Color unwound

— Glauber gluons and factorization in azimuthal asymmetries in Drell-Yan

Tomas Kasemets

JGU Mainz

D. Boer, T. van Daal, J.R. Gaunt, TK, P.J. Mulders; arXiv:1709.04935

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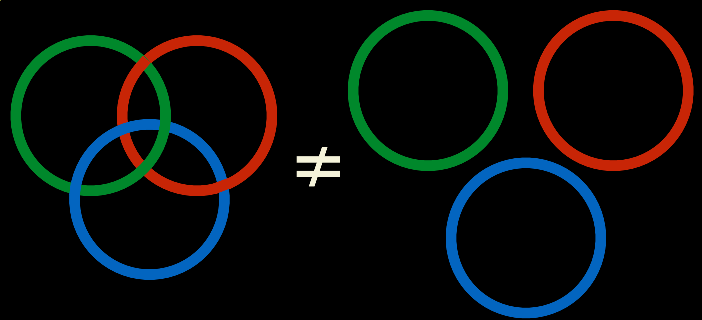
The What and the Why?

- **Contradiction**: *CSS factorization* vs *color entanglement* for *Boer-Mulders (BM) function* in Drell-Yan (DY)
  - Color entanglement suggested by Buffing and Mulders in DY for double *T-odd contributions* in low transverse momentum region

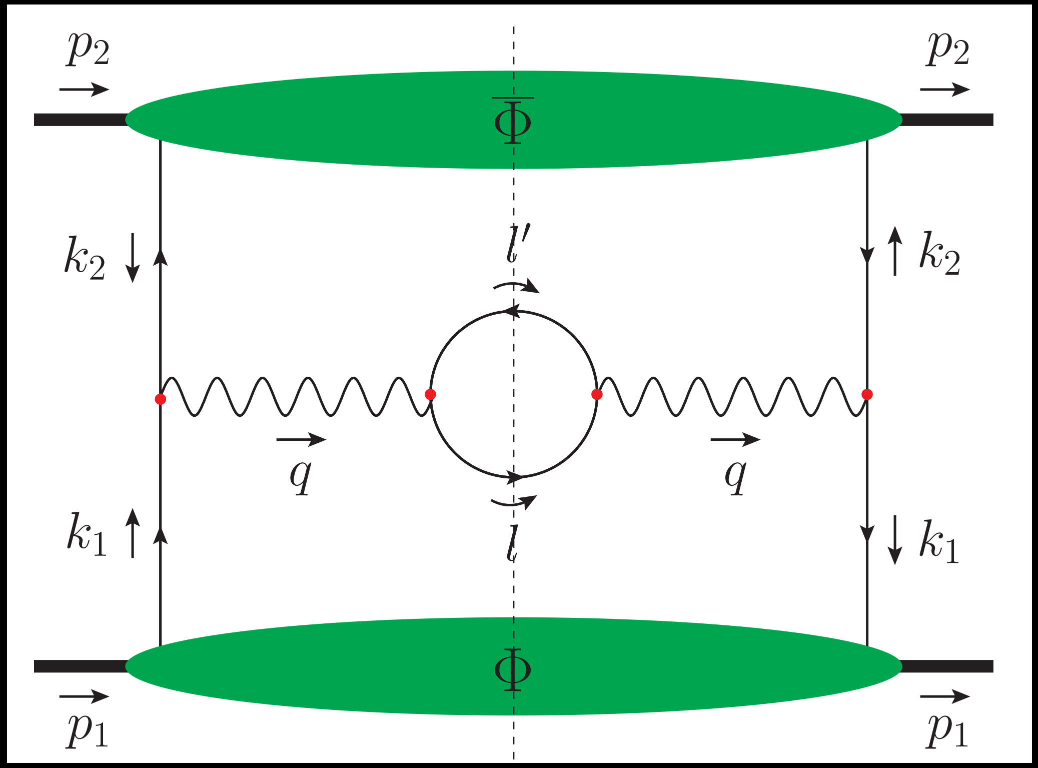


  - Polarization gives loop hole in the general proof?
  - How far does the process dependence of BM function go?
    - Important for experimental efforts
- Better understand factorization (violation) and Glauber gluons
  - Factorization violation due to Glauber gluons. When and where?
  - Asymmetries useful in isolating Glauber contribution (as we shall see)

Drell-Yan with measured q_T

- Cross section $d\sigma(pp \to Z/\gamma^* \to l\bar{l})$:
  - Hard interaction

    $q(k_1)\,\bar{q}(k_2) \to Z/\gamma^*(q) \to l\bar{l}$
  - Hadronic correlators
    - Transverse momentum dependent PDFs (TMDs)



- Factorization theorem:

hard coefficient - short distance

$$\frac{d\sigma}{dx_1 dx_2 d^2 q_T} = H \times [\, f_i(x_1, k_{1T}) \otimes \, f_j(x_2, k_{2T})\,] + \mathcal{O}(q_T^2/Q^2)$$

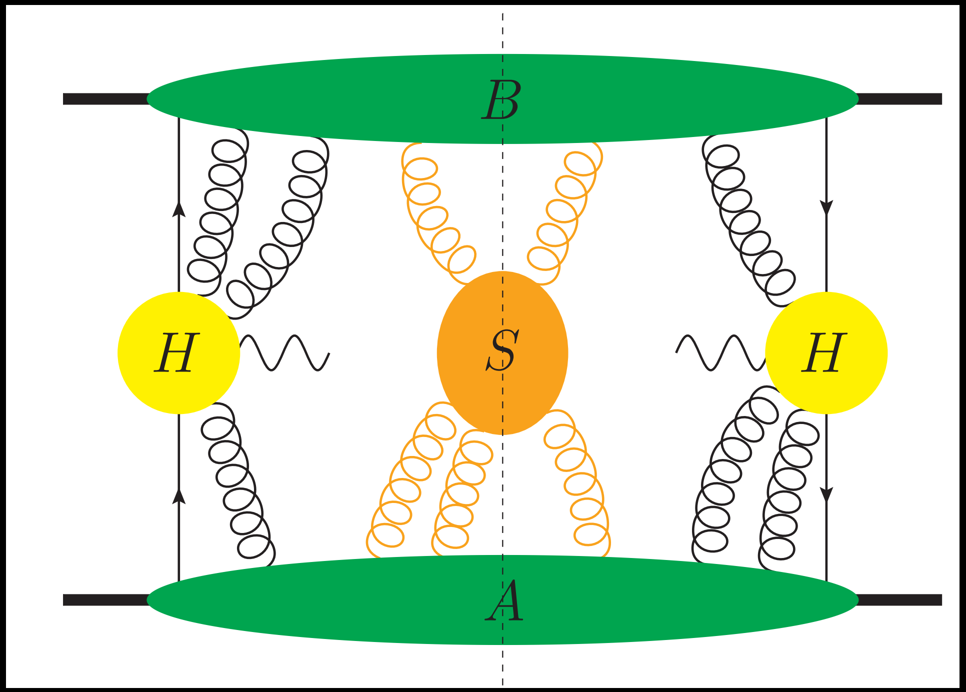
Transverse momentum dependent PDFs - long distance

