FeynHiggs

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Introduction

Higgs mass calculation

Other observables

Running the code

Conclusions
Purpose of **FeynHiggs**

Calculation of masses, mixings etc. in the MSSM at highest level of accuracy.

- works with real and complex parameters
- written in **Fortran**
- standard tool for masses, couplings and some decays in the LHCHXSWG
- current version: **2.13.0**

**FeynHiggs team**

HB, Sven Heinemeyer, Thomas Hahn, Wolfgang Hollik, Sebastion Paßehr, Heidi Rzehak, Georg Weiglein
Core of FeynHiggs: Higgs mass calculation

- MSSM includes two Higgs doublets

\[ \Phi_1 = \left( v_1 + \frac{1}{\sqrt{2}}(\phi_1 + i\chi_1) \right), \quad \Phi_2 = \left( v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \right) \]

→ five physical Higgs states: \( h, H, A, H^\pm \)

- Higgs potential:

\[ V_H = m_1^2 H_{1i}^* H_{1i} + m_2^2 H_{2i}^* H_{2i} - \epsilon^{ij}(m_{12} H_{1i} H_{2j} + m_{12}^* H_{1i}^* H_{2j}^*) \]

\[ + \frac{1}{8}(g^2 + g'^2)(H_{1i}^* H_{1i} - H_{2i}^* H_{2i})^2 + \frac{1}{2}g^2 |H_{1i}^* H_{2i}|^2 \]

- Minimization of potential → \( m_1^2 \) and \( m_2^2 \) eliminated
- Reexpress \( m_{12}^2 \) through mass of \( A \) boson

→ Higgs sector at tree-level determined by only two variables: \( M_A \) and \( \tan \beta = v_2/v_1 \)

→ Mass of SM-like Higgs can be predicted
▶ tree-level bound on SM-like Higgs boson mass: $M_{h}^{2} \leq M_{Z}^{2}$
▶ at loop level mixing between $h$, $H$ and $A$
▶ loop-corrections can be large (up to 100%)
→ For precision studies higher order corrections are essential!
**Fixed-order calculation**

**Straightforward approach**

Calculate self-energy corrections!

For $M_A^2 \gg M_Z^2$ mixing negligible: Solve $p^2 - \hat{\Sigma}_{hh}(p^2) = 0$!

- Full 1L and partial 2L results included
- Renormalization scheme: OS or DR
- Resummation of bottom Yukawa coupling for large $\tan \beta$

Includes all corrections at given order
- Precise for not too much separated scales
- For high SUSY scale, large logarithms spoil convergence

E.g. $M_h^2 \mathcal{O}(\alpha_t) m_h^2 + 12k \frac{M_t^4}{v^2} (\ln(M_S^2/M_t^2) + ...)$
Code generation

Full 1L and $\mathcal{O}(\alpha_t^2)$ corrections can be generated automatically

- relies on tools FeynArts, FormCalc and TwoCalc
- bash scripts run the tools and output compile-ready Fortran files

Toolchain for $\mathcal{O}(\alpha_t^2)$ corrections: [Hahn & Paßehr]

1. Generate diagrams with FeynArts
2. Prepare for tensor reduction
3. Tensor reduction with TwoCalc and FormCalc
4. Simplify expressions
5. Calculate renormalization constants with FeynArts/FormCalc
6. Combine everything and simplify
7. Generate code
EFT calculation

Alternative approach

If all SUSY particles are heavy, integrate them out!
\[ M_h^2 = 2\lambda(M_t)v^2 \]

State of the art EFT calculation:

- Full LL+NLL resummation
- \( \mathcal{O}(\alpha_s, \alpha_t) \) NNLL resummation
- separate chargino/neutralino threshold

EFT calculation resums large logarithms
→ precise prediction for high scales
→ misses however terms suppressed by SUSY scale \( \propto v/M_S \)
Effect of resummation

**Graph:**
- **2-loop fixed order**
- **+3-loop logs**
- **+4-loop logs**
- **+5-loop logs**
- **+6-loop logs**
- **Logs resummed**

