# All-order resummations and global PDF analyses

**Juan Rojo**
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# Disclaimer

- I am **not** a SCET-ian!

- My single paper involving SCET was limited **to calculations in 1+1 dimensions** 😜

- However I believe many **synergies** could be built **between the SCET and PDF-fitting communities**

- Here I will suggest some directions, and looking forward to the discussions afterwards

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**Heavy meson semileptonic differential decay rate in two dimensions in the large $N_c$**

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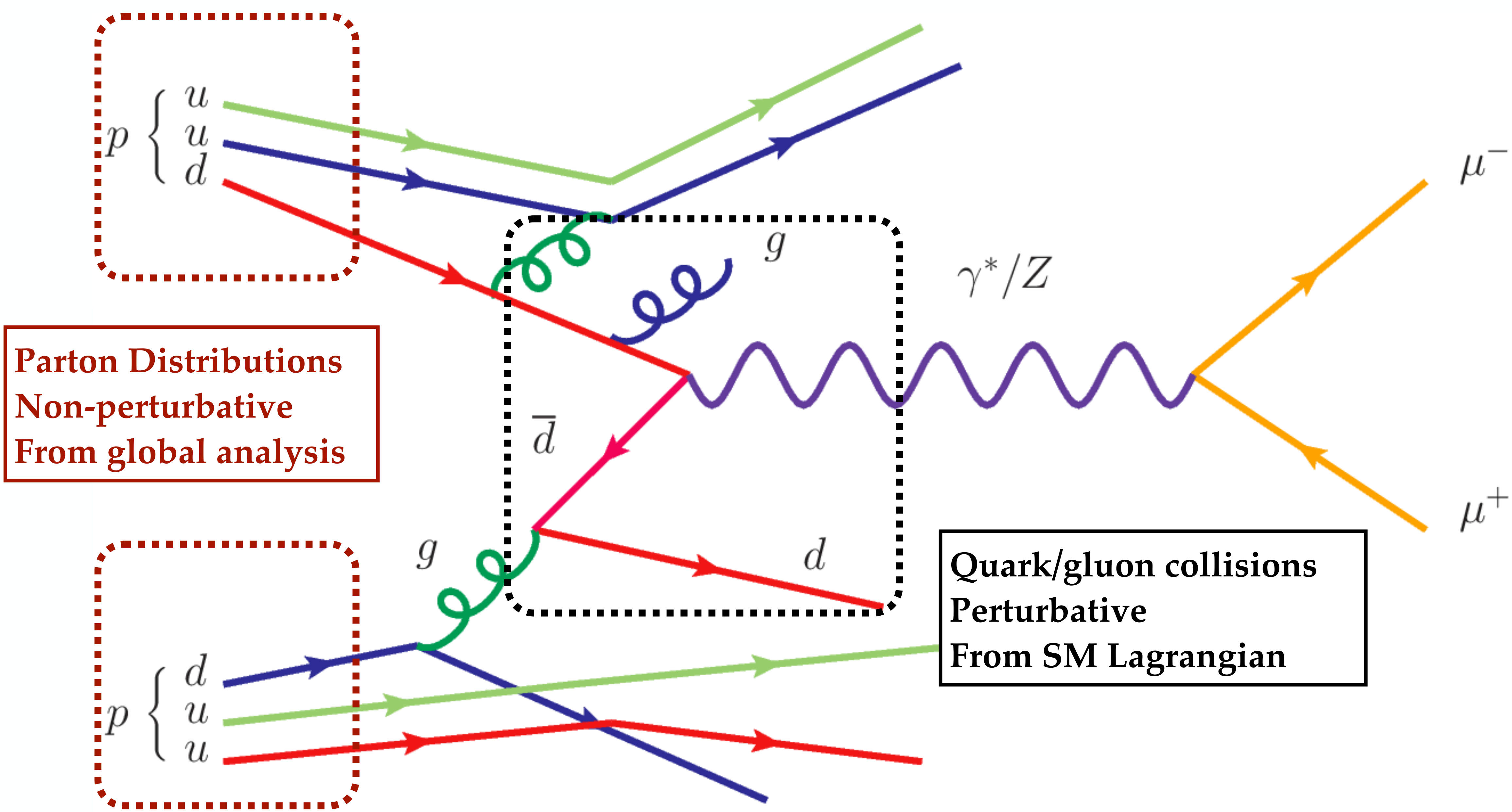
Abstract: We study QCD in 1+1 dimensions in the large $N_c$ limit using light-front Hamiltonian perturbation theory in the $1/N_c$ expansion. We use this formalism to exactly compute hadronic transition matrix elements for arbitrary currents at leading order in $1/N_c$. We compute the semileptonic differential decay rate of a heavy meson, $d\Gamma/dx$, and its moments, $M_N$, using the hadronic matrix elements obtained previously. We put some emphasis in trying to understand parity invariance. We also study with special care the kinematic region where the operator product expansion ($1/N \sim 1-x \sim 1$) or non-local effective field theories ($1/N \sim 1-x \sim \Lambda_{QCD}/m_Q$) can be applied. We then compare with the results obtained using an effective field theory approach based on perturbative factorization, with the focus to better understand quark-hadron duality. At the end of the day, using effective field theories, we have been able to obtain expressions for the moments with relative accuracy of $O(\Lambda^2_{QCD}/m^2_Q)$ in the kinematic region where the operator product expansion can be applied, and with relative accuracy of $O(\Lambda_{QCD}/m_Q)$ in the kinematic region where non-local effective field theories can be applied. These expressions agree,

# Anatomy of hadronic collisions

In high-energy **hadron colliders** the collisions involve **composite particles** (protons) with internal substructure (quarks and gluons): the LHC is actually a quark/gluon collider!




Calculations of **cross-sections** in hadron collisions require the combination of **perturbative cross-sections** with **non-perturbative parton distribution functions (PDFs)**

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*DGLAP evolution kernel (perturbative)*

$$\sigma^{(\mathrm{th})}\left(\{a_i\}\right) = \widehat{\sigma}_{ij}(Q^2) \otimes \Gamma_{ij,kl}(Q^2, Q_0^2) \otimes q_k\left(Q_0, \{a_i\}\right) \otimes q_l\left(Q_0, \{a_i\}\right)$$

*Hard-scattering partonic cross-sections (perturbative)*

*PDFs at input parametrisation scale (non-perturbative)*

Calculations of **cross-sections** in hadron collisions require the combination of **perturbative cross-sections** with **non-perturbative parton distribution functions (PDFs)**

# Anatomy of hadronic collisions

In high-energy **hadron colliders** the collisions involve **composite particles** (protons) with internal substructure (quarks and gluons): the LHC is actually a quark/gluon collider!

***Perturbative resummations relevant here!***

***DGLAP evolution kernel (perturbative)***

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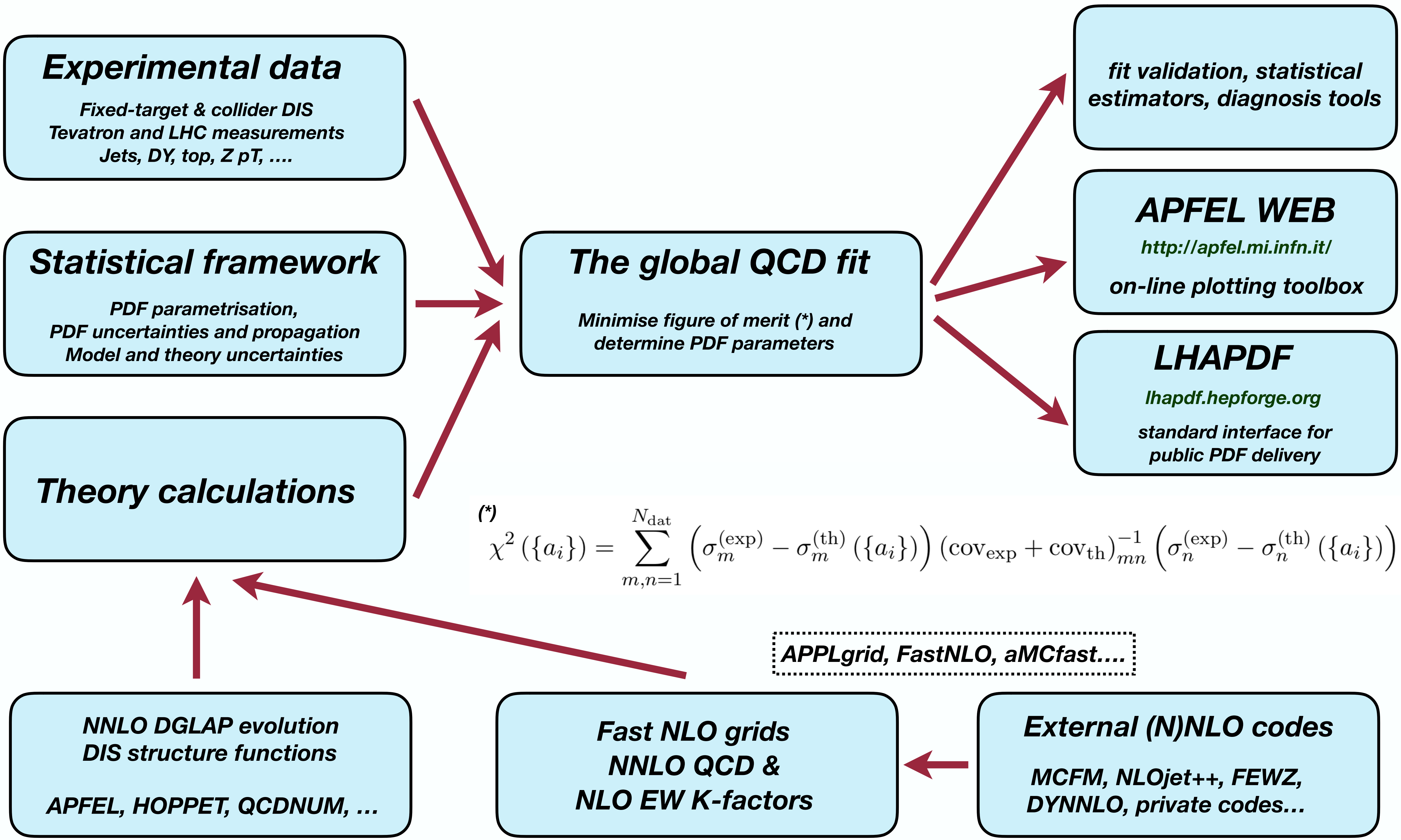
***Hard-scattering partonic cross-sections (perturbative)***

***PDFs at input parametrisation scale (non-perturbative)***

Calculations of **cross-sections** in hadron collisions require the combination of **perturbative cross-sections** with **non-perturbative parton distribution functions (PDFs)**

# The global QCD fit machinery




(*)

$$\chi^2\left(\{a_i\}\right) = \sum_{m,n=1}^{N_{\rm dat}} \left(\sigma_m^{(\rm exp)} - \sigma_m^{(\rm th)}\left(\{a_i\}\right)\right) \left({\rm cov}_{\rm exp} + {\rm cov}_{\rm th}\right)^{-1}_{mn} \left(\sigma_n^{(\rm exp)} - \sigma_n^{(\rm th)}\left(\{a_i\}\right)\right)$$

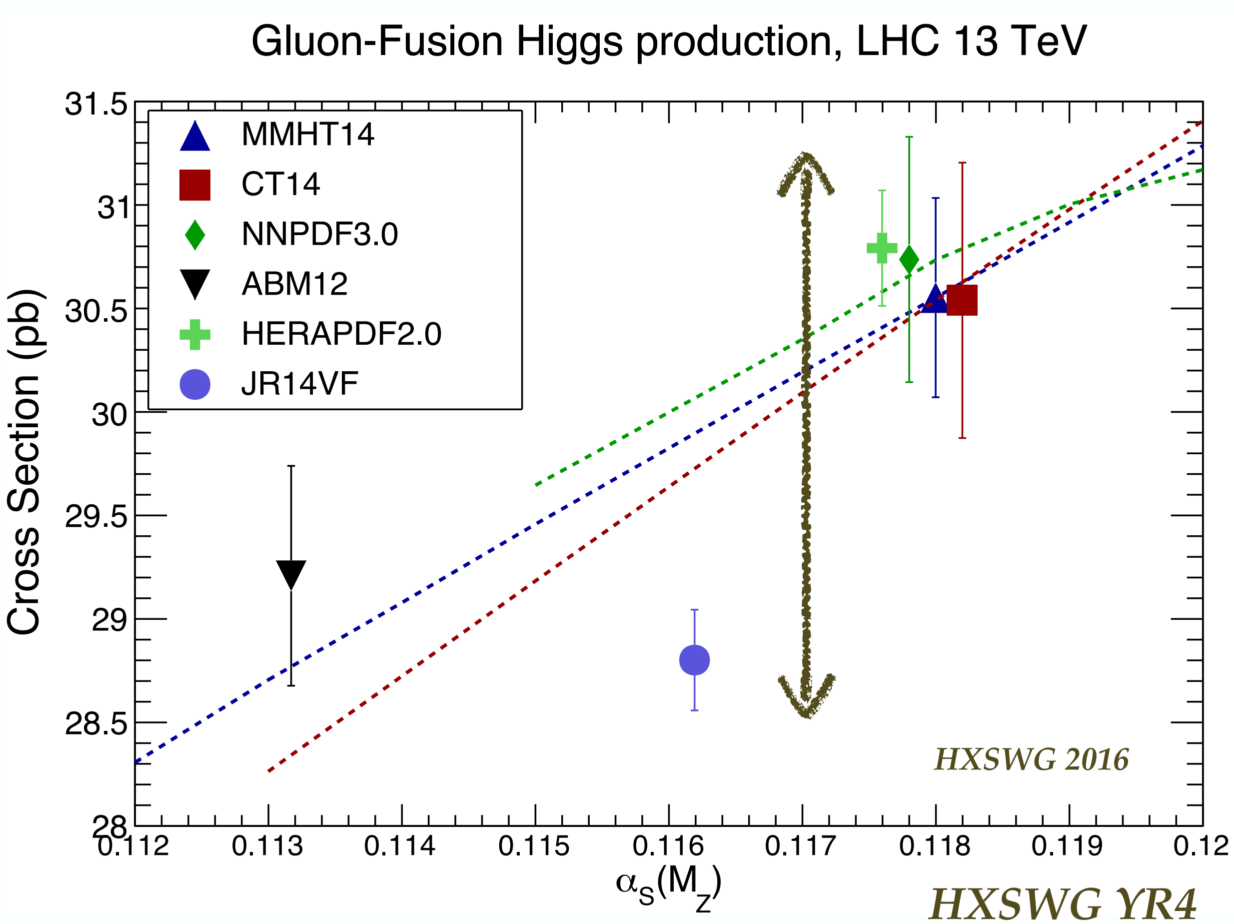
# Why better PDFs?

