The NNLO soft function for top-quark pair production

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Outline

- Introduction and motivation
- Factorization at threshold and soft function
- Calculation of soft function
- Renormalization of soft function
- Conclusion

Introduction and motivation

The heaviest particle in SM

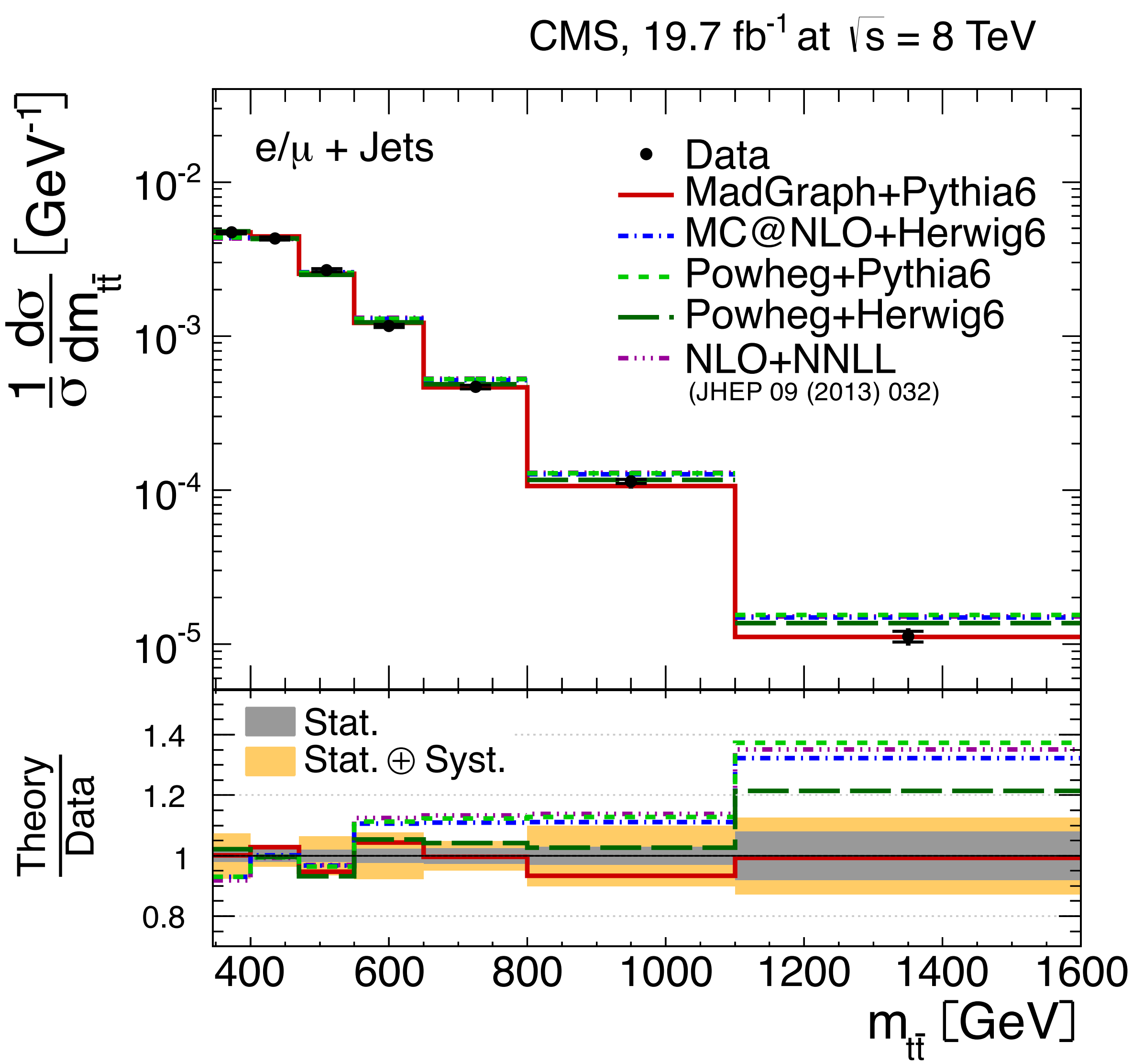
- Perturbative QCD and EWSB
- New physics
- Large background to rare processes

