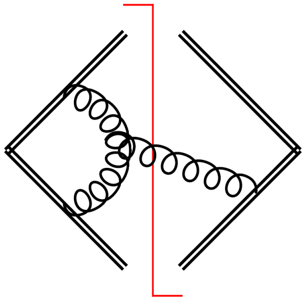
5. Isolate singularities with standard subtraction techniques:

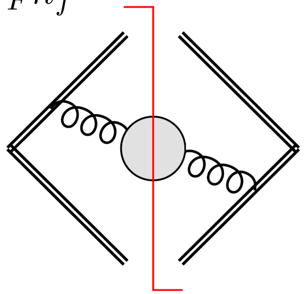
$$\int_0^1 dx\;\, x^{-1+n\varepsilon}\, f(x) = \int_0^1 dx\;\, x^{-1+n\varepsilon} \left[\, \underbrace{f(x) - f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\varepsilon}\, \right]$$

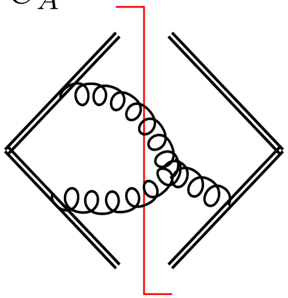
Dijet soft functions at NNLO

Bell, Rahn, Talbert (2015)

- ✓ Virtual corrections scaleless
- ✓ Real-Virtual contribution: follow the same strategy of NLO $|\mathscr{A}_{RV}(k)|^2 \sim k_+^{-1-\varepsilon} k_-^{-1-\varepsilon}$
- ✓ Double real corrections: soft q$\bar{\text{q}}$ and gg emissions (assume non-abelian exponentiation for C_F^2)







...

- ✓ Soft function at NNLO:

$$S_2^{RR} \sim \int d^d k \left(\frac{\nu}{k_+ + k_-}\right)^{\alpha} \delta(k^2)\,\theta(k^0) \int d^d l \left(\frac{\nu}{l_+ + l_-}\right)^{\alpha} \delta(l^2)\,\theta(l^0)\;\mathcal{M}(\tau; k, l)\;|\mathcal{A}(k, l)|^2$$

- ➢ Non-trivial matrix element

$$|\mathcal{A}(k,l)|^2\Big|_{C_F T_F n_f} \sim \frac{2k\cdot l\,(k_- + l_-)\,(k_+ + l_+) - (k_- l_+ - k_+ l_-)^2}{(k_- + l_-)^2\,(k_+ + l_+)^2 \boxed{(2k\cdot l)^2}} \longrightarrow \text{overlapping divergence}$$

Setup of calculation at NNLO

Bell, Rahn, Talbert (2015)

Strategy

1. Parametrization: collective and relative variables related to a two body system

$$p_T = \sqrt{(k_+ + l_+)(k_- + l_-)} \qquad a = \sqrt{\frac{k_- \, l_+}{k_+ \, l_-}} = \sqrt{\frac{y_l}{y_k}}$$

$$y = \frac{k_+ + l_+}{k_- + l_-} \qquad b = \sqrt{\frac{k_- \, k_+}{l_- \, l_+}} = \frac{k_T}{l_T}$$

2. Generic form of the measurement function

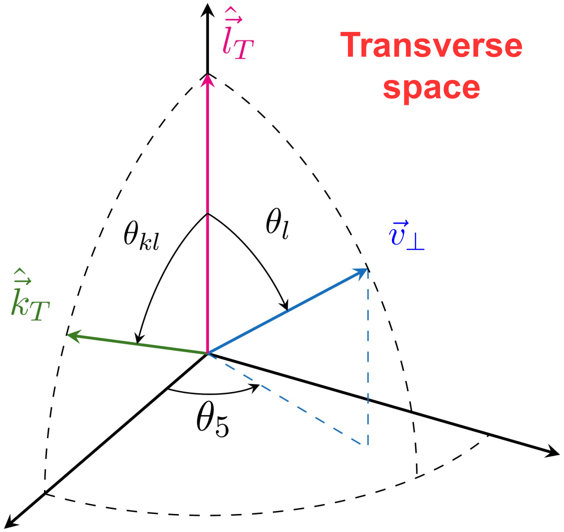
$$\boxed{\mathcal{M}(\tau; k, l) = \exp\Big(- \tau \, p_T \, y^{n/2} \, F(a, b, y, \theta_k, \theta_l, \theta_{kl}) \Big)}$$

- p_T dependence fixed on dimensional grounds
- three angles in transverse plane: $\theta_k \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$, $\theta_l \sphericalangle (\vec{l}_\perp, \vec{v}_\perp)$, $\theta_{kl} \sphericalangle (\vec{k}_\perp, \vec{l}_\perp)$
- $F(a, b, y, \theta_k, \theta_l, \theta_{kl})$ finite and non-zero for $y \to 0$

Setup of calculation at NNLO

Bell, Rahn, Talbert (2015)

3&4. Integrate p_T analytically and obtain the master formula



$$S_{C_F T_F n_f}(\tau,\mu) \sim \frac{\overbrace{\Gamma(-4\epsilon-2\alpha)}^{\text{Soft divergence}}}{\Gamma(-\epsilon)\Gamma(1/2-\epsilon)} \left(\tau e^{\gamma_E}\mu\right)^{4\epsilon}$$

$$\times \int_0^1 \mathrm{d}y\,\mathrm{d}a\,\mathrm{d}b \int_{-1}^1 \mathrm{d}\cos\theta_{kl}\ \mathrm{d}\cos\theta_l\ \mathrm{d}\cos\theta_5\ \sin^{-1-2\,\epsilon}\theta_{kl}\ \sin^{-1-2\,\epsilon}\theta_l\ \sin^{-2-2\,\epsilon}\theta_5$$

$$\times \underbrace{\frac{y^{-1+2n\epsilon+\alpha}}{\left(1+a^2-2\,a\ \cos\theta_{kl}\right)^2}}_{\text{Collinear divergences}} \underbrace{\Big[F(a,b,y,\theta_{kl},\theta_l,\theta_5)\Big]^{4\epsilon+2\alpha}}_{\text{Measurement function}} \underbrace{\mathcal{J}(a,b,y,\epsilon,\alpha)}_{\substack{\text{Matrix element}\\ \text{Jacobian}\\ \text{Rapidity regulator}}}$$

N-jet soft functions

N-jet soft functions at NLO

- Soft functions with **multiple soft Wilson lines S_n**

$$S(\tau,\mu) = \frac{1}{d_R}\sum_X \mathcal{M}(\tau,\{k_i\})\, Tr\, \langle 0|\left(S_{n_1}\, S_{n_2}\, S_{n_3}\dots\right)^\dagger |X\rangle\langle X|\left(S_{n_1}\, S_{n_2}\, S_{n_3}\dots\right)|0\rangle$$

d_R the dimension of color representation and **S_n are matrices in color space**

✓ One-loop: Virtual corrections scaleless, real emissions diagrams contribute

✓ N-jet soft function at NLO: $S_N = \sum_{a\neq b} T_a\cdot T_b\, S_{ab}$

Catani, Grazzini (2000)
Catani, Seymour (1996)

$$S_{ab} \sim \int \mathrm{d}^d k\, \delta(k^2)\,\theta(k^0)\left(\frac{n_a\cdot n_b}{2}\,\frac{\nu}{n_a\cdot k + n_b\cdot k}\right)^\alpha \mathcal{M}(\tau,\{k_i\})\,|\mathcal{A}_{ab}(k)|^2$$

dipole matrix element

$$|\mathcal{A}_{ab}(k)|^2 \sim \frac{n_a\cdot n_b}{2\, n_a\cdot k\, n_b\cdot k}$$

