Quasi transverse-momentum dependent PDFs

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Introduction: Quasi PDFs

Collinear PDFs

Reminder: Quark PDF

- Operator definition of quark PDF

$$f_q(x) = \int \frac{\mathrm{d}\xi^+}{4\pi} e^{\mathrm{i}\xi^+ (xP_n^-)} \left\langle P(P_n) \middle| \bar{q}(\xi^+) W_n(\xi^+, 0) \gamma^- q(0) \middle| P(P_n) \right\rangle$$

- Quark fields separated by lightlike Wilson line

$$W_n(\xi^+, 0) = P \exp\left[\mathrm{i}g \int_0^{\xi^+} \mathrm{d}s \, \bar{n} \cdot \mathcal{A}(s\bar{n}) \right]$$

- Gauge invariant and boost invariant lightcone correlation

Phenomenological importance:

- Crucial input for theory predictions
- Interesting probe of proton structure by itself

Can we calculate PDFs from first principle?

Lattice determination of PDFs

Lattice QCD

- Only practical tool for nonperturbative calculations
- Based on the path integral with imaginary time $t = \mathrm{i}t_E$:

$$\langle\mathcal{O}\rangle = \int D\psi D\bar{\psi} D\mathcal{A}\ \mathcal{O}\ e^{\mathrm{i}S} \to \int D\psi D\bar{\psi} D\mathcal{A}\ \mathcal{O}\ e^{-S_E}$$

- In general: can only calculate Euclidean-time dependence
  - Requires analytical continuation to Minkowski time

Obstacles to calculating PDFs from lattice

- PDF is time-dependent correlation function
- No lightlike kinematics on lattice: $n_E^2 = 0 \ \Leftrightarrow \ n_E^\mu = 0$
- Many proposals on PDF determination in recent years
  - Focus of this talk: Quasi PDF [Ji '13, '14]

Quasi PDFs

- PDF:

$$f_q(x,\mu) = \int \frac{\mathrm{d}\xi^+}{4\pi} e^{\mathrm{i}\xi^+(xP_n^-)} \Big\langle P(P_n) \Big| \bar q(\xi^+) W_n(\xi^+,0)\, (\gamma_0+\gamma_3)\, q(0) \Big| P(P_n) \Big\rangle$$

- Quasi PDF: Drop time dependence [Ji '13, '14]

$$\tilde f_q(x,P_z,\mu) = \int \frac{\mathrm{d}z}{4\pi} e^{\mathrm{i}z(xP_z)} \big\langle P(P_z) \big| \bar q(z) W_z(z,0) \gamma_3 q(0) \big| P(P_z) \big\rangle$$

  - Describes same IR physics as PDF
  - Differs in UV (partially due to P_z)

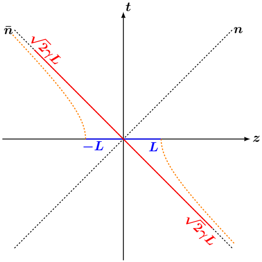
- Factorization theorem: [Xiong, Ji, Zhang, Zhao '13 (one loop)]
[Ji, Jin, Stewart, Zhao '18 (full proof)]

$$\underbrace{\boxed{\tilde f_i(x,P_z,\tilde\mu)}}_{\text{Simulation on lattice}} = \int_0^1 \frac{\mathrm{d}y}{y} \underbrace{\boxed{C_{ij}\left(\frac{x}{y},\frac{\tilde\mu}{P_z},\frac{\mu}{yP_z}\right)}}_{\text{Perturbative matching coefficient}} \underbrace{\boxed{f_j(y,\mu)}}_{\text{PDF}} + \underbrace{\boxed{\mathcal{O}\left(\frac{M^2}{P_z^2},\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}\right)}}_{\text{Higher-twist correction}}$$

Quasi PDFs

Physical interpretation

- Start with equal-time correlator
  - Calculable on lattice
- Boost along z direction
  - Spatial Wilson line approaches lightlike Wilson line
  - Equal-time correlation approaches lightlike correlation
  - Corrections suppressed by $\mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$

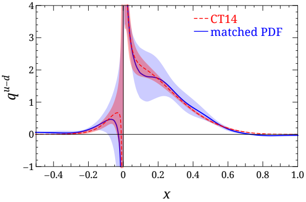


Current state:

- Quasi PDFs actively studied by lattice community

  [Lin et al. (LP^3 collaboration) '15 '16 '17]

  [Alexandrou et al. (ETMC collaboration) '15 '16 '17]
- This talk: How to formulate quasi TMDPDFs?



