Fermionic Glauber Operators and Quark Reggeization

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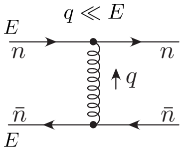
Based on the work in collaboration with I. Moult, M.P. Solon, and I.W. Stewart,
hep-ph/1709.09174

QCD at high energy



Study QCD at high energy $s \gg t$

For amplitudes:

- Resummation dresses the t-channel exchange, leading to $(s/|t|)^{\omega}$ behavior of amplitudes, where ω is called the *Regge trajectory*.
- This behavior is referred to as **Reggeization**
- Predicts higher order terms in perturbative expansion of amplitudes.
- Places important **constraints on structure** of amplitudes (e.g. bootstrapping)

[Dixon et al.], [Lipatov et al.]

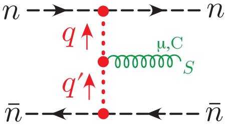
Constraint	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$
0. Functions	(10,10)	(82,88)	(639,761)	(5153,6916)	(???,???)
1. Steinmann	(7,7)	(37,39)	(174,190)	(758,839)	(3105,3434)
2. Symmetry	(3,5)	(11,24)	(44,106)	(174,451)	(???,???)
3. Final-entry	(2,2)	(5,5)	(19,12)	(72,32)	(272,83)
4. Collinear	(0,0)	(0,0)	(1,1)	(3,5)	(9,15)
5. Regge	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

TABLE I. Free parameters remaining after applying each constraint, for the 6-point (MHV,NMHV) amplitude at L loops.

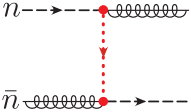
At the cross section level:

- **Large logarithms**, $\log(s/|t|)$, appear in the cross sections at small $\frac{t}{s} \equiv x$: **small-x physics** (see also Adi's talk)
- Small-x logs are resummed via BFKL equation
- Saturation effects

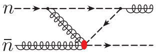
Outline

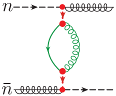


- Review of forward scattering in SCET
  - ◇ Leading power Glauber Lagrangian
  - ◇ Gluon reggeization

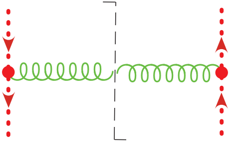


- Quark-mediated forward scattering in SCET
  - ◇ Relevant Lagrangians for quark reggeization
  - ◇ Quark reggeization



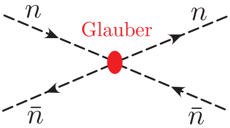


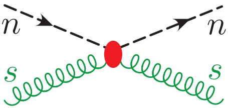
  - ◇ BFKL for $q\bar{q} \to \gamma\gamma$

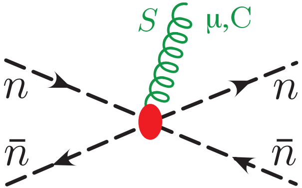


- Conclusion and Outlook

Review of forward scattering in SCET







EFT for forward scattering

SCET can be used to treat forward limit:

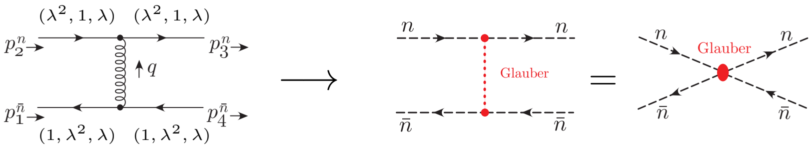
- Take $2 \to 2$ scattering with forward condition:

$$\bar{n} \cdot p_2^n = \bar{n} \cdot p_3^n \qquad \text{and} \qquad n \cdot p_1^{\bar{n}} = n \cdot p_4^{\bar{n}}$$

- Exchanged **gluon** in the t channel has **Glauber scaling**:

$$q^\mu \sim Q(\lambda^2, \lambda^2, \lambda) \implies t = q^2 = q_\perp^2 + \underbrace{\ldots}_{\text{higher orders in } \lambda}$$

- Integrate out Glauber modes, get Glauber potentials/operators



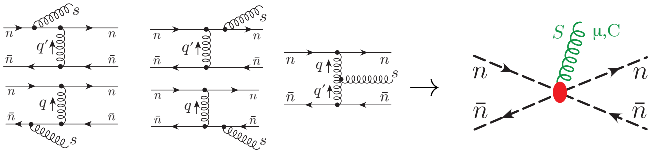
"An Effective Field Theory for Forward Scattering and Factorization Violation"
I.Rothstein and I.W.Stewart [hep-ph/1601.04695]

Leading power Glauber Lagrangians

The complete set of Glauber operators at LP gives **Glauber Lagrangian**

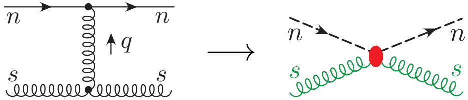
$$\mathcal{L}_G^{\text{II}(0)} = \sum_{n,\bar{n}} \mathcal{O}_{n\bar{n}s} + \sum_{n} \mathcal{O}_{ns}$$

- **3-rapidity** *sector operators*: $\mathcal{O}_{n\bar{n}s} = \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$

e.g. 