Dijet matrix element

$$|\mathcal{A}(k)|^2 \sim \frac{n_+\cdot n_-}{2\, k_-\, k_+}$$

Setup of calculation at NLO

Strategy

1. Boost invariant parametrization: use the **transverse momentum** and **rapidity** measure in the frame where each pair of **dipoles are back to back**

$$k_T = \sqrt{\frac{2\,k_a\,k_b}{n_{ab}}} \qquad y = \frac{k_a}{k_b} \qquad \begin{aligned} n_{ab} &\equiv n_a \cdot n_b \\ k_X &\equiv n_X \cdot k \end{aligned}$$

- **Parameterizing the solid angle:** Sudakov decomposition is a **Lorenz covariant relation**

$$k^\mu = k_b\,\frac{n_a^\mu}{n_{ab}} + k_a\,\frac{n_b^\mu}{n_{ab}} \underbrace{- k_{x_3} n_{x_3}^\mu - k_{x_4} n_{x_4}^\mu + \ldots}_{k_\perp^\mu}$$

$$\begin{aligned} k_{x_3} &= -k_T\,\cos(\theta_1) \\ k_{x_4} &= -k_T\,\cos(\theta_2)\,\sin(\theta_1) \\ k_{x_d} &= -k_T\,\cos(\theta_{d-2})\,\sin(\theta_{d-3})\ldots\sin(\theta_1) \end{aligned}$$

Kasemets, Waalewijn, Zeune (2016)

Setup of calculation at NLO

2. Generic measurement function (inspired by **Laplace space**)

$$\boxed{\mathcal{M}(\tau;k) = \exp\Big(-\tau\, k_T\, y^{n/2}\ \sqrt{n_{ab}/2}\ f(y,\theta_1,\theta_2)\Big)}$$

- Factorized part of kinematic dependences on n_{ab} : improves numerical convergence
- External kinematics are limited to 4-dim → **2 angles** for N-jet processes

3&4. Master formula for N-jet soft function at NLO

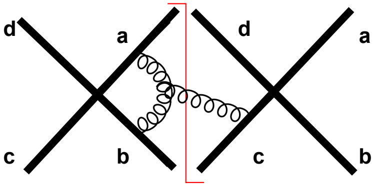
$$S_{ab}(\tau,\mu) \sim \frac{\Gamma(-2\epsilon-\alpha)}{\Gamma(-\epsilon)}\big(\sqrt{n_{ab}/2}\,\tau e^{\gamma_E}\mu\big)^{2\epsilon}$$

$$\times \int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\cos\theta_1\, \mathrm{d}\cos\theta_2\ \sin^{-1-2\,\epsilon}\theta_1\ \sin^{-2-2\,\epsilon}\theta_2\ \frac{y^{-1+n\epsilon+\alpha/2}}{(1+y)^{\alpha}}\Big[f(y,\theta_1,\theta_2)\Big]^{2\epsilon+\alpha}$$

- For three light like directions (n_a, n_b, n_c):

align $n_{c,T}$ with one of the coordinate axes → recover the master formula for dijet soft function

Setup of calculation at NNLO



- ✓ Two-Loop: Virtual corrections scaleless
- ✓ Real-Virtual contribution Catani, Grazzini (2000)

$$S_{\mathrm{RV}} = \sum_{a\neq b} T_a \cdot T_b \, S^R_{ab} + \underbrace{\sum_{a\neq b\neq c} (\lambda_{ab} - \lambda_{ak} - \lambda_{bk})\, f_{ABC}\, T^A_a\, T^B_b\, T^C_c\, S^{Im}_{abc}}_{\text{Three-parton correlation (process dependent)}}$$

$$\lambda_{XY} = \begin{cases} +1 & \text{if X and Y are both incoming/outgoing} \\ 0 & \text{otherwise} \end{cases}$$

Three-parton correlation
(process dependent)

- ➢ **dipole contribution :** follow the same strategy of NLO

$$|\mathcal{A}^R_{ab}(k)|^2 \sim \left(\frac{n_{ab}}{2\, k_a\, k_b}\right)^{1+\epsilon}$$

Dijet matrix element

$$|\mathcal{A}(k)|^2 \sim \left(\frac{n_+ \cdot n_-}{2\, k_+\, k_-}\right)^{1+\epsilon}$$

- ➢ **tripole contribution:**
  - ✓ only present in processes with four or more hard partons
  - ✓ choose dipole n_a- n_c and follow the same strategy of NLO

$$|\mathcal{A}^{Im}_{abc}(k)|^2 \sim \left(\frac{n_{ac}}{2\, k_a\, k_c}\right)\left(\frac{n_{ab}}{2\, k_a\, k_b}\right)^{\epsilon}$$

Setup of calculation at NNLO

- ✓ Double real corrections:

Catani, Grazzini (2000)

I) radiation of soft q$\bar{\text{q}}$ pair
$$S_N^{q\bar{q}} = T_F\, n_f \sum_{a\neq b} T_a \cdot T_b\, S_{ab}^{T_F n_f}$$

II) radiation of double-real gluons
$$S_N^{gg} = C_A \sum_{a\neq b} T_a \cdot T_b\, S_{ab}^{C_A}$$

III) tripole and quadrupole contributions are accounted for by non-abelian exponentiation

- ➢ T_F n_f structure

$$S_{ab}^{T_F nf} \sim \int \mathrm{d}^d k\, \delta(k^2)\, \theta(k^0) \left(\frac{n_{ab}\ \nu}{2(k_a + k_b)}\right)^{\alpha} \int \mathrm{d}^d l\, \delta(l^2)\, \theta(l^0) \left(\frac{n_{ab}\ \nu}{2(l_a + l_b)}\right)^{\alpha} \mathcal{M}(\tau; k, l) \left|\mathcal{A}_{ab}(k,l)\right|^2_{T_F nf}$$

matrix element

$$\left|\mathcal{A}_{ab}(k,l)\right|^2_{T_F nf} \sim \frac{2\, k\cdot l(k_i + l_i)(k_j + l_j) - (k_i\, l_j - l_i\, k_j)^2}{(k_i + l_i)^2\, (k_j + l_j)^2\, (2\, k\cdot l)^2}$$

$$\left|\mathcal{A}(k,l)\right|^2_{C_F T_F nf} \sim \frac{2\, k\cdot l(k_- + l_-)(k_+ + l_+) - (k_-\, l_+ - l_-\, k_+)^2}{(k_- + l_-)^2\, (k_+ + l_+)^2\, (2\, k\cdot l)^2}$$

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Setup of calculation at NNLO

Strategy

1. Parametrization: collective and relative variables (similar to dijet case)

2. Generic form of the measurement function: five angles in transverse plane

$$\boxed{\mathcal{M}(\tau;k,l)=\exp\Big(-\tau\,p_T\,y^{n/2}\;\sqrt{n_{ab}/2}\;F(a,b,y,\theta_{kl},\theta_{nk_1},\theta_{nk_2},\theta_{nl_1},\theta_{nl_2})\Big)}$$