- Rigorously proven for Drell-Yan

Bodwin, 1985; Collins, Soper, Sterman, 1985; Collins, 2011

Factorization and relevant modes

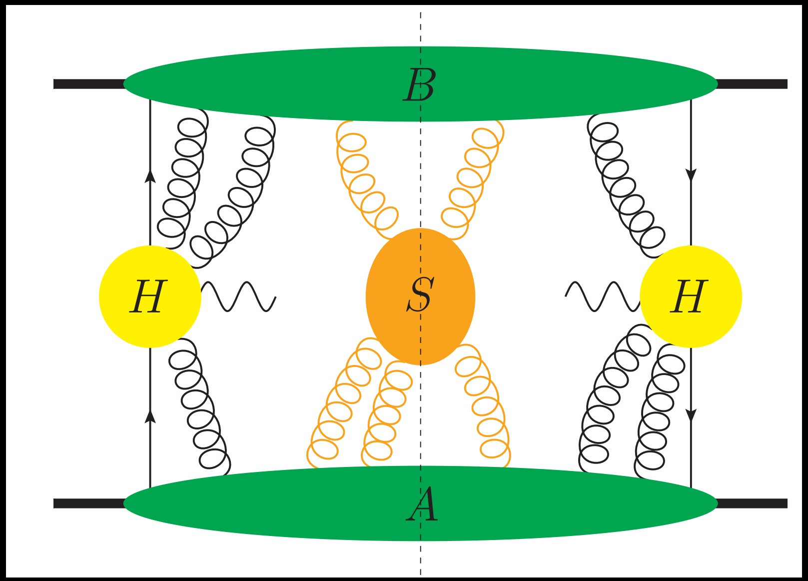
- Take possible Feynman graphs for DY production
- Identify leading-power infrared regions of the diagrams
  - Pinched-singular-surfaces — classically allowed processes
- Power counting analysis: does integration in neighborhood give leading contribution?
- Leading regions ($\lambda \sim |q_T|/Q$): $\ell \sim (+, -, \perp)$
  - hard (H) $\ell \sim (1, 1, 1)Q$
  - right-moving collinear (C1) $\ell \sim (1, \lambda^2, \lambda)Q$
  - left-moving collinear (C2) $\ell \sim (\lambda^2, 1, \lambda)Q$
  - (central) soft $\ell \sim (\lambda, \lambda, \lambda)Q$
  - Glauber $|\ell^+ \ell^-| \ll \ell_T^2 \ll Q^2$



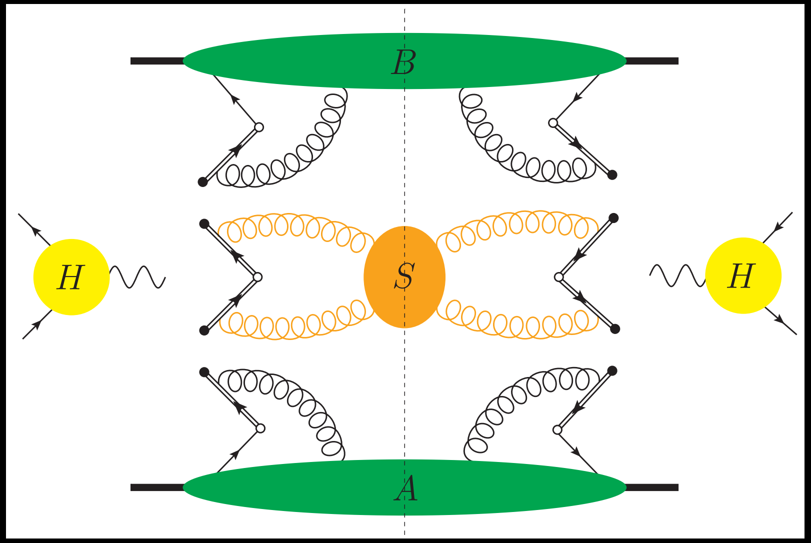
Factorization for Drell-Yan

- Glauber scalings
  - right-moving Glauber (G1) $\ell \sim (\lambda, \lambda^2, \lambda)Q$
  - left-moving Glauber (G2) $\ell \sim (\lambda^2, \lambda, \lambda)Q$
  - central Glauber (G) $\ell \sim (\lambda^2, \lambda^2, \lambda)Q$
- Regions and subtractions of a graph Γ
  - Contribution from region R $C_R\Gamma = T_R\Gamma - \sum_{R'<R} T_R C_{R'}\Gamma$
  - Adding all regions $\Gamma = \sum_R C_R T_R$
- Eikonal approximation
  - Propagator denominator $\frac{1}{(p_1+l)^2+i\epsilon} \approx \frac{1}{2\,p\cdot l+i\epsilon}$
- Ward identities
  - Remove soft attachments from collinear subgraphs, after sum over attachments

Factorization for Drell-Yan



- Similarly: unphysical polarized collinear attachments into hard subgraph
- No Glaubers = Factorization
  - soft, collinear and hard
- Glauber gluons ⇒ No Eikonal approximation
- Final state poles cancel = only initial state poles.
  - Deform ⇒ Eikonalize
  - Deform back to real axis