(Lipatov vertex)

- **2-rapidity** *sector operators*: $\mathcal{O}_{ns} = \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$

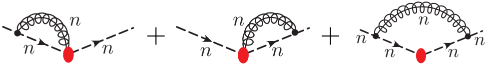
e.g. 

Gluon Reggeization

These operators contain *rapidity divergencies* $\Longrightarrow$

- They have RRG anomalous dimensions γ
- Their RGE gives rise to the **gluon reggeization**.

<u>Example:</u> $\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{\not{n}}}{2} \chi_n$:



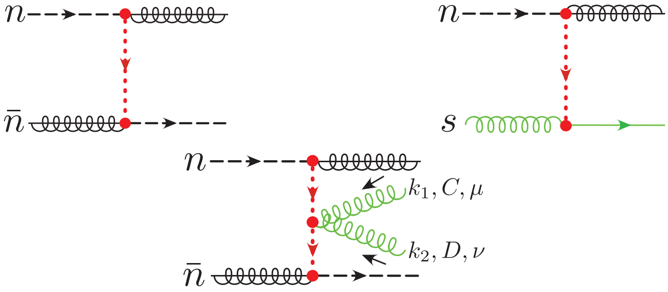
$+$ w.fn. renorm

$$= \mathcal{S}^{nq}\, \frac{\alpha_s C_A}{4\pi} \left[w^2 \frac{2\, h\left(\epsilon, \frac{\mu^2}{m^2}\right)}{\eta} + w^2 \frac{2}{\epsilon} \ln\left(\frac{\nu}{\bar{n}\cdot p}\right) + \frac{3}{2\epsilon} \right] = \mathcal{S}^{nq}\, \delta V_n^{qq}\,,$$

- Regulate collinear Wilson lines
- Rapidity divergencies show up as $\frac{1}{\eta}$ poles
- Derive RRG equation for the operator
- Evolution resums logs

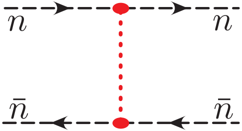
$$\ln(s/-t) \;\Longrightarrow\; \left(\frac{s}{-t}\right)^{\gamma} \;\Longrightarrow\; \textbf{Reggeization}$$

Forward scattering at subleading powers in SCET

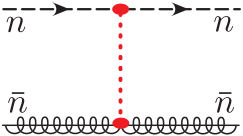


Forward scattering processes

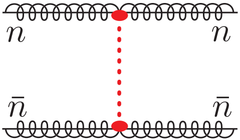
Up to now we've seen



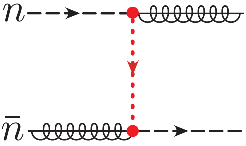
By changing i, j labels in $\mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$ we also get



and



But what about processes like



?

Power counting formulae

The leading power Glauber Lagragian scales as

$$\mathcal{L}_G^{(0)} \sim \underbrace{\mathcal{O}_n^{iB}}_{\lambda^2} \overbrace{\frac{1}{\mathcal{P}_\perp^2}}^{\lambda^{-2}} \underbrace{\mathcal{O}_s^{BC}}_{\lambda^2} \overbrace{\frac{1}{\mathcal{P}_\perp^2}}^{\lambda^{-2}} \underbrace{\mathcal{O}_{\bar{n}}^{jC}}_{\lambda^2} + \underbrace{\mathcal{O}_n^{iB}}_{\lambda^2} \overbrace{\frac{1}{\mathcal{P}_\perp^2}}^{\lambda^{-2}} \underbrace{\mathcal{O}_s^{j_n B}}_{\lambda^3}$$

and from p.c. theorem the measure scales as

$$\mathrm{d}^4 x_{n\bar{n}} \sim \lambda^{-2} \qquad \text{and} \qquad \mathrm{d}^4 x_{ns} \sim \lambda^{-3}$$

$$\begin{aligned}\delta &= 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + 2u \\ &\quad + \sum_k (k-8)V_k^{us} + (k-4)\left(V_k^n + V_k^{\bar{n}} + V_k^S\right) + (k-3)V_k^{nS} + (k-2)V_k^{n\bar{n}}\end{aligned}$$

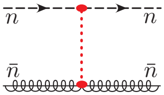
Expected power counting for glauber quark potentials

- **Gluon** Glauber propagators $\frac{1}{\mathcal{P}_\perp^2} \sim \lambda^{-2}$ enhance the counting

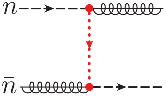
- If the exchanged Glauber particle is a **quark** the propagator is going to be $\frac{1}{\not{\mathcal{P}}_\perp} \sim \lambda^{-1}$

- This explains why the Glauber quark potential comes in **power suppressed** Lagrangians