- External kinematics are limited to 4-dim → **5 angles** for N-jet processes

3&4. Integrate k_T analytically and obtain the master formula

$$S_{ab}^{T_F n_f}(\tau,\mu)\sim\frac{\Gamma(-4\epsilon-2\alpha)}{\Gamma(-\epsilon)\Gamma(-1/2-\epsilon)}\big(\sqrt{n_{ab}/2}\;\tau e^{\gamma_E}\mu\big)^{4\epsilon}\int_0^1 \mathrm{d}y\,\mathrm{d}a\,\mathrm{d}b$$

$$\times\int_{-1}^{1}\mathrm{d}\cos\theta_{kl}\;\mathrm{d}\cos\theta_{nk_1}\;\mathrm{d}\cos\theta_{nk_2}\;\sin^{-1-2\epsilon}\theta_{kl}\;\sin^{-2-2\epsilon}\theta_{nk_1}\;\sin^{-3-2\epsilon}\theta_{nk_2}$$

$$\times\int_{-1}^{1}\mathrm{d}\cos\theta_{nl_1}\;\mathrm{d}\cos\theta_{nl_2}\;\sin^{-1-2\epsilon}\theta_{nl_1}\;\sin^{-2-2\epsilon}\theta_{nl_2}$$

$$\times\;\frac{y^{-1+2n\epsilon+\alpha}}{\left(1+a^2-2\,a\,\cos\theta_{kl}\right)^2}\Big[F(a,b,y,\theta_{kl},\theta_{nk_1},\theta_{nk_2},\theta_{nl_1},\theta_{nl_2})\Big]^{4\epsilon+2\alpha}\mathcal{J}(a,b,y,\epsilon,\alpha)$$

Applications:

N-jettiness soft function

N-jettiness soft function

- N-jettiness variable: $\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2\, q_i \cdot p_k}{Q_i} \right\}$

i runs over a and b for the beams and 1,...,N for the final-state jets and $q_i^\mu = \omega_i \, n_i^\mu$

For simplicity here we consider $\mathcal{T}_N = \sum_k \min_i \{ n_i \cdot p_k \}$

where $Q_i = 2\, \omega_i$

Two approaches

- pySecDec (results shown in this talk) Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk,Zirke (2017)

  general implementation of sector decomposition algorithm
  Cuba library for numerical integrations

- SoftSERVE (in progress)

  C++ implementation for N-jet soft function
  Cuba library for numerical integrations

Solving RGE

- RGE for the renormalized soft function and the counterterm

$$\mu\frac{\mathrm{d}\,S(\tau,\mu)}{\mathrm{d}\mu}=\frac{1}{2}\gamma_s\,S(\tau,\mu)+\frac{1}{2}S(\tau,\mu)\,\gamma_s^\dagger$$

$$\mu\frac{\mathrm{d}\,Z_S(\tau,\mu)}{\mathrm{d}\mu}=-\frac{1}{2}\gamma_s S(\tau,\mu)$$

$$\boxed{i\pi\alpha_s^2\left[\sum_{a\neq b}T_a\cdot T_b\,\ln(\sqrt{2\,n_{ab}})\,,\sum_{c\neq d}T_c\cdot T_d\,\Delta_{cd}\right]}$$

- Soft anomalous dimension given by consistency relation

$$\gamma_s=\Gamma_{\text{cusp}}\Big[-2\sum_{a\neq b}T_a\cdot T_b\,\ln\left(\sqrt{2\,n_{ab}}\,\mu\,\bar{\tau}\right)+i\pi\sum_{a\neq b}T_a\cdot T_b\Delta_{ab}\Big]+\gamma_s^{\text{non}-\text{cusp}}$$

$$\Delta_{ab}=\begin{cases}+1 & \text{if a and b are both incoming/outgoing}\\ 0 & \text{otherwise}\end{cases}$$

related to the anomalous dimension of hard Wilson Coefficient from matching QCD to SCET

Solve iteratively for the bare soft function (provides a cross check for the poles)

$$S^{\text{bare}}(\tau)=Z_S(\tau,\mu)\,S(\tau,\mu)\,Z_S^\dagger(\tau,\mu)$$

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N-jettiness soft function

The soft function in Laplace space

$$Z_\alpha = 1 - \left(\frac{\alpha_s}{4\,\pi}\right)\frac{\beta_0}{\epsilon}$$

$$\bar{\tau} = \tau\, e^{\gamma_E}$$

$$\begin{aligned} S(\tau,\mu) = 1 &+ \left(\frac{Z_\alpha\,\alpha_s}{4\,\pi}\right) \sum_{a\neq b} \mathbf{T_a}\cdot\mathbf{T_b}\left(\sqrt{2\,n_{ab}}\,\mu\,\bar{\tau}\right)^{2\epsilon} S^{(1)}_{ab}(\epsilon) \\ &+ \left(\frac{Z_\alpha\,\alpha_s}{4\,\pi}\right)^2 \Bigg[\sum_{a\neq b} \mathbf{T_a}\cdot\mathbf{T_b}\left(\sqrt{2\,n_{ab}}\,\mu\,\bar{\tau}\right)^{4\epsilon} S^{(2)}_{ab}(\epsilon) + \sum_{a\neq b\neq c} \mathbf{f_{ABC}}\,\mathbf{T_a^A}\,\mathbf{T_b^B}\,\mathbf{T_c^C}\left(\mu\,\bar{\tau}\right)^{4\epsilon} S^{(2,Im)}_{ab}(\epsilon) \\ &+ \frac{1}{2}\sum_{a\neq b, c\neq d} \mathbf{T_a}\cdot\mathbf{T_b}\,\mathbf{T_c}\cdot\mathbf{T_d}\left(2\,\sqrt{n_{ab}\,n_{cd}}\,\mu^2\,\bar{\tau}^2\right)^{2\epsilon} S^{(1)}_{ab}(\epsilon) S^{(1)}_{cd}(\epsilon) \Bigg] + \mathcal{O}(\alpha_s^3) \end{aligned}$$

known results for any number of jets Jouttenus, Stewart, Tackmann, Waalewijn (2011)

This work: Preliminary Results

$$S^{(1)}_{ab}(\epsilon) = \overbrace{\frac{2}{\epsilon^2} + \frac{0}{\epsilon}} + \mathbf{I^1_{ab}} + \epsilon\, K^1_{ab}$$

$$\begin{aligned} S^{(2)}_{ab}(\epsilon) = \Bigg(& T_F n_f \left[-\frac{2}{3\,\epsilon^3} - \frac{10}{9\,\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{56}{7} + \frac{\pi^2}{9} - \frac{4}{3}\,\mathbf{I^1_{ab}} \right) + I^{T_F n_f}_{ab} \right] \\ & + C_A \left[\frac{0}{\epsilon^4} + \frac{11}{6\,\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{67}{18} - \frac{\pi^2}{6} \right) + \frac{1}{\epsilon}\left(\frac{202}{27} - \frac{11\,\pi^2}{36} - 7\zeta_3 + \frac{11}{3}\,\mathbf{I^1_{ab,c}} \right) + I^{C_A}_{ab} \right] \Bigg) \end{aligned}$$

poles are known from RGE

Numerical checks

One-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\,\mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