*Dominant TH unc for $M_W$ measurements at LHC*

*ATLAS 2017*

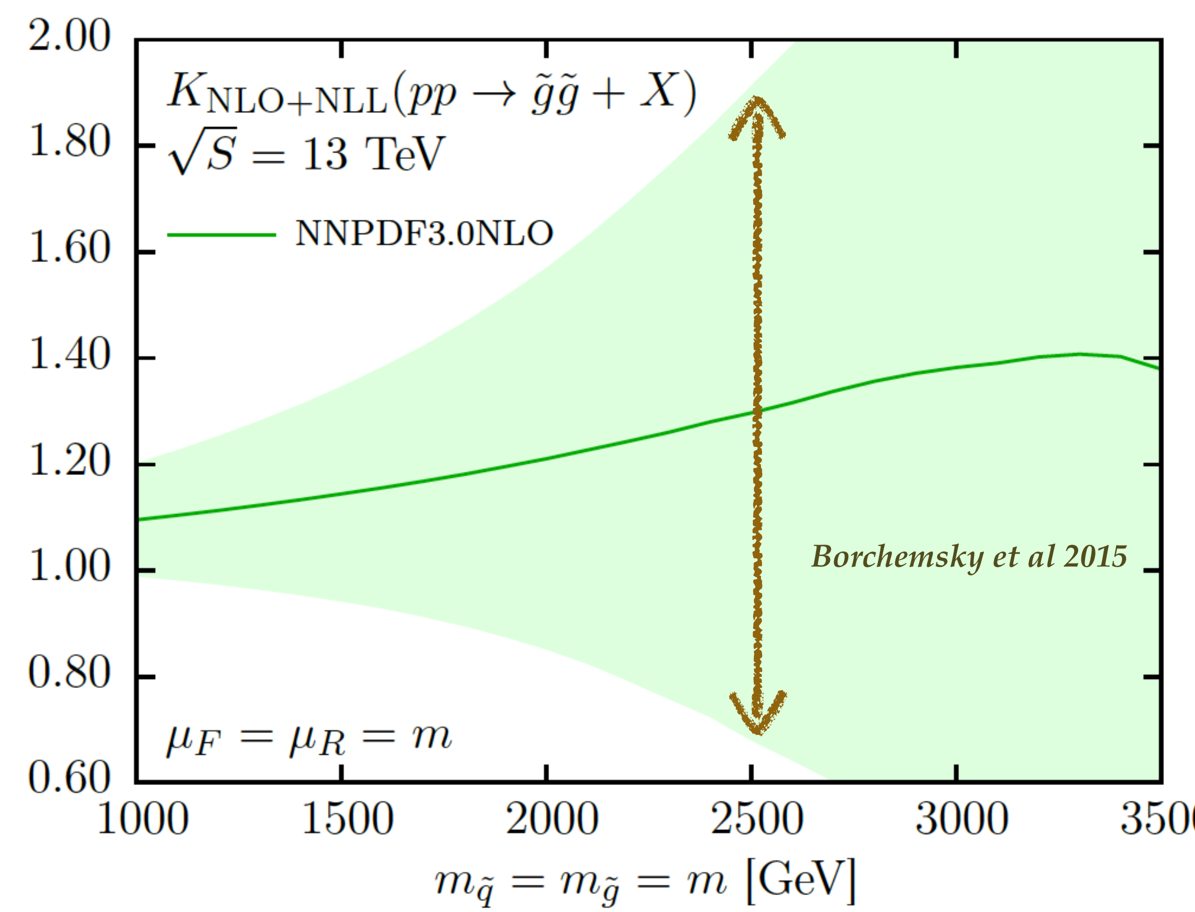
| Channel | $m_{W^+} - m_{W^-}$ [MeV] | Stat. Unc. | Muon Unc. | Elec. Unc. | Recoil Unc. | Bckg. Unc. | QCD Unc. | EW Unc. | PDF Unc. | Total Unc. |
|---|---|---|---|---|---|---|---|---|---|---|
| $W \to e\nu$ | −29.7 | 17.5 | 0.0 | 4.9 | 0.9 | 5.4 | 0.5 | 0.0 | 24.1 | 30.7 |
| $W \to \mu\nu$ | −28.6 | 16.3 | 11.7 | 0.0 | 1.1 | 5.0 | 0.4 | 0.0 | 26.0 | 33.2 |
| Combined | −29.2 | 12.8 | 3.3 | 4.1 | 1.0 | 4.5 | 0.4 | 0.0 | 23.9 | 28.0 |

*Higgs coupling measurements*



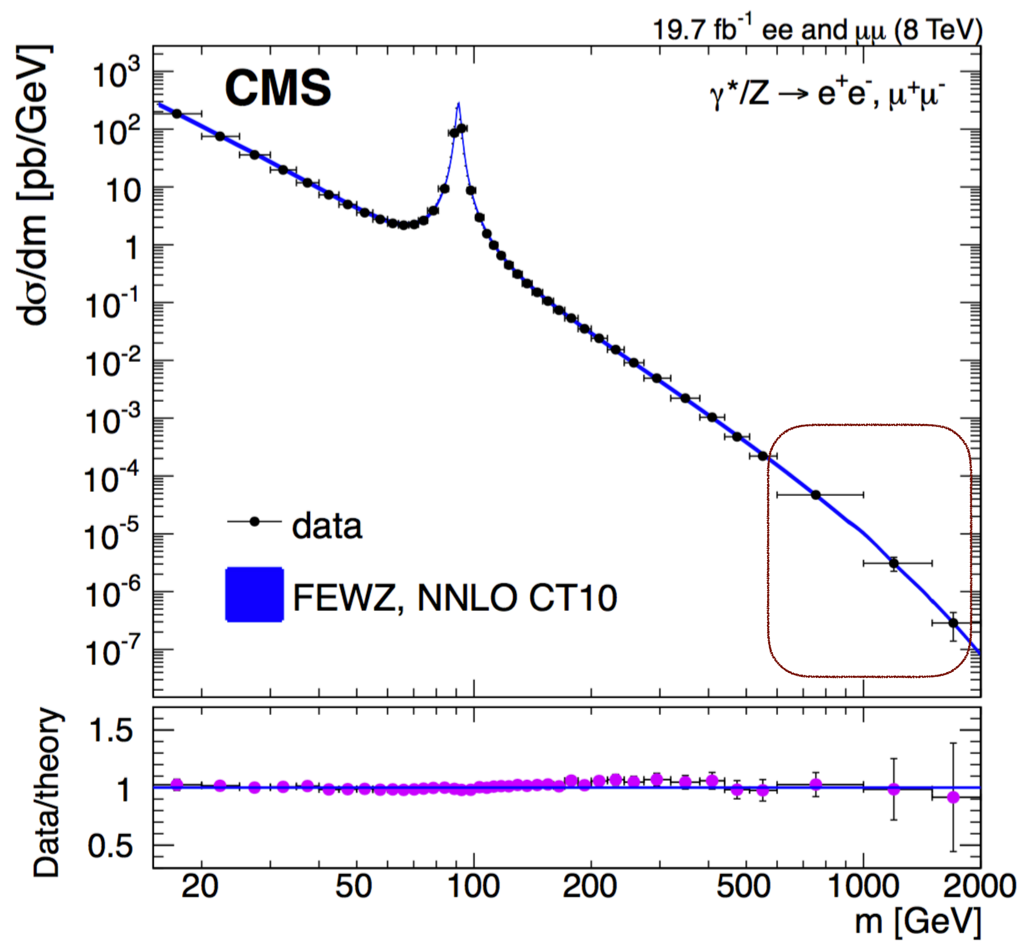

*HXSWG YR4*

*High-mass BSM cross-sections*



# Why better PDFs?

- BSM physics could manifest as **subtle deviations** wrt to the Standard Model predictions
- Even for high-mass resonances, **PDF uncertainties degrade or limit many BSM searches**
- The robustness of **global stress-tests of the SM** (electroweak fit, SM Effective Field Theory analysis) relies crucially in high-precision theoretical calculations




*SMEFT expansion*

$$\sigma(E) = \sigma_{SM}(E)\left(1 + \epsilon\frac{m_{SM}^2}{m_W^2} + \boxed{\epsilon\frac{E^2}{m_W^2}} + \ldots\right)$$

For $\mathbf{E} \simeq \mathbf{1}$ **TeV**, a measurement with $\delta\sigma/\sigma \simeq \mathbf{10\%}$ is sensitive to $\epsilon \simeq \mathcal{O}(\mathbf{0.1\%})$!

*BSM physics might very well hiding itself in the tails of LHC distributions, but need to make sure first that PDF uncertainties are under control*

arXiv:1709.04922v1 [hep-ph] 14 Sep 2017

# The Structure of the Proton in the LHC Precision Era

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**Abstract**

We review recent progress in the determination of the parton distribution functions (PDFs) of the proton, with emphasis on the applications for precision phenomenology at the Large Hadron Collider (LHC). First of all, we introduce the general theoretical framework underlying the global QCD analysis of the quark and gluon internal structure of protons. We then present a detailed overview of the hard-scattering measurements, and the corresponding theory predictions, that are used in state-of-the-art PDF fits. We emphasize here the role that higher-order QCD and electroweak corrections play in the description of recent high-precision collider data. We present the methodology used to extract PDFs in global analyses, including the PDF parametrization strategy and the definition and propagation of PDF uncertainties. Then we review and compare the most recent releases from the various PDF fitting collaborations, highlighting their differences and similarities. We discuss the role that QED corrections and photon-initiated contributions play in modern PDF analysis. We provide representative examples of the implications of PDF fits for high-precision LHC phenomenological applications, such as Higgs coupling measurements and searches for high-mass New Physics resonances. We conclude this report by discussing some selected topics relevant for the future of PDF determinations, including the treatment of theoretical uncertainties, the connection with lattice QCD calculations, and the role of PDFs at future high-energy colliders beyond the LHC.

*Keywords:* Parton Distributions, Quantum Chromodynamics, Large Hadron Collider, Higgs boson, Standard Model, Electroweak theory

*166 pages, 82 figures, > 500 references, to appear in Physics Reports*

*Only time for a brief snapshot here!*

# Resummation in PDF fits

Large-$x$ (threshold) resummation and high-mass BSM physics

Small-$x$ resummation and BFKL dynamics

Shower resummation and MC event generators

# Resummation in PDF fits

Large-$x$ (threshold) resummation and high-mass BSM physics

**Small-$x$ resummation and BFKL dynamics**

Shower resummation and MC event generators

# Parton distributions with BFKL resummation

- **Perturbative fixed-order QCD calculations** have been extremely successful in describing a wealth of data from proton-proton and electron-proton collisions

- There are theoretical reasons that eventually we need to go beyond DGLAP: at small-x, **logarithmically enhanced terms in *1/x* become dominant** and need to be resummed to all orders

- ***BFKL/high-energy/small-x resummation*** can be matched to the **DGLAP collinear framework**, and thus be included into a standard PDF analysis

**DGLAP Evolution in $Q^2$**

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x,\mu^2) = \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(\mu^2)\right) f_j(z,\mu^2),$$

**BFKL Evolution in x**

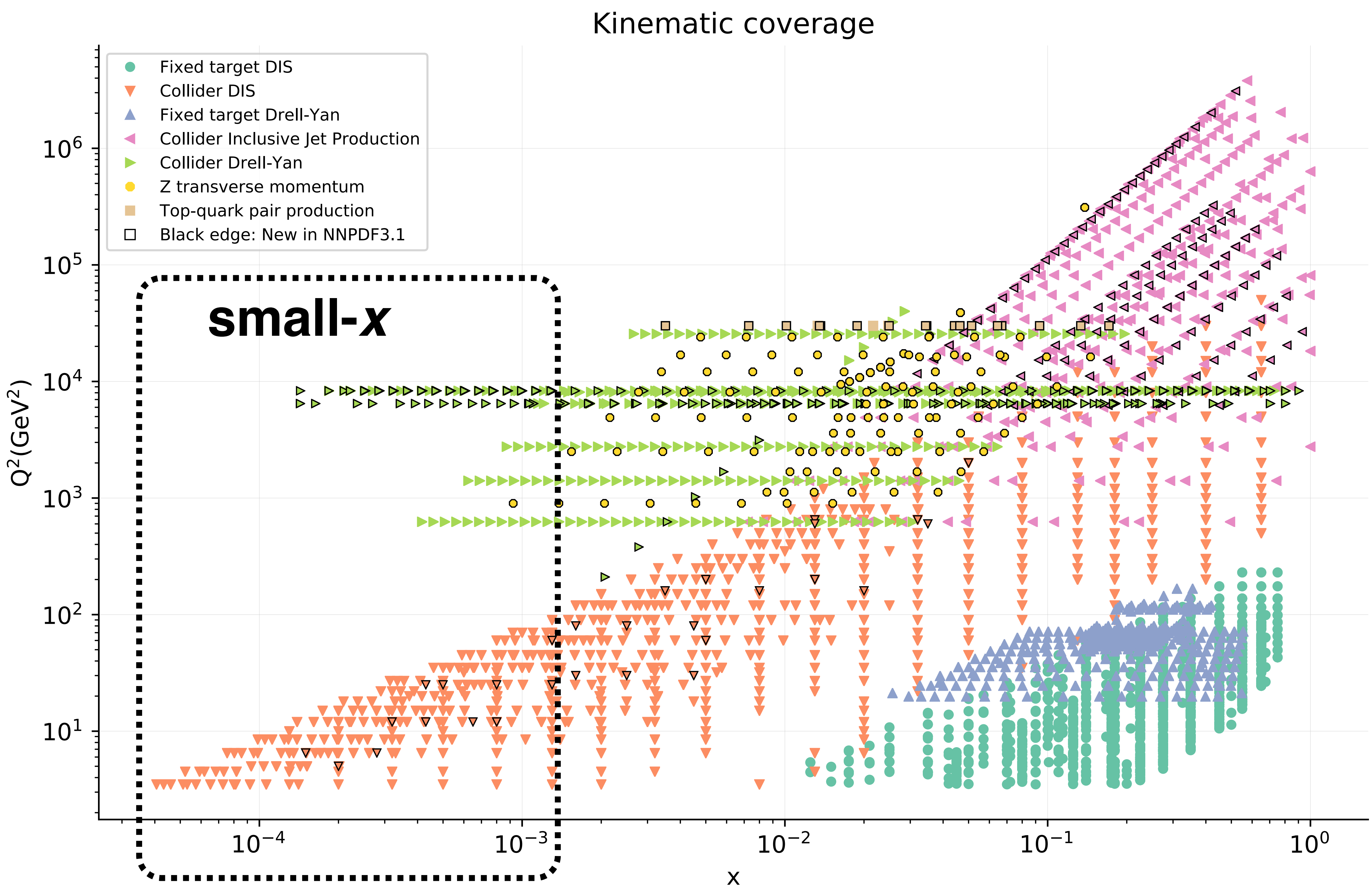
$$-x\frac{d}{dx} f_+(x,\mu^2) = \int_0^\infty \frac{d\nu^2}{\nu^2} K\left(\frac{\mu^2}{\nu^2}, \alpha_s\right) f_+(x,\nu^2)$$

Within small-$x$ resummation, the **$\mathrm{N}^k$LO fixed-order DGLAP splitting functions** are complemented with the **$\mathrm{N}^h$LL$x$ contributions from BKFL**

*ABF, CCSS, TW + others, 94-08*

$$P_{ij}^{\mathrm{N}^k\mathrm{LO}+\mathrm{N}^h\mathrm{LL}x}(x) = P_{ij}^{\mathrm{N}^k\mathrm{LO}}(x) + \Delta_k P_{ij}^{\mathrm{N}^h\mathrm{LL}x}(x),$$