- Looking at operators with fermion number in each collinear sector



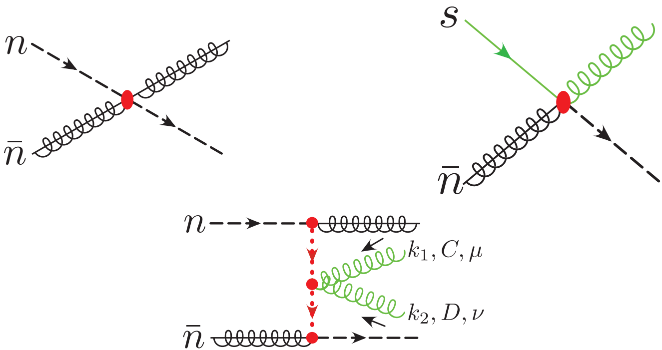
vs



What was previously known about it

- Resummation of quark exchange in the t-channel firstly studied in QED (where photon doesn't reggeize)
- In QCD, much less is known. 1-loop quark regge trajectory was firstly worked out in 1976 by V. S. Fadin and V. E. Sherman in *"Fermion Reggeization in Nonabelian Calibration Theories"*
- More recently, assuming reggeization, A. V. Bogdan, et al. [hep-ph/0201240] extracted info about quark regge trajectory from 2 loop $2 \to 2$ scattering in QCD
- Lack of systematical framework in which quark reggeization can be studied

Fermionic Glauber Lagrangian

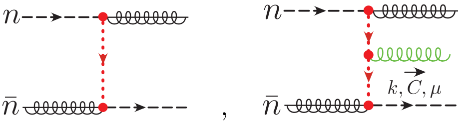


Glauber quark Lagrangians

The complete set of Glauber operators responsible for LL quark Reggeization are

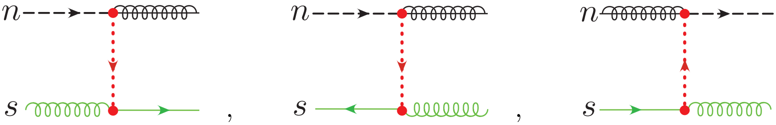
- **3-rapidity** *sector operators*: $\mathcal{L}^{\mathrm{II}(1)} \supset \sum_{n,\bar{n}} \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^{n\bar{n}} \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_{\bar{n}} \equiv \mathcal{O}_{n\bar{n}}$

e.g.



(call it Fadin-Sherman vertex)

- **2-rapidity** *sector operators*: $\mathcal{L}^{\mathrm{II}(1/2)} \supset \sum_{n} \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^{n}$ + h.c. $\equiv \mathcal{O}_{ns}$



Glauber quark Lagrangian operators

$$\sum_{n,\bar n} \underbrace{\bar{\mathcal{O}}_n}_{\lambda^2} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_s^{n\bar n}}_{\lambda} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_{\bar n}}_{\lambda^2} + \sum_n \underbrace{\bar{\mathcal{O}}_n}_{\lambda^2} \frac{1}{\mathcal{P}\!\!\!/_\perp} \underbrace{\mathcal{O}_s^n}_{\lambda^{3/2}} + \text{h.c.}$$

Forward scattering **collinear** operators with fermion number:

$$\mathcal{O}_n = \mathcal{B}\!\!\!/_{\perp n}\chi_n \qquad \mathcal{O}_{\bar n} = \mathcal{B}\!\!\!/_{\perp \bar n}\chi_{\bar n}$$
$$\bar{\mathcal{O}}_n = \bar\chi_n \mathcal{B}\!\!\!/_{\perp n} \qquad \bar{\mathcal{O}}_{\bar n} = \bar\chi_{\bar n}\mathcal{B}\!\!\!/_{\perp \bar n}$$

Single index **soft** operators (2-rapidity sector Lagrangian):

$$\mathcal{O}_s^n = -4\pi\alpha_s \mathcal{B}\!\!\!/_{\perp S}^n \psi_S^n \qquad \bar{\mathcal{O}}_s^n = -4\pi\alpha_s \bar\psi_S^n \mathcal{B}\!\!\!/_{\perp S}^n\,.$$

Two index **soft** operator (3-rapidity sector Lagrangian):

$$\mathcal{O}_s^{n\bar n} = -2\pi\alpha_s \Big[S_n^\dagger S_{\bar n} \mathcal{P}\!\!\!/_\perp + \mathcal{P}\!\!\!/_\perp S_n^\dagger S_{\bar n} - S_n^\dagger S_{\bar n} g \mathcal{B}\!\!\!/_{S\perp}^{\bar n} - g\mathcal{B}\!\!\!/_{S\perp}^n S_n^\dagger S_{\bar n} \Big]$$
$$= -4\pi\alpha_s \Big[S_n^\dagger S_{\bar n}\mathcal{P}\!\!\!/_\perp - g\mathcal{B}\!\!\!/_{S\perp}^n S_n^\dagger S_{\bar n}\Big] \qquad \text{via } \mathcal{B}\!\!\!/_{S\perp}^n \text{ definition}$$

