Quasi-radial instability of differentially rotating relativistic stars

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Abstract

The stability against gravitational collapse of the remnant left by a merger of binary neutron stars is of great interest in gravitational-wave astronomy. This property can be explored with simulations in full general relativity, which are often computational extremely demanding. A well-established result in this landscape is that the rotation of the remnant is a crucial factor in determining its stability. In the case of uniform rotation, the turning-point method by Friedman, Ipser and Sorkin (1988) provides a shortcut to infer physical properties regarding the stability of neutron stars in a more computationally affordable way. This method is based on the study of the *turning points*, which are particular equilibrium models that satisfy a specific condition and that can be found without performing full simulations. The turning-point method detects the onset of secular instability, which is, in general, close to the dynamical instability to collapse. Here, we applied the turning-point method to differentially rotating neutron stars, obtaining an estimate of the location of the instability region in the parameter space for different equations of state and rotation laws. To validate this approach we performed three-dimensional simulations of select models to locate the onset of the actual dynamical instability. Despite finding that more refined methods are required to obtain an accurate estimation, we claim that the turning points are reliable indicators of the dynamical instability region. Furthermore, we found universal relations among some of the physical properties of interest along the sequence of turning-point models.

Introduction

The outcome of a binary neutron star merger is the formation of a hot, massive and rapidly spinning compact object. The lifespan of this remnant not only greatly affects the characteristics and the strength of both the gravitational and the electromagnetic signals, but also influences phenomena like gamma-ray-bursts or heavy-elements nucleosynthesis. Therefore, **the stability of the remnant against gravitational collapse is a crucial feature in the understanding of binary neutron star mergers**.

It is a well-established result that the lifespan of the remnant is mainly determined by its rotational properties. A key-tool to study *uniformly* rotating neutron stars is the **turning-point method** by Friedman, Ipser and Sorkin (1988). In this case, the turning-point method provides a convenient way to locate the secular-instability region: the onset can be found by constructing sequences of equilibrium models with constant rest mass M_0 or angular momentum J and by finding their stationary points, which the *turning points*. The interpolation of these points is known as *turning-point* line. Furthermore, dynamical instability is usually close to secular one, thus the turning-point line effectively separates stable models from collapsing ones. However, binary neutron star merger remnants are differentially rotating, and no investigations on the turning-point method in this latter case are available in literature. Notice that the turning point theorem does not strictly apply to differentially rotating stars, so one can only obtain indications, but not proof, for instability.

For a more extensive discussion about equilibrium models and instabilities see Stergioulas and Friedman (2003).

Main Goal

The main objective of our research is to **study the turning-point lines for *differentially* rotating stars and estimate the location of the instability region in the parameter space.**

Method and Tools

Methodology

We construct cold, axisymmetric, equilibrium sequences of models with constant rest mass (M_0) or angular momentum (J) and varying central energy density (ϵ_c). The two turning-point lines, one from J-constant and one from M_0-constant sequences, are found with an accuracy (in the central energy) of $\approx 0.5\,\%$ by locating the stationary point of the sequences. In the case of uniform rotation, the turning-point criterion states that the two lines must coincide and that they separate secularly stable models from unstable ones. In the case of differential rotation the two lines do not overlap anymore, but their location can be still used as a guess about where the instability region is. This estimation depends on the gap between the J-constant turning-point line and the M_0-constant one: the smaller it is, the more accurate the guess should be.

work with three different rotation laws and sixteen equations of state (fourteen tabulated and two polytropic). After finding all the turning-point lines we focus on the polytropic model where the two lines spread the most, which, being the worst-case-scenario, is used to validate the method.

Computational Tools

All our work is in full general relativity and we use the following codes:

- `RNS` to generate equilibrium models and prepare the initial data for the dynamical simulations
- `Einstein Toolkit` to perform the simulations (up to $\approx 6\,\mathrm{ms}$)
- `Carpet` to set up the computational grid (with fixed mesh refinement and resolution of the innermost grid $\Delta x = 0.0625\,M_\odot$)
- `GRHydro` to evolve the hydrodynamics (with the HLLE Riemann solver, WENO5 reconstruction method and the RK4 integrator)
- `AHFinderDirect` to find the black hole that results from a collapse

The verification of our numerical setup is performed by recovering results in the well-known uniformly rotating case.