$$n_{12} = 2$$
$$n_{13} = 1 - \cos(\theta)$$
$$n_{23} = 2 - n_{13}$$

1-Loop

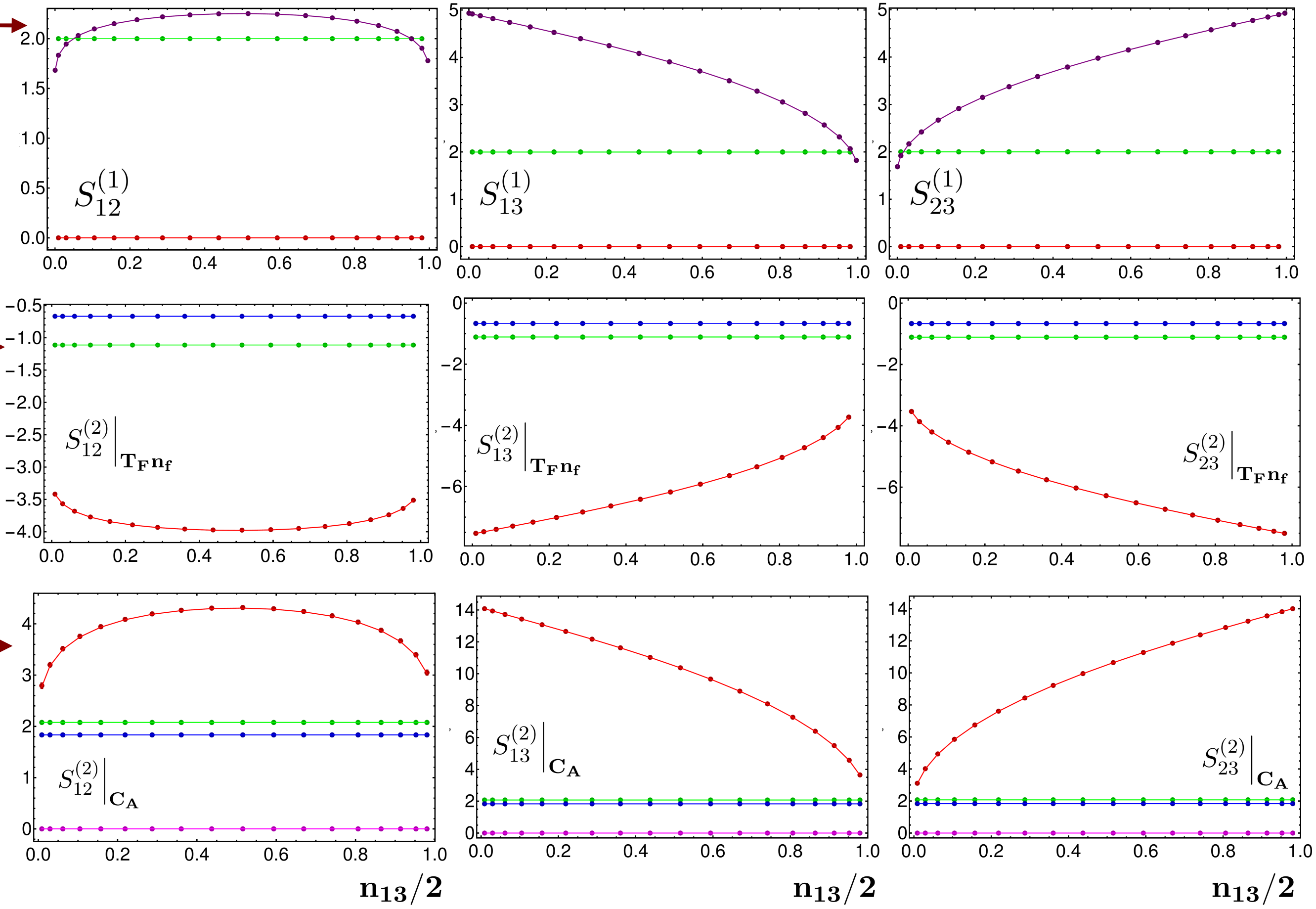
Relative Uncertainty < 0.01%

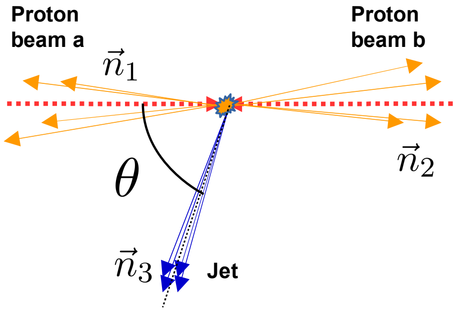
2-Loop $T_F\, n_f$

Relative Uncertainty < 0.1%

2-Loop C_A

Relative Uncertainty < 0.4%





Our numerical results using VEGAS ***(dots)*** *agree within the uncertainty with the known* ***results at NLO*** *and the* ***divergent terms at NNLO (lines)****.*

Preliminary Results

Our predictions

One-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\, \mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$