Building the 3-rapidity soft operator

At LP for the gluon we had $\quad \mathcal{O}_s^{BC} \sim \lambda^2\,, \quad \left[\mathcal{O}_s^{BC}\right] = [m]^2$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s\Big\{\mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n}\mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}\mathcal{S}_n^T\mathcal{S}_{\bar n} - \mathcal{S}_n^T\mathcal{S}_{\bar n}\, g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}\mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}\mathcal{S}_n^T\mathcal{S}_{\bar n}\, g\widetilde{\mathcal{B}}^{\bar n}_{S\perp\mu} - \frac{n_\mu\bar n_\nu}{2}\mathcal{S}_n^T\, ig\widetilde{G}_s^{\mu\nu}\mathcal{S}_{\bar n}\Big\}$$

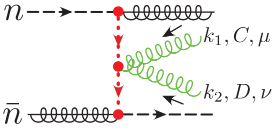
While for the Glauber quark two index soft operator we have

$$\mathcal{O}_s^{n\bar n} = -4\pi\alpha_s\left[S_n^\dagger S_{\bar n}\not{\mathcal{P}}_\perp - g\not{\mathcal{B}}_{S\perp}^n S_n^\dagger S_{\bar n}\right]$$

$\mathcal{O}_s^{n\bar n} \sim \lambda \quad$ and has mass dimension 1

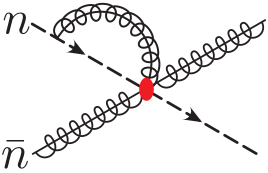
- Easier than LP case because of lower p.c. and mass dim.
- As for the gluon case, $\mathcal{O}_s^{n\bar n}$ has Feynman rule at all orders in g. For example:

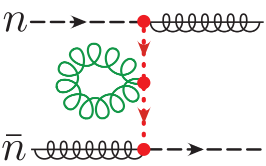
Two soft gluon emission



$$= \quad \mathcal{O}_s^{BC}\Big|_{\mathcal{O}(g^4)}$$

Quark Reggeization





Renormalization of soft and collinear operators

Renormalize $\mathcal{O}_n$ and $\mathcal{O}_s \implies$ renormalization at the amplitude level.

- Consider all virtual diagrams in the EFT at one loop.
- Find counterterm δV from $\frac{1}{\eta}$ poles. $\mathcal{O}^{\text{bare}} = V_{\mathcal{O}} \cdot \mathcal{O}(\nu, \mu)$
- Get **rapidity anomalous dimension** $\gamma_{\mathcal{O}}^{\nu}$ from

$$\gamma_{\mathcal{O}}^{\nu} = -V_{\mathcal{O}}^{-1} \cdot \nu \frac{\partial}{\partial \nu} V_{\mathcal{O}}$$

- Gives RRG equation

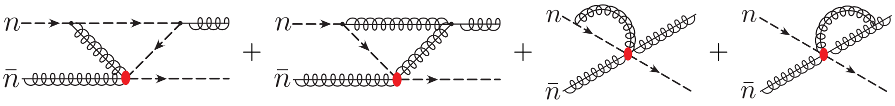
$$\nu \frac{\partial}{\partial \nu} \mathcal{O}(\nu, \mu) = \gamma_{\mathcal{O}}^{\nu} \cdot \mathcal{O}(\nu, \mu)$$

δV, $\gamma_{\mathcal{O}}^{\nu}$ can depend on IR regulator (to be removed at the XS level).

Renormalization of collinear operators

Collinear operators: $\mathcal{O}_n = \mathcal{B}\!\!\!/_{\perp n}\chi_n\,, \qquad \bar{\mathcal{O}}_n = \bar{\chi}_n\mathcal{B}\!\!\!/_{\perp n}$

One loop rapidity divergent virtual diagrams:



- C_A terms cancel
- **Simple** RG structure: no mixing $\implies$ $\mathcal{O}_n^{\text{bare}} = V_n\mathcal{O}_n$
- Doing the above procedure we get

$$\gamma^\nu_{\mathcal{O}_n} = \frac{\alpha_s C_F}{2\pi}\left[g(\epsilon,\mu^2/t) + h(\epsilon,\mu^2/m^2)\right] \xrightarrow{\epsilon\to 0} \frac{\alpha_s C_F}{2\pi}\ln\left(\frac{-t}{m^2}\right)\,.$$

- $\bar{n}$-operators are obtained from $\mathcal{O}_n$'s by simply $n\to\bar{n}$
- Anomalous dim for $\mathcal{O}_s$ can be derived from consistency relations

$$\gamma^\nu_{\mathcal{O}_n} = \gamma^\nu_{\mathcal{O}_{\bar{n}}}\,, \qquad \gamma^{\text{dir}}_{s\nu} + \gamma^T_{s\nu} = -\gamma^\nu_{\mathcal{O}_n} - \gamma^\nu_{\mathcal{O}_{\bar{n}}}\,, \qquad \gamma^\nu_{\mathcal{O}_s^n} = -\gamma^\nu_{\mathcal{O}_n}\,.$$

