Leading log resummation in high-energy nuclear collisions

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Outline

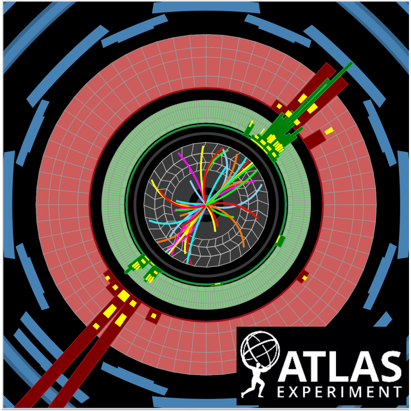
1. **Motivations**
2. **Double log from jet-underlying-event scattering**
3. **Medium-induced and Sudakov double logs**
4. **Summary and Perspectives**

Liou, Mueller and BW, Nucl. Phys. A 916 (2013) 102-125;

Mueller, BW, Xiao and Yuan, PLB763 (2016) 208-212; PRD95 (2017) 034007 ;

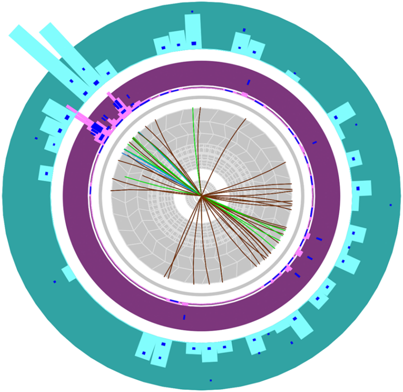
Iancu, Taels and BW, in preparation.

1.1 Jets in proton-proton collisions



a dijet event recorded by ATLAS at the LHC

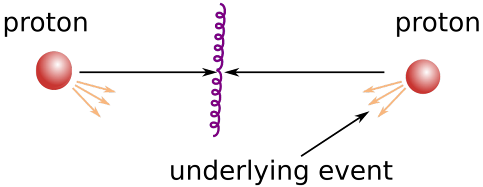
1.2 Jets in nucleus-nucleus collisions



an asymmetric dijet event in a PbPb collision

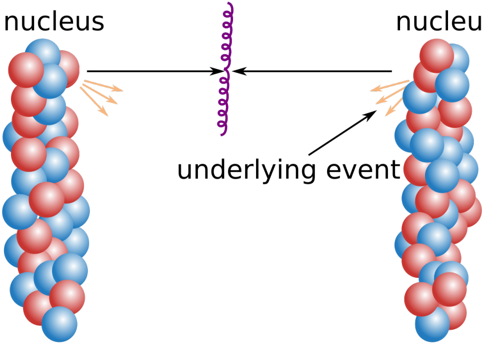
ATLAS, Phys. Rev. Lett. **105**, 252303 (2010).