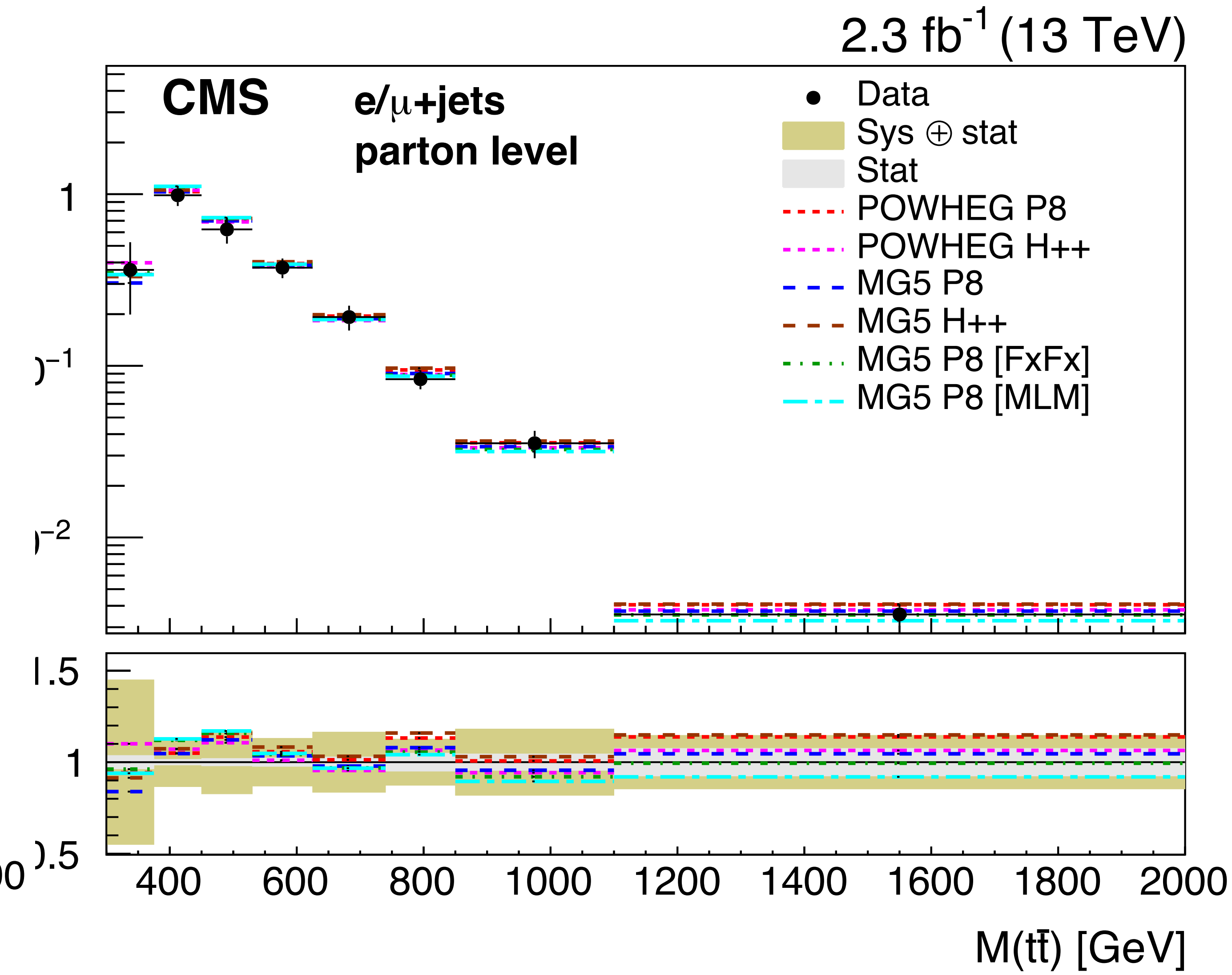
Experiments

- $\delta\sigma_{t\bar{t}}$: 2014($\sqrt{s}$ = 8TeV) $\sim$ 3.5%, RUN 2(2015) $\sim$ 4.4%
- Differential cross section