Results

Universal Relations

We find that the turning-point models satisfy some universal relations which are independent on the degree of differential rotation and the equation of state. In particular, there are relations with an accuracy of $2\,\%$ between *rest mass, gravitational mass* and *angular momentum*, expressed in suitable units defined by the non-rotating turning point.

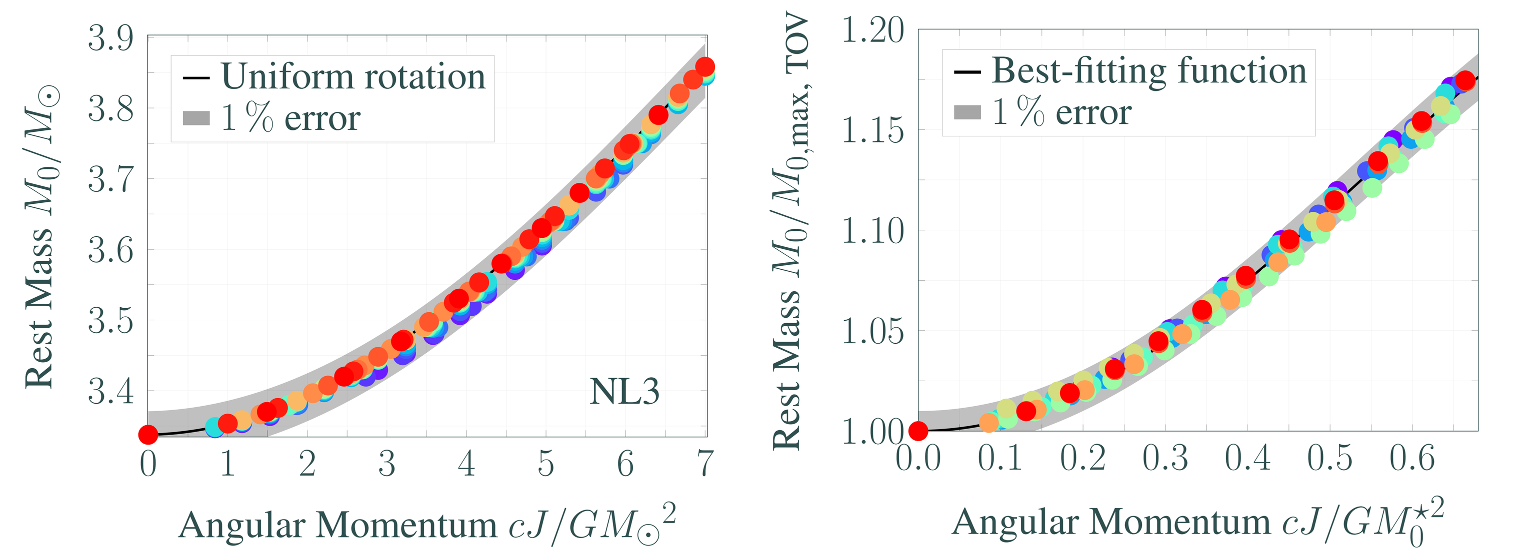

Figure 1: Left: Relation independent of the degree of differential rotation between rest mass and angular momentum for the equation of state NL3 (different colors are different degrees of differential rotation). The uniformly rotating sequence approximates every other sequence with accuracy of $1\,\%$. **Right:** EOS-independent relation between rest mass and angular momentum (scaled by the non-rotating turning-point rest mass). Each color is a different equation of state, the black line is the best-fitting approximation, which has accuracy of $1\,\%$.

The relations hold for:

- Tabulated cold equations of state and polytropes with index $n = 1.0$ or $n = 0.5$
- Uniform, one-parameter and three-parameters (piecewise extension of the one-parameter law) rotation laws
- Turning points with gravitational mass smaller than the maximum allowed by uniform rotation

Strange stars (with usual MIT-bag model) **do not** satisfy one of aforementioned laws.

Instability Line

We focus on the case where the J-constant turning-point line and the M_0-constant one have the widest gap, which is $6\,\%$ in energy instability line.