# A new world at small-x



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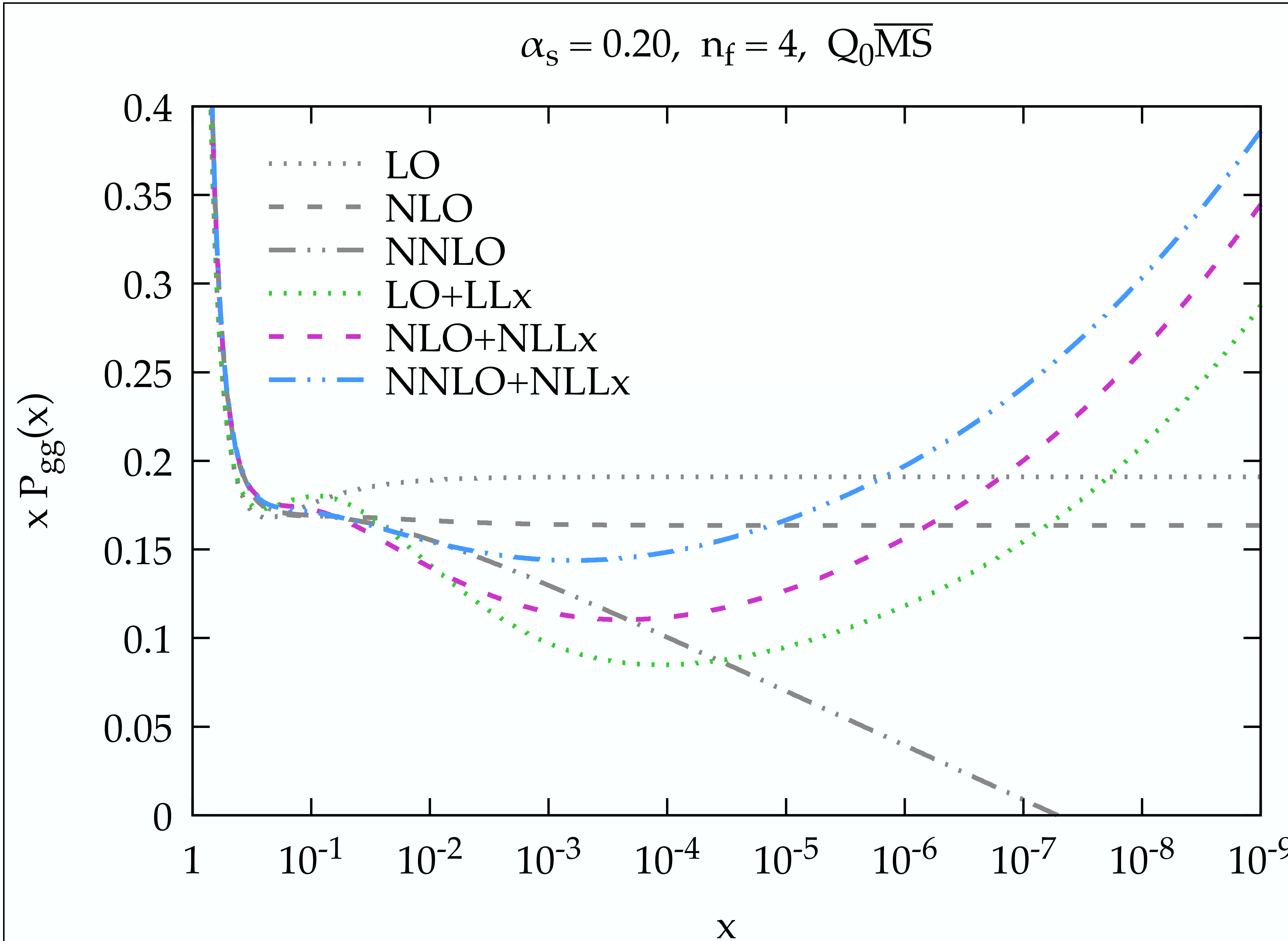
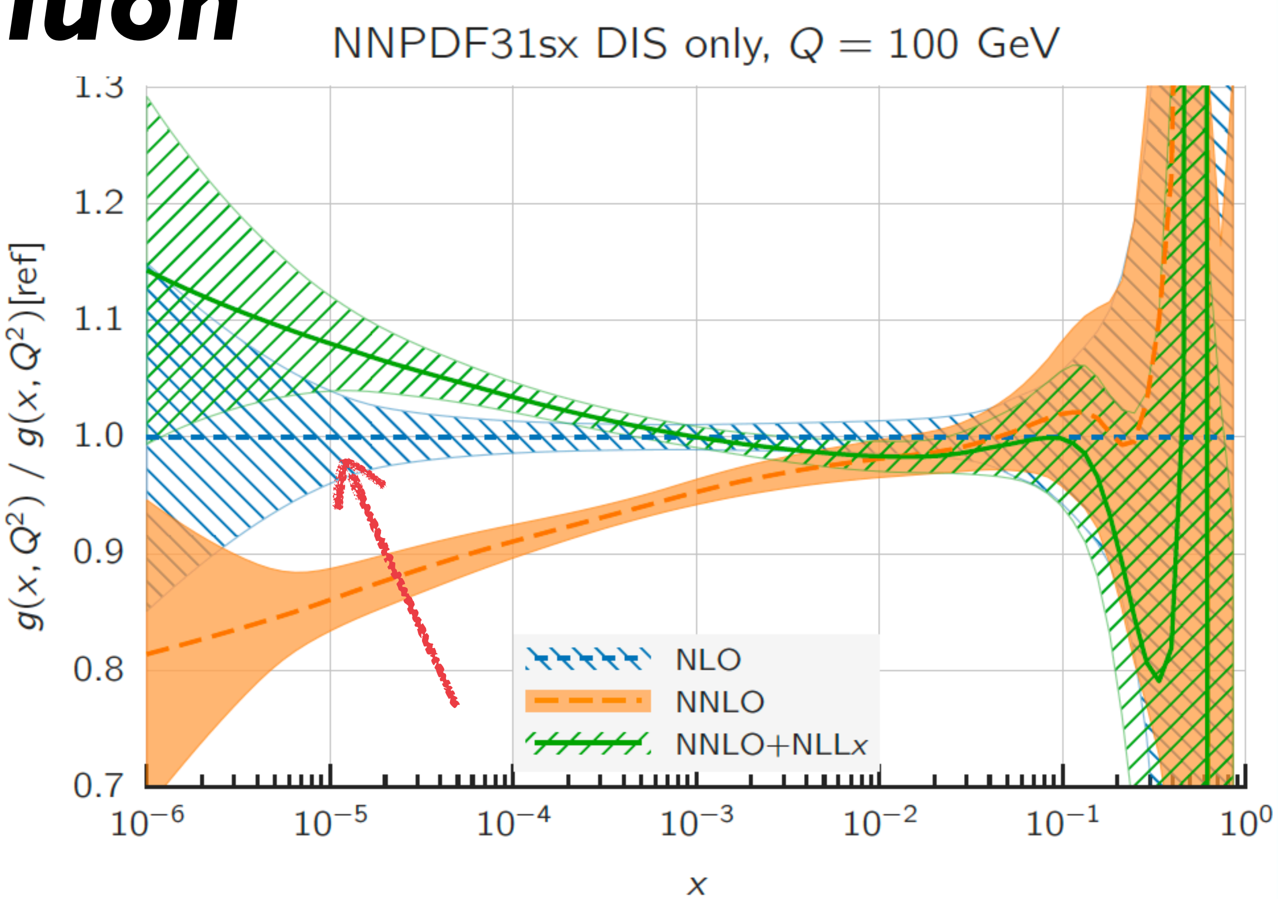


# A new world at small-x

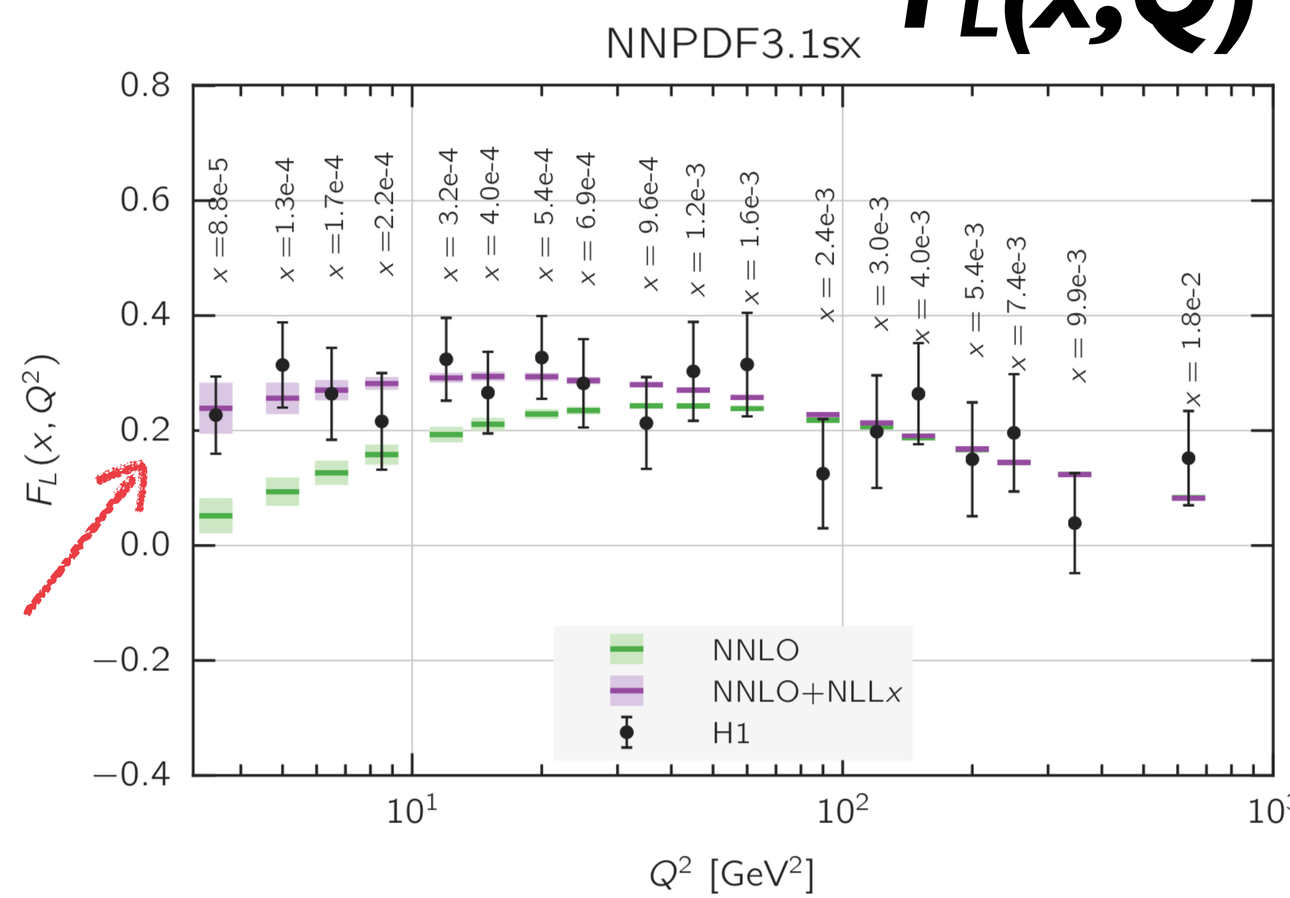
- Ultimately, the need for (or lack of) BKFL resummation in **ep and pp collider data** can only be assessed by performing a **global PDF analysis based on (N)NLO+NLLx theory**

- NNPDF3.1 (N)NLO+NLL fits **stabilise the perturbative PDF expansion at small-*x***, in particular for the gluon, and markedly improve the **fit quality to the small-*x* HERA data**

***gluon***




$\boldsymbol{F_L(x,Q)}$

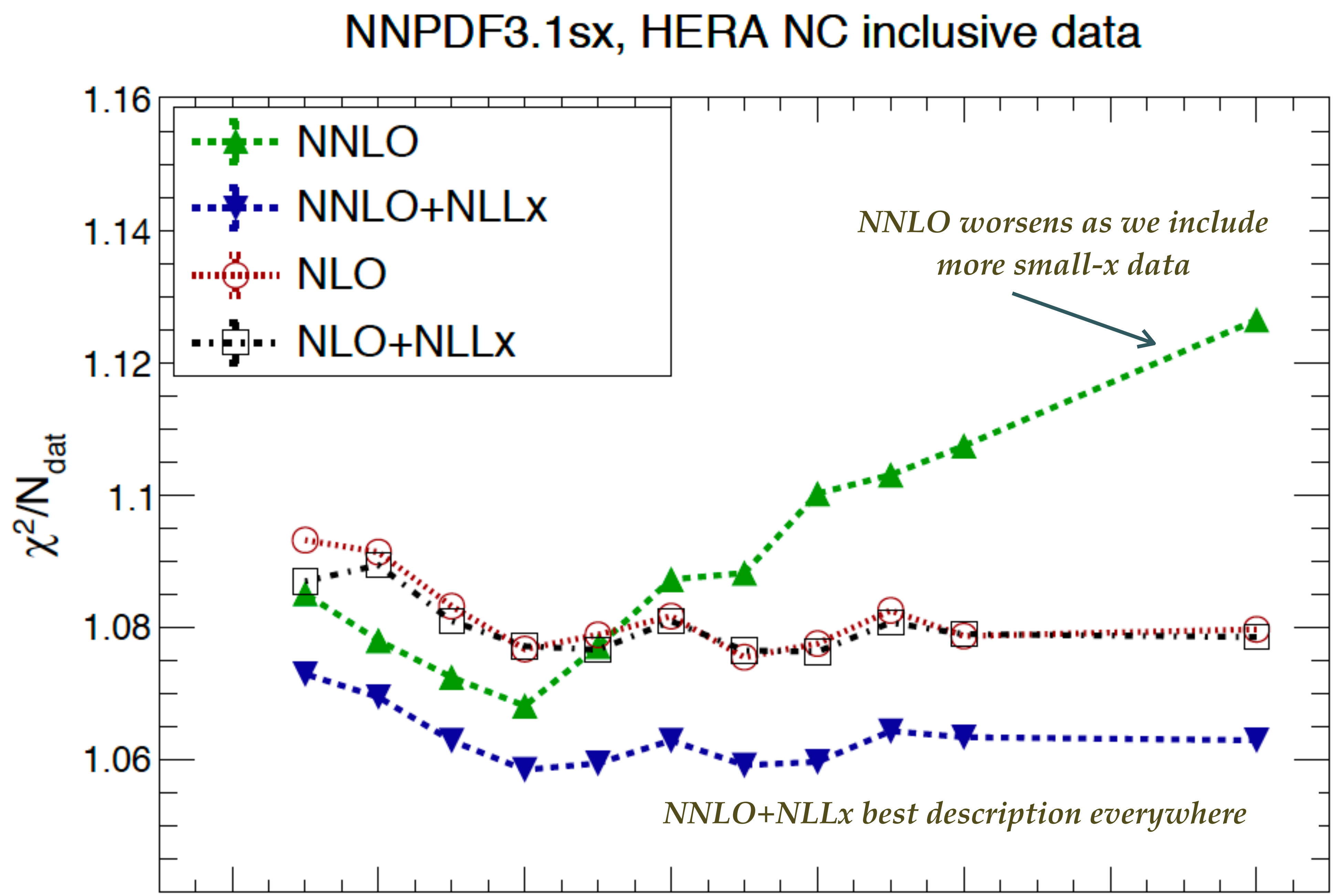



*Ball, Bertone, Bonvini, Marzani, JR, Rottoli 16*

# Evidence for BFKL dynamics in HERA data

Using **NNLO+NLL$x$ theory, the NNLO instability at small-$x$ of the $\chi^2$ disappears**

Excellent fit quality to **inclusive and charm HERA** data achieved in the **entire (x,Q²) region**




*Including more low-x, low Q data*

# Nunca es tarde si la dicha es buena

**Science**
Life and Physics

## After 40 years of studying the strong nuclear force, a revelation

**This was the year that analysis of data finally backed up a prediction, made in the mid 1970s, of a surprising emergent behaviour in the strong nuclear force**