Ulrich Husemann. ArXiv:1704.01356, CMS. ArXiv:1505.04480,1610.04191

Introduction and motivation





Ulrich Husemann. ArXiv:1704.01356, CMS. ArXiv:1505.04480,1610.04191

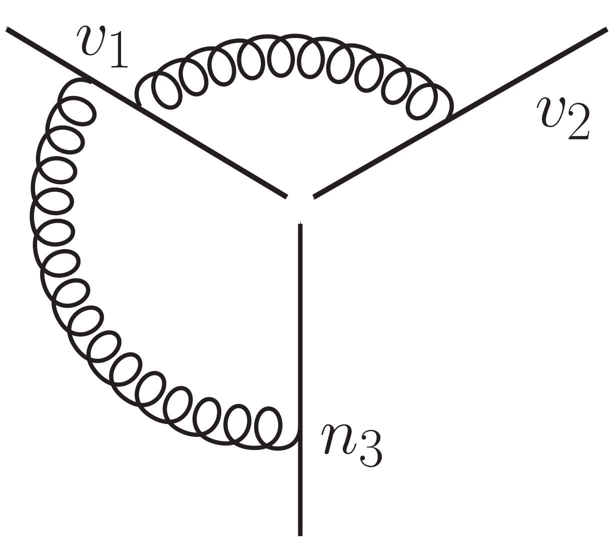
Introduction and motivation

Phenomenology

- Logarithms: $\ln(\frac{m_t}{s}),\ \ln(\beta)$
- Bottleneck in extending threshold resummation to N^3LL

Theoretical interests

Non-trivial correlation among three partons



$$\Gamma_s \sim \sum_{I,J,K} i f^{abc} T_I^a T_J^b T_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI})$$

$$+ \sum_{I,J} \sum_{k} i f^{abc} T_I^a T_J^b T_k^c f_2(\beta_{IJ}, ln\frac{w_{Jk}\sqrt{v_I^2}}{w_{Ik}\sqrt{v_J^2}})$$

$$+ \ldots$$

$$w_{IJ} = v_I \cdot v_J + i0, \beta_{IJ} = arccosh(-\frac{v_I \cdot v_J}{\sqrt{v_I^2}\sqrt{v_J^2}} - i0)$$

Factorization at threshold and soft function

Kinematics of the process $N(P_1) + N(P_2) \to t(p_3) + \bar{t}(p_4) + X_s(p_s)$

$$s = (P_1 + P_2)^2, \hat{s} = (p_1 + p_2)^2, M^2 = (p_3 + p_4)^2,$$

$$z = \frac{M^2}{\hat{s}}, \tau = \frac{M^2}{s}, \beta = \sqrt{1 - \frac{4m_t^2}{M^2}}, t_1 = -\frac{M^2}{2}(1 - \beta \cos\theta)$$

QCD factorization theorem

$$\frac{d\sigma}{dM\, d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu) C_{ij}(z, M, m_t, \cos\theta, \mu)$$

At threshold region $z \to 1$

$$C_{ij} = Tr(\mathbf{H}_{ij}(M, m_t, \cos\theta, \mu)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1 - z), m_t, \mu)) + \mathcal{O}(1 - z)$$

John C. Collins, *et al.* ArXiv:hep-ph/0409313

V. Ahrens, *et al.* ArXiv:1003.5827

Factorization at threshold and soft function

Wilson line

$$\mathbf{S}_i = \mathbf{P} exp(i g_s \int_{-\infty}^{0} ds v_i \cdot A^a(x + s v_i) \mathbf{T}_i^a)$$

For massless particles $v_i^2 = 0$, and massive particles $v_i^2 = 1$.