$$n_{12} = 2$$
$$n_{13} = 1 - \cos(\theta)$$
$$n_{23} = 2 - n_{13}$$

1-Loop

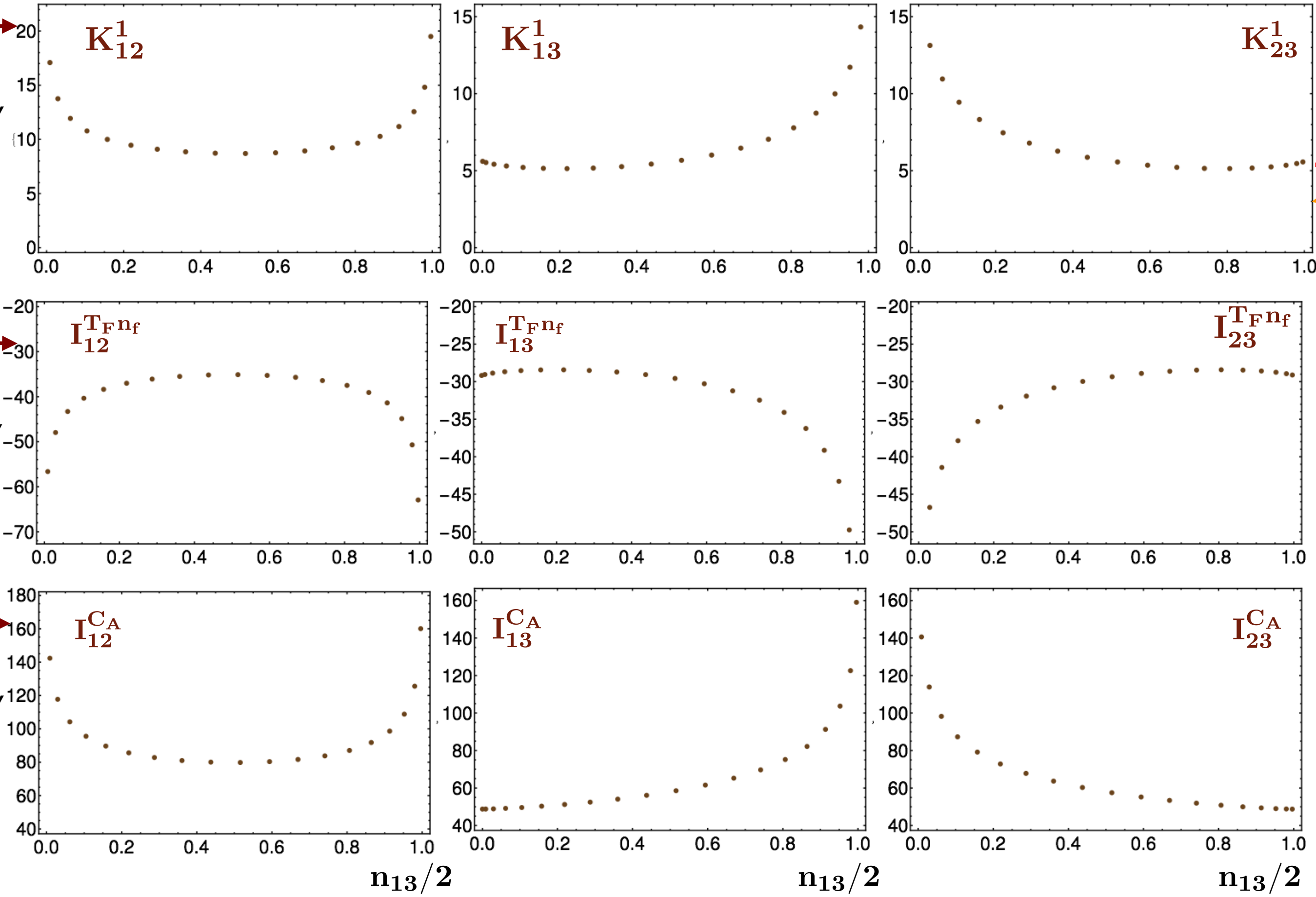
Relative Uncertainty < 0.01%

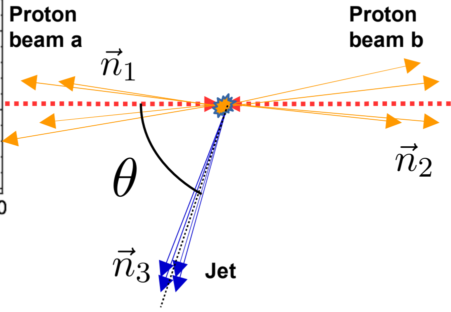
2-Loop $T_F n_f$

Relative Uncertainty < 0.1%

2-Loop C_A

Relative Uncertainty < 0.3%





Preliminary Results

One-jettiness in pp collision

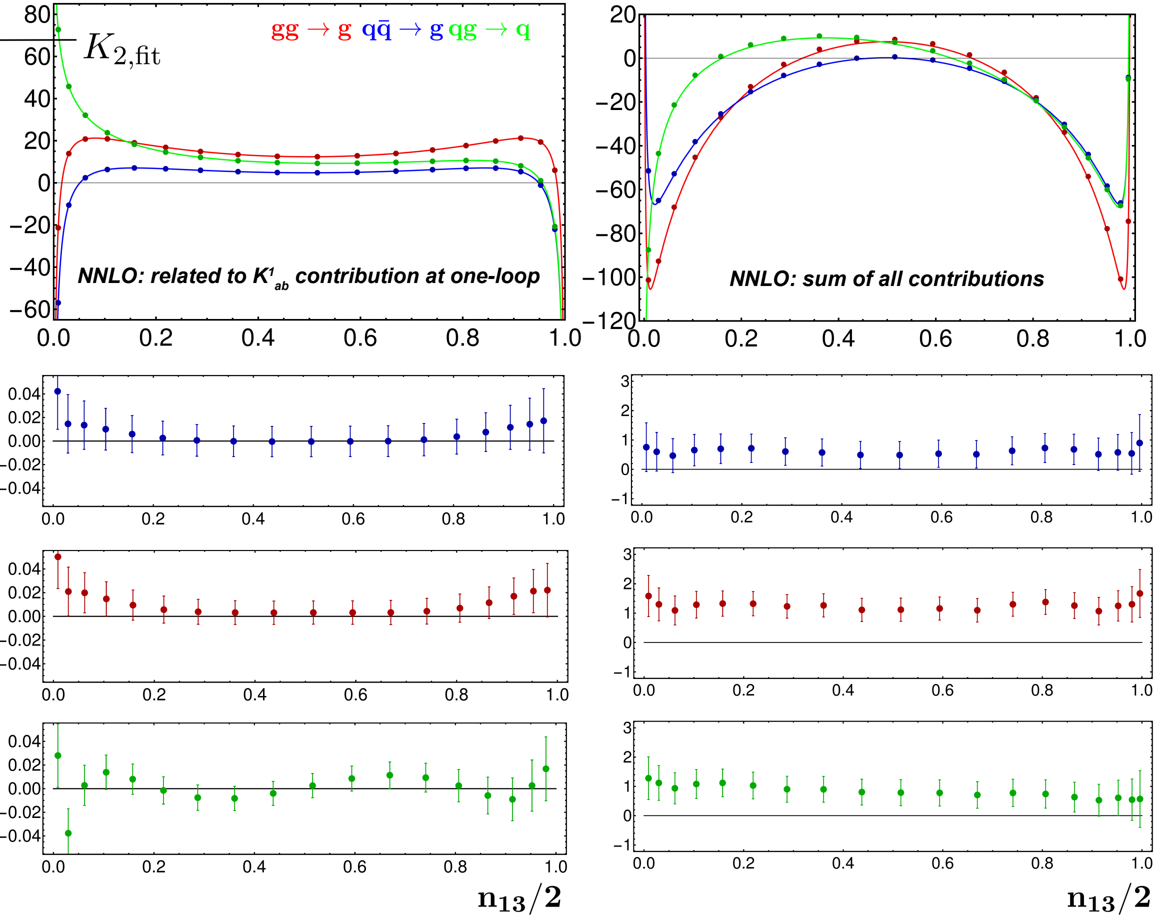
Sum of the dipole contributions and **color factors** at NNLO for different partonic channels $gg \rightarrow g$, $q\bar{q} \rightarrow g$, $qg \rightarrow q$ in the distribution space (coefficients of $\delta(\mathcal{T}_1)$). **Our results (dots)** vs. fit result in **Ref.[2] (lines)** .

$n_{12} = 2$

$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

See Ref [2]



Difference between our results and the fit result in Ref [2]

Ref. [1]: Boughezal, Liu, Petriello (2015)

Ref. [2]: Campbell, Ellis, Mondini, Williams (2017)

Ref. [1]: provides one plot for qg → q

Ref. [2] provides useful fits to their numerical results. However we could not reconstruct their uncertainties!

Our numerical error estimates (w.i.p)

Preliminary Results

Numerical checks

Two-jettiness in pp collision

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C}^{1}_{-2}}{\epsilon^2} + \frac{\mathbf{C}^{1}_{-1}}{\epsilon} + \mathbf{I}^{1}_{\mathbf{ab}} + \epsilon\, \mathbf{K}^{1}_{\mathbf{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C}^{2}_{-3}}{\epsilon^3} + \frac{\mathbf{C}^{2}_{-2}}{\epsilon^2} + \frac{\mathbf{C}^{2}_{-1}}{\epsilon} + \mathbf{I}^{\mathbf{T_F n_f}}_{\mathbf{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C}^{2}_{-4}}{\epsilon^4} + \frac{\mathbf{C}^{2}_{-3}}{\epsilon^3} + \frac{\mathbf{C}^{2}_{-2}}{\epsilon^2} + \frac{\mathbf{C}^{2}_{-1}}{\epsilon} + \mathbf{I}^{\mathbf{C_A}}_{\mathbf{ab}}\right]\right)$$