1.2 Jets in nucleus-nucleus collisions



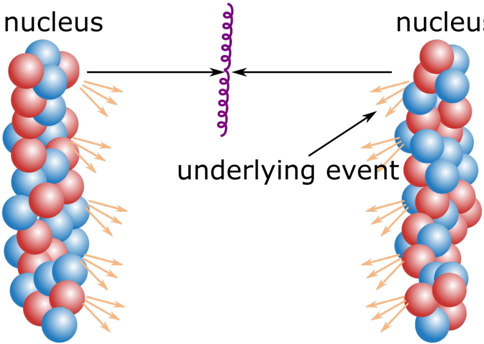
compare to a proton-proton collision

1.2 Jets in nucleus-nucleus collisions



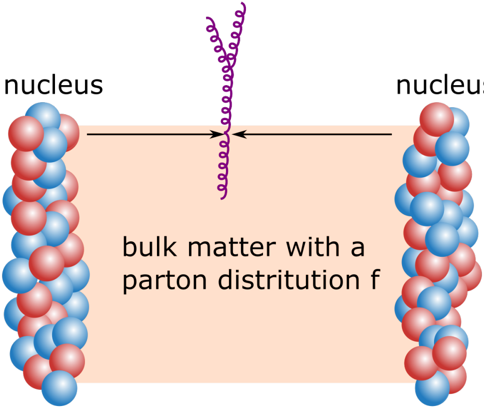
partons in a hard scattering process

1.2 Jets in nucleus-nucleus collisions



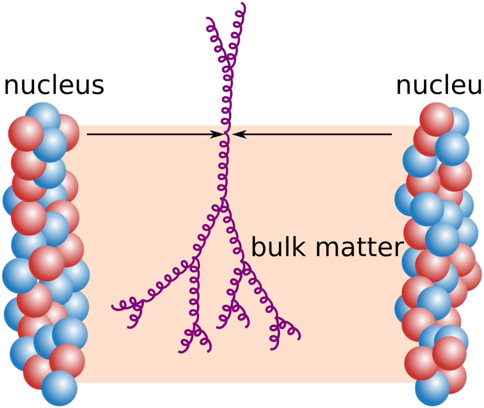
partons in a hard scattering process

1.2 Jets in nucleus-nucleus collisions



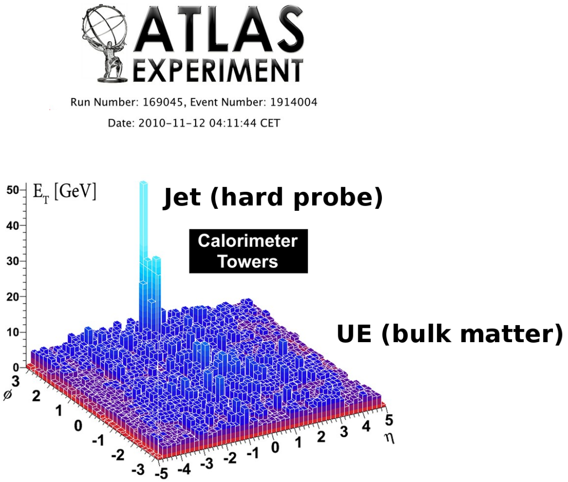
underlying event $\rightarrow$ bulk QCD matter

1.2 Jets in nucleus-nucleus collisions



jet evolution in bulk QCD matter

1.2 Jets in nucleus-nucleus collisions

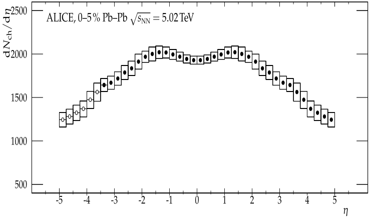


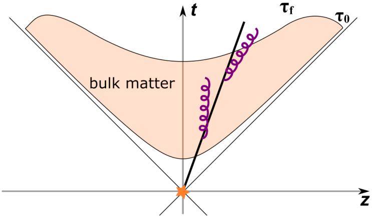
Most central collision: $\frac{dN_{ch}}{d\eta} \approx 1{,}600$ and $\langle p_T \rangle \sim 0.7$ GeV

1.2 Jets in nucleus-nucleus collisions

1. **Underlying events**

longitudinally boost invariant



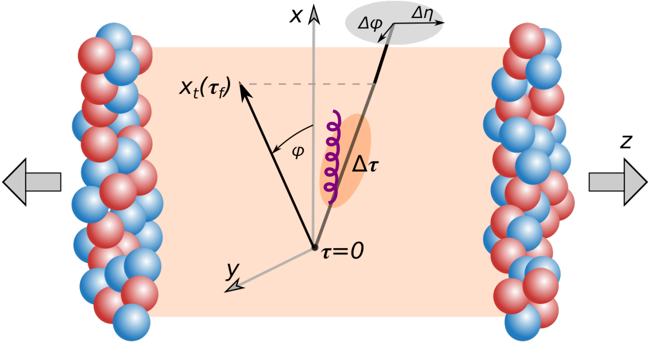


Bjorken, Phys. Rev. D **27**, 140 (1983).