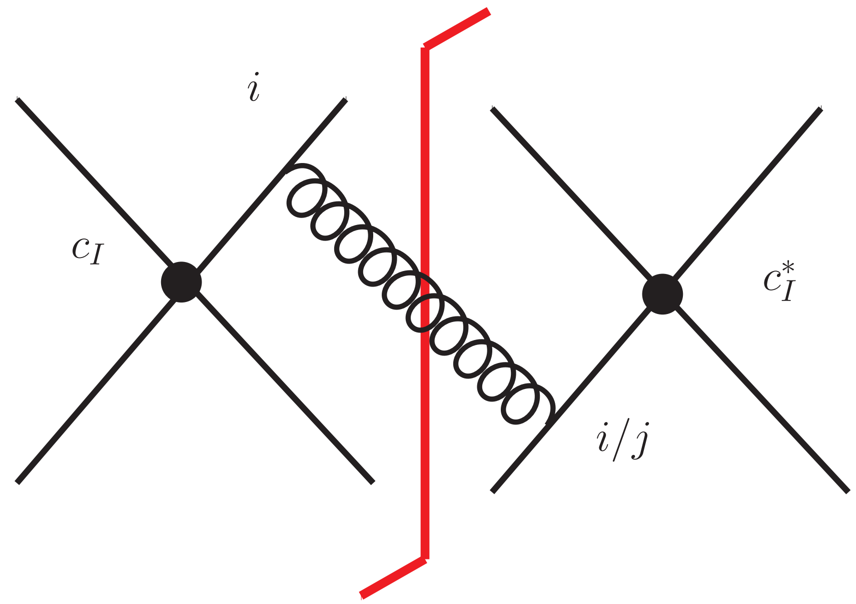
Soft function

$$S = \frac{1}{d_R} \sum_{X_S} \langle 0 | \mathbf{O}_S^\dagger(0) | X_S \rangle \langle X_s | \mathbf{O}_S(0) | 0 \rangle \delta(w - v_0 \cdot p_{X_S})$$

$$\mathbf{O}_s(x) = [\mathbf{S}_{v_1} \mathbf{S}_{v_2} \mathbf{S}_{v_3} \mathbf{S}_{v_4}](x)$$

Calculation of soft function

Diagrams of NLO soft function



The NLO soft function

$$\mathbf{S}^{(1)}_{bare} \sim \sum_{ij} w^{(1)}_{ij} \int [dk] \frac{v_i \cdot v_j}{v_i \cdot k v_j \cdot k} \delta(w - v_0 \cdot k)$$

Calculation of soft function

Integration by parts(IBP)

$$\int (\prod_i d^d k_i) \frac{\partial}{\partial k_j^\mu} q_\mu \prod_l \frac{1}{D_l^{a_l}} = 0$$

$$\delta^+(k^2) = \frac{1}{(k^2)}_{disc} = \frac{1}{2\pi i}\left(\frac{1}{k^2 + i0} - \frac{1}{k^2 - i0}\right)$$

Differential equation of canonical basis

$$\partial_x \vec{f}(\epsilon, x) = \epsilon A(x) \cdot \vec{f}(\epsilon, x)$$

$$\vec{f}^{(i+1)}(x) = \int^x dx' A(x') \cdot \vec{f}^{(i)}(x')$$

Calculation of soft function

Only one integral family at NLO

$$G_{a_1,a_2} \equiv \int [dk] \frac{\delta(w - v_0 \cdot k)}{(v_1 \cdot k)^{a_1} (v_3 \cdot k)^{a_2}}$$

Canonical basis and differential equations

$$\vec{f}(x,\epsilon) = \left\{(1-2\epsilon)G_{0,0}, \beta w \epsilon G_{0,1}, w^2\epsilon(1-\beta y)G_{1,1}\right\}$$

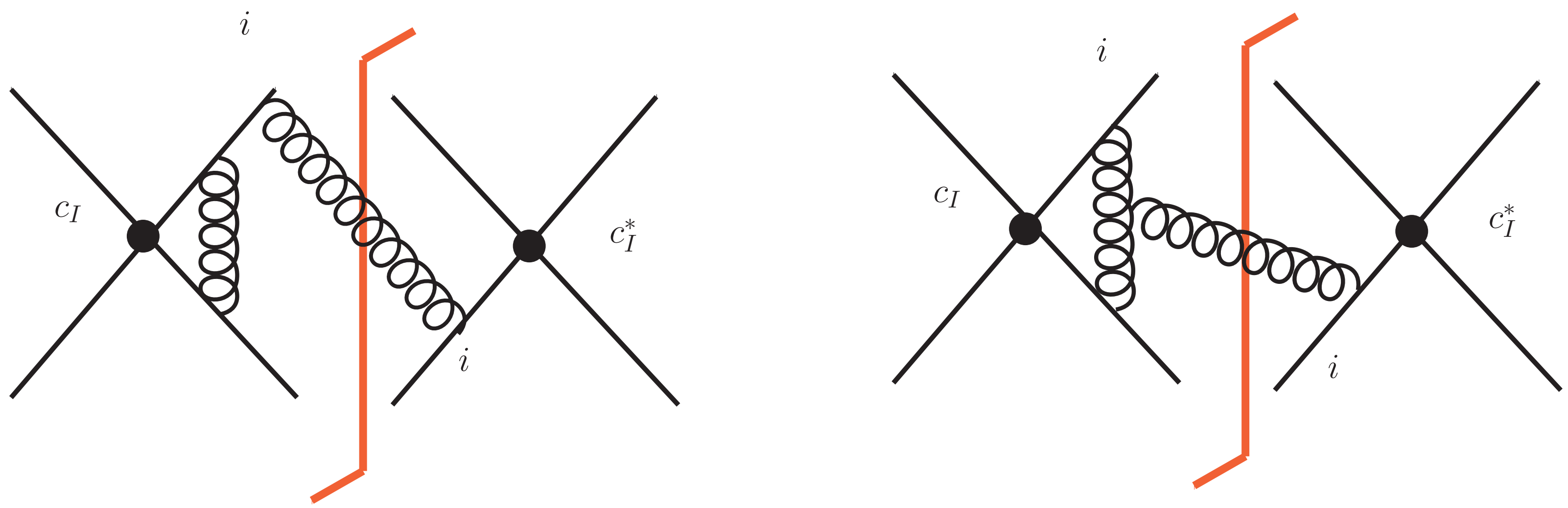
$$A = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\beta+1} - \frac{1}{\beta-1} & \frac{2}{\beta} - \frac{1}{\beta+1} - \frac{1}{\beta-1} & 0 \\ \frac{4}{\beta} - \frac{2}{\beta+1} - \frac{2}{\beta-1} & \frac{2}{\beta+1} - \frac{2}{\beta-1} & \frac{2}{\beta} - \frac{2y}{y\beta-1} \end{pmatrix}$$