- Glaubers cancelled and/or absorbed

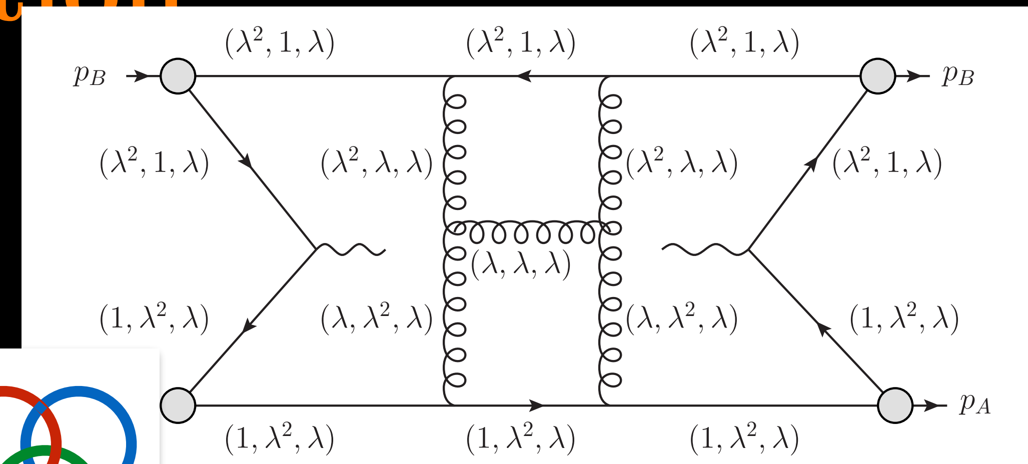
$$\frac{d\sigma}{dp_T} \sim \mathrm{H} \times (\mathrm{C1} \otimes \mathrm{S} \otimes \mathrm{C2})$$

- Absorb soft: $\frac{d\sigma}{dp_T} \sim \mathrm{Hard} \times (\mathrm{TMD} \otimes \mathrm{TMD})$

$$\mathrm{TMD} \sim (\text{Matching coeff.} \otimes \mathrm{PDF}) \times \text{Non-pert.}$$

Glaubers violating factorization

Fig. from Gaunt, 2014



- Factorization violation (Glauber gluons):
  - MPI sensitive obse
    (event shapes in ha
  - TMD factorization
  - super-leading logs
  - Strict collinear factorization
  - **color entanglement in Drell-Yan?**

QCD

Glauber Gluon

Senior Factorization Breaker

Hadron Collider Street 1
Geneva, Switzerland

, 2014; Schwartz, Yan, Zhu. 2018

Mulders, Rogers, 2014

orshaw, Kyrieleis, Seymour, 2006

Catani, de Florian, Rodrigo, 2012

Buffing, Mulders, 2013

- Off-shell scaling (cannot be integrated out as soft and collinear modes in SCET)
  - Included in SCET through potential insertions Rothstein, Stewart, 2016
- Most difficult part of deriving factorization

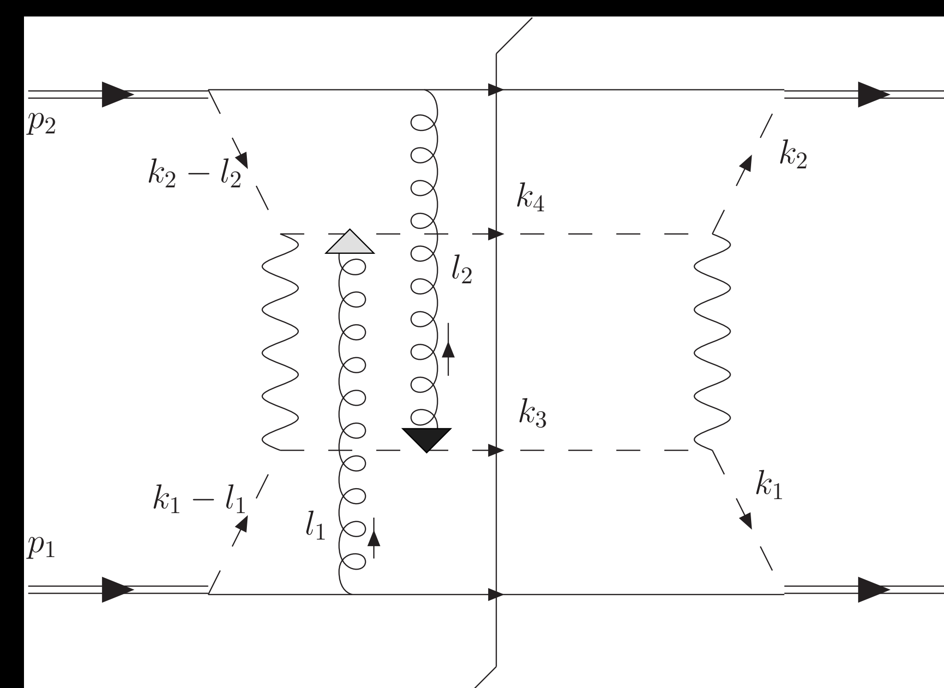
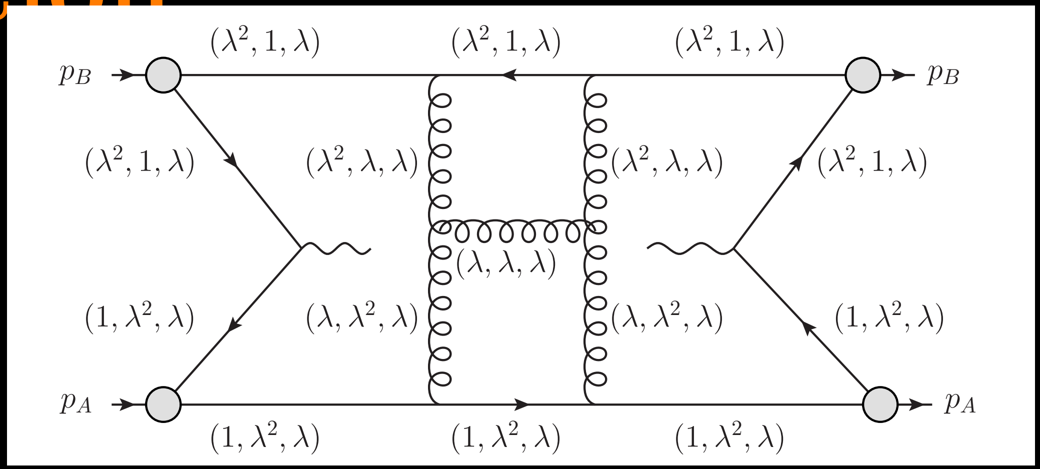


