A Markov chain Monte Carlo approach to the relativistic inverse stellar problem

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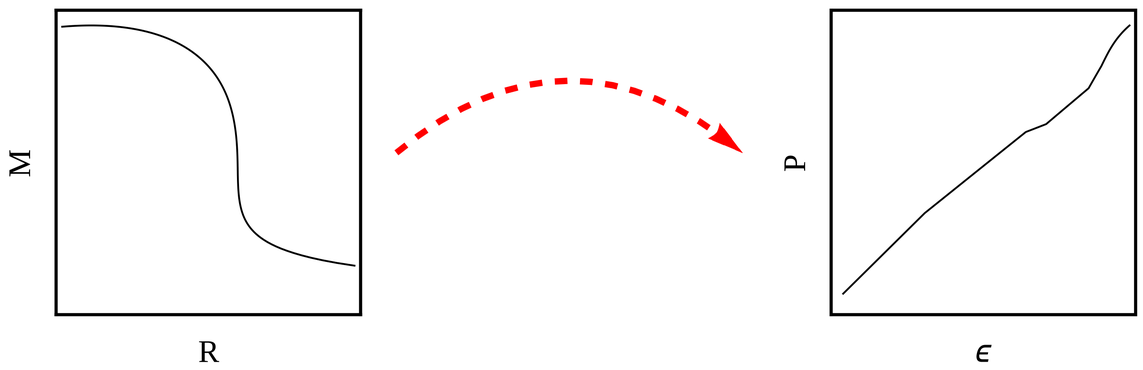
In this work we propose a method to determine the neutron star equation of state features using gravitational wave signals produced by coalescing binary systems. We parametrize the equation of state using phenomenological piecewise polytropic models. Adopting a Bayesian scheme of inference, we determine, through Markov chain Monte Carlo simulations, the posterior probability distribution of the equation of state parameters, for a given set of neutron star masses and tidal deformabilities, within their experimental uncertainties measured by gravitational wave detections.

Introduction

The lack of information on the nature of the neutron star core at supra-nuclear densities has so far prevented a unique description of its equation of state (EoS). Future gravitational wave detections by second and third-generation of ground-based interferometers will shed new light on this open problem.

The inverse stellar problem

The complete knowledge of the neutron star EoS makes possible the theoretical calculation of macroscopic quantities such as mass, radius, etc. Viceversa, from the measurement of macroscopic observables, it's possible to invert this map and reconstruct the EoS: this is the so-called *inverse stellar structure problem* [1, 2].



Tidal Love numbers

A static, spherically symmetric star perturbed by an external stationary tidal field develops a multipolar response. The main contribution is the quadrupolar one [3]

$$Q_{ij} = \lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3} k_2 R^5$$

$$\mathcal{E}_{ij} \equiv R_{i0j0} \xrightarrow[\text{limit}]{\text{Newtonian}} \frac{\partial^2 \phi}{\partial x^i \partial x^j}$$

Q_{ij}: mass quadrupole moment

$\mathcal{E}_{ij}$: quadrupolar tidal field

λ: tidal deformability

k_2: Love number

The tidal deformability encodes the properties of the neutron star internal structure, i.e. it depends on the equation of state.

Tidal effects enter in the waveform of gravitational signals emitted by a coalescing compact binary only by means of tidal deformabilities. They introduce 5PN order corrections to the phase of the gravitational waveform [4]

$$h(f) = A(f) e^{i[\psi_{PP}(f) + \psi_T(f)]} \qquad \psi_T = -\psi_N \frac{117 v^{10}}{8 \eta M^5} \Lambda$$

$$\Lambda = \frac{1}{26} \left(\frac{m_1 + 12 m_2}{m_1} \lambda_1 + \frac{m_2 + 12 m_1}{m_2} \lambda_2 \right)$$

Equation of state

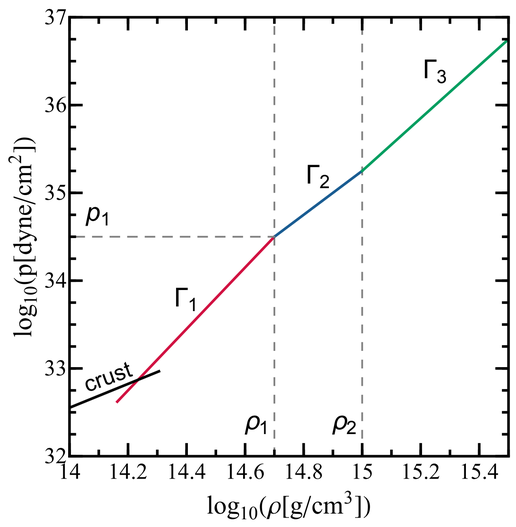
We parametrize the EoS using the phenomenological piecewise polytropic representation developed by Read et al. [5]. This model fits a large class of realistic EoS, including pure nucleonic matter, hyperons, condensates and deconfined quarks.