[Chen et al (LP3) '18]

Reminder: TMD factorization

TMD factorization

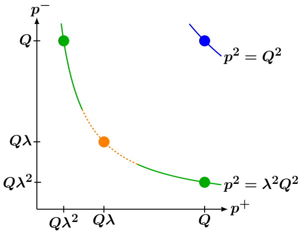
Collinear factorization:

$$\sigma(Q) = \hat{\sigma} \otimes f_n(x_1) \otimes f_{\bar{n}}(x_2)$$

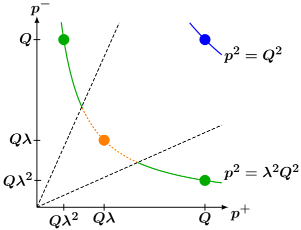
TMD factorization:

$$\sigma(Q, \vec{q}_T) = \hat{\sigma} \otimes B_n(x_1, \vec{k}_1) \otimes B_{\bar{n}}(x_2, \vec{k}_2) \otimes S_{n\bar{n}}(\vec{k}_s)$$

- Beam functions $B_n, B_{\bar{n}}$ measure collinear radiation
  - Factorize into n and $\bar{n}$ functions
- Soft function $S_{n\bar{n}}$ measures soft radiation
  - Depends on *both* n and $\bar{n}$
  - Universal: same soft function for DIS
- SCET$_{\mathrm{II}}$ measurement:
  - $p_n \sim Q(1,\, \lambda^2, \lambda)$
  - $p_{\bar{n}} \sim Q(\lambda^2,\, 1, \lambda)$
  - $p_s \;\sim Q(\lambda\,,\, \lambda,\, \lambda)$



Rapidity divergences



- Collinear / soft modes have same virtuality
- Loop integrals not regulated by dim reg
- Need additional rapidity regulator
  - Wilson lines off light cone [Collins '11]
  - analytic regulator [Becher, Bell '11]
  - δ regulator [Echevarria, Idilbi, Scimemi '11]
  - η regulator [Chiu, Jain, Neill, Rothstein '12]
  - exponential regulator [Yi, Neill, Zhu '16]
- Final results are *independent* of choice of regulator
- Rapidity divergences cancel in cross section
- RGE allows to resum rapidity logarithms:

$$\sigma(Q, \vec{q}_T) \sim \int \mathrm{d}^2\vec{b}_T \, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T} \, e^{\ln(Q^2 b_T^2)\gamma(\vec{b}_T)} \ldots$$

- $\gamma(\vec{b}_T) = \gamma^{\text{(pert)}}(\vec{b}_T) + \gamma^{\text{(non-pert)}}(\vec{b}_T)$ has nonperturbative contribution
  - TMDPDF from lattice $\leftrightarrow$ $\gamma^{\text{(non-pert)}}(\vec{b}_T)$ from lattice

Soft subtraction and TMDPDF

- Factorization theorem in Fourier space:

$$\sigma(\vec{q}_T) = \hat{\sigma} \int \mathrm{d}^2\vec{b}_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\, B_n(\vec{b}_T) \times B_{\bar{n}}(\vec{b}_T) \times S_{n\bar{n}}(\vec{b}_T)$$

- Can absorb soft function into TMDPDFs:

$$\sigma(\vec{q}_T) = \hat{\sigma} \int \mathrm{d}^2\vec{b}_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\, f_n^{\mathrm{TMD}}(\vec{b}_T) \times f_{\bar{n}}^{\mathrm{TMD}}(\vec{b}_T)$$

$$f_n^{\mathrm{TMD}}(\vec{b}_T) = B_n(\vec{b}_T)\sqrt{S_{n\bar{n}}(\vec{b}_T)}$$

  - Motivation: f^{TMD} experimentally accessible

- B_n are pure collinear matrix elements
  - Requires zero-bin subtraction $S^0_{n\bar{n}}$
    typically: $S^0_{n\bar{n}} = S_{n\bar{n}}$
  - f_n^{TMD} also depends on $\bar{n}$ (and vice versa)
    ("collinear anomaly" [Becher, Neubert '10])

$$\boxed{f_n^{\mathrm{TMD}}(\vec{b}_T) \sim \frac{B_n^{(\mathrm{naive})}(\vec{b}_T)}{\sqrt{S_{n\bar{n}}(\vec{b}_T)}}}$$

Towards Quasi TMDPDFs

Towards Quasi TMDPDFs

Roadmap:

1. Wilson line structure of quasi TMDPDF / quasi soft function on the lattice
   - Lattice size limits length L of Wilson lines
2. Rapidity divergences
   - Affected by L
   - Affected by equal-time correlation
3. Match (naive) quasi beam function and soft function onto lightlike case
4. Perform zero-bin (soft) subtraction:

$$f_n^{\mathrm{TMD}}(\vec{b}_T) \sim \frac{B_n^{(\mathrm{naive})}(\vec{b}_T)}{\sqrt{S_{n\bar{n}}(\vec{b}_T)}}$$

(Quasi) beam function at finite length

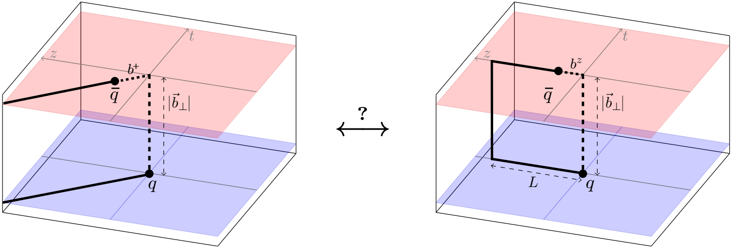
- Beam function definition:

$$B_n(b^+, \vec{b}_T) = \langle P(P_n) \big| \bar{q}(b^+, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^+,\vec{b}_T)} \frac{\gamma^-}{2} q(0) \big| P(P_n) \rangle$$

- Quasi beam function: (drop time dependence)

$$\tilde{B}_n(b^z, \vec{b}_T) = \langle P(P_z) \big| \bar{q}(b^z, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^z,\vec{b}_T)} \frac{\gamma^3}{2} q(0) \big| P(P_z) \rangle$$

- Path:



- Finite length L requires to close Wilson line in transverse direction
  - For $L \to \infty$: required only in singular gauges [Ji, Ma, Yuan '05; Idilbi, Scimemi '10]

(Quasi) soft function at finite length

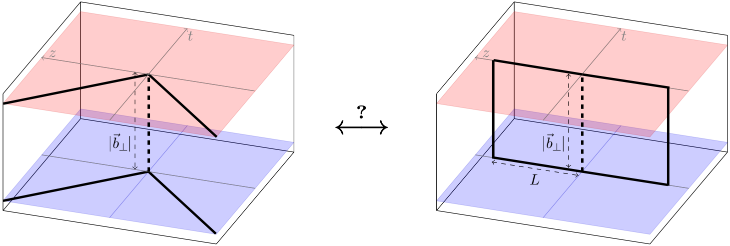
- Soft function definition:

$$S_{n\bar{n}}(\vec{b}_T) = \langle 0 \big| [S^\dagger_{\bar{n}} S_n](\vec{b}_T)[S^\dagger_n S_{\bar{n}}](\vec{0}_T) \big| 0 \rangle$$

- Equal-time soft function definition: (drop time dependence)

$$\tilde{S}_{\hat{z},-\hat{z}}(\vec{b}_T) = \langle 0 \big| [S^\dagger_{-\hat{z}} S^\dagger_{n_\perp} S_{n_\perp} S_{\hat{z}}](\vec{b}_T)[S^\dagger_{\hat{z}} S_{n_\perp} S_{n_\perp} S^\dagger_{n_\perp} S_{-\hat{z}}](\vec{0}_T) \big| 0 \rangle$$

- Path:



- Finite length L requires to close Wilson line in transverse direction
  - For $L \to \infty$: required only in singular gauges [Ji, Ma, Yuan '05; Idilbi, Scimemi '10]

Rapidity divergences at finite L

- Rapidity divergences arise from integrals of type

$$\int_0^\infty \mathrm{d}k^+ \mathrm{d}k^- \frac{f(k^+k^-)}{(k^+k^-)^{1+\epsilon}}$$

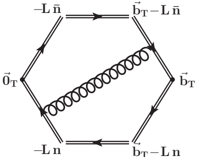
    - Integrand depends only on product k^+k^-

- Eikonal propagator for $L < \infty$:

$$\frac{1}{k^\pm + \mathrm{i}0} \to \frac{1 - e^{\mathrm{i}k^\pm L}}{k^\pm}$$

    - Finite L fully regulates $k^\pm \to 0$
    - No rapidity divergences for finite L