Fig. from Mulders, Rogers, 2014

Glaubers violating factorization

Fig. from Gaunt, 2014



Gaunt, 2014; Schwartz, Yan, Zhu. 2018

- Factorization violation (Glauber gluons):
  - MPI sensitive observables (event shapes in hadron collisions)
  - TMD factorization for h1 + h2 —> h3 + h4

Mulders, Rogers, 2014

  - super-leading logs

Forshaw, Kyrieleis, Seymour, 2006

  - Strict collinear factorization

Catani, de Florian, Rodrigo, 2012

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  - **color entanglement in Drell-Yan?**

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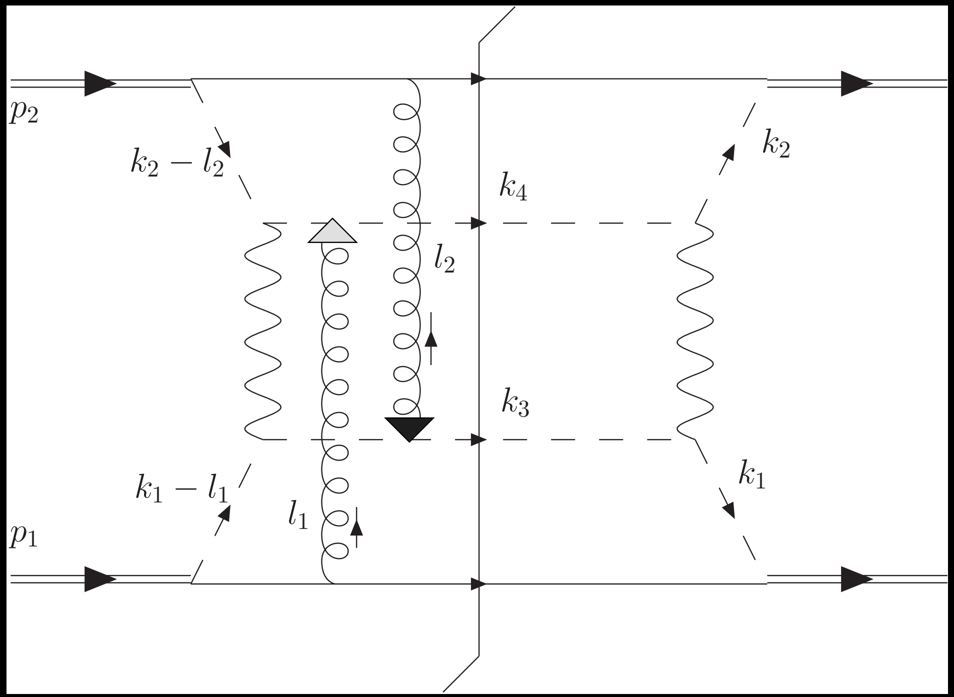


Fig. from Mulders, Rogers, 2014

Color factors for double Boer-Mulders effect

- Factorization theorem closer look:

Different non-perturbative functions!

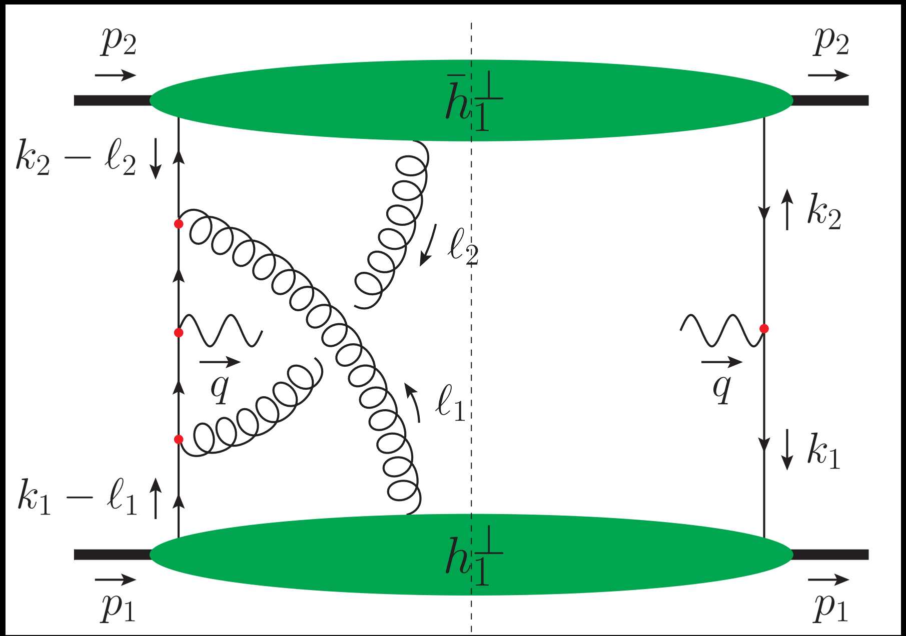
$$\frac{d\sigma}{d\Omega\, dx_1 dx_2\, d^2 q_T} = \frac{\alpha^2}{N_c\, q^2} \left\{ A(\theta) \left[f_1 \otimes \bar{f}_1 \right] + B(\theta) \cos(2\phi)\, C_{\text{ent.}} \left[w(\boldsymbol{k}_1, \boldsymbol{k_2}) \otimes h_1^\perp \otimes \bar{h}_1^\perp \right] \right\}$$

Boer, Brodsky, Huang, 2003;
Boer, 1999

  - Azimuthal asymmetry = Double Boer-Mulders effect (dBM)
  - Correlation: lepton decay plane $\leftrightarrow$ vector boson $\boldsymbol{q_T}$
- Contradicting color factor of dBM effect

Buffing, Mulders 2013

  - CSS factorization: $C_{\text{ent.}} = 1$
  - Color entanglement: $C_{\text{ent.}} = -\frac{1}{N_c^2 - 1}$
- Color entangled type of diagram:
  - First order giving a non-zero dBM effect
    $\text{Tr}\left\{t^a t^b t^a t^b\right\} \neq \text{Tr}\left\{t^a t^a\right\} \text{Tr}\left\{t^b t^b\right\}$