**Jon Butterworth**
@jonmbutterworth
Thu 28 Dec 2017 17.30 GMT

529 59

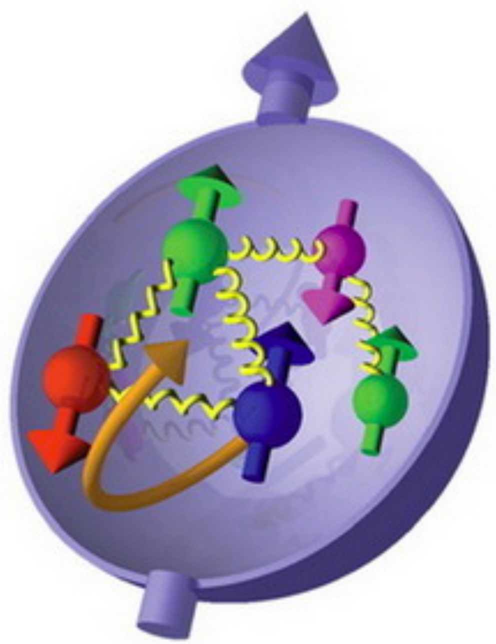
*Jon Butterworth, the Guardian, 28/12/2018*

In the mid 1970s, four Soviet physicists, Batlisky, Fadin, Kuraev and Lipatov, made some predictions involving the strong nuclear force which would lead to their initials entering the lore. "BFKL" became a shorthand for a difficult-to-

# Resummation in PDF fits

**Large-$x$ (threshold) resummation and high-mass BSM physics**

Small-$x$ resummation and BFKL dynamics



Shower resummation and MC event generators

# Why threshold resummation?

- Several methods for threshold (large-$x$) resummation exist
- *i.e.* start from a **factorised cross-section** and transform it to **Mellin (conjugate) space**

$$\sigma(x,Q^2) = x \sum_{a,b} \int_x^1 \frac{dz}{z} \mathcal{L}_{ab}\left(\frac{x}{z},\mu_{\rm F}^2\right) \frac{1}{z}\hat{\sigma}_{ab}\left(z,Q^2,\alpha_s(\mu_{\rm R}^2),\frac{Q^2}{\mu_{\rm F}^2},\frac{Q^2}{\mu_{\rm R}^2}\right)$$

$$\sigma(N,Q^2) = \int_0^1 dx\, x^{N-2}\sigma(x,Q^2) = \sum_{a,b} \mathcal{L}_{ab}(N,Q^2)\hat{\sigma}_{ab}\left(N,Q^2,\alpha_s\right)$$

- Compute a **resummed coefficient function** that includes terms or the type $\boldsymbol{\alpha_S^k \ln^p N}$ ( corresponding to $\boldsymbol{\alpha_S^k \ln^r (1\text{-}x)}$ ) to **all orders in perturbation theory that** are large near the **partonic threshold x -> 1 ( N ->** $\boldsymbol{\infty}$**)**

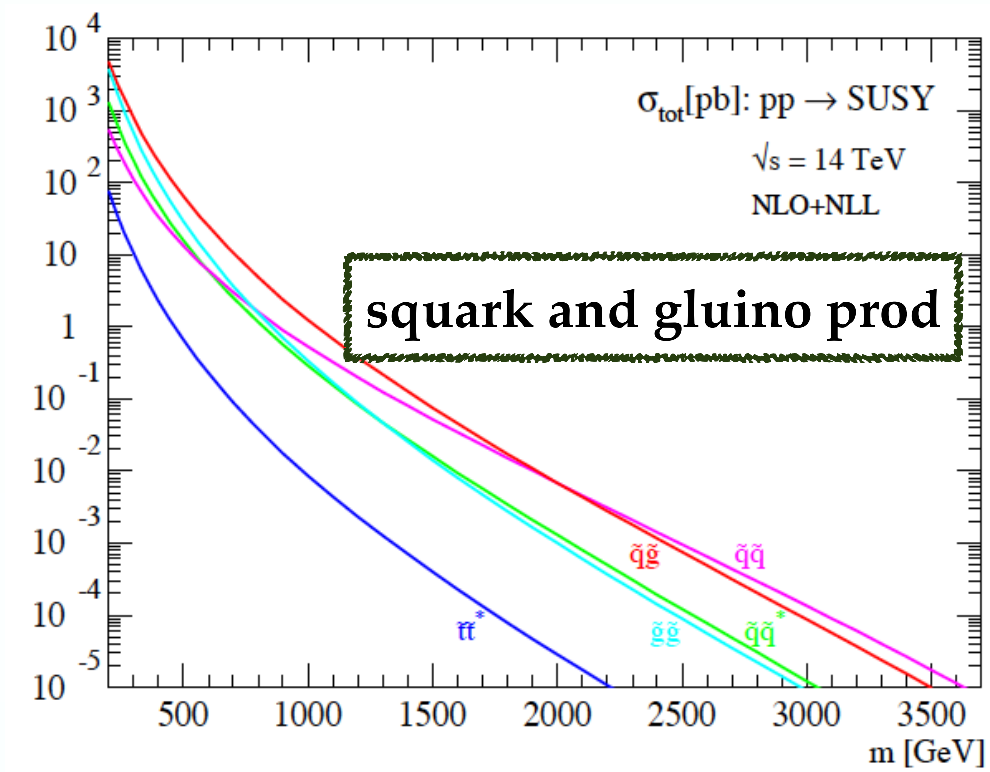
$$\hat{\sigma}_{ab}^{(\rm res)}(N,Q^2,\alpha_s) = \sigma_{ab}^{(\rm born)}(N,Q^2,\alpha_s)\, C_{ab}^{(\rm res)}(N,\alpha_s)$$

$$C^{(N\text{-soft})}(N,\alpha_s) = g_0(\alpha_s)\exp \mathcal{S}(\ln N,\alpha_s),$$
$$\mathcal{S}(\ln N,\alpha_s) = \left[\frac{1}{\alpha_s}g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right]$$

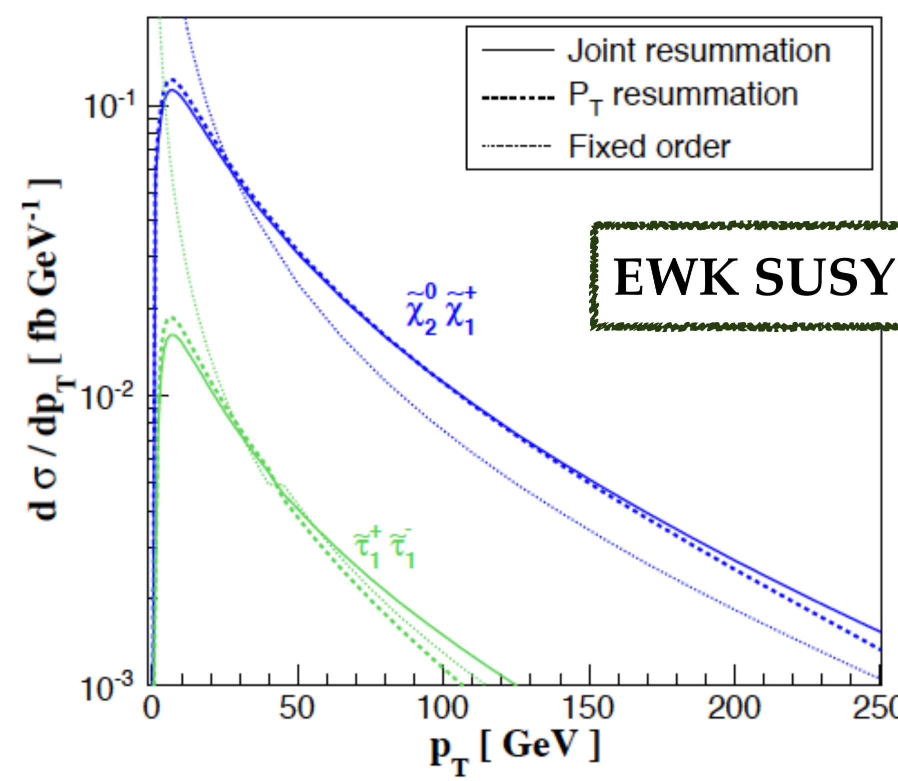
- Threshold resummation **improves the perturbative expansion**, reduces scale uncertainties and allows to **construct approximate higher-order results**
- Note that threshold resummation effects are absent from the DGLAP splitting functions

# Why threshold resummation?

- **Threshold resummation of partonic cross-sections** extensively used in precision LHC phenomenology ….
- but only for partonic cross-sections, the **fitted PDFs could also be affected by resummation**

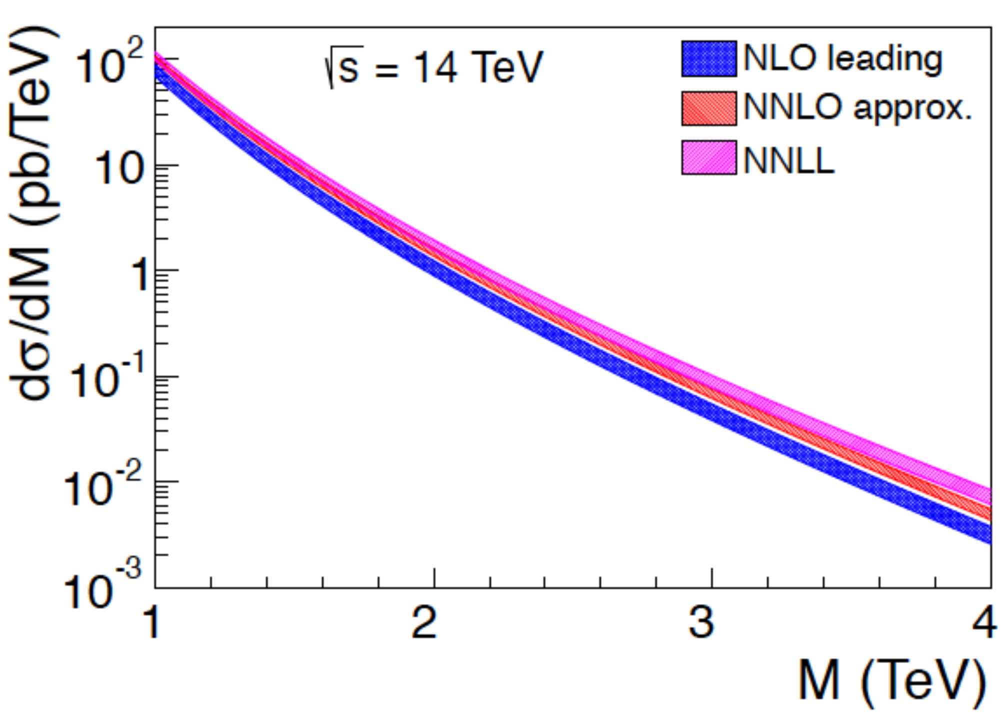



**squark and gluino prod**

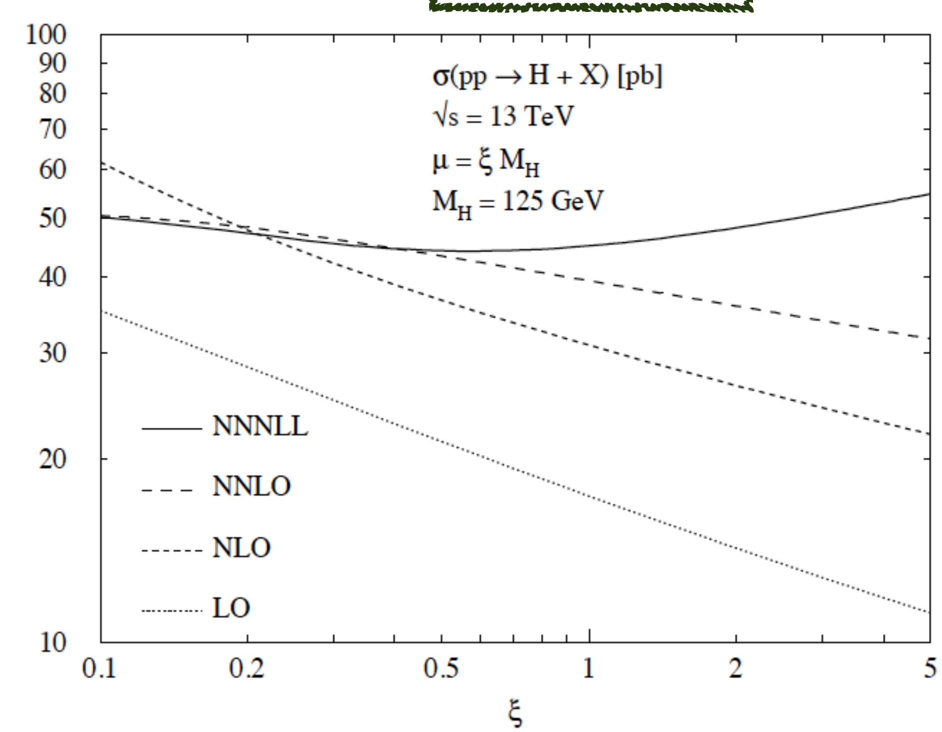



**EWK SUSY prod**

**large mass top quark pair production**




**gg -> Higgs**



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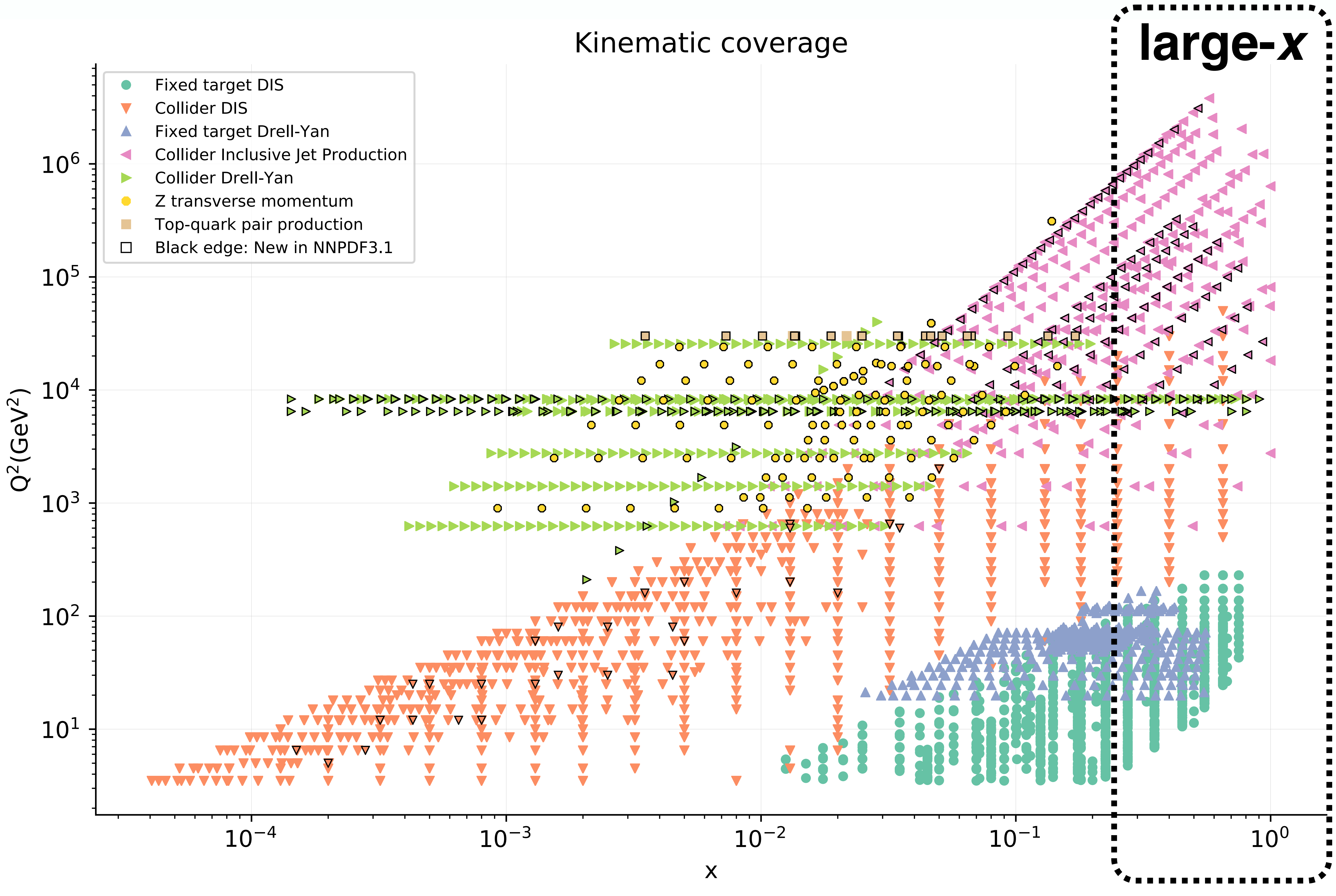