Boundary condition at $\beta = 0$

$$\vec{f} = \frac{2^{2\epsilon-3}\pi^{\epsilon-\frac{3}{2}} w^{1-2\epsilon}}{\Gamma\left(\frac{3}{2} - e\right)} \{1, 0, -2\}$$

Calculation of soft function

Typical diagrams of NNLO-RV soft function



General Feynman integral for real-virtual case

$$G_{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9} \equiv \int [dk] d^d l \prod_{i=2}^{8} (D_i)^{-a_i} \delta(w - v_0 \cdot k)$$

Simplify integrals using relations like that

$$\frac{1}{v_i \cdot k} \frac{1}{v_i \cdot (k+l)} + \frac{1}{v_i \cdot l} \frac{1}{v_i \cdot (k+l)} = \frac{1}{v_i \cdot k} \frac{1}{v_i \cdot l}$$

Calculation of soft function

There are totally *4* integral families

$$\{l^2, (k+l)^2, v_1 \cdot k, v_1 \cdot (k+l), v_2 \cdot (-l), v_3 \cdot k, v_3 \cdot (-l)\}$$
$$\{l^2, (k+l)^2, v_1 \cdot (k+l), v_2 \cdot k, v_2 \cdot (-l), v_3 \cdot k, v_3 \cdot (-l)\}$$
$$\{l^2, (k+l)^2, v_1 \cdot k, v_1 \cdot (-l), v_3 \cdot k, v_3 \cdot (k+l), v_4 \cdot (-l)\}$$
$$\{l^2, (k+l)^2, v_1 \cdot k, v_1 \cdot (-l), v_3 \cdot (k+l), v_4 \cdot k, v_4 \cdot (-l)\}$$

The master integrals of the third integral family

$$\{G_{1,0,1,0,0,0,0,1,1}, G_{1,-1,1,0,0,0,0,1,1}, G_{1,1,0,0,0,0,1,0,1}, G_{1,1,-1,0,0,0,1,0,1},$$
$$G_{1,1,0,1,0,0,1,0,1}, G_{1,0,1,0,1,0,1,0,1}, G_{1,0,1,0,1,1,1,0,1}, G_{1,-1,1,0,1,1,1,0,1},$$
$$G_{1,1,1,-1,1,0,1,0,1}, G_{1,1,1,0,1,-1,1,0,1}, G_{1,1,1,0,1,0,1,0,1}, G_{1,0,1,0,0,1,0,1,1},$$
$$G_{1,1,0,0,0,0,1,1,1}, G_{1,1,0,0,0,0,2,1,1}, G_{1,0,1,0,0,0,1,1,1}, G_{1,0,1,0,0,0,1,2,1},$$
$$G_{1,0,1,0,0,1,1,1,1}, G_{1,1,1,0,0,0,1,1,1}, G_{1,1,0,1,0,0,1,1,1}\}$$