- Explicit check: lightlike soft function with finite L is free of divergences



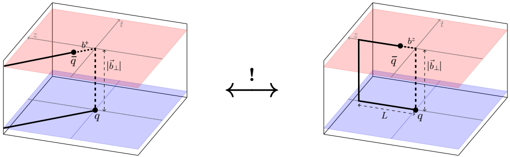
Matching the naive beam function

- (Quasi) beam function definition:

$$B_n(b^+, \vec{b}_T) = \langle P(P_n) \big| \bar{q}(b^+, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^+,\vec{b}_T)} \frac{\gamma^-}{2} q(0) \big| P(P_n) \rangle$$

$$\tilde{B}_n(b^z, \vec{b}_T) = \langle P(P_z) \big| \bar{q}(b^z, \vec{b}_T) W^{(0,\vec{0}_T)}_{(b^z,\vec{b}_T)} \frac{\gamma^3}{2} q(0) \big| P(P_z) \rangle$$

- Quasi beam function approaches beam function after Lorentz boost



  - Transverse separation not affected by boost
  - Standard boost argument still applies

$$\tilde{B}_i(x, \vec{b}_T, \tilde{\mu}, L) \sim \int_0^1 \frac{\mathrm{d}y}{y} C_{ij}\left(x, y, \tilde{\mu}, \mu, L, \nu\right) B_j(y, \vec{b}_T, \mu, \nu)$$

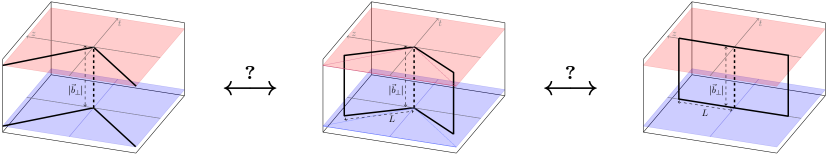
Obstructions in matching the soft function

Obstructions in matching the soft function

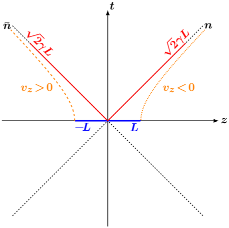
- Soft function and quasi soft function differ only by path:

$$S_{n\bar n}(\vec b_T) = \langle 0 \big| [S^\dagger_{\bar n} S_n](\vec b_T)[S^\dagger_n S_{\bar n}](\vec 0_T) \big| 0\rangle$$

$$\tilde S_{\hat z,-\hat z}(\vec b_T) = \langle 0 \big| [S^\dagger_{-\hat z} S^\dagger_{n_\perp} S_{n_\perp} S_{\hat z}](\vec b_T)[S^\dagger_{\hat z} S_{n_\perp} S_{n_\perp} S^\dagger_{n_\perp} S_{-\hat z}](\vec 0_T) \big| 0\rangle$$

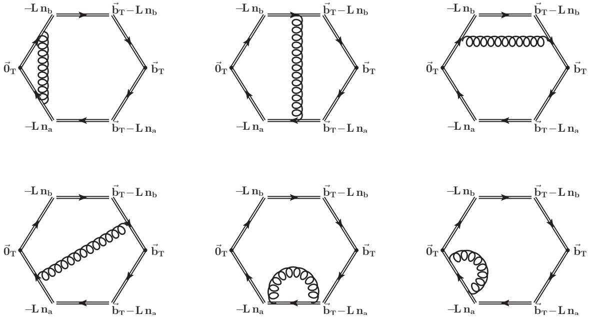


- Soft function depends on both n and $\bar n$
- Can not simultaneously boost $\hat z \to n^\mu$, $-\hat z \to \bar n^\mu$
  - Can we still derive a matching relation?



Perturbative comparison of (quasi) soft function

- $S_{n\bar{n}}$ and $\tilde{S}_{\hat{z},-\hat{z}}$ must describe the same IR physics
  - $\vec{b}_T$ dependence must be identical
- Compare at one loop for finite L, but take $L \gg b_T \sim \Lambda_{\rm QCD}^{-1}$
- Diagrams:



Perturbative comparison of (quasi) soft function

- $S_{n\bar{n}}$ and $\tilde{S}_{\hat{z},-\hat{z}}$ must describe the same IR physics
  - $\vec{b}_T$ dependence must be identical
- Compare at one loop for finite L, but take $L \gg b_T \sim \Lambda_{\mathrm{QCD}}^{-1}$
- Result:

$$S_{n\bar{n}} = -\frac{\alpha_s C_F}{\pi}\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}(L_L - 1) - \frac{1}{2}{L_b}^2 + L_b(L_L - 1) + \cdots\right]$$

$$\tilde{S}_{\hat{z},-\hat{z}} = \frac{\alpha_s C_F}{\pi}\left[+ \frac{2}{\epsilon} + L_b + 2\pi\frac{L}{b_T}\right]$$