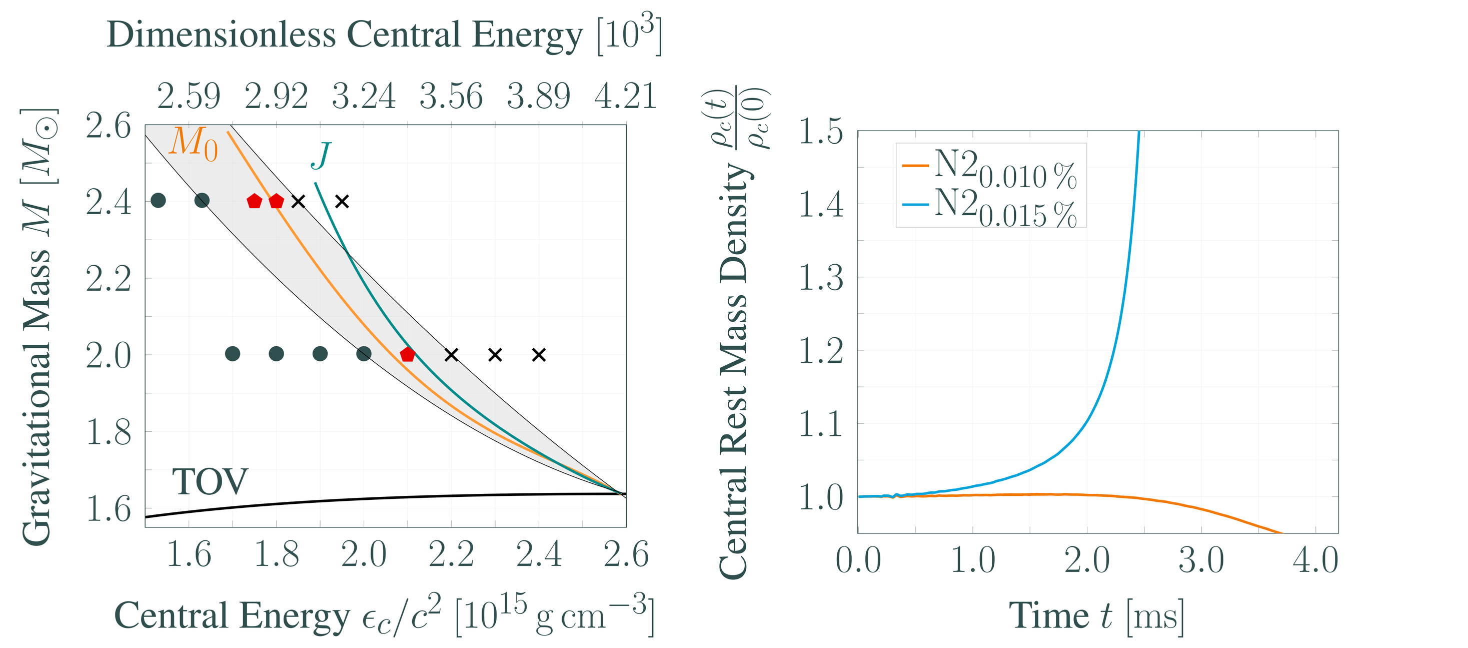

Figure 2: Left: Time evolutions of models around the turning-point lines with the widest gap are performed with the Einstein Toolkit to find the actual dynamical instability line. Circles are stable models and crosses unstable ones when the perturbation amplitude is $0.1\,\%$ of the central rest-mass density. Red pentagons are models that cannot be simulated correctly with our setup. **Right:** Time evolution of the central rest-mass density of the same model, but with different perturbation amplitude. Changing this parameter dramatically changes the outcome of the simulation near the instability line.

Near the turning-point line the truncation error becomes important to control because it might artificially stabilize models. We notice that with the methods used in this work it is important to control the amplitude and sign of the truncation errors, in order to arrive at an accurate determination of the instability line. Tentatively, we find that the dynamical instability line is in the same region as the two turning-point lines, with an error of less than $12\,\%$, which can be further reduced by additional high-resolution simulations.

Conclusions

- The turning-point line for J-constant and M_0-constant sequences are always close (the widest gap is $6\,\%$ in energy density)
- Turning-point models satisfy relations that are not dependent on the equation of state nor the degree of differential rotation with an accuracy of $2\,\%$
- The turning-point line is a reliable indicator of dynamical instability with error below $12\,\%$, and additional high-resolution simulation, providing more control on the amplitude of the truncation error, could further reduce this error

Forthcoming Research

Future works will focus on performing more accurate simulations (possibly exploiting the axisymmetry of the system) and using different methods to have full control over the truncation error.

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