**Axes:**
- **$M_h$ [GeV]**
- **$M_S$ [GeV]**

**Ranges:**
- $M_h$ ranges from 110 to 150 GeV.
- $M_S$ ranges from 5000 to 20000 GeV.
Hybrid approach

Idea

Combine EFT and fixed-order approach to allow for precise prediction for all scales.

\[
\hat{\Sigma}_{hh}(m_h^2) \rightarrow \hat{\Sigma}_{hh}(m_h^2) - [2v^2\lambda(M_t)]_{\log} - [\hat{\Sigma}_{hh}(m_h^2)]_{\log} = \\
= [\hat{\Sigma}_{hh}(m_h^2)]_{\text{nolog}} - [2v^2\lambda(M_t)]_{\log}
\]

- Have to avoid double-counting of 1L and 2L logarithms
- Have to avoid double-counting of non-log terms
- EFT uses DR, fixed-order calculation can be OS
  → parameter conversion needed

Benefits:
→ precise prediction for all scales
Comparison to pure EFT calculation

\[ \frac{X_t^{\text{DR}}}{M_{\text{SUSY}}} = 2 \]

\[ \frac{X_t^{\text{DR}}}{M_{\text{SUSY}}} = 0 \]
Summary of available self-energy corrections

Need to find complex poles ($\mathcal{M}^2 = M^2 - iM\Gamma$) of inverse propagator matrix $\Delta^{-1}$:

\[
\begin{pmatrix}
 p^2 - m_h^2 + \Sigma_{hh} & \Sigma_{hH} & \Sigma_{hA} \\
 \Sigma_{Hh} & p^2 - m_H^2 + \Sigma_{HH} & \Sigma_{HA} \\
 \Sigma_{Ah} & \Sigma_{AH} & p^2 - m_A^2 + \Sigma_{AA}
\end{pmatrix}
\]

and $\Sigma_{H^\pm H^\pm}$, \((\Sigma = \Sigma(p^2))\)

- **●**: full one-loop corrections (all phases, $p^2$ dependence, NMFV)
- **○**: $O(\alpha_s \alpha_t)$ corrections (all phases, $p^2$ dependence), $O(\alpha_t^2)$ corrections (all phases, $p^2 = 0$)
- **□**: $O(\alpha_t \alpha_b, \alpha_b^2)$ corrections (phases interpolated, $p^2 = 0$)
- **△**: resummed logarithms using EFT
Numerical determination of the poles?

For $M_A \gg M_Z$,

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}(m_h^2) - \hat{\Sigma}_{hh}(m_h^2) + \hat{\Sigma}_{hh}^{(2)}(m_h^2)\hat{\Sigma}_{hh}(m_h^2) + \ldots$$

- Non-SM contributions to $\hat{\Sigma}_{hh}^{(1)'}(m_h^2)\hat{\Sigma}_{hh}(m_h^2)$ are cancelled by subloop-renormalization in $\hat{\Sigma}_{hh}^{(2)}(m_h^2) \to \text{vev-CT}$
- Holds generally at 2L (probably also at higher orders)
- But FH includes $\hat{\Sigma}_{hh}^{(2)}$ only for vanishing electroweak couplings $\to$ incomplete cancellation

$\downarrow$

Numerical determination of poles spoils calculation!

$\rightarrow$ Solution easy for $M_A \gg M_Z$, but what to do for $M_A \sim M_Z$?
Procedure for general $M_A$

At 1L level $M_h^2 = m_h^2 - \hat{\Sigma}_{hh}(m_h^2)$ → expand around 1L solution

⇒ determine poles of

$$\Delta_{hh}(p^2) = p^2 - m_h^2 + \hat{\Sigma}_{hh}(m_h^2) + \hat{\Sigma}_{hh}(0) - \left[ \hat{\Sigma}_{hh}'(m_h^2) \hat{\Sigma}_{hh}(m_h^2) \right]_{g=g_Y=0}$$

$$\Delta_{hH}(p^2) = + \hat{\Sigma}_{hH}(m_h^2) + \hat{\Sigma}_{hH}(0) - \left[ \hat{\Sigma}_{hH}'(m_h^2) \hat{\Sigma}_{hh}(m_h^2) \right]_{g=g_Y=0}$$

$$\Delta_{HH}(p^2) = p^2 - m_H^2 + \hat{\Sigma}_{HH}(m_h^2) + \hat{\Sigma}_{HH}(0) - \left[ \hat{\Sigma}_{HH}'(m_h^2) \hat{\Sigma}_{hh}(m_h^2) \right]_{g=g_Y=0}$$