The high density core is represented by three polytropic segments

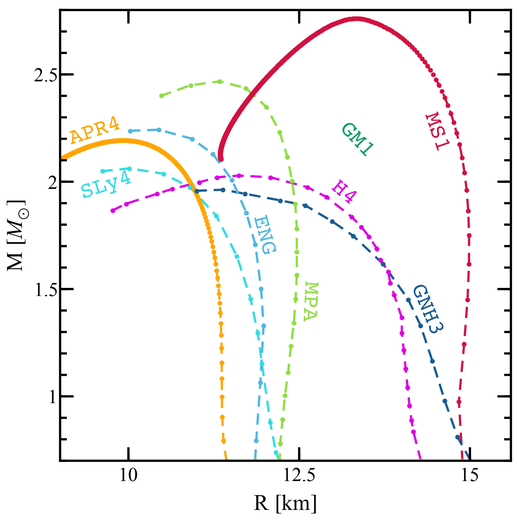
$$p(\rho) = K_i \rho^{\Gamma_i} \qquad \rho_{i-1} \leq \rho \leq \rho_i$$

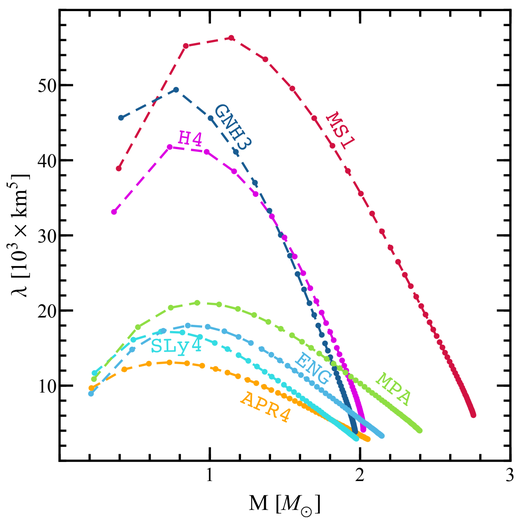
specified by the dividing rest-mass densities $\rho_1 = 10^{14.7}$ g/cm^3 and $\rho_2 = 10^{15}$ g/cm^3, with adiabatic constant and index given by K_i and Γ_i, respectively.

At low densities, the last interface is matched dynamically to a fixed crust, which is chosen to be a parametrized four-piece polytropic version of the SLy EoS.



The EoS representation is completely specified by four independent parameters, namely $\{\mathbf{p_1}, \mathbf{\Gamma_1}, \mathbf{\Gamma_2}, \mathbf{\Gamma_3}\}$, where $p_1 \equiv p(\rho_1)$.





Markov chain Monte Carlo simulation

We estimate the EoS parameters starting from the neutron star masses and tidal deformabilities provided by gravitational wave detections. We determine the posterior probability distribution of the parameters $\vec{\theta}$, given the experimental data $\vec{d}$, using Bayes theorem

$$\mathcal{P}(\vec{\theta}|\vec{d}) \propto \mathcal{P}_0(\vec{\theta})\, \mathcal{L}(\vec{d}|\vec{\theta})$$

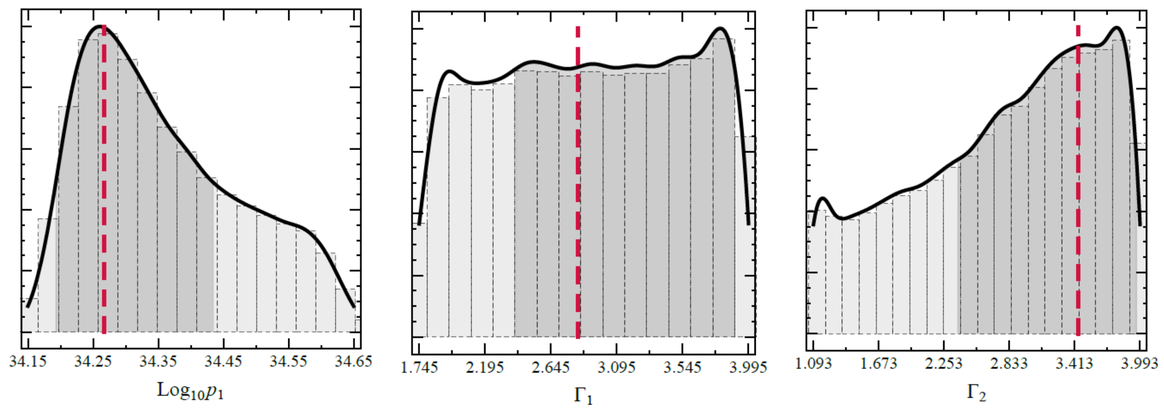
where $\mathcal{P}_0(\vec{\theta})$ describes the prior information on the parameters, and $\mathcal{L}(\vec{d}|\vec{\theta})$ is the likelihood function. Under the assumption that the experimental data are independent and Gaussian distributed, the likelihood can be written as $\mathcal{L} \propto e^{-\chi^2}$, where the chi-square variable reads

$$\chi^2 = \frac{1}{2} \sum_{i=1}^{N} \left\{ \frac{\left[M^{EoS}(p_1, \Gamma_1, \Gamma_2, \Gamma_3) - M_i^{obs} \right]^2}{\sigma_{M_i}^2} + \frac{\left[\lambda^{EoS}(p_1, \Gamma_1, \Gamma_2, \Gamma_3) - \lambda_i^{obs} \right]^2}{\sigma_{\lambda_i}^2} \right\}$$