$$n_{12} = 2$$

$$n_{13} = 1 - \cos(\theta)$$

$$n_{23} = 2 - n_{13}$$

1-Loop

Relative Uncertainty < 0.01%

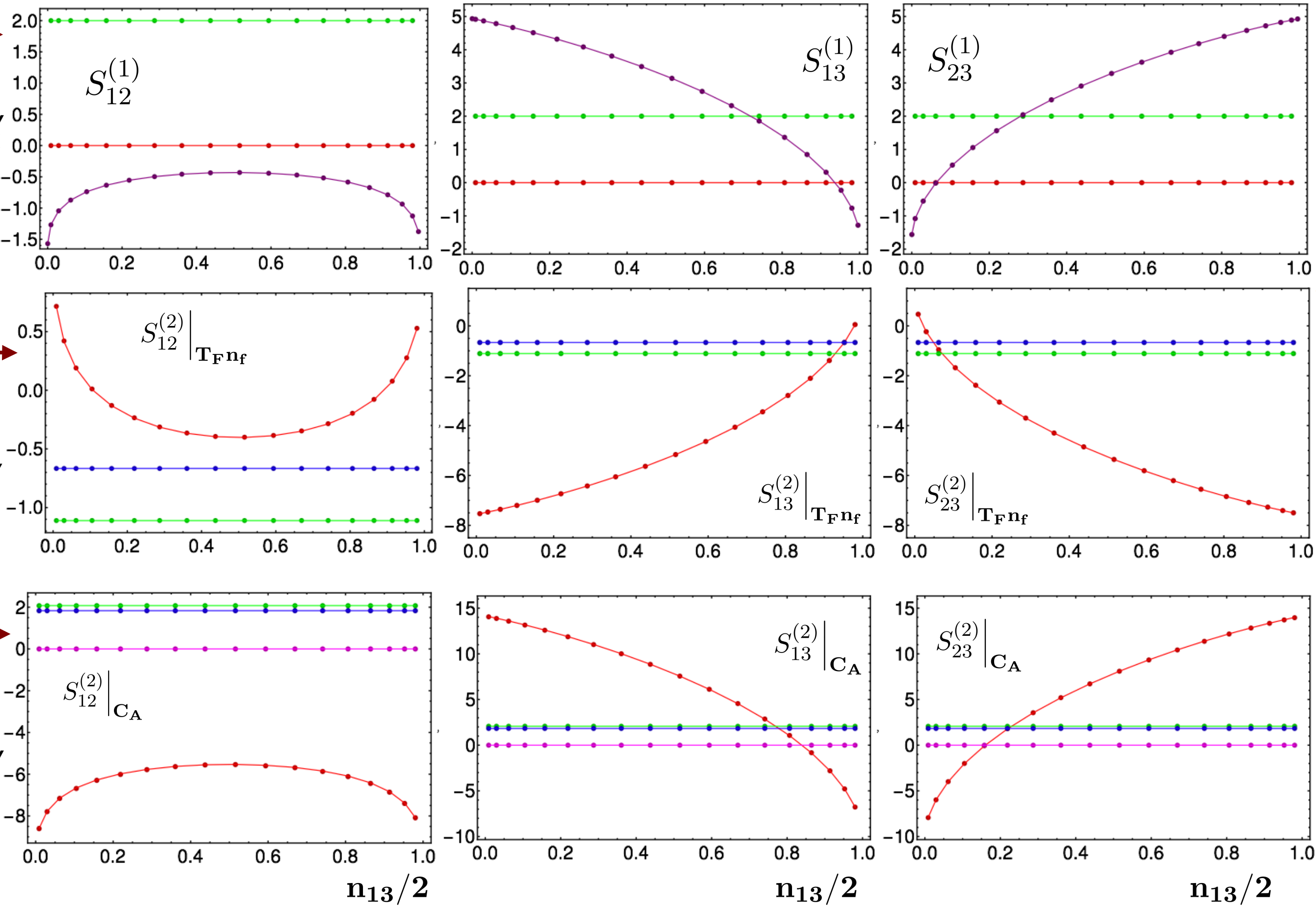
The first Purple line < 1%

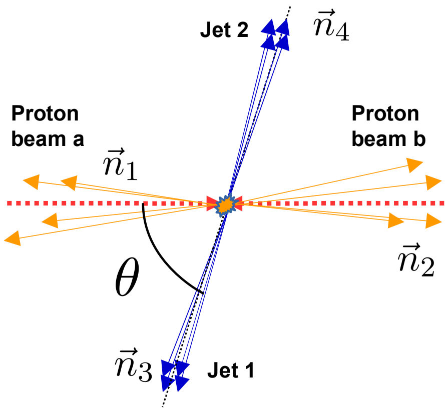
2-Loop $T_F n_f$

Relative Uncertainty < 0.1%

2-Loop C_A

Relative Uncertainty < 0.5%





*Our numerical results using VEGAS **(dots)** agree within the uncertainty with the known **results at NLO** and the **divergent terms at NNLO (lines)**.*

Preliminary Results

New predictions

Two-jettiness in pp collision

$$S_{ab}^{(1)}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\, \mathbf{K^1_{ab}}$$

$$S_{ab}^{(2)}(\epsilon) = \left(\mathbf{T_F n_f} \left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}} \right] + \mathbf{C_A} \left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}} \right] \right)$$