Boer-Mulders function

- TMD for transversely polarized quark in unpolarized proton

$$\frac{\widetilde{k}_{1T}^{j}}{M}\, h_{1,q}^{\perp}(x_1, \boldsymbol{k}_1^2) \equiv \int \frac{d\xi^- d^2\boldsymbol{\xi}}{(2\pi)^3}\, e^{ik_1\cdot\xi}\, \langle p_1|\, \overline{\psi}_q(0)\, W_{[0,\xi]}\, \Gamma_T^j\, \psi_q(\xi)\, |p_1\rangle \Big|_{\xi^+=0}$$

Boer, Mulders 1997

$$\Gamma_T^j \equiv \tfrac{1}{2} i\sigma^{j+}\gamma^5$$

  - + (part of) Soft function — will not be relevant
- Existence possible due to Wilson lines
  - Function is T-odd
- TMDs can be process dependent (in a calculable way)
  - BM function changes sign between SIDIS and DY

Model calculation to test factorization

- Essential features of QCD necessary for color entangled result
- Simple enough to calculate explicitly
- Spectator model, proton couple to
  - spin-1/2 quark (color triplet)
  - scalar spectator (color anti-triplet)
- QCD corrections: gluons couple to quarks and scalars via standard (fermion and scalar) QCD Feynman rules
  - Obeys physical principles, such as unitarity

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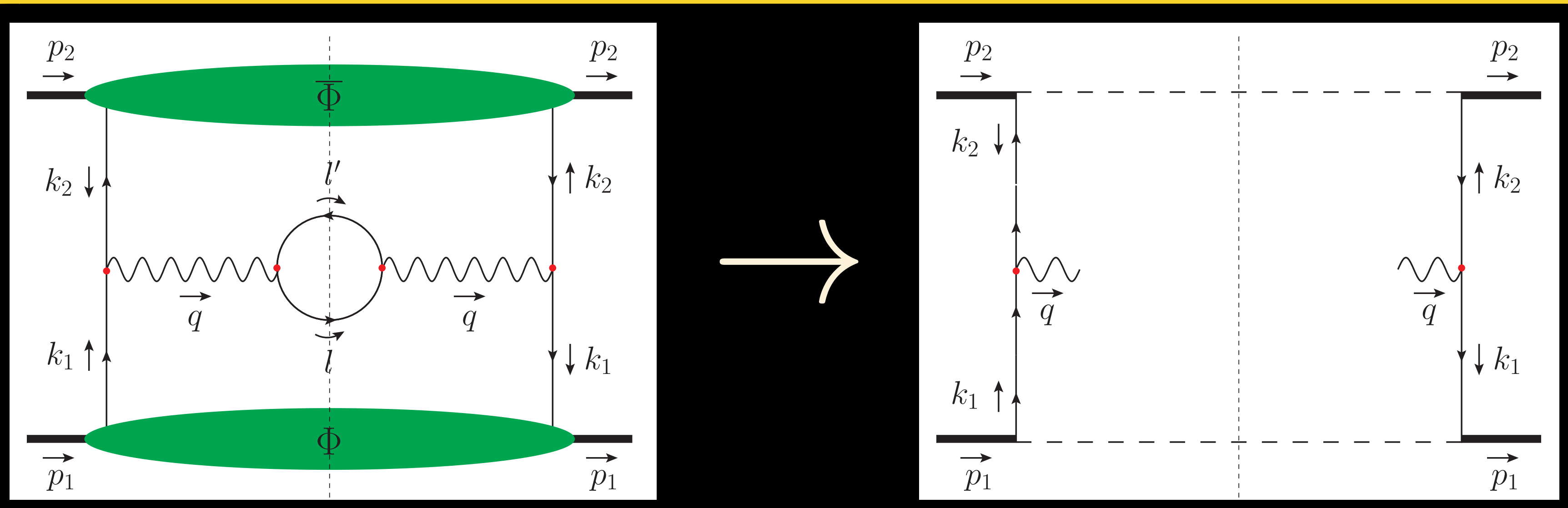
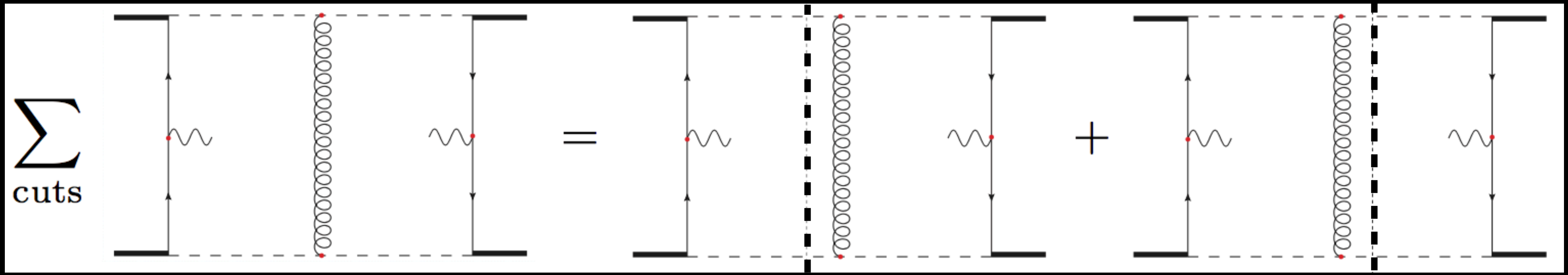
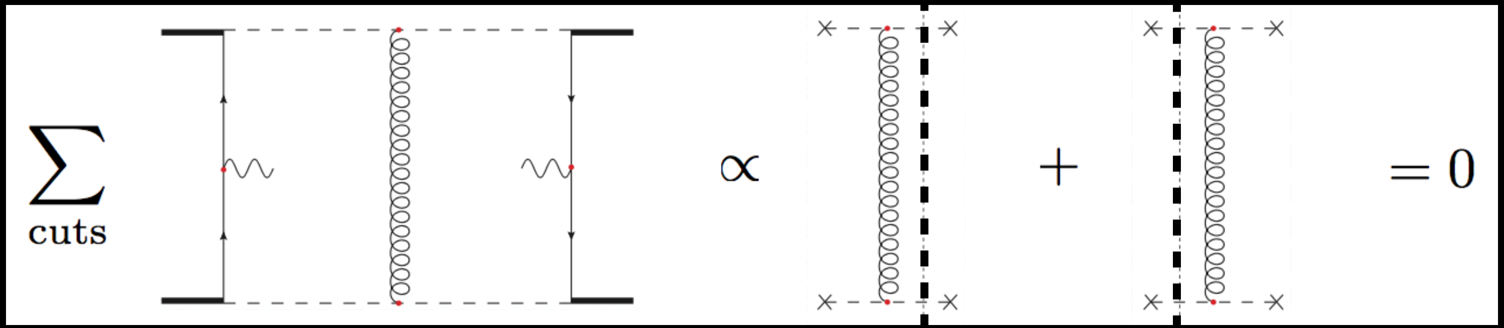