For determination of $M_H$ expand around $M_H^2 = m_H^2 - \hat{\Sigma}_{HH}(m_H^2)$

→ will be available in the next release (FH2.14.0)
Numerical impact of improved pole determination

\[ \frac{X_t^{\text{DR}}}{M_{\text{SUSY}}} = 2 \]

\[ \frac{X_t^{\text{DR}}}{M_{\text{SUSY}}} = 0 \]
Observables

- Higgs masses: $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}$
- $Z_{ij}$-factors for calculating processes involving external Higgs bosons
- effective mixing angle $\alpha_{\text{eff}}$

For all observables theory uncertainty is estimated by

- change of renormalization scheme
- scale variation
- switching off the resummation of the bottom Yukawa coupling
Neutral Higgs decays

- total decay width: $\Gamma_{h_i}^{\text{tot}}$, $\Gamma_{H^+}^{\text{tot}}$
- Branching ratios of $h_i$
  - SM fermions: $h_i \rightarrow f \bar{f}$
  - gauge bosons: $h_i \rightarrow \gamma\gamma, Z(\ast)Z(\ast), W(\ast)W(\ast), gg$
  - gauge and Higgs boson: $h_i \rightarrow Z(\ast)h_j$
  - two Higgs bosons: $h_i \rightarrow h_jh_k$
  - sfermions: $h_i \rightarrow \tilde{f}_i\tilde{f}_j$
  - charginos/neutralinos: $h_i \rightarrow \tilde{\chi}_j^\pm \tilde{\chi}_k^\mp, \tilde{\chi}_j^0\tilde{\chi}_k^0$
- For comparison also SM branching ratios are calculated
- Branching ratios of $H^+$
  - SM fermions: $H^+ \rightarrow f(\ast)\bar{f}'$
  - gauge and Higgs boson: $H^+ \rightarrow W^+(\ast)h_i$
  - sfermions: $h_i \rightarrow \tilde{f}_i\tilde{f}_j'$
  - chargino and neutralino: $h_i \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$
Higgs production

Available cross-sections:

- $\bar{b}b, gg \rightarrow h + X$
- $\bar{q}q \rightarrow \bar{q}qh + X$
- $\bar{q}q, gg \rightarrow \bar{t}th + X$
- $\bar{q}q \rightarrow Wh + X$
- $\bar{q}q \rightarrow Zh + X$
- $pp \rightarrow \tilde{t}_1\tilde{t}_1 h + X$
- $gb \rightarrow tH^- + X$
- $t \rightarrow H^+\bar{b}$ for $M_{H^\pm} \leq M_t$
- For comparison also SM cross-sections are calculated.
EWPO and flavour observables

Electroweak precision observables:

- $W$ boson mass $M_W$
- Effective weak mixing angle $\sin\theta_{\text{eff}}$
- $\Delta r$, $\Delta \rho$
- Anomalous magnetic moment of the muon $g_\mu - 2$
- Electric dipole moments of the electron, neutron and mercury

Flavour observables:

- $b \rightarrow s\gamma$
- $\Delta M_s$
- $B_s \rightarrow \mu^+\mu^-$
Getting and running the code

- Download latest version at feynhiggs.de
- Install via: “./configure, make, make install”
- 4 ways to run the code
  - Command line
  - Call from Fortran/C++ code
  - Mathematica interface
  - Web interface feynhiggs.de/fhucc
Command line I

Inputfile:

MT 173.32
MSusy 2000
MA0 400
TB 10
Abs(At) 500
Arg(At) 0.8

Screen output:

- HIGGS MASSES -
  | Mh0 = 121.50245316
  | MHH = 1002.92887238
  | MA0 = 1000.00000000
  | MHp = 1005.78424242

- ESTIMATED UNC. -
  | DeltaMh0 = 1.06229069
  | DeltaMHH = 0.40235972
  | DeltaMA0 = 0.00000000
  | DeltaMHp = 0.24339873

▶ Possible to define loops over parameters
▶ Possible to use interpolation tables
▶ Alternatively use SLHA files as input
Command line II

Example bash script

```bash
#!/bin/sh

FeynHiggs - ${4:-4002423110} «- _EOF_ > FH.out
MT 173.32
MSusy $2
Xt $3
TB 10
MA0 1000
MUE 1000
M_2 1000
M_3 1000
_EOF_

cat FH.out | table $1 FH.out 2>/dev/null
```
Call from Fortran/C++ code

- Link static Fortran library FHlib.a
- For C/C++ prototype file available: CFeynHiggs.h
- Most important routines
  - input
    - FHSetFlags  - options (accuracy, approximations, ...)
    - FHSetPara   - MSSM input parameters
  - output
    - FHHiggsCorr - Higgs masses and mixings
    - FHUncertainties - theory uncertainty estimate for Higgs masses and mixings
    - FHCouplings - Higgs couplings and BRs
    - FHHiggsProd - Higgs production cross-sections
    - FHEWPO - electroweak precision observables
    - FHFlavour - flavour constraints
    - FHConstraints - additional constraints
Call from Mathematica

- make all to generate MathLink executable
- uses it via:
  - input:
    - `Install["MFeynHiggs"];
    - `FHSetFlags[...];`
    - `FHSetPara[...];`
    - `FHHiggsCorr[]`
  - output
    - `{MHiggs -> {124.495, 999.552, 1000., 1003.24},
      SAeff -> -0.101772,
      UHiggs -> ..., ZHiggs -> ...}`
- allows to use Mathematica functions (ContourPlot, ...)
**The FeynHiggs User Control Center (2.13.0)**

You can still access older versions: [FH2.10.2] [FH2.9.5] [FH2.8.6] [FH2.7.4] [FH2.5.1] [FH2.3.2]

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### Flags

<table>
<thead>
<tr>
<th>Flag Description</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scope of the 1-loop part</td>
<td>full MSSM</td>
</tr>
<tr>
<td>1-loop field renormalization</td>
<td>DRbar</td>
</tr>
<tr>
<td>1-loop tan(beta) renormalization</td>
<td>DRbar</td>
</tr>
<tr>
<td>Mixing in the neutral Higgs sector</td>
<td>$2 \times 2$ (h0-HH) mixing = real parameters</td>
</tr>
<tr>
<td>Approximation for the 1-loop result</td>
<td>no approx., UHiggs evaluated at $p^2 = 0$</td>
</tr>
<tr>
<td>Higher-order corrections</td>
<td>2-loop corrections</td>
</tr>
<tr>
<td>Inclusion of log resummation</td>
<td>no log resummation</td>
</tr>
<tr>
<td>$m_t$ in the 1/2-loop corrections</td>
<td>running top mass (SM MSbar 2L)</td>
</tr>
<tr>
<td>$m_b$ in the 1/2-loop corrections</td>
<td>resummed MB</td>
</tr>
<tr>
<td>Complex parameters in the 2-loop corrections</td>
<td>all corrections evaluated in the rMSSM</td>
</tr>
<tr>
<td>$p^2$-dependent terms in the 2-loop corrections</td>
<td>not included</td>
</tr>
</tbody>
</table>

In a hurry? [Go straight to the results]

Or, choose a predefined scenario: [mh_max] [no-mixing] [small alpha_eff] [gluophobic Higgs] [CPX]
Summary:

- **FeynHiggs** calculates Higgs masses, mixings, decays, production cross-sections etc. in the MSSM
- Includes combined state-of-the-art fixed-order and EFT calculations
- Allows for precise predictions for low and high SUSY scales

Near future outlook (version 2.14.0):

- Improved pole mass determination
- Optional $\overline{\text{DR}}$ renormalization of stop sector

To come later...

- Complete revamp of 2L corrections
- Improved resummation for low $M_A$ (eff. THDM)
- NMSSM extension