$n_{12} = 2$

$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

1-Loop

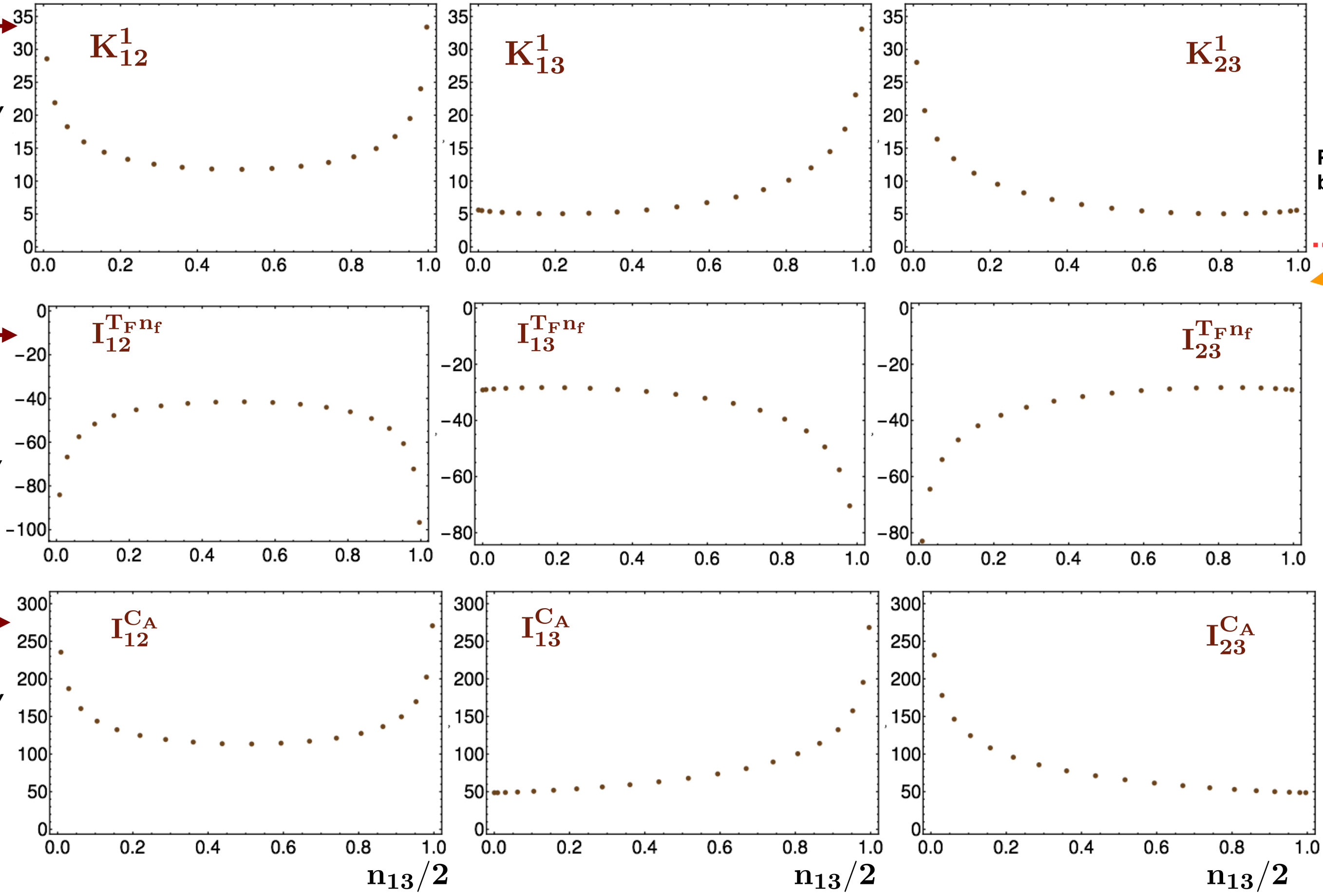
Relative Uncertainty < 0.2%

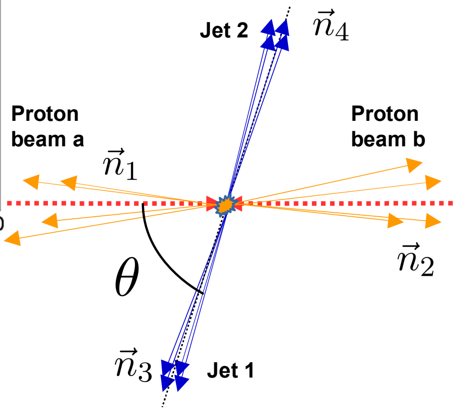
2-Loop $T_F n_f$

Relative Uncertainty < 0.1%

2-Loop C_A

Relative Uncertainty < 0.3%





Preliminary Results

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Two-jettiness in pp collision

$$\sum_{a\neq b\neq c} f_{ABC}\, T_a^A\, T_b^B\, T_c^C\, S_{ab}^{(2,Im)}(\epsilon) = f_{ABC}\, T_1^A\, T_2^B\, T_3^C\, I^{(2,Im)}(\epsilon)$$

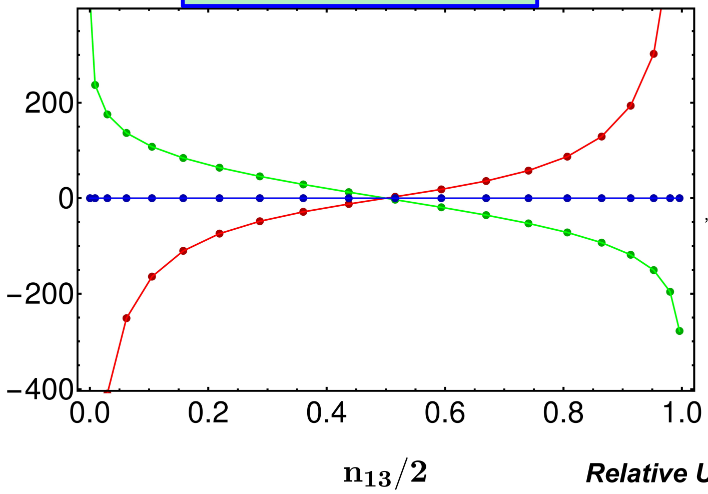
$$I^{(2,Im)}(\epsilon) = \frac{C_{-3}^{(2,Im)}}{\epsilon^3} + \frac{C_{-2}^{(2,Im)}}{\epsilon^2} + \frac{C_{-1}^{(2,Im)}}{\epsilon} + I_{ab}^{(2,Im)}$$

$n_{12} = 2$

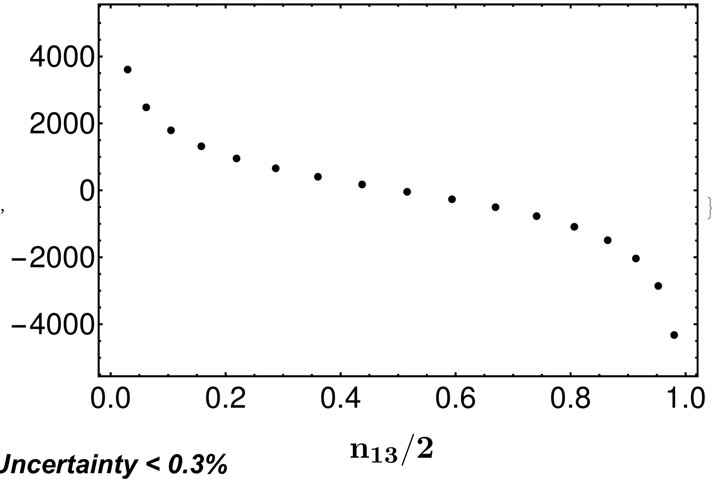
$n_{13} = 1 - \cos(\theta)$

$n_{23} = 2 - n_{13}$

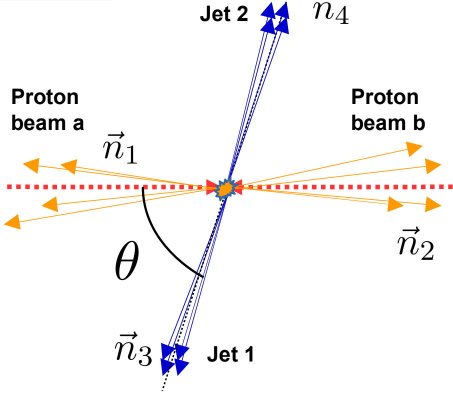
Numerical checks



New predictions



Relative Uncertainty < 0.3%



- ✓ *Our numerical results using VEGAS* ***(dots)*** *agree within the uncertainty with the known* ***results at NLO*** *and the* ***divergent terms at NNLO (lines)****.*

Preliminary Results

Conclusions and outlook

Conclusions

✓ Systematic extension of our framework for automated calculations of N-jet soft functions

- First step assumes non-abelian exponentiation and SCET-1 type observable

✓ First NNLO results

- Numerical results for 1-jettiness soft function
- **First numerical results for 2-jettiness soft function**
- A reliable error estimate needs further studies (w.i.p)

Outlook

➢ Other observables on the horizon (angularities, boosted-tops, hadronic event shapes, etc) (w.i.p)

- may trigger new ideas for subtraction techniques

➢ N-jet implementation in SoftSERVE (w.i.p)