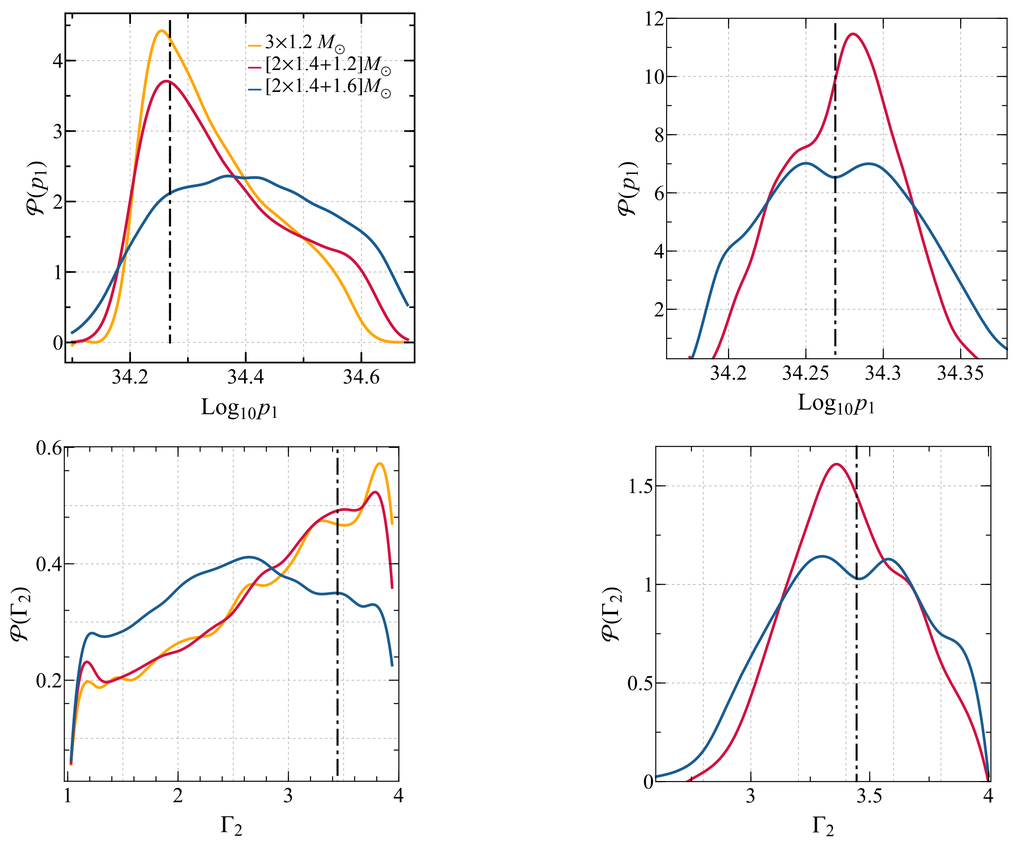
We sample the posterior probability distribution using Markov chain Monte Carlo simulations based on the Metropolis-Hastings algorithm. We adopt an adaptive framework, in which the proposal function is continuously updated through a Gaussian adaptation algorithm (GaA) [6].

Preliminary results

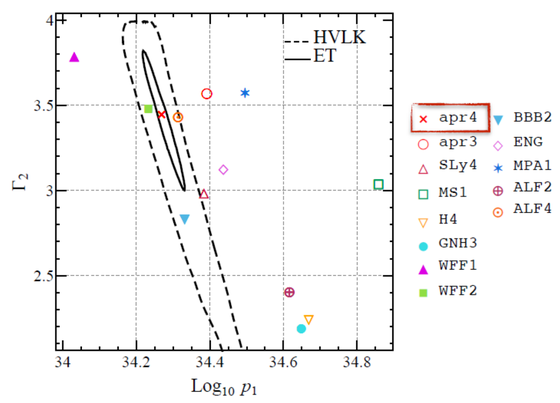
EoS: APR4 $\mathbf{p_1 = 34.269}$, $\mathbf{\Gamma_1 = 2.830}$, $\mathbf{\Gamma_2 = 3.445}$, $\mathbf{\Gamma_3 = 3.348}$



Marginalized probability distributions of the EoS parameters reconstructed from the detection of three neutron stars, with masses $(1.4, 1.4, 1.2) M_\odot$*, by a network of four second-generation interferometers (HLVK: 2 LIGO + Virgo + KAGRA).*



Mass dependence of the posterior distributions: HLVK (left) and the Einstein Telescope (right).



Discriminating the EoSs: 1σ *confidence intervals.*

References

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