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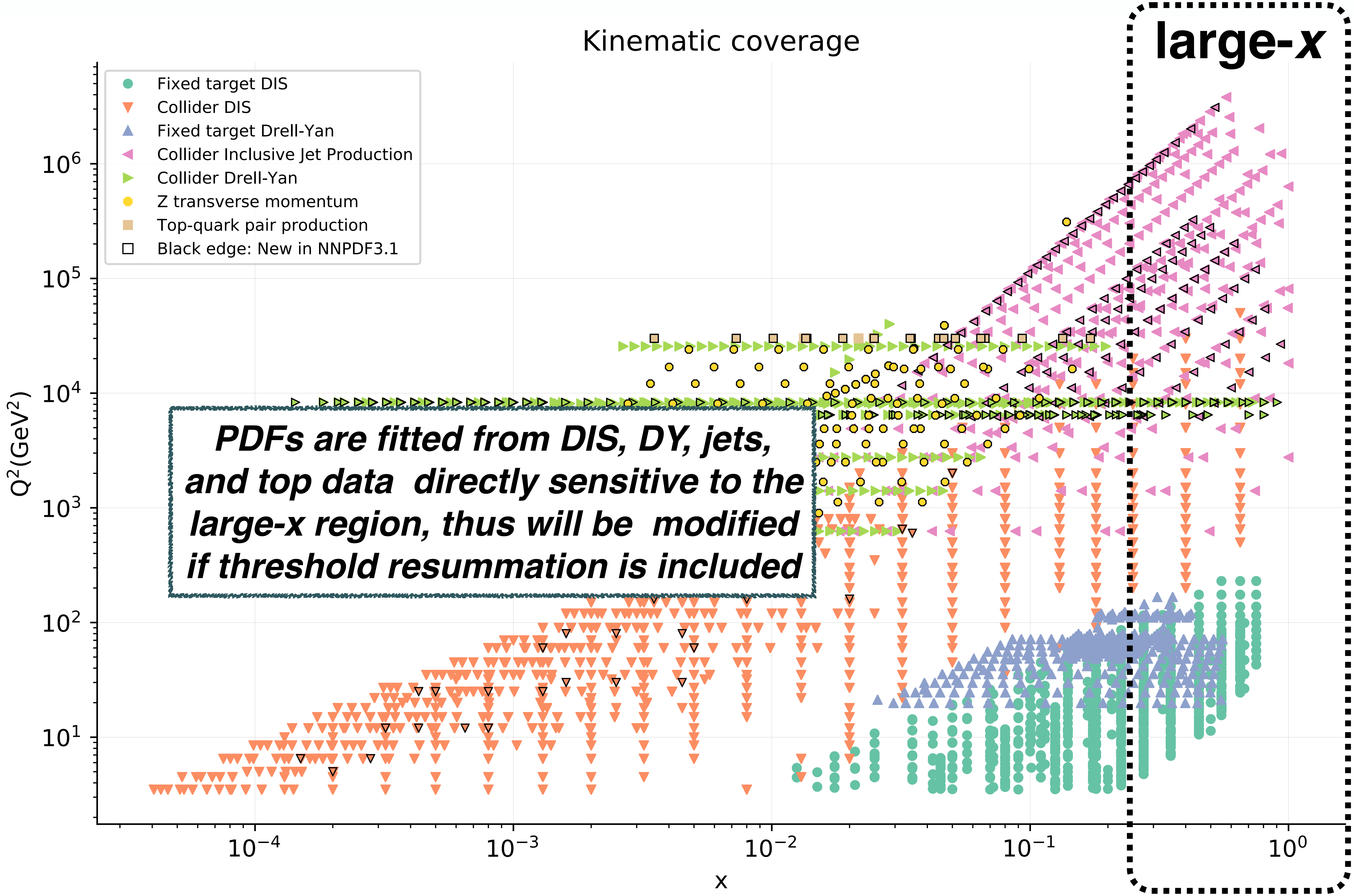
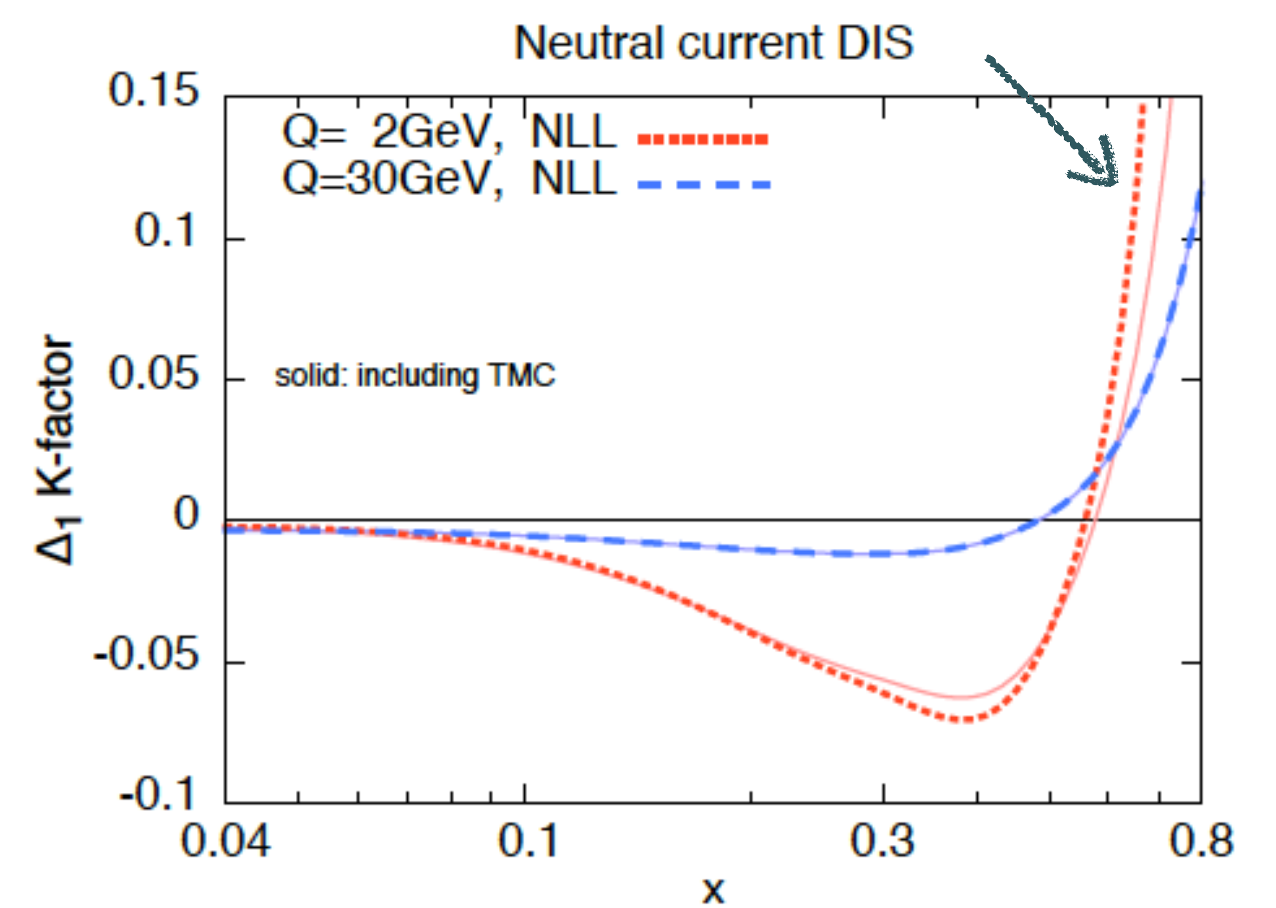


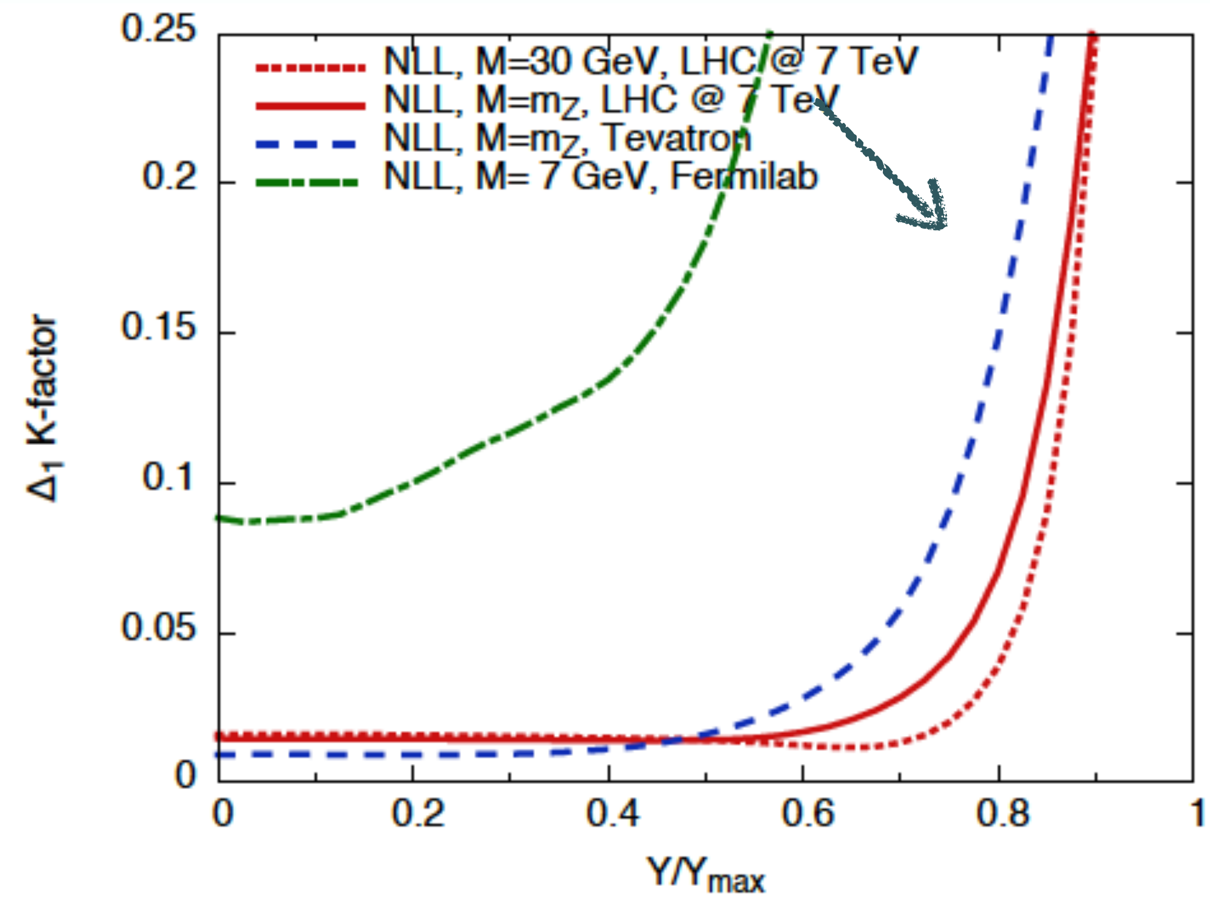

***PDFs are fitted from DIS, DY, jets, and top data directly sensitive to the large-x region, thus will be modified if threshold resummation is included***

# threshold resummation of DIS and DY

☑ **Enhancement due to NLL** resummation in DIS structure functions and DY distributions will lead to a suppression of the fitted PDFs

$$\sigma_{\mathrm{N}^j\mathrm{LO}+\mathrm{N}^k\mathrm{LL}} = \sigma_{\mathrm{N}^j\mathrm{LO}} + \sigma_{\mathrm{LO}} \times \Delta_j K_{\mathrm{N}^k\mathrm{LL}}$$