Single gluon exchange

- dBM effect should be zero as this order, lets check:
  - Non-zero graph has gluon exchange between the two scalar spectators
  - Must have central Glauber scaling for leading power contribution
    - Two places to put the final state cut



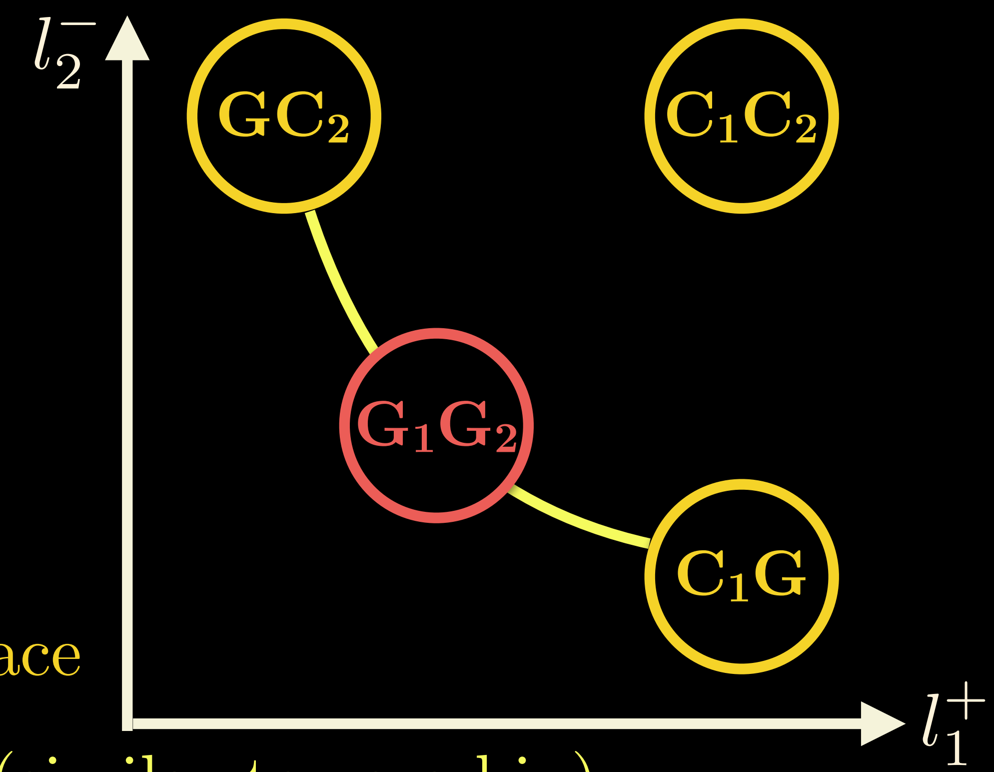
    - Sum over these two cuts gives zero (Cutkosky)



$$i\left[\mathcal{M}(i \to f) - \mathcal{M}^*(f \to i)\right] = -\sum_{\text{int. cuts}} \int d\Phi_X \mathcal{M}(i \to X)\mathcal{M}^*(f \to X)$$

    - Physical reason: unitarity

Two gluon exchange



- Most relevant regions AB (scaling of $l_1\ l_2$):
- Regions and Collins subtractions
  - with explicit Glauber region!
  - Sizes of regions:
    - G$_1$G$_2$ point, GC$_2$ and C$_1$G line, C$_1$C$_2$ surface
  - Calculate each region and subtract overlap (similar to zero-bin)

$$C_{G_1G_2}\Gamma = T_{G_1G_2}\Gamma,$$
$$C_{C_1G}\Gamma = T_{C_1G}(1 - T_{G_1G_2})\Gamma,$$
$$C_{C_1C_2}\Gamma = T_{C_1C_2}(1 - T_{C_1G} - T_{GC_2})(1 - T_{G_1G_2})\Gamma.$$

- Rapidity regulator (compare SCET II)
  - Regulator inspired by η regulator (CMU regulator)

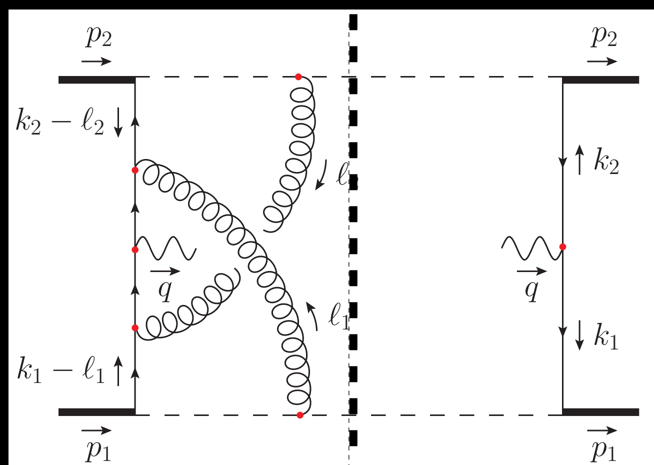
$$\left|\frac{\ell_1^+}{\nu}\right|^{-\eta_1}\left|\frac{\ell_2^-}{\nu}\right|^{-\eta_2}$$ careful with order of limits