  - Different UV physics
    → taken care by matching
  - Different dependence on
    rapidity renormalization parameter
    → taken care by matching
  - Different IR physics ($\vec{b}_T$)
    → No nonperturbative matching!

$$L_L = \ln\frac{L^2\mu^2}{e^{-2\gamma_E}}$$

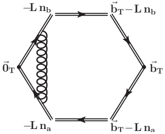
$$L_b = \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}$$

Almost-lightlike soft function

- Study smooth transition from quasi soft function to soft function

$$\begin{aligned} n^\mu &= (1,0,0,+1) \quad \longrightarrow \quad n_v = (v,0,0,+1) \\ \bar n^\mu &= (1,0,0,-1) \quad \longrightarrow \quad \bar n_v = (v,0,0,-1) \end{aligned}$$

  - $v = 1$: soft function
  - $v = 0$: quasi soft function
- Study limit $v \to 1$
  - $v < 1$ not lightlike → may be accessible on lattice
- No smooth behavior of UV divergences:
  - $v = 1$: $\quad S \supset -\frac{\alpha_s C_F}{\pi}\left[\frac{1}{\epsilon^2} + \cdots\right]$
  - $v \to 1$: $\quad S \supset -\frac{\alpha_s C_F}{\pi}\left[\frac{1}{2\epsilon v}\ln\frac{1+v}{1-v} + \cdots\right]$
  - Different UV physics is absorbed in matching coefficient



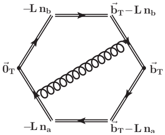
Almost-lightlike soft function

- Study smooth transition from quasi soft function to soft function

$$\begin{aligned} n^\mu &= (1,0,0,+1) \quad \longrightarrow \quad n_v = (v,0,0,+1) \\ \bar{n}^\mu &= (1,0,0,-1) \quad \longrightarrow \quad \bar{n}_v = (v,0,0,-1) \end{aligned}$$

  - $v = 1$: soft function
  - $v = 0$: quasi soft function
- Study limit $v \to 1$
  - $v < 1$ not lightlike → may be accessible on lattice
- Real corrections smoothly approach lightlike limit:

$$S \supset \frac{\alpha_s C_F}{2\pi} (n_v \cdot \bar{n}_v) \left[-\mathrm{Li}_2\left(\frac{-4L^2}{b_T^2} \right) + \mathcal{O}(v-1) \right]$$



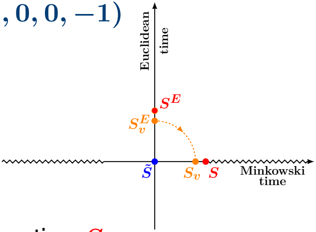
- $v \to 1$ can be matched onto lightlike result

Almost-lightlike soft function from lattice

- Study smooth transition from quasi soft function to soft function

$$n^\mu = (1,0,0,+1) \quad \longrightarrow \quad n_v = (v,0,0,+1)$$
$$\bar{n}^\mu = (1,0,0,-1) \quad \longrightarrow \quad \bar{n}_v = (v,0,0,-1)$$

  - $v = 1$: soft function
  - $v = 0$: quasi soft function



- Lattice can only calculate in Euclidean time
  - Analytical continuation required
- Illustration:
  Euclidean soft function $S_E \leftrightarrow$ Minkowski soft function S

$$S_E \supset \frac{2\alpha_s C_F}{\pi}\left[\qquad +\frac{1}{\epsilon} + 2\frac{\sqrt{2}L}{b_T}\arctan\frac{\sqrt{2}L}{b_T} + \cdots\right]$$

$$S_{n\bar{n}} \supset \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln(L^2\mu^2) + \frac{1}{2}\ln^2(b_T^2\mu^2) + \cdots\right]$$

- No simple analytical continuation from lattice to Minkowski soft function

Conclusion

Conclusion

Quasi PDFs

- Promising approach to calculating PDFs from lattice QCD
- Relates equal-time to lightcone correlation through Lorentz boost
- Factorization theorem has been established