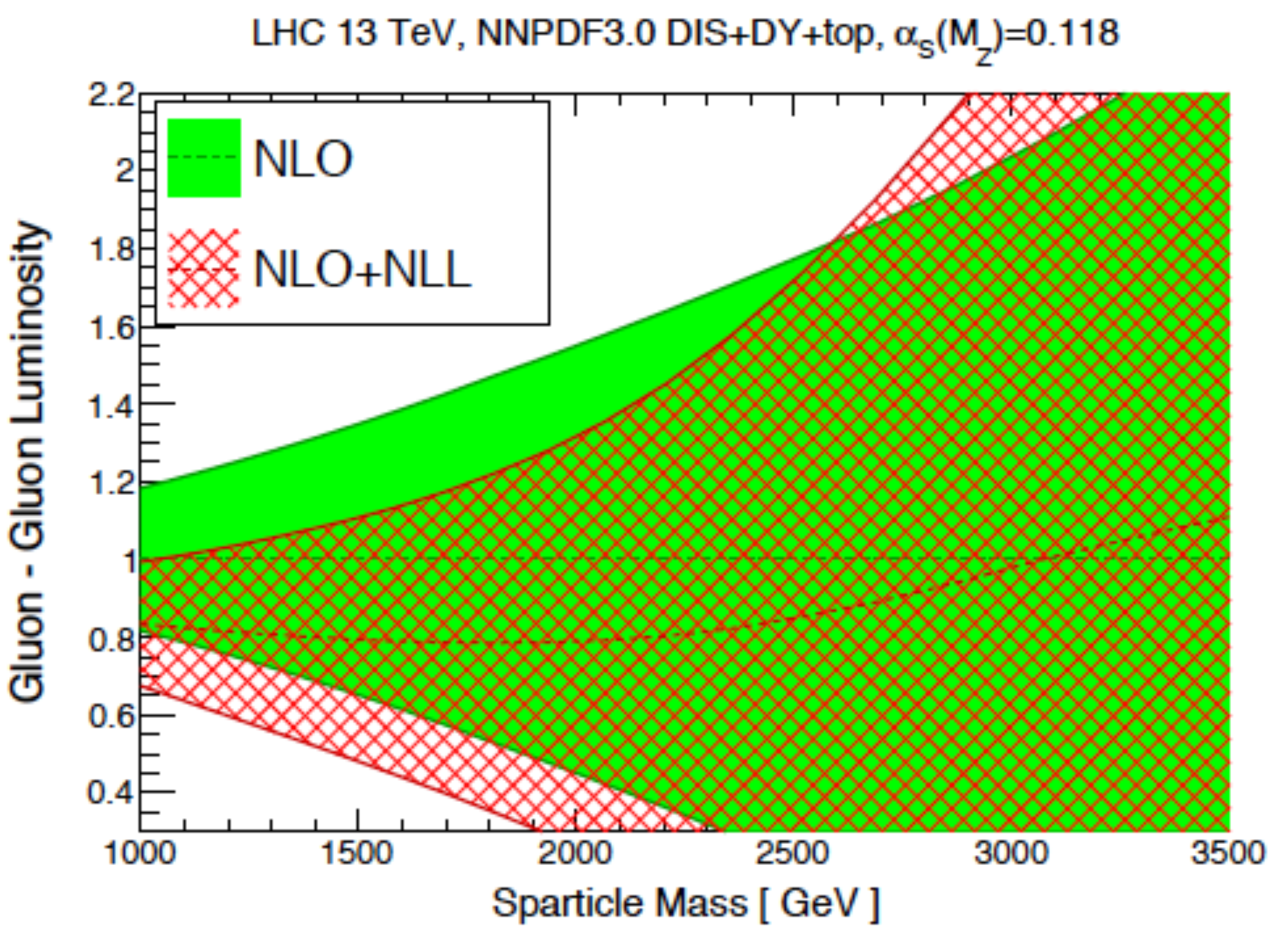



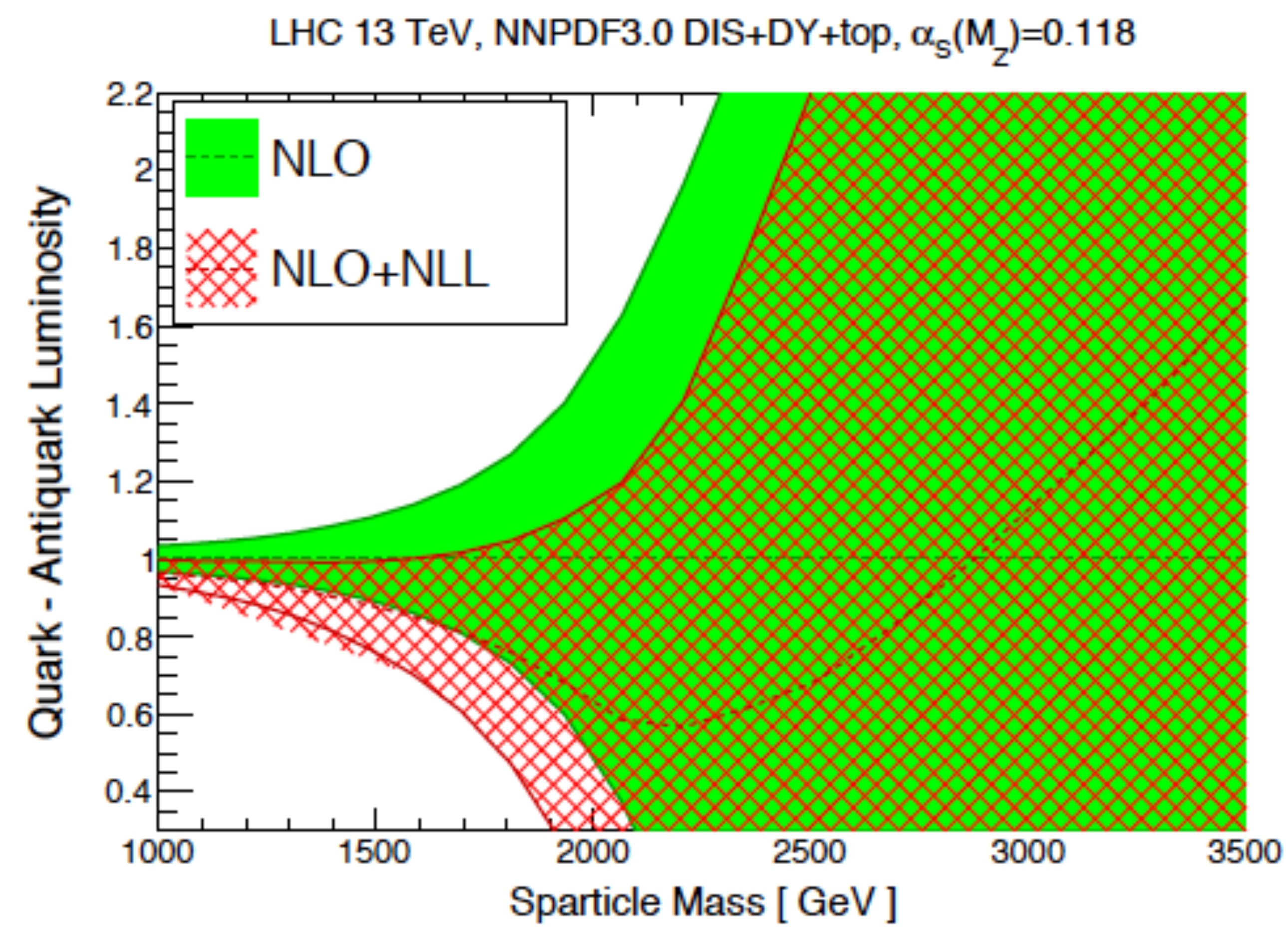

*Bonvini, Marzani, JR, Rottoli, Ubiali , Ball, Bertone, 15*

☑ Effects of resummation are reduced when **matched to NNLO calculations**, as well as at **high energies**

# PDFs with threshold resummation

**Threshold-improved PDFs** can differ substantially wrt fixed-order PDFs: **up to -20% for gg luminosity and -40% for quark-antiquark luminosity,** in the high-mass region relevant for new BSM heavy particles

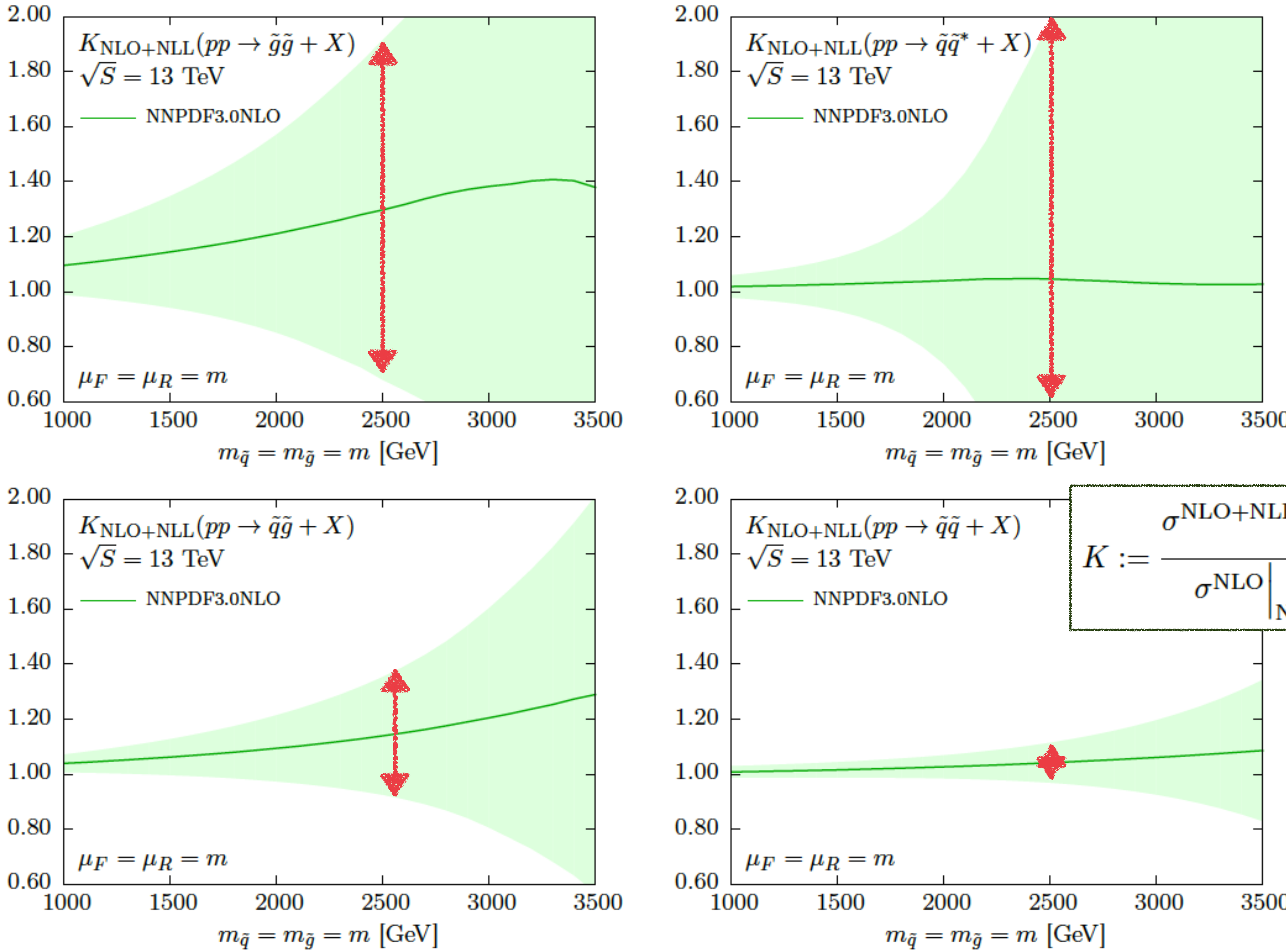



**Phenomenologically most relevant:** this suppression **will partially compensate enhancements** in partonic cross-sections for processes not included in the fit (SUSY, Higgs, ttbar differential, …..)

# Updated NLO+NLL cross-sections with NNPDF3.0NLO

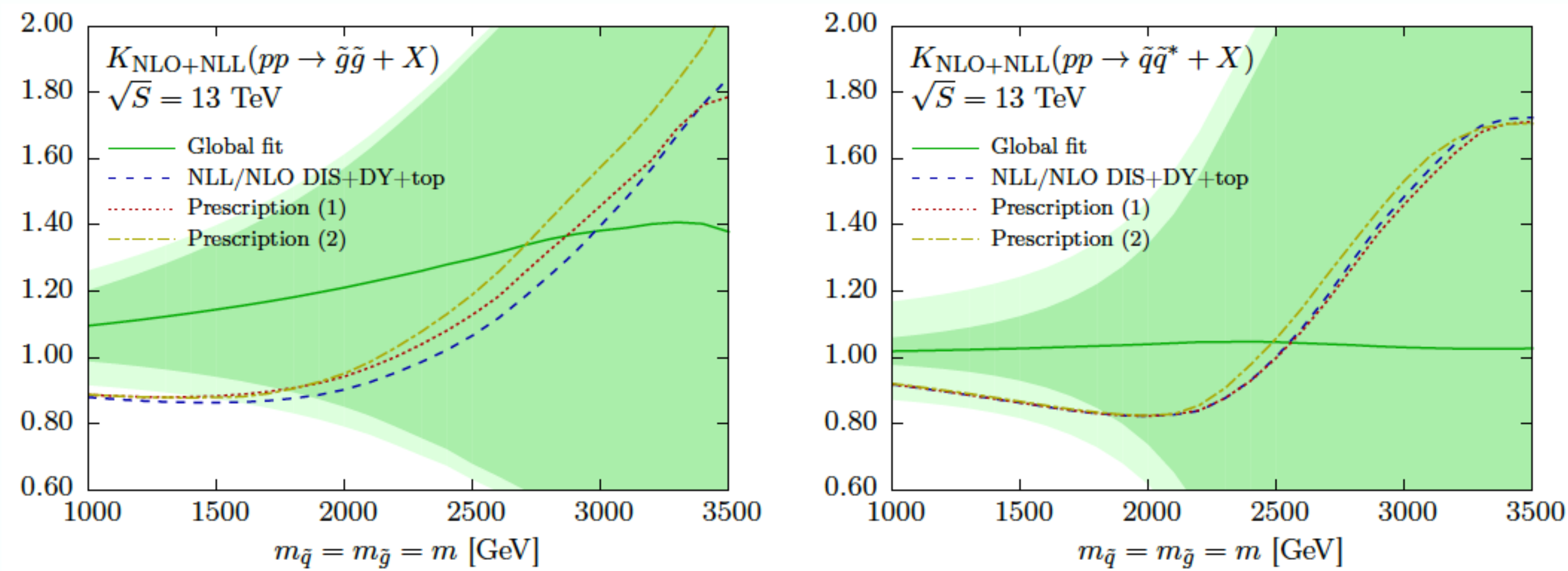
- **Previous NLL-fast calculations** at 13 TeV based on CTEQ6.6 and MSTW08 sets
- **NLL-fast version 3.1** has now been updated to **NLO+NLL cross-sections with NNPDF3.0NLO**




$$K := \frac{\sigma^{\mathrm{NLO+NLL}}\Big|_{\mathrm{NLO\ global}}}{\sigma^{\mathrm{NLO}}\Big|_{\mathrm{NLO\ global}}}\,.$$

# NLO+NLL SUSY xsecs with threshold-improved PDFs

- Now include the effect of **NLO+NLL threshold-improved PDF**
- The resulting shift **changes qualitatively and quantitatively** the behaviour of NLO+NLL SUSY xsecs
- Shift within total theory band so **current exclusion limits unaffected**
- But will become crucial if we ever need to **characterise SUSY particles from LHC data**, much in the same way as we do for the Higgs boson




*To be extended to N(N)LO+NNLL calculations!*

*Beernakker, Borchemsky, Kramer, Kulesza, Laenen, Marzani, JR 15*

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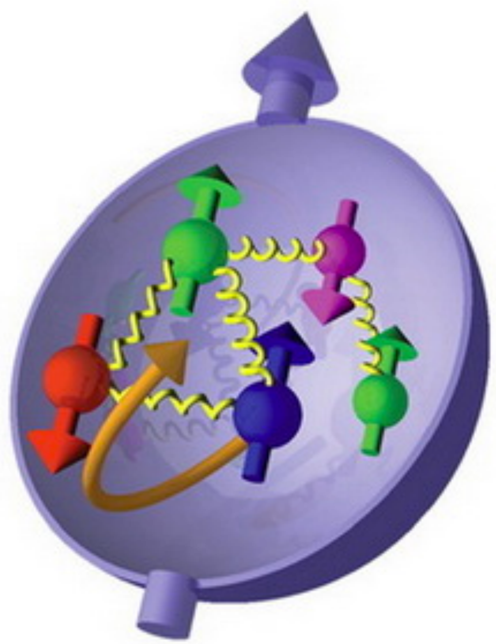