Solving the RRG: Quark reggeization

- Look at renormalized forward scattering operator

$$\mathcal{O}_{n\bar{n}} = \bar{\mathcal{O}}_n(\sqrt{-t})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_s^{n\bar{n}}(\sqrt{-t})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_{\bar{n}}(\sqrt{-t})$$

$$\nu\frac{d}{d\nu}\mathcal{O}_n(\nu) = \gamma^\nu_{\mathcal{O}_n}\mathcal{O}_n(\nu) \qquad \text{run to } \nu = \sqrt{s} \quad \text{(minimize logs)}$$

$$\gamma^\nu_{\mathcal{O}_n} = \frac{\alpha_s C_F}{2\pi}\ln\left(\frac{-t}{m^2}\right)$$

- No ν dependence in $\gamma^\nu_{\mathcal{O}_n}$ $\implies$ RG is easy to solve

$$\nu\frac{d}{d\nu}\mathcal{O}_n(\nu) = \gamma^\nu_{\mathcal{O}_n}\mathcal{O}_n(\nu) \implies \mathcal{O}_n(\sqrt{-t}) = \left(\frac{s}{-t}\right)^{-\frac{1}{2}\gamma^\nu_{\mathcal{O}_n}}\mathcal{O}_n(\sqrt{s})$$

- Plug into $\mathcal{O}_{n\bar{n}}$ and get

$$\boxed{\mathcal{O}_{n\bar{n}} = \left(\frac{s}{-t}\right)^{-\frac{\alpha_s(\mu)C_F}{2\pi}\log\left(\frac{-t}{m^2}\right)}\bar{\mathcal{O}}_n(\sqrt{s})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_s(\sqrt{-t})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_{\bar{n}}(\sqrt{s})}$$

Quark reggeization

$$\mathcal{O}_{n\bar{n}} = \left(\frac{s}{-t}\right)^{-\frac{\alpha_s(\mu)C_F}{2\pi}\log\left(\frac{-t}{m^2}\right)} \bar{\mathcal{O}}_n(\sqrt{s})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_s(\sqrt{-t})\frac{1}{\mathcal{P}_\perp}\mathcal{O}_{\bar{n}}(\sqrt{s})$$

Regge theory predicts that in the forward limit the amplitude follows *Regge Trajectory* ω, i.e.

$$\mathcal{M} \sim \left(\frac{s}{-t}\right)^{\omega}$$

At 1-loop our scattering amplitude, in the forward limit, is

$$\mathcal{M} \sim \langle \mathcal{O}_{n\bar{n}} \rangle \sim \left(\frac{s}{-t}\right)^{-\frac{\alpha_s(\mu)C_F}{2\pi}\log\left(\frac{-t}{m^2}\right)}$$

Hence we derived the **one-loop Regge trajectory** for the quark from RG equations

$$\boxed{\omega_q = -\frac{\alpha_s(\mu)C_F}{2\pi}\log\left(\frac{-t}{m^2}\right)}$$

Matches V. S. Fadin and V. E. Sherman, see also A. V. Bogdan, et al. [hep-ph/0201240]

Color decomposition

Color structure of quark-gluon scattering can be decomposed as

$$3 \otimes 8 = 3 \oplus \bar{6} \oplus 15$$

- Decompose amplitude as

$$\mathcal{M} = 2\mathcal{A}(T^A T^B)_{ij} + 2\mathcal{B}(T^B T^A)_{ij} + \mathcal{C}\delta^{AB}\delta_{ij}$$

- Contributions from different channels can be written as

$$\mathcal{M}_3 = 2C_F\mathcal{A} - \frac{1}{N}\mathcal{B} + \mathcal{C}$$
$$\mathcal{M}_{\bar{6}} = -\mathcal{B} + \mathcal{C}$$
$$\mathcal{M}_{15} = \mathcal{B} + \mathcal{C}$$

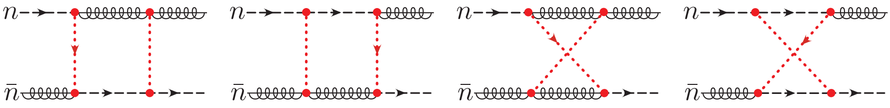
Bogdan, Del Duca, Fadin, and Glover

[hep-ph/0201240]

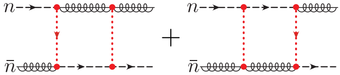
- What we have seen so far is Reggeization for 3 channel
- Contributions from the $\bar{6}$ and 15 start at $\mathcal{O}(\alpha_s^2)$

Glauber Boxes

- In the EFT, contributions from $\mathcal{M}_{\bar{6}}$ and $\mathcal{M}_{15}$ are easily generated by **T-products** of Glauber quark and Glauber gluon Lagrangians.
- These T-products reproduce **quark-gluon** **Glauber boxes**



- **Cross boxes vanish** in the EFT. Boxes give $i\pi$ (ϵ is IR regulator):



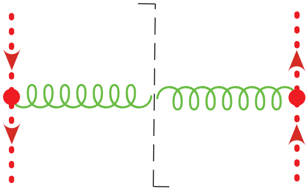
$$+ \qquad \sim \delta^{AB}\delta_{ij}\,\frac{\alpha_s}{4\pi}\left[-\frac{1}{\epsilon}-\log\frac{\mu^2}{-t}\right](-i\pi)$$