Calculation of soft function

Loop integrals in the master integrals

$$M_1 \equiv \int d^d l \frac{1}{l^2(k+l)^2(-v_j \cdot l)}$$

$$M_2 \equiv \int d^d l \frac{1}{l^2(v_i \cdot (k+l))(-v_j \cdot l)}$$

$$M_3 \equiv \int d^d l \frac{1}{l^2(k+l)^2(v_i \cdot (k+l))(-v_j \cdot l)}$$

If $v_i = v_3, v_j = v_4, M_2 \sim \beta^\epsilon$. So some integrals are singular in the limit $\beta = 0$. For example

$$\int [dk] d^d l \frac{\delta(w - v_0 \cdot k)}{l^2 v_3 \cdot (k+l) v_4 \cdot (-l)}$$

Isabella Bierenbaum, *et al.* ArXiv:1107.4384

Calculation of soft function

Boundary condition of the differential equations

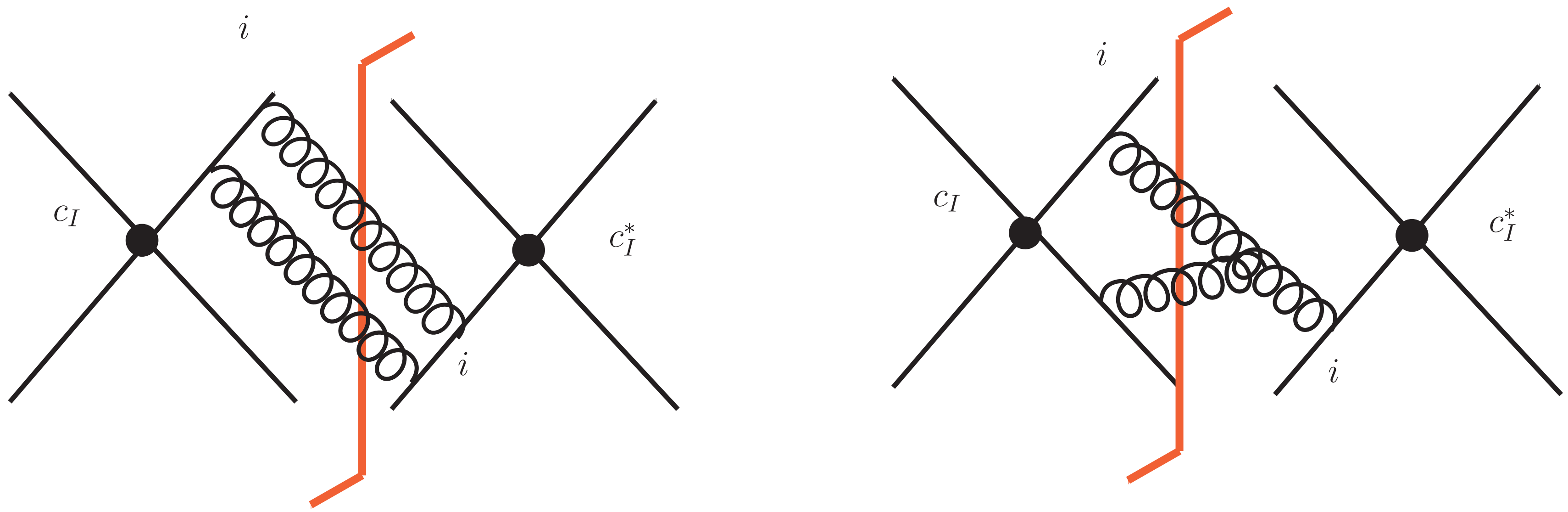
- Set the boundary condition at $\beta = 0$

- If the integrals are not singular at the boundary , set $s = 4m_t^2$ and do reduction to simplify the boundary condition

- As for the singular parts ,have to compute the integral then determine the constants