**Shower resummation and MC event generators**

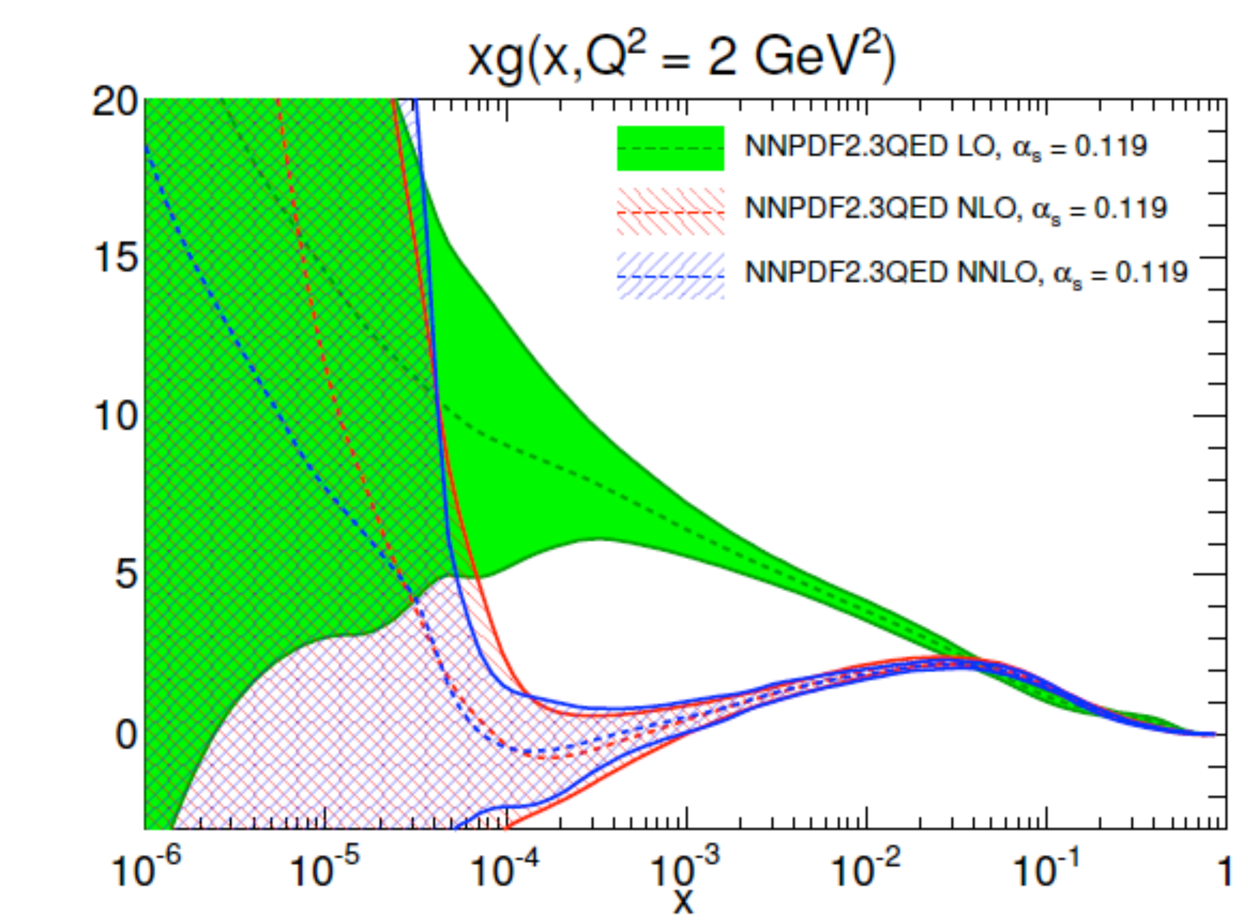
# Shower resummation and MC generators

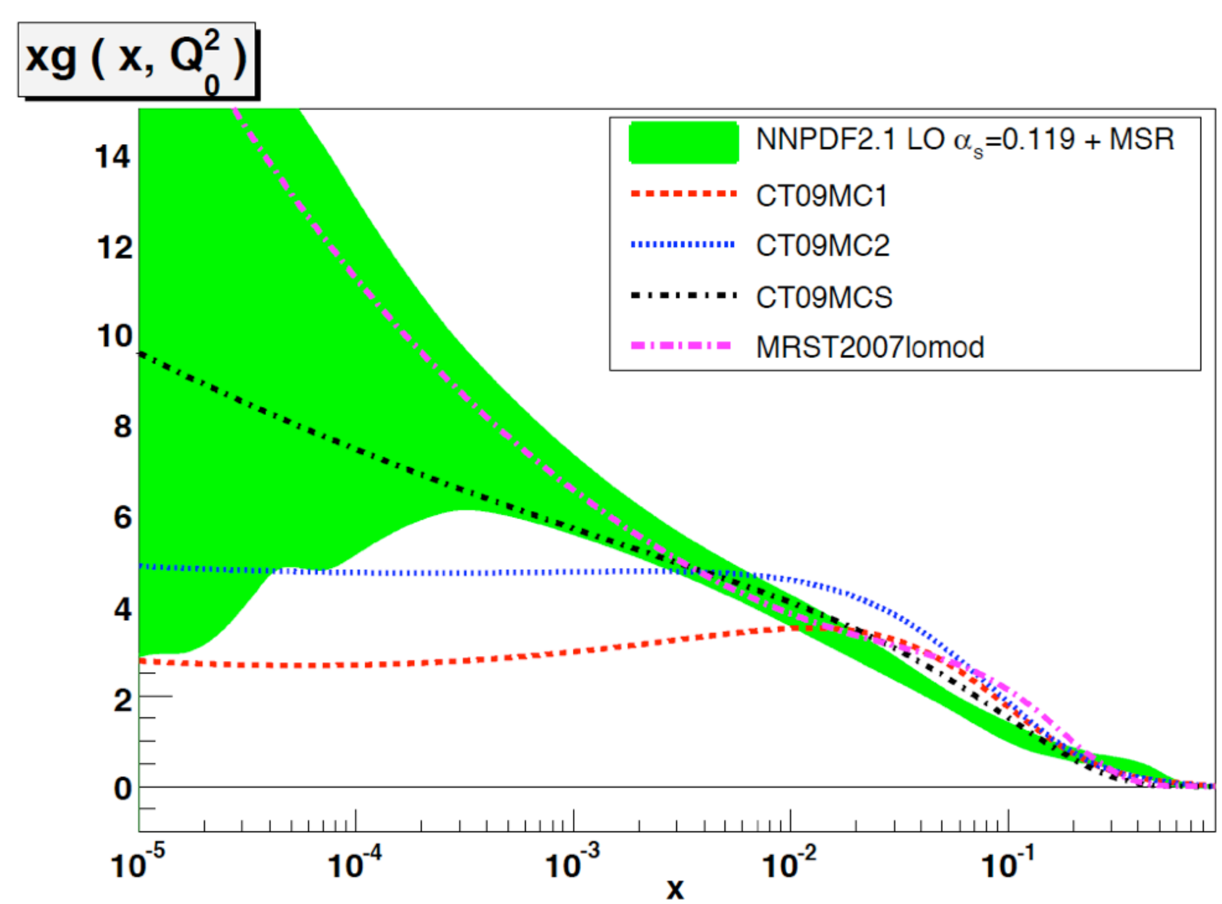
- Parton Distributions are also essential ingredient of **Event Generation at the LHC, where they are relevant** to describe both **perturbative and non-perturbative physics** such as UE/MPI/….

- **Leading order PDFs** are sometimes advocated, since the steep rise of the gluon at small-$x$ fits better **soft and semi-hard data** keeping other parameters of the tune with reasonable values (in Pythia8 at least …)

- **LO PDFs** are very different from NLO and NNLO PDFs, in particular for the **gluon PDF,** due to missing higher-order corrections when fitting the HERA structure function data

- LO **gluons differ a lot between groups:** direct impact on **MC tuning studies**

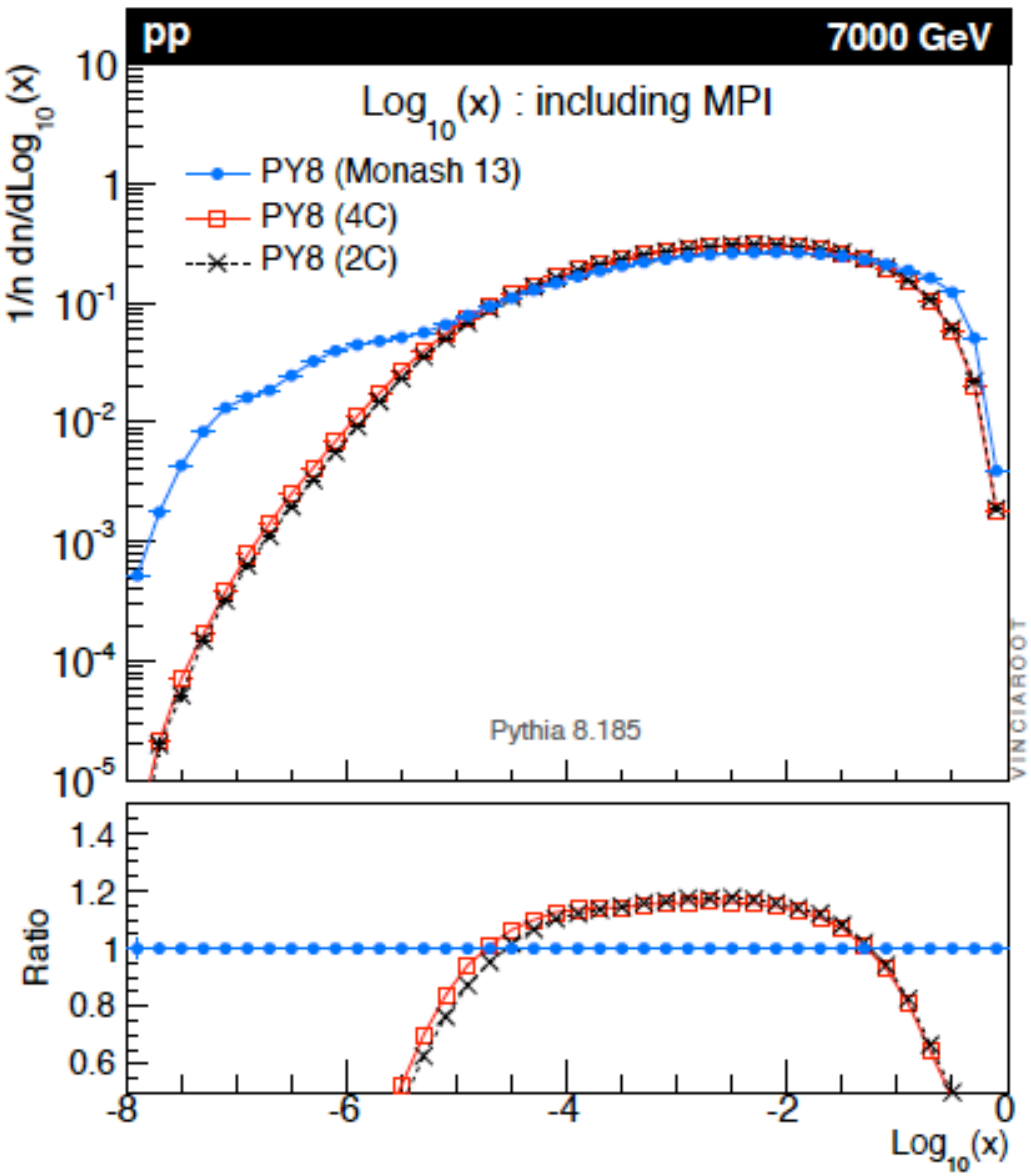


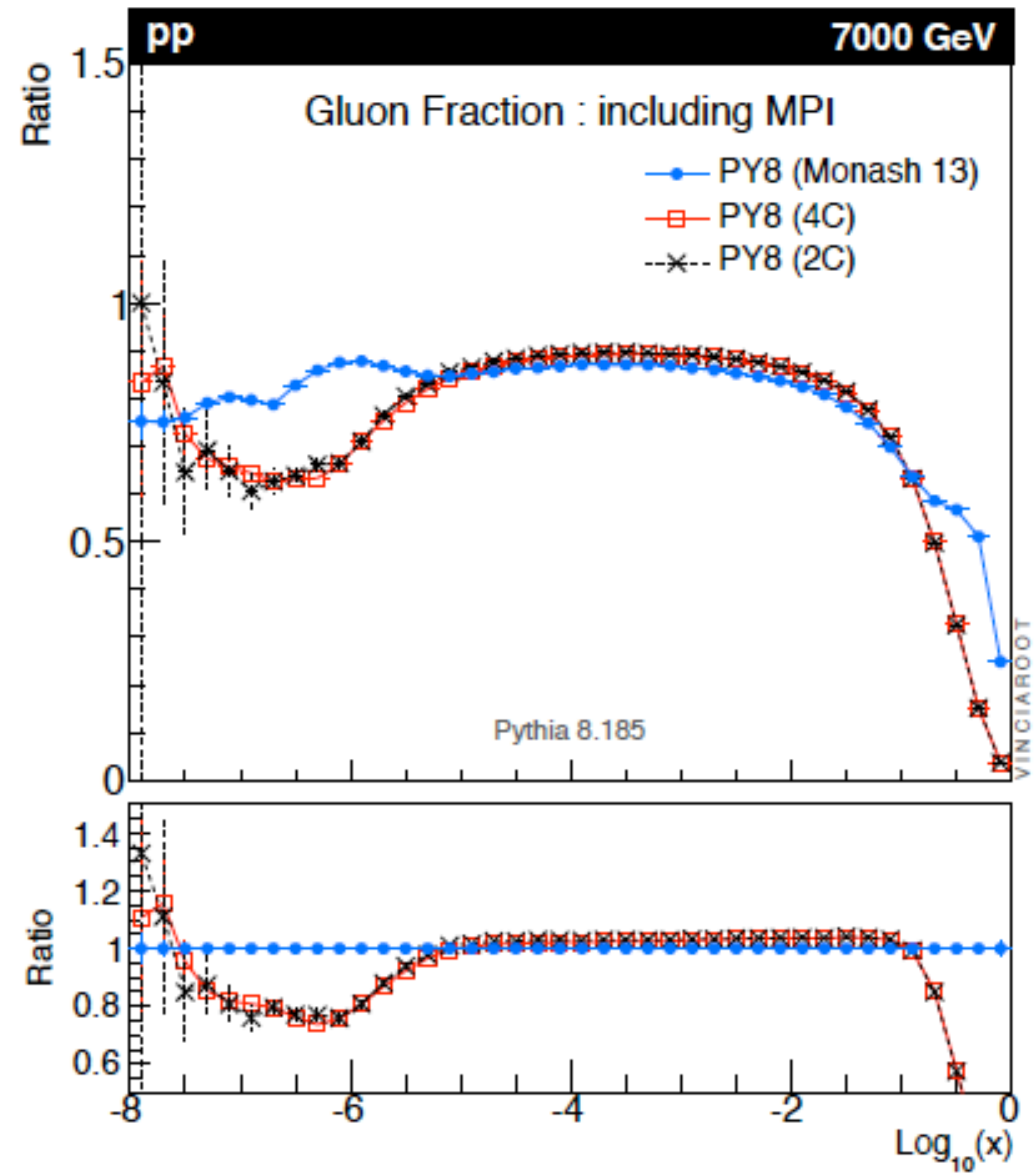



# PDFs and soft QCD modelling

- The distribution of values of **Bjorken-x** probed in Minimum Bias events, including MPI, is directly sensitive to the **shape of the gluon PDF**

- In the Monash 2013 tune, based on NNPDF2.3LO, we find a harder distribution at large and small-x as compared to previous tunes, based on CTEQ6L, consistently with the **differences between gluon PDFs**