- From color structure we can read the amplitudes for the $\bar{6}$ and 15

$$\mathcal{M}_{\bar{6}} = \mathcal{M}_{15} = \left[-i4\pi\alpha_s \bar{u}_{\bar{n}}\gamma_\perp^\mu \frac{\not{q}_\perp}{q_\perp^2}\gamma_\perp^\nu u_n\right]\frac{\alpha_s}{4\pi}\left[-\frac{1}{\epsilon}-\log\frac{\mu^2}{-t}\right](-i\pi)$$

Matches Bogdan, Del Duca, Fadin, and Glover [hep-ph/0201240]

BFKL equation for $q\bar{q} \to \gamma\gamma$



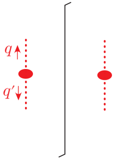
$q\bar{q} \to \gamma\gamma$: Bare factorization

- Repeat everything at the **cross section level**.
- Take a process where soft function is given by squared matrix element of $\mathcal{O}_s^n$ and $\mathcal{O}_s^{n\bar{n}}$: $q\bar{q} \to \gamma\gamma$.
- At LL the forward scattering cross section is

$$T^q_{(1,1)} = \int d^2q_\perp d^2q'_\perp C^q_n(q_\perp, p^-) S^q(q_\perp, q'_\perp) C^q_{\bar{n}}(q'_\perp, p'^+) \,,$$

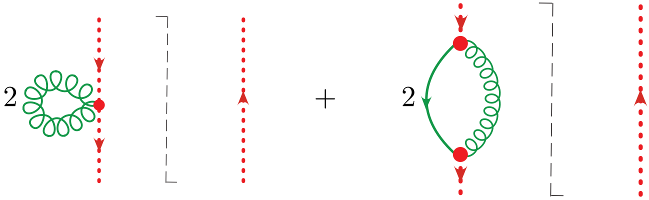
- **Soft function** $S^q(q_\perp, q'_\perp)$ is

$$S^q(q_\perp, q'_\perp) \sim \sum_{X_s} \langle 0| \left(O_s^{n\bar{n}} + T[O_s^n, O_s^{\bar{n}}] \right) |X_s\rangle\langle X_s| \left(O_s^{n\bar{n}} + T[O_s^n, O_s^{\bar{n}}] \right)^\dagger |0\rangle$$

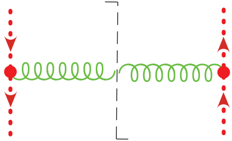
- LO Soft Function: $S_0^q \sim \langle 0|O_s^{n\bar{n}}|0\rangle\langle 0| \left(O_s^{n\bar{n}}\right)^\dagger |0\rangle =$ 

$q\bar{q} \to \gamma\gamma$: Soft function renormalization

- Repeat renormalization procedure at cross section level
- For virtual corrections: **flower** and **eye** graphs as for quark Reggeization

$S_V^q =$ 

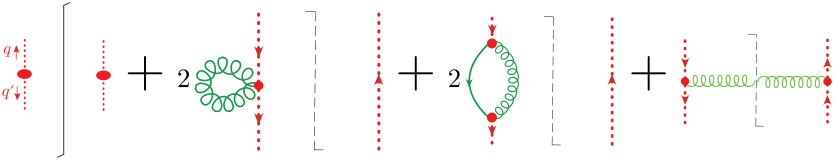
- Now we also have real corrections from **Fadin-Sherman vertex** in $\mathcal{O}_s^{n\bar{n}}$

$S_R^q =$ 

BFKL for $q\bar{q} \to \gamma\gamma$

- Put everything together

$S^q_{\text{1-l}} =$



- Subtract rapidity divergences via counterterm $Z_{S^q}(q_\perp, k_\perp)$.
- Bare Soft function is independent of $\nu \implies$ derive RGE

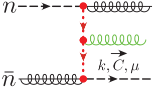
$$0 = \nu \frac{d}{d\nu} S^q_{\text{bare}}(q_\perp, q'_\perp) = \nu \frac{d}{d\nu} \int d^2k_\perp \, Z^{-1}_{S^q}(q_\perp, k_\perp) \, S^q(k_\perp, q'_\perp, \nu)$$

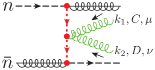
- RGE for S^q is BFKL equation with C_F instead of C_A

$$\nu \frac{d}{d\nu} S^q(q_\perp, q'_\perp, \nu) = \frac{2C_F \, \alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S^q(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} S^q(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

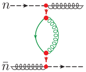
Summary

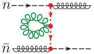
- Derived Lagrangians for t-channel exchange of Glauber quarks in the Regge limit

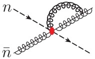




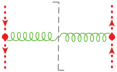
- Obtained Quark reggeization by Rapidity Renormalization Group Evolution



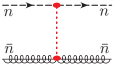


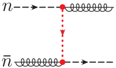


- Resummed small-x logs in the cross section for $q\bar{q} \to \gamma\gamma$ via BFKL equation



- Now leading Lagrangians for both Glauber gluon and Glauber quark exchange available in SCET.
  Many interesting directions!