2. **Variables**

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right).$$

1.3 Purpose of this talk

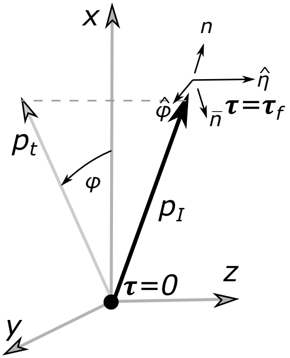


Resummation of double log in the jet broadening due to **multple scattering between a jet and UE**

2.1 Broadening in ϕ and η

Vector decomposition:

Denote the jet momentum by p_I^μ



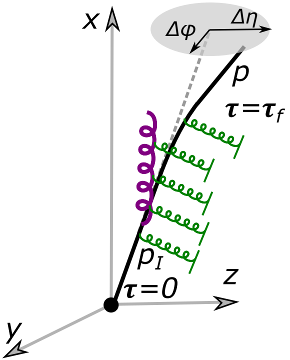
$$
\begin{aligned}
n^\mu &= \frac{1}{\sqrt{2}p_t} p_I^\mu \\
&= \frac{1}{\sqrt{2}} (\cosh \eta_I, \hat{p}_t, \sinh \eta_I), \\
\bar{n}^\mu &= \frac{1}{\sqrt{2}} (\cosh \eta_I, -\hat{p}_t, \sinh \eta_I), \\
\hat{\phi}^\mu &= (0, -\sin \phi_I, \cos \phi_I, 0), \\
\hat{\eta}^\mu &= (\sinh \eta_I, 0, 0, \cosh \eta_I).
\end{aligned}
$$

with $\hat{p}_t = (\cos \phi_I, \sin \phi_I)$.

Iancu, Taels and BW, in preparation.

2.1 Broadening in ϕ and η

Observable: jet broadening



$$
\frac{dN}{dp^\phi dp^\eta} \equiv \frac{1}{\sigma_J}\frac{d\sigma_J}{dp^\phi dp^\eta} \approx \frac{1}{p_t^2}\frac{dN}{d\Delta\phi d\Delta\eta}.
$$

where

$$
\begin{aligned}
p^\mu &= (p\cdot\bar{n})n^\mu + (p\cdot n)\bar{n}^\mu \\
&\quad -(p\cdot\hat{\phi})\hat{\phi}^\mu - (p\cdot\hat{\eta})\hat{\eta}^\mu \\
&\equiv (p^+, p^-, \underbrace{p^\phi, p^\eta}_{p_\perp})
\end{aligned}
$$

$$
p^\phi = p_t \sin(\underbrace{\phi - \phi_I}_{\Delta\phi}) \approx p_t\Delta\phi
$$

$$
p^\eta = -p_t \sinh(\underbrace{\eta - \eta_I}_{\Delta\eta}) \approx -p_t\Delta\eta.
$$

2.1 Broadening in ϕ and η

One example: $pp \to$ dijet+X

$$\frac{d^4\sigma}{dy_1 dy_2 dk_{1\perp}^2 d^2k_{2\perp}} = \sum_{ab} \sigma_0 \int \frac{d^2\vec{x}_\perp}{(2\pi)^2} e^{-i(\vec{k}_{1\perp}+\vec{k}_{2\perp})\cdot\vec{x}_\perp} \times x_1 f_a(x_1, \mu_b) x_2 f_b(x_2, \mu_b) S(Q^2, x_\perp)$$

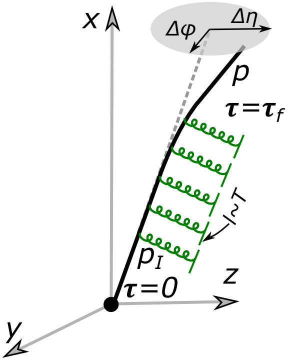
where σ_0 represents normalization of the differential cross section and y_1 and y_2 are rapidities of the two jets.

Mueller, BW, Xiao and Yuan, PLB763 (2016) 208-212.

Focus: contributions to $S(Q^2, x_\perp)$ from multiple scattering

2.1 Broadening in ϕ and η

Jet broadening at tree-level



- Scaling

$$\text{Jet:} \quad p^\mu \sim p_t(1, \lambda^2, \lambda),$$
$$\text{UV:} \quad l^\mu \sim p_t(\lambda, \lambda, \lambda)$$

with $\lambda = T/p_t$.

- Wilson lines

$$M(x_\perp) = W_n(x_\perp) \equiv P_n e^{ig\int dx^+ n\cdot A_s(x_\perp)}$$

with A_s gauge field of the constituents of UV.