*Skands, Carrazza, Rojo 14*

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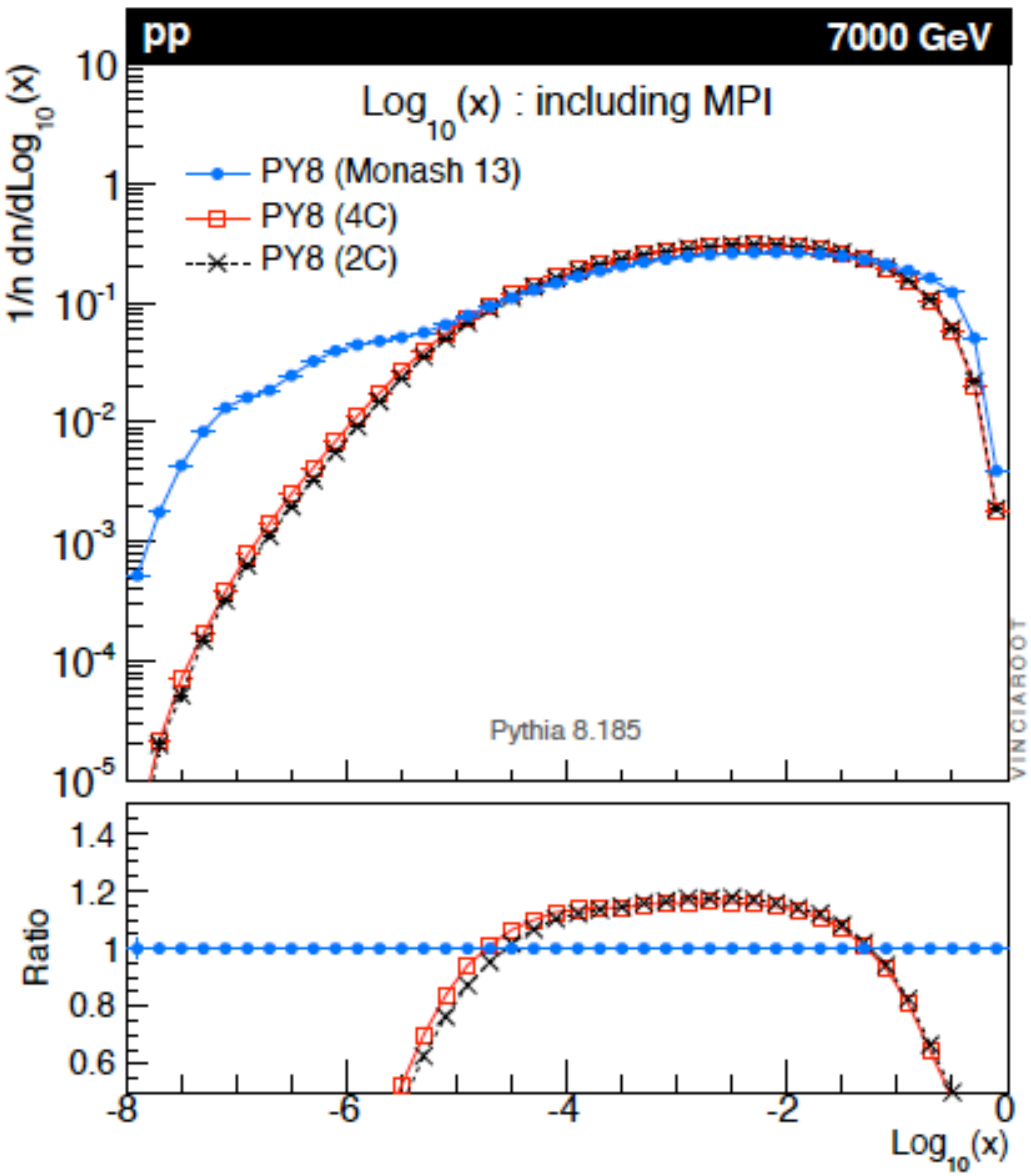
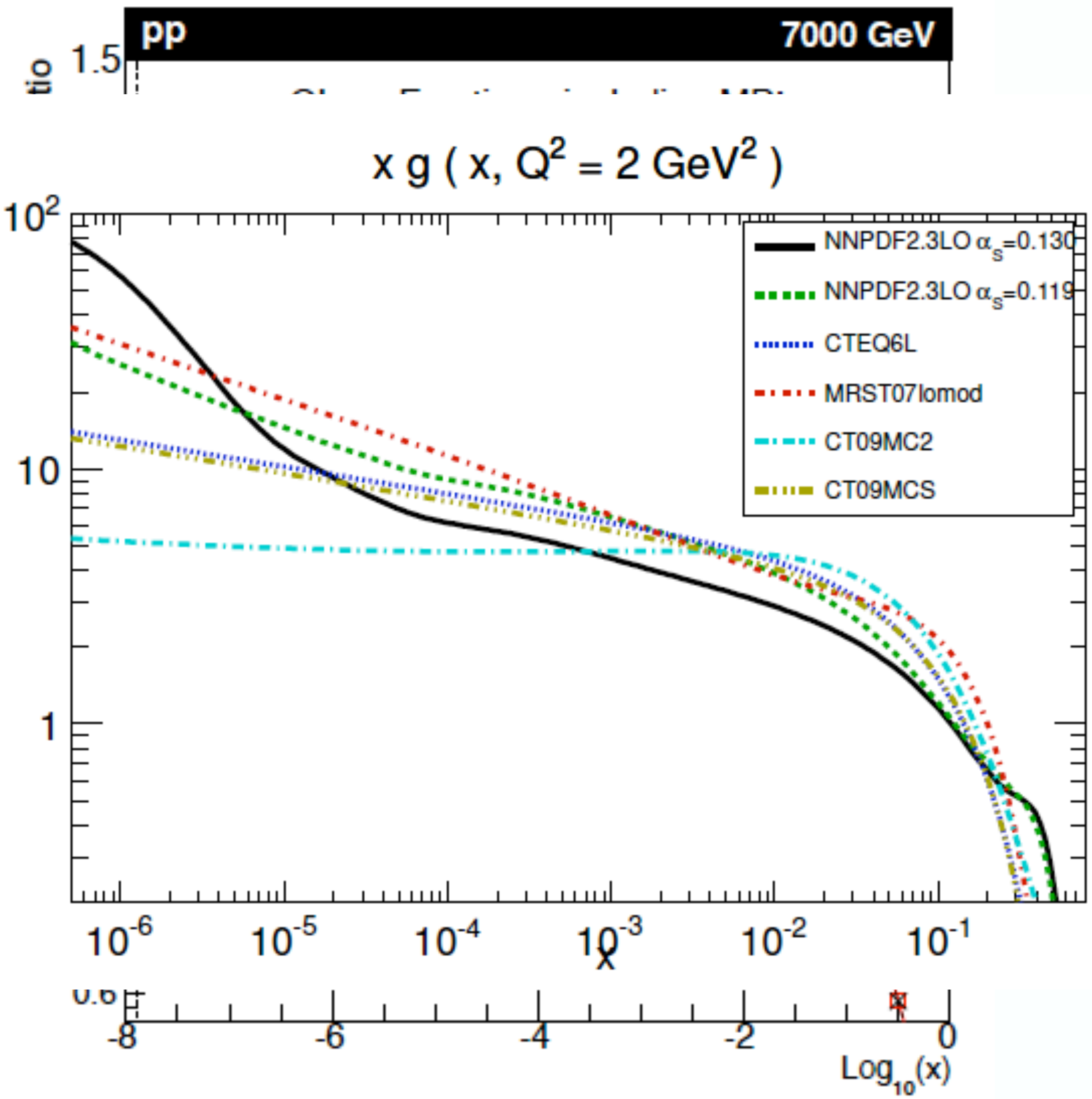
- In the Monash 2013 tune, based on NNPDF2.3LO, we find a harder distribution at large and small-x as compared to previous tunes, based on CTEQ6L, consistently with the **differences between gluon PDFs**



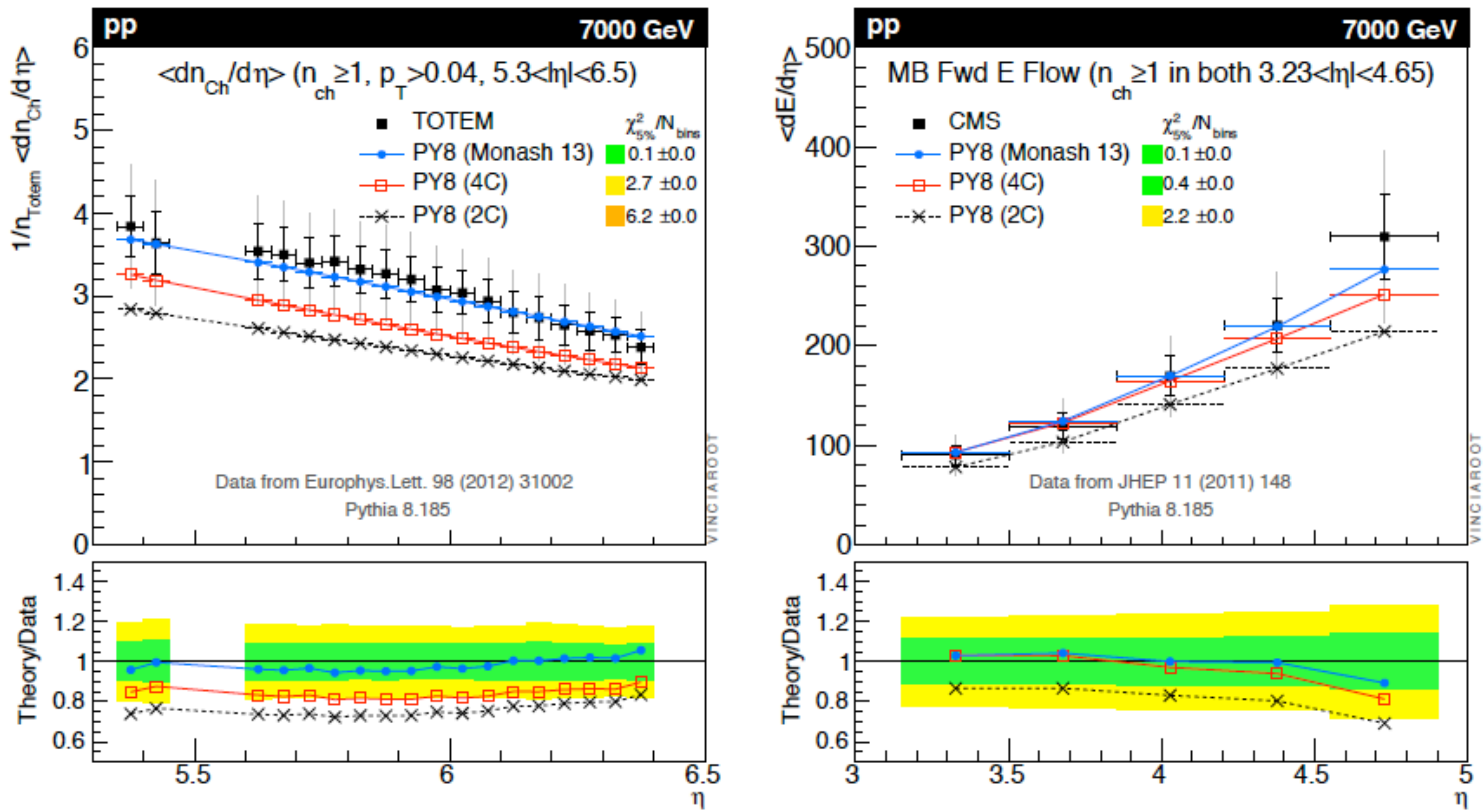




*Skands, Carrazza, Rojo 14*

# Tuning forward data

- The **precise forward measurements** from **TOTEM** and **CMS** are important in the context of tuning Event Generators, since they provide **direct access to the ``gluon PDF"** in a region with scarce constraints

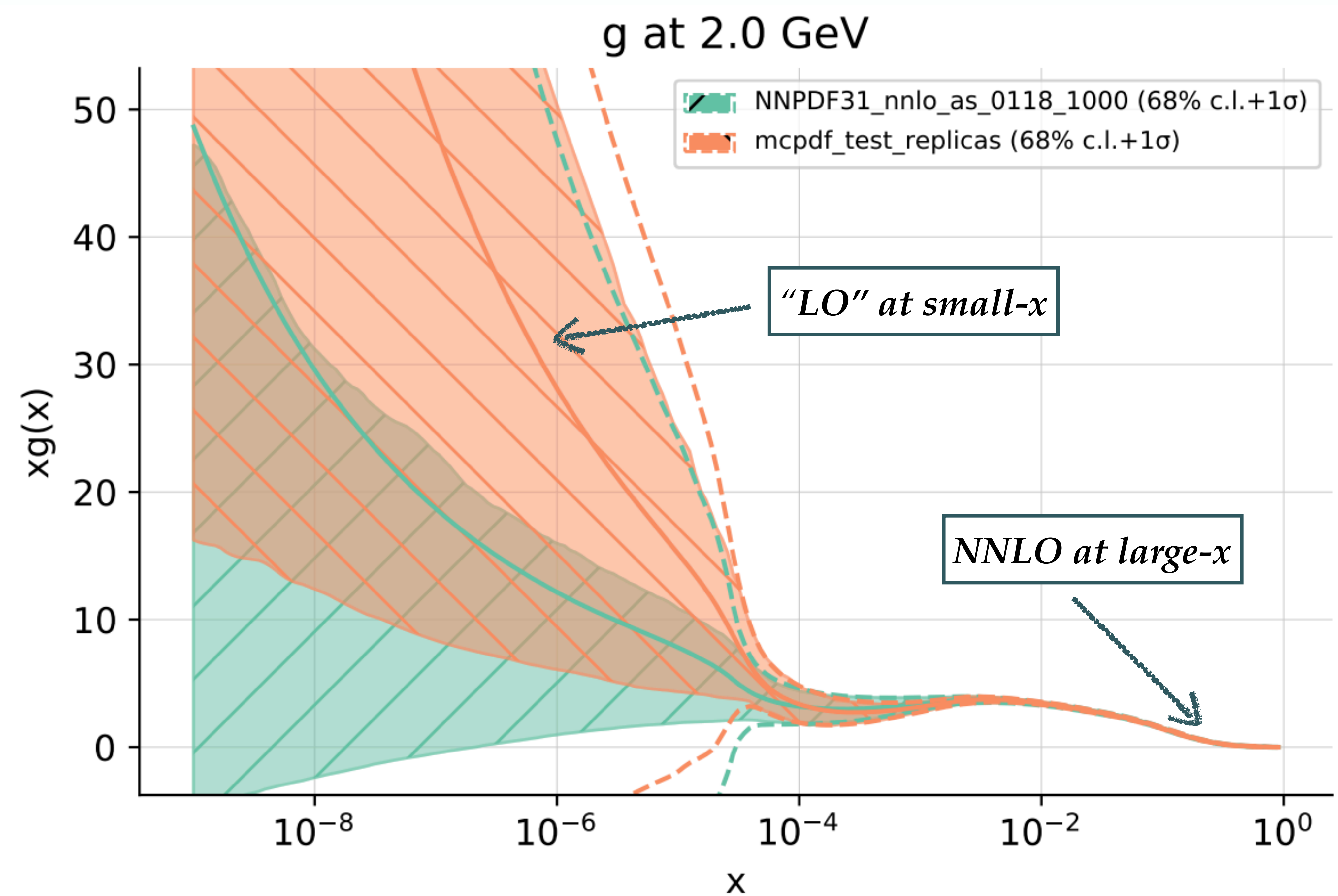
- The Monash2013 tune describes **better the forward data**, specially for **TOTEM large rapidity data**, where previous tunes clearly undershoot the data: again mostly consequence of the steeper small-x gluon



# Towards better "MC PDFs"

- Work in progress towards producing better **"MC PDFs"** that **interpolate between NNLO PDFs** (that provide the best fit to data in the medium and large-x region) and **LO PDFs** (helpful for MC tunes)

- Ask me if you want more details!




*Kassabov PDF4LHC Sept 17*

# Resummation and PDF fits

- **All-orders resummations** are by now an integral part of the global PDF fitting toolbox!

  - ☑ **Small-x resummation**: stabilises perturbative convergence of small-$x$ PDFs, improves descriptive of small-$x$ HERA data, could be relevant also for LHC cross-sections

  - ☑ **Large-x resummation**: extends the validity of the perturbative expansion in the large-$x$ region, allows threshold-improved PDFs for tailored LHC applications (*e.g*. SUSY)

  - ☑ **MC resummation**: PDFs relevant for MC tuning, details depend on how much and how perturbative information is encoded into the shower algorithms

- Several **resummation formalisms have been developed within SCET**, perhaps some of them could be used within the framework of global PDF fits?

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Thanks for your attention!

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