Summary

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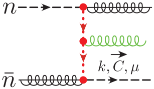

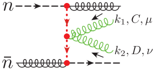

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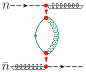

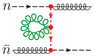

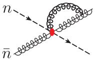

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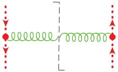

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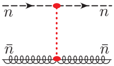
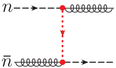
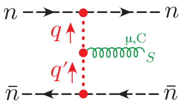




Thank you!

Backup slides

Momentum scaling



mode	fields	scaling $(n\cdot p, \bar{n}\cdot p, p_\perp)$	physical objects
onshell			
n-collinear	χ_n, $\mathcal{B}^\mu_n$	$p^\mu_n \sim Q(\lambda^2, 1, \lambda)$	n-collinear jet
$\bar{n}$-collinear	$\chi_{\bar{n}}$, $\mathcal{B}^\mu_{\bar{n}}$	$p^\mu_{\bar{n}} \sim Q(1, \lambda^2, \lambda)$	$\bar{n}$-collinear jet
	λ, $(\lambda^2, 1, \lambda)$		
soft	ψ_S, $\mathcal{B}^\mu_S$	$p^\mu_s \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation
	$\lambda^{3/2}$, λ		
offshell			
n-$\bar{n}$ Glauber	–	$q^\mu_G \sim Q(\lambda^2, \lambda^2, \lambda)$	forward scattering potential
n-s Glauber	–	$q^\mu_G \sim Q(\lambda^2, \lambda, \lambda)$	forward scattering potential
$\bar{n}$-s Glauber	–	$q^\mu_G \sim Q(\lambda, \lambda^2, \lambda)$	forward scattering potential
hard	–	$p^2 \gtrsim Q^2$	hard scattering

$$q^\mu_G = Q(\lambda^a, \lambda^b, \lambda) \qquad \text{with } a + b > 2$$

$$q^2_G = \underbrace{q^2_\perp}_{\lambda^2} + \underbrace{n\cdot q_G\, \bar{n}\cdot q_G}_{\lambda^{a+b>2}} = q^2_\perp + \text{ higher orders in } \lambda$$

Leading Power Glauber Lagrangian operators

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

Collinear quark operators

$$\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{\not{n}}}{2} \chi_n$$

$$\mathcal{O}_{\bar{n}}^{qB} = \overline{\chi}_{\bar{n}} T^B \frac{\not{n}}{2} \chi_{\bar{n}}$$

Collinear gluon operators

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^{C} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^{C} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{\bar{n}\perp}^{D\mu}$$

Single index soft operators (2-rapidity sector Lagrangian)

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^{n} \, T^B \frac{\not{n}}{2} \psi_S^{n} \right)$$

$$\mathcal{O}_s^{q_{\bar{n}} B} = 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} \, T^B \frac{\bar{\not{n}}}{2} \psi_S^{\bar{n}} \right)$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

$$\mathcal{O}_s^{g_{\bar{n}} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)$$

Two index soft operator (3-rapidity sector Lagrangian) : $\mathcal{O}_s^{BC}$ *non-trivial !*

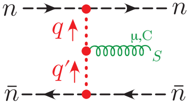
Two index soft operator (3-rapidity sector Lagrangian)

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s\Big\{\mathcal{P}^\mu_\perp \mathcal{S}_n^T\mathcal{S}_{\bar n}\mathcal{P}_{\perp\mu} - \mathcal{P}^\perp_\mu g\widetilde{\mathcal{B}}^{n\mu}_{S\perp}\mathcal{S}_n^T\mathcal{S}_{\bar n} - \mathcal{S}_n^T\mathcal{S}_{\bar n} g\widetilde{\mathcal{B}}^{\bar n\mu}_{S\perp}\mathcal{P}^\perp_\mu$$
$$- g\widetilde{\mathcal{B}}^{n\mu}_{S\perp}\mathcal{S}_n^T\mathcal{S}_{\bar n} g\widetilde{\mathcal{B}}^{\bar n}_{S\perp\mu} - \frac{n_\mu\bar n_\nu}{2}\mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu}\mathcal{S}_{\bar n}\Big\}^{BC}$$

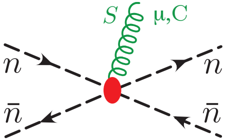
$$\mathcal{O}_s^{BC} \sim \lambda^2\,, \qquad [\mathcal{O}_s^{BC}] = [m]^2$$

- Operators appearing constrained by symmetries, p.c. and mass dim.
- Matching at 1 and 2-gluon emissions completely fix it.
- The operator $\mathcal{O}_s^{BC}$ is not corrected at higher order
- $\mathcal{O}_s^{BC}$ has feynman rule at any order in g.
  For example:

Lipatov vertex

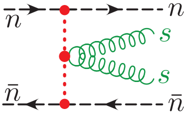


$=$



$$= \mathcal{O}_s^{BC}\Big|_{\mathcal{O}(g^3)}$$

Two soft gluon vertex



$$= \mathcal{O}_s^{BC}\Big|_{\mathcal{O}(g^4)}$$

Anomalous dimension consistency

The anomalous dim. $\gamma^\nu_{\mathcal{O}_s}$ and $\gamma^\nu_{\mathcal{O}_n}$ are related by **consistency**.