2.1 Broadening in ϕ and η

Jet quenching parameter: $\hat{q}$

- Scaling

$$S(x_\perp) = \left\langle W_n^\dagger(x_\perp) W_n(0_\perp) \right\rangle = e^{-\frac{1}{4}x_\perp^2 \int d\tau \hat{q}(\tau)} \equiv e^{-\frac{1}{4}x_\perp^2 \mathcal{Q}_\perp^2}$$

$$\Rightarrow \frac{1}{p_t^2} \frac{dN}{d\Delta\phi d\eta} = e^{-\frac{p_t^2}{\mathcal{Q}_\perp^2}\left(\Delta\phi^2 + \Delta\eta^2\right)}$$

with $\langle \cdots \rangle$ integration over f for UV.

- Physical meaning of $\hat{q}$

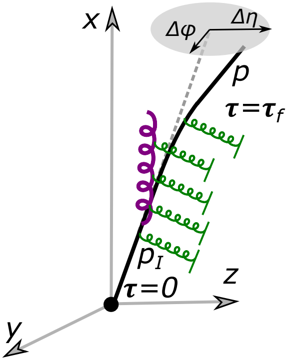
$$\hat{q} \quad \propto \quad g^2 \int d^2 q_\perp d^2 y_\perp dy^+ e^{-iq^- y^+ + iq_\perp \cdot y_\perp} \left\langle F_i^{a+}(y^+, y_\perp) F_i^{a+}(0) \right\rangle ,$$

that is, it is **proportional to the UE gluon distribution**.

Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B **484**, 265 (1997).

2.2 Medium-induced double log

There exists a double log $\propto \hat{q}$



- From one-gluon radiation

$$\mathcal{Q}_\perp^2(\tau)_{rad} = \frac{\alpha_s N_c}{2\pi} \ln^2(\tau T)\, \mathcal{Q}_\perp^2(\tau).$$

- Resummation

$$\mathcal{Q}_\perp^2(\tau)_{rad} = \frac{\mathcal{Q}_\perp^2(\tau)}{\kappa} I_1(2\kappa)$$

with $\kappa = \frac{\alpha_s N_c}{\pi} \ln(\tau T)$.

Liou, Mueller and BW, Nucl. Phys. A 916 (2013) 102-125.

2.2 Medium-induced double log

Some details of our calculation

$$S(\tau,\vec{x}_\perp) = -\frac{\alpha_s C_R}{4\pi\omega D^3}\mathrm{Re}\; i\int_{\tau_0}^{\tau} d\tau_2 \int_{\tau_0}^{\tau_2} d\tau_1$$

$$\int dk^n \left[\left(x_\perp^2\omega(c_1+1)(c_2+1) - 8iD\right) e^{\frac{ix_\perp^2\omega(c_1+c_2+2)}{8D}} + \left(x_\perp^2\omega(c_1-1)(c_2-1) + 8iD\right) e^{\frac{ix_\perp^2\omega(c_1+c_2-2)}{8D}}\right],$$

where $c_1 = c(\tau_2,\tau_1)$, $c_2 = c(\tau_1,\tau_2)$ and D and c are given by

$$D(\tau_2,\tau_1) = \pi\nu\sqrt{\tau_1\tau_2}\left[J_\nu\left(2\nu\Omega_1\tau_1\right)Y_\nu\left(2\nu\Omega_2\tau_2\right) - J_\nu\left(2\nu\Omega_2\tau_2\right)Y_\nu\left(2\nu\Omega_1\tau_1\right)\right]$$

$$c(\tau_2,\tau_1) = \frac{\pi\nu\sqrt{\tau_1\tau_2}\Omega_2}{\sin(\pi\nu)}\left[J_{\nu-1}\left(2\nu\Omega_2\tau_2\right)J_{-\nu}\left(2\nu\Omega_1\tau_1\right) + J_{1-\nu}\left(2\nu\Omega_2\tau_2\right)J_\nu\left(2\nu\Omega_1\tau_1\right)\right]$$

