



# Mass and radius constraints for neutron stars from pulse shape modeling

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## Abstract

We present a framework that can be used to constrain masses and radii of neutron stars. The method is suitable for accreting millisecond pulsars, where a rapidly rotating neutron star accretes matter from a relatively low mass companion star onto the magnetic poles of the neutron star. We model the exact shape of the resulting pulses using Schwarzschild-Doppler approximation, which takes the general and special relativistic effects into account. We consider also the geometrical effects due to the oblate shape of the neutron star. By using Bayesian analysis and a Monte Carlo sampling method, called ensemble sampler, we obtain probability distributions for the different parameters of our model, especially for the mass and the radius. To test the robustness of our method, we have generated synthetic data and fitted the resulting pulse profiles. In the same way, simulations are currently being performed also for the real observations of SAX J1808.4 - 3658.



Figure 1: Geometry of the rotating pulsar with one hot spot. The angles shown are observer inclination *i*, spot colatitude  $\theta$ , and phase angle of the rotation  $\phi$ . Our pulse profiles are computed taking into account the special relativistic effects (Doppler boost, relativistic aberration) as well as general relativistic effects, such as gravitational redshift and light bending in the Schwarzschild geometry (see e.g., Poutanen & Beloborodov (2006) and Poutanen & Gierliński (2003)). The geometrical effects of the oblate shape of the star, due to the fast rotation, are also taken into account (see e.g., Morsink et al. (2007) and AlGendy & Morsink (2014)).





Figure 2: We approximate the spectrum with an empirical Comptonization model called SIMPL (Steiner et al. 2009). In this model a fraction of photons in a seed blackbody spectrum (red curve) is scattered into a power-law component (blue curve). Some photons remain thermal (orange curve). Figure shows the corresponding spectra with temperature of the seed spectrum T = 2.0 keV, scattered fraction of photons  $X_{sc} = 0.8$ , and photon spectral index of the Comptonized component  $\Gamma = 1.8$ .





Figure 4: Angular distribution of radiation (beaming pattern)  $I(\alpha)$ , where  $\alpha$  is the emission angle relative to the surface normal. Distributions are shown with different linear beaming parameters  $a_{bb}$ , defined as  $I(\alpha) = I_0(1 + a_{bb} \cos \alpha)$ (normalized so that  $2 \int_0^1 \mu I(\mu) d\mu = 1$ , where  $\mu = \cos \alpha$