- There is no overall ν dependence in n-$\bar{n}$ scattering and n-s scattering amplitude.
- At one-loop, this simplifies considerably, and we have

$$\nu \frac{\partial}{\partial \nu}\left(\mathcal{O}_{\bar{n}n} + i\int d^4x\; T\; \mathcal{O}_{\bar{n}s}(x)\cdot \bar{\mathcal{O}}_{ns}(0)\right) = 0\,.$$

Using $\mathcal{O}_{n\bar{n}} = \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^{n\bar{n}} \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_{\bar{n}}$, $\mathcal{O}_{ns} = \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^n$ + h.c. and

$$\nu \frac{\partial}{\partial \nu}\mathcal{O}(\nu,\mu) = \gamma^\nu_{\mathcal{O}} \cdot \mathcal{O}(\nu,\mu)$$

we get the consistency eqn for the anomalous dimensions

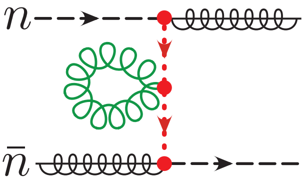
$$\gamma^\nu_{\mathcal{O}_n} = \gamma^\nu_{\mathcal{O}_{\bar{n}}}\,, \qquad \gamma^{\text{dir}}_{s\nu} + \gamma^T_{s\nu} = -\gamma^\nu_{\mathcal{O}_n} - \gamma^\nu_{\mathcal{O}_{\bar{n}}}\,, \qquad \gamma^\nu_{\mathcal{O}^n_s} = -\gamma^\nu_{\mathcal{O}_n}\,.$$

Renormalization of soft operators: diagrams

Soft operators:

$$\mathcal{O}_s^{n\bar{n}} = -4\pi\alpha_s \left[S_n^\dagger S_{\bar{n}} \slashed{\mathcal{P}}_\perp - g \slashed{\mathcal{B}}_{S\perp}^n S_n^\dagger S_{\bar{n}} \right] , \qquad \mathcal{O}_s^n = -4\pi\alpha_s \slashed{\mathcal{B}}_{\perp S}^n \psi_S^n$$

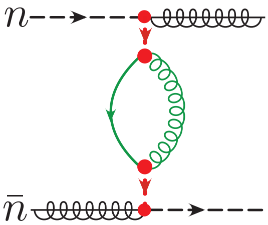
Flower



$\mathcal{O}_s^{n\bar{n}}$ self-contraction

One insertion of $\mathcal{O}(\lambda)$ operators

Eye



$T[\mathcal{O}_s^{\bar{n}}, \bar{\mathcal{O}}_s^n]$

2 insertions of $\mathcal{O}(\lambda^{1/2})$ operators

Renormalization of soft operators: Mixing

Mixing between $\mathcal{O}_s^{n\bar{n}}$ and $T[\mathcal{O}_s^{\bar{n}}, \bar{\mathcal{O}}_s^{n}]$ $\Longrightarrow$ <u>Anomalous dim. is a matrix</u>

$$\vec{\mathcal{O}}_s^{\,\mathsf{bare}} = \hat{V}_{\mathcal{O}_s} \cdot \vec{\mathcal{O}}_s \,, \quad \vec{\mathcal{O}}_s = \begin{pmatrix} \mathcal{O}_s^{n\bar{n}} \\ i\int d^4x \; T \; \mathcal{O}_s^{\bar{n}}(x)\bar{\mathcal{O}}_s^{n}(0) \end{pmatrix}$$

$$\hat{V}_{\mathcal{O}_s} = \begin{pmatrix} 1+\delta V_s & 0 \\ \delta V_s^T & V_{\mathcal{O}_s^{\bar{n}}} V_{\bar{\mathcal{O}}_s^{n}} \end{pmatrix}, \quad \hat{\gamma}^{\nu}_{\mathcal{O}_s} = \begin{pmatrix} \gamma_{s\nu}^{\mathsf{dir}} & 0 \\ \gamma_{s\nu}^{T} & \gamma^{\nu}_{\mathcal{O}_s^{\bar{n}}} \gamma^{\nu}_{\bar{\mathcal{O}}_s^{n}} \end{pmatrix}.$$

After the summing diagrams and taking poles in η we find

$$\delta V_s = -\frac{\alpha_s}{\pi} C_F \frac{h(\epsilon, \mu^2/m^2)}{\eta} \,, \qquad \delta V_s^T = -\frac{\alpha_s}{\pi} C_F \frac{g(\epsilon, \mu^2/t)}{\eta} \,,$$

$$\gamma_{s\nu}^{\mathsf{dir}} = -\frac{\alpha_s}{\pi} C_F h(\epsilon, \mu^2/m^2) \,, \qquad \gamma_{s\nu}^{T} = -\frac{\alpha_s}{\pi} C_F g(\epsilon, \mu^2/t) \,,$$