Calculation of soft function

Diagrams of the NNLO-RR soft function



The integrals are of the form

$$G_{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9} \equiv \int [dk_1][dk_2] \prod_{i=3}^{8} (D_i)^{-a_i} \delta(w - v_0 \cdot (k_1 + k_2))$$

$$\{(k_1 + k_2)^2, v_1 \cdot k_1, v_1 \cdot (k_1 + k_2), v_2 \cdot k_2, v_3 \cdot k_1, v_3 \cdot k_2\}$$

$$\{(k_1 + k_2)^2, v_1 \cdot (k_1 + k_2), v_2 \cdot k_1, v_2 \cdot k_2, v_3 \cdot k_1, v_3 \cdot k_2\}$$

$$\{(k_1 + k_2)^2, v_1 \cdot k_1, v_1 \cdot k_2, v_3 \cdot k_1, v_3 \cdot (k_1 + k_2), v_4 \cdot k_2\}$$

$$\{(k_1 + k_2)^2, v_1 \cdot k_1, v_1 \cdot k_2, v_3 \cdot (k_1 + k_2), v_4 \cdot k_1, v_4 \cdot k_2\}$$

Calculation of soft function

- The strategy of calculating integrals of RR part is the same to RV part

- The integrals are not singular at the boundary, so we can do reduction at boundary

- The contributions of three Wilson-lines can be factorized by using the identity

$$\frac{1}{v_i \cdot k_1} \frac{1}{v_i \cdot (k_1 + k_2)} + \frac{1}{v_i \cdot k_2} \frac{1}{v_i \cdot (k_1 + k_2)} = \frac{1}{v_i \cdot k_1} \frac{1}{v_i \cdot k_2}$$

Renormalization of soft function

Bare soft function is expanded in the coupling constant

$$S_{bare} = S_{bare}^{(0)} + (\frac{Z_\alpha \alpha_s}{4\pi})S_{bare}^{(1)} + (\frac{Z_\alpha \alpha_s}{4\pi})^2 S_{bare}^{(2)} + \cdots$$

$$Z_\alpha \alpha_s \mu^{2\epsilon} = e^{-\epsilon\gamma_E}(4\pi)^\epsilon \alpha_s^{(0)}$$

$$Z_\alpha = 1 - \frac{\beta_0 \alpha_s}{4\pi\epsilon}$$

Define the renormalized soft function and get NNLO renormalized soft function

$$\tilde{s} = Z_s^\dagger \tilde{s}_{bare} Z_s$$

$$\tilde{s}^{(2)}(L, t_1/M^2, \mu) = \tilde{s}_{bare}^{(2)} + Z_s^{\dagger(2)}\tilde{s}^{(0)} + \tilde{s}^{(0)} Z_s^{(2)} +$$

$$Z_s^{\dagger(1)}\tilde{s}^{(1)} + \tilde{s}^{(1)} Z_s^{(1)} + Z_s^{\dagger(1)}\tilde{s}^{(0)} Z_s^{(1)} - \frac{\beta_0}{\epsilon}\tilde{s}_{bare}^{(1)}$$

Renormalization of soft function

The renormalization factor

$$lnZ_s = \frac{\alpha_s}{4\pi}(-\frac{A_0}{2\epsilon^2} + \frac{A_0 L + \gamma_0^s}{2\epsilon})+$$
$$(\frac{\alpha_s}{4\pi})^2(-\frac{3A_0\beta_0}{8\epsilon^3} + \frac{-A_1 - 2\beta_0(A_0 L + \gamma_0^s)}{8\epsilon^2} + \frac{A_1 L + \gamma_1^s}{4\epsilon})+$$

Where $A_0 = C_F\gamma_{cusp}$ for the $q\bar{q}$ channel and $A_0 = C_A\gamma_{cusp}$ for gg channel

The infrared divergence is canceled

Andrea Ferroglia, *et al.* ArXiv:0907.4791

Renormalization of soft function

The property of soft function in $\beta \to 1$

$$\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), m_t, t_1, \mu) = \tilde{S}_{ij}(\sqrt{\hat{s}}(1-z), t_1, \mu) \otimes S_D(m_t(1-z), \mu) \otimes S_D(m_t(1-z), \mu)$$

The property of soft function in $\beta \to 0$

- The singlet part of soft function is identical to the soft function in Drell-Yan and Higgs production process in [2]
- For the octet part, our result is identical to result in [3]

[1]Andrea Ferroglia, *et al.* ArXiv:1205.3662, [2]A.V. Belinsky, ArXiv:9808389, [3]M. Czakon, *et al.* ArXiv:1311.2541

Conclusion

- Here we calculate soft function using IBP and differential equation method
- The IR structure of bare soft function are coincidence with the RG equation
- The soft function combined with known pieces in the factorization theorem can provide higher precision of theoretical prediction