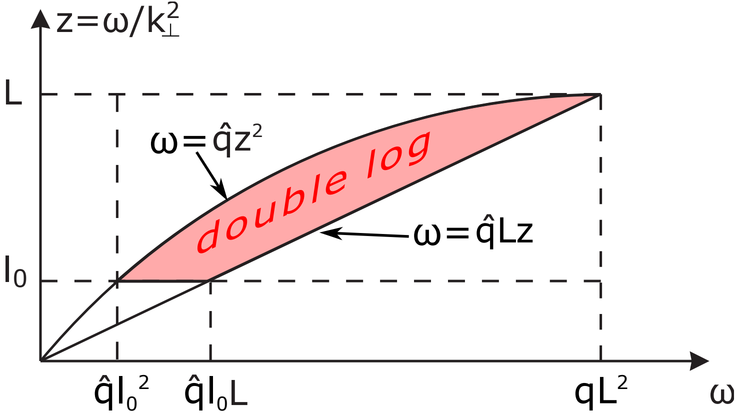
with

$$\Omega(\tau_0) = \frac{1-i}{2}\left(\frac{N_c\hat{q}(\tau_0)}{C_R k^n}\right)^{\frac{1}{2}} = \frac{1-i}{2}\left(\frac{\hat{q}_A(\tau_0)}{k^n}\right)^{\frac{1}{2}}, \qquad \Omega(\tau) = \Omega(\tau_0)(\tau_0/\tau)^{1-\frac{1}{2\nu}}.$$

and the shorthand notation $\Omega_{1,2} = \Omega(\tau_{1,2})$ is used.

2.2 Medium-induced double log

Double log phase space



with $L = \tau$ and $l_0 = 1/T$.

2.2 Medium-induced double log

Renormalization of $\hat{q}$

The double log is a consequence the evolution of UE PDF.

$$S(x_\perp) = e^{-\frac{1}{4} x_\perp^2 \mathcal{Q}_\perp^2(\tau)_{rad}}$$

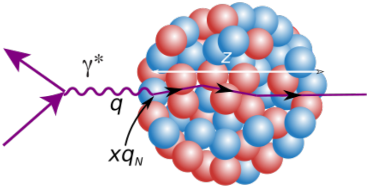
or

$$\hat{q}_{rad}(\tau) = \frac{\hat{q}(\tau)}{\kappa} I_1(2\kappa)$$

Blaizot & Mehtar-Tani (2014); Iancu (2014).

3.1 Double logs in nucleus DIS

For $\frac{1}{q^-} \ll L$, the size of nuclei:



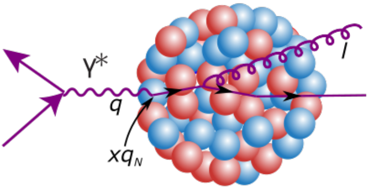
Mueller, BW, Xiao and Yuan, PRD95 (2017) 034007.

$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right)$$

$$\times \int_0^L dz \exp\left[-\frac{1}{4}\hat{q} \quad x_\perp^2 z \qquad\qquad\qquad\qquad \right].$$

3.1 Double logs in nucleus DIS

For $\frac{1}{q^-} \ll L$:



$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_{rad} x_\perp^2 z}_{\text{medium-induced}}\right].$$

3.1 Double logs in nucleus DIS

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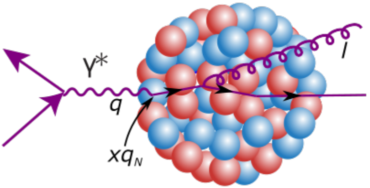
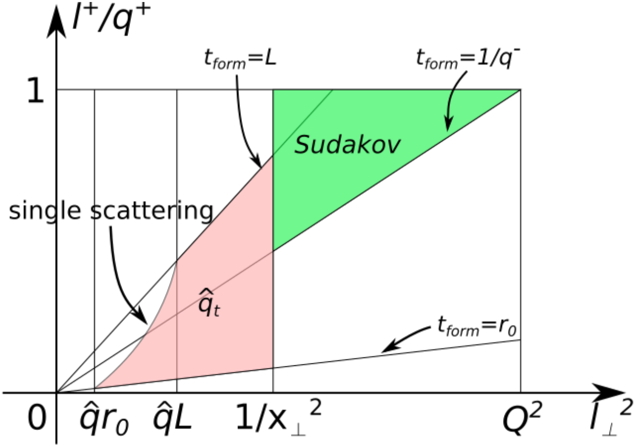


$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_{rad} x_\perp^2 z}_{\text{medium-induced}} \underbrace{-\frac{\alpha_s C_F}{2\pi} \ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}}\right].$$

See, for resummation, Collins, Soper & Sterman (1985); Mueller, Xiao & Yuan (2013).