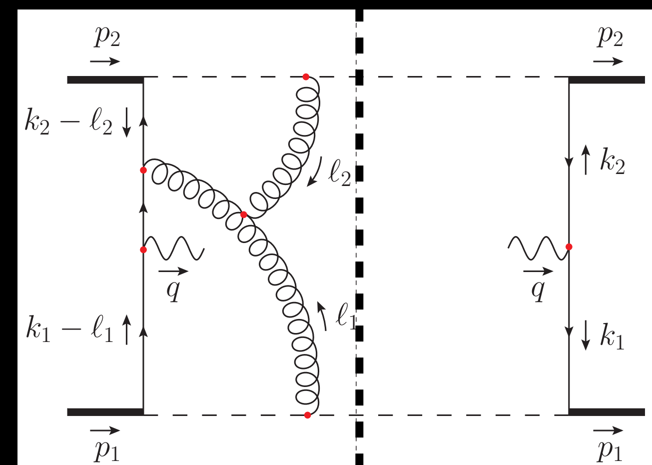
Chiu, Jain, Neill, Rothstein, 2012

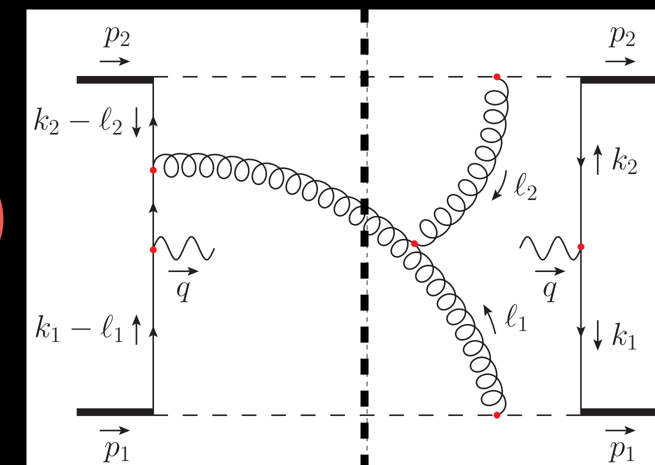
$\mathrm{G_1G_2}$ region

$$\ell_1 \sim (\lambda, \lambda^2, \lambda)Q$$
$$\ell_2 \sim (\lambda^2, \lambda, \lambda)Q$$

- Most interesting diagrams:

a) 

b) 

c) 

- Cross section contribution:

$$d\sigma_{\mathrm{dBM}} \sim C_{(\mathrm{a})}\left[I_{(\mathrm{a})} - N_c^2\left(I_{(\mathrm{b})} + I_{(\mathrm{c})}\right)\right]$$

- Integrals over larger (λ scaled) gluon momentum light cone components
- $$I_{(\mathrm{a})} \equiv \int \frac{d\ell_1^+}{2\pi}\,\frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi}\,\frac{\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{\ell_2^- + i\epsilon}$$
    - only initial state poles

Final state pole cancellation

- $$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+ \nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\ell_{1T} + \ell_{2T})^2 + i\epsilon}$$

$$I_{(\mathrm{c})} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \theta(-\ell_1^+)\, \ell_1^+\, \delta[2\ell_1^+\ell_2^- - (\ell_{1T} + \ell_{2T})^2]\, \nu^{\eta_2}|\ell_2^-|^{-\eta_2}$$

$$2\pi i\, \delta(x) = \frac{1}{x - i\epsilon} - \frac{1}{x + i\epsilon}$$

  - initial and final state poles

- Sum over cuts cancel the final state poles

$$I_{(\mathrm{b})} + I_{(\mathrm{c})} = -\frac{i}{2} \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} + \mathcal{O}(\eta_2) = I_{(\mathrm{a})}$$

- Result agree with factorization theorem

$$d\sigma_{\mathrm{dBM}} \sim C_{(\mathrm{a})} \left[I_{(\mathrm{a})} - N_c^2 \left(I_{(\mathrm{b})} + I_{(\mathrm{c})} \right) \right] = \frac{1}{N_c} C_\Phi^2\, I_{(\mathrm{a})}$$

GC_2, C_1G and C_1C_2 region

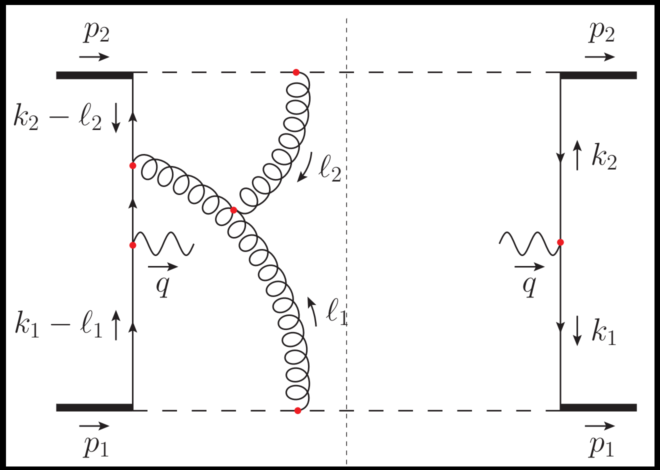
- GC_2 and C_1G separately disentangle (can fix one of l_1^+ and l_2^-)
- C_1C_2: Calculate contribution with approximations of collinear gluons

$$C_{C_1C_2}\Gamma = T_{C_1C_2}(1 - T_{C_1G} - T_{GC_2})(1 - T_{G_1G_2})\Gamma.$$

  - Only need fixed values of l_1^+ and l_2^-.
    - Subtractions remove regions where they go to zero
    - Can ignore the i-epsilon in the denominators
    - Leads directly to a color disentangled result
- Adding non-entangled diagrams, get zero for the C_1C_2, GC_2 and C_1G
- Implies that collinear result in Glauber region is equal to Glauber contribution
  - Glaubers can be absorbed in the collinear
    - But, the i-epsilons of Collinear result then matters!