Quasi TMDPDFs

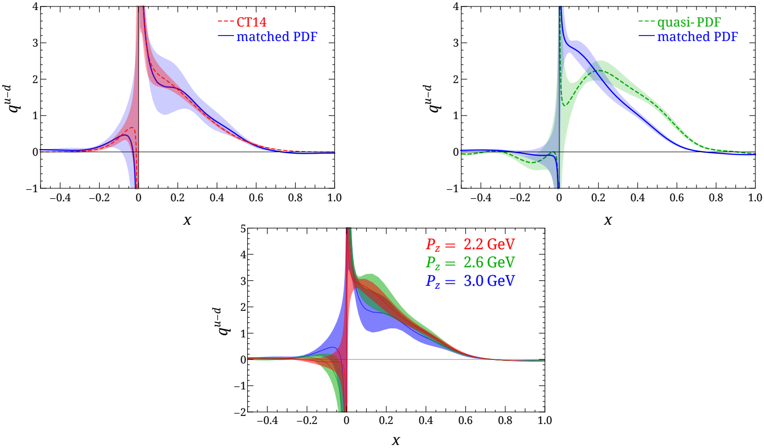
- Effect of lattice size L on Wilson lines / rapidity divergences understood
- Naive quasi beam TMD function can be matched through Lorentz boost
- Quasi soft function can *not* be matched onto TMD soft function
  - Depends on both n and $\bar{n}$
  - Inherently different IR physics than soft function
- Prohibits construction of quasi TMDPDFs ...
- ... but ratios of quasi TMDPDFs work!
  - Soft function is universal (spin and flavor blind)
  - For example, spin and flavor dependent ratios can be studied

Backup slides

Status of quasi PDFs from lattice

PDF determination from [Chen, Jin, Lin, Liu, Yang, Zhang, Zhao (LP3) ’18]

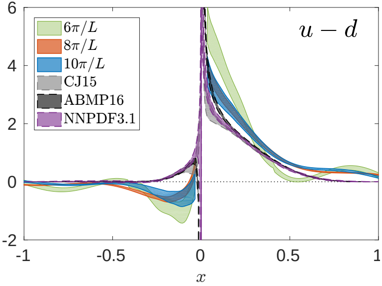
- Lattice: $a = 0.09$ fm, $L = 64 \approx 5.8$ fm, $m_\pi \approx 135$ MeV, $P_z \approx 3.0$ GeV
- $N_f = 2 + 1 + 1$ quarks
- $\Gamma = \gamma^t$, one-step matching from RI/MOM quasi PDF to $\overline{\text{MS}}$ PDF at $\mu = 3$ GeV

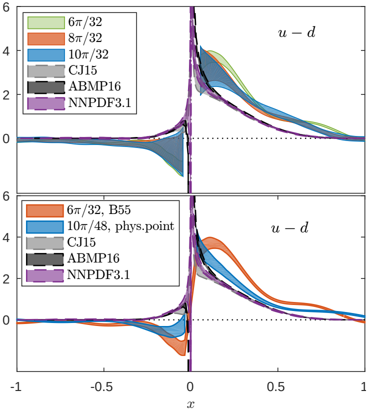


Status of quasi PDFs from lattice

PDF determination from [Alexandrou, Cichy, Constantinou, Jansen, Scapellato, Steffens (ETMC) ’18]

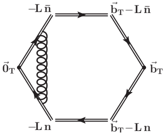
- Lattice: $a = 0.09\text{ fm}, L = 48a \approx 4.5\text{ fm}, m_\pi \approx 130\text{ MeV}, P_z \approx 1.4\text{ GeV}$
- $N_f = 2 + 1 + 1$ quarks
- $\Gamma = \gamma^t$, two-step matching from RI/MOM quasi PDF to $\overline{\text{MS}}$ PDF at $\mu = 2\text{ GeV}$
- Right: Comparison to $L = 32a, m_\pi \approx 375\text{ MeV}$



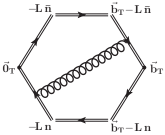


Consequences of finite L

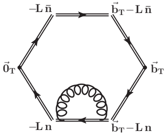
Illustration for soft function



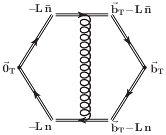
Vertex correction



Real exchange



Perp selfenergy



Perp exchange

- Transverse self energy $S^{\perp}_{\mathrm{SE}} = \dfrac{\alpha_s C_F}{\pi}\left[\dfrac{1}{\epsilon} + \ln\dfrac{\mu^2 b_T^2}{4e^{-2\gamma_E}} + 2\right]$
  - *Independent* of L
  - Does not vanish for $L \to \infty$
  - Cancels with self energy in TMDPDF (see also [Ji, Jin, Yuan, Zhang, Zhao '18])
  - Zero-bin subtraction crucial