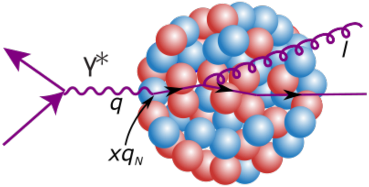
3.1 Double logs in nucleus DIS

Two double logs are factorized:



3.1 Double logs in nucleus DIS

To study nuclear effects, Q should not be large!



$$\frac{dN}{d^2b\,d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right)$$

$$\times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_{rad}x_\perp^2 z}_{\text{medium-induced}} \underbrace{-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}}\right].$$

Mueller, BW, Xiao and Yuan, PRD95 (2017) 034007.

3.2 Applications in dijet production

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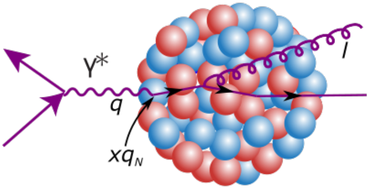

Otherwise,

$$\frac{dN}{d^2b\,d^2k_\perp} \approx \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, xq_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}} \right].$$

3.2 Applications in dijet production

Dijet azimuthal angular distributions ($Q \to p_\perp$)

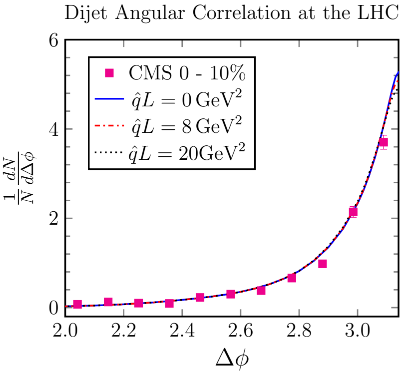

Figure: $p_\perp$=120 GeV and 50 GeV in PbPb collisions at the LHC.

Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016).

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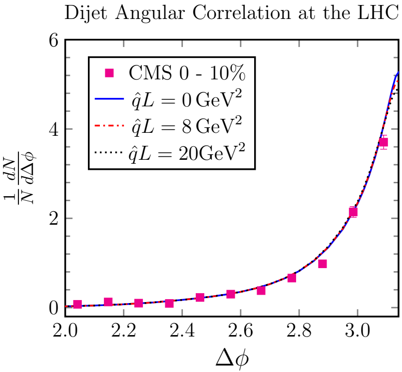

Figure: $p_\perp$=120 GeV and 50 GeV in PbPb collisions at the LHC.

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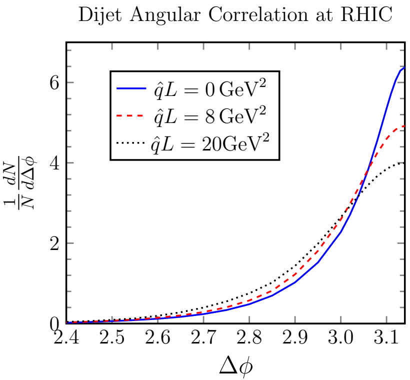
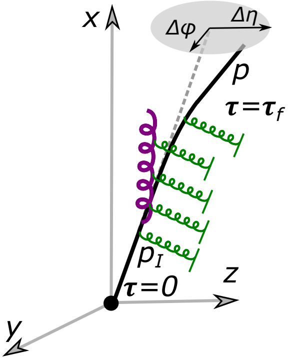


Figure: $p_\perp$=35 GeV in AuAu collisions at RHIC.

Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016).

Summary and Perspectives



For $\frac{dN}{d\Delta\phi d\Delta\eta}$,

1. Resummation of medium-induced double log
2. Interplay with Sudakov double log
3. A new way to measure $\hat{q}$

SCET for high-energy nuclear collisions?