Rapidity regulator dependence

- Different regulators for different diagram
- Diagrams depends on choice of rapidity regulator
  - example: the triple gluon (G_1 - G_2 - S) vertex diagram



- Our default choice, $\left|\ell_1^+/\nu\right|^{-\eta_1} \left|\ell_2^-/\nu\right|^{-\eta_2}$ with $\eta_1 \gg \eta_2$ (avoid that η_2) after integration over l_2^- anti-regulates the l_1^+ integral.
- Theta functions (from Collinear region) gives same result after sum over cuts

$$\theta\left[\min(k_1^+, p_1^+ - k_1^+) - |l_1^+|\right],\ \theta\left[\min(k_2^-, p_2^- - k_2^-) - |l_2^-|\right]$$

- Using instead $|(l_1^+ - l_2^-)/\nu|^{-\eta}$, triple gluon vertex diagram vanishes in G_1G_2 region.
  - Colors disentangle between momentum regions.

Boer-Mulders function = a Glauber function?

- At this order, full contribution from Glauber region

$$C_{G_1}\Gamma + C_{C_1}\Gamma = T_{G_1}\Gamma + T_{C_1}\left(1 - T_{G_1}\right)\Gamma = T_{G_1}\Gamma$$

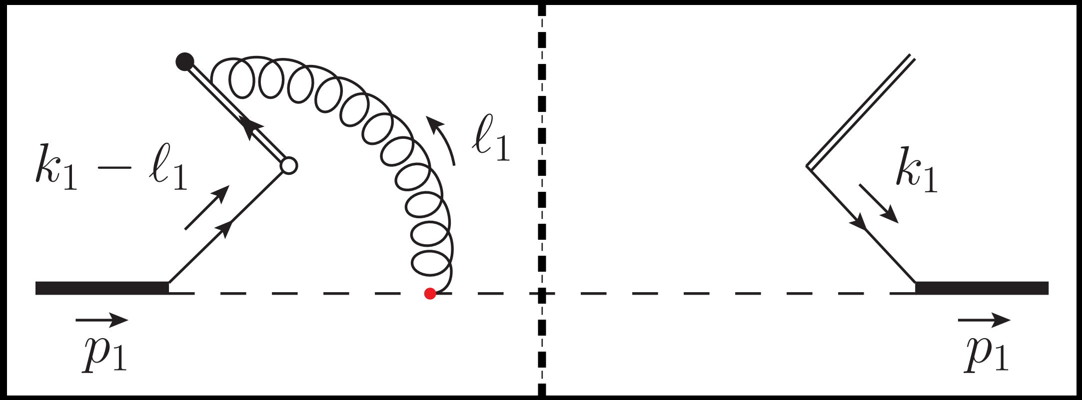
- Boer-Mulders function:

$$\frac{\widetilde{k}^j_{1T}}{M}\, h_1^{\perp}(x_1, \boldsymbol{k}_1^2) = -\,2i\, C_\Phi\,(1 - x_1)\, p_1^+\, \chi^j(x_1, \boldsymbol{k}_1) \int \frac{d\ell_1^+}{2\pi}\, \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} + \text{h.c.}$$

  - Color factor

$$C_\Phi = C_A C_F = \frac{N_c^2 - 1}{2}$$

  - Initial state Wilson line



- Sign change for SIDIS (final state Wilson line)

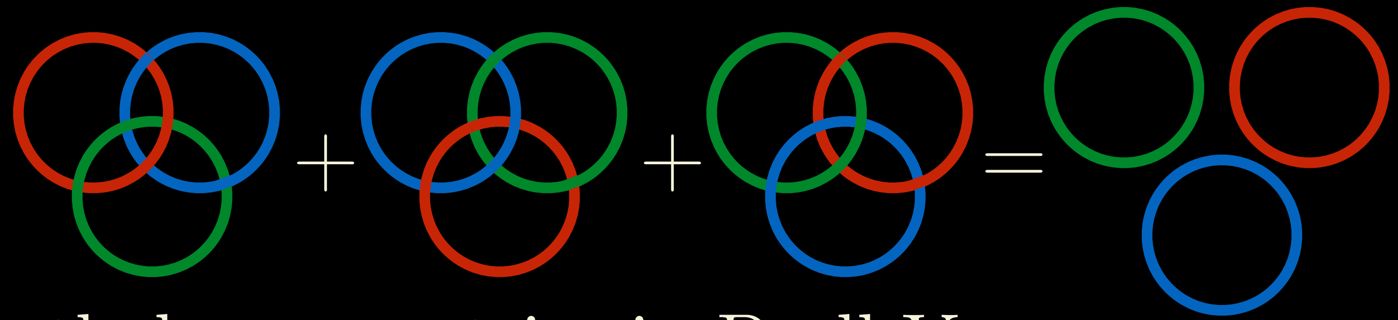
Calculation summary

- **Colors disentangle separately in each region**
  - G_1G_2 , C_1G, GC_2, C_1C_2
- Adding non-entangled diagrams, the result reproduces the factorization theorem at this order
- **Contribution to dBM entirely from Glauber region**
- Underlying reason: **after sum over cuts** of diagrams one component of each gluon loop momentum ($l_1^+$and l_2^-) **not trapped in Glauber region** and can be **deformed into the collinear regions**.
- Glaubers absorbed in other region consistent with factorization
  - Note: Not possible to deform to make absorption into soft region. Not even after summing over diagrams and cuts.
- CSS/SCET factorization does not specify which region absorbs which type of Glaubers — interesting to examine further

Glaubers and (spin) asymmetries

- Reduce complexity, move effects to lower orders
  - Remember: higher orders does not mean small effect (small scale)
- Double unpolarized contribution:
  - More diagrams contributing
  - Contributions already at lower orders
    - Color entanglement between diagrams (a)-(c) in the G_1G_2 region proceeds the same way as for the polarized example
- BM: Given by Glauber region at $\mathcal{O}(\alpha_s)$
- Unpolarized TMD: imaginary Glauber contribution cancel at $\mathcal{O}(\alpha_s)$ non-zero at $\mathcal{O}(\alpha_s^2)$

Conclusions



- Colors disentangle for azimuthal asymmetries in Drell-Yan
  - Factorization formula exactly recovered
    - Unitarity to cancel the final-state poles after sum over cuts on a graph
    - Non-Abelian Ward identity to give the predicted color factor
  - Surviving Glauber contribution can be absorbed by collinear and soft
- Glauber gluons absorbed in
  - Transverse momentum dependent PDFs/Beam functions
  - Soft and Collinear Wilson lines
- Interesting to examine further when and where Glauber regions are absorbed by collinear and soft functions
- Better understand when, where and how much Glauber gluons violate factorization