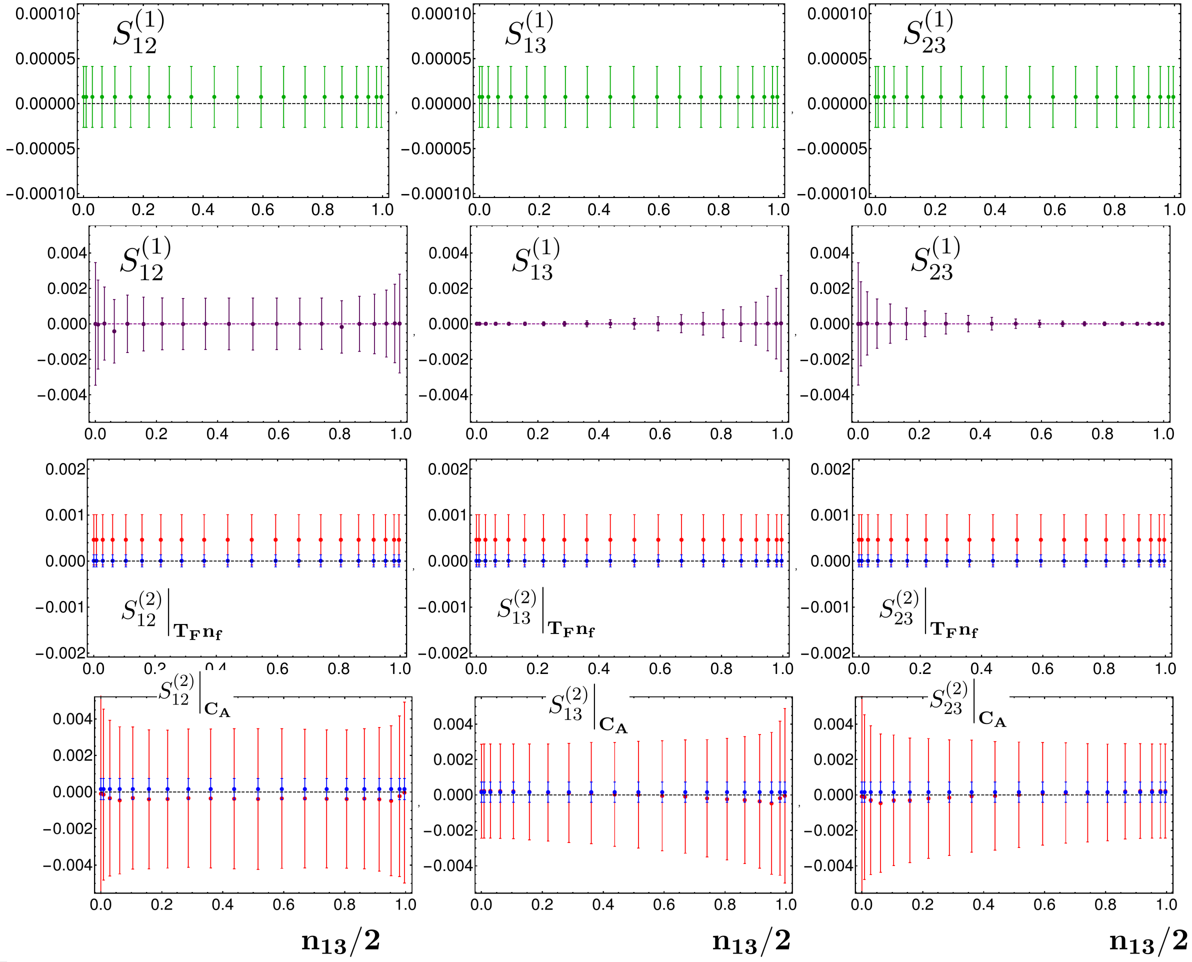
Thank you for your attention!

Back up slides

One-jettiness (RGE vs Numerics)

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\, \mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$



Preliminary Results

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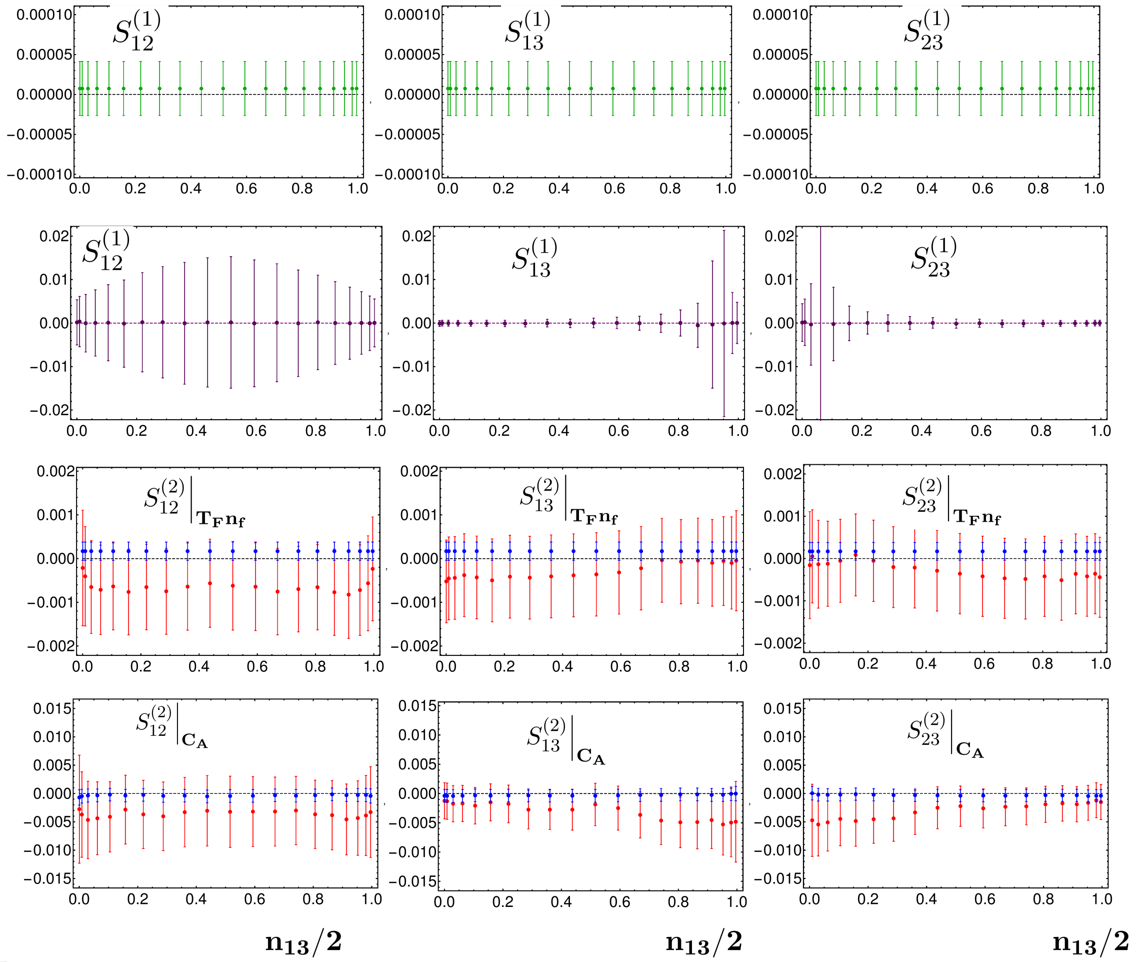
Two-jettiness (RGE vs Numerics)

$$S^{(1)}_{ab}(\epsilon) = \frac{\mathbf{C^1_{-2}}}{\epsilon^2} + \frac{\mathbf{C^1_{-1}}}{\epsilon} + \mathbf{I^1_{ab}} + \epsilon\, \mathbf{K^1_{ab}}$$

$$S^{(2)}_{ab}(\epsilon) = \left(\mathbf{T_F n_f}\left[\frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{T_F n_f}_{ab}}\right] + \mathbf{C_A}\left[\frac{\mathbf{C^2_{-4}}}{\epsilon^4} + \frac{\mathbf{C^2_{-3}}}{\epsilon^3} + \frac{\mathbf{C^2_{-2}}}{\epsilon^2} + \frac{\mathbf{C^2_{-1}}}{\epsilon} + \mathbf{I^{C_A}_{ab}}\right]\right)$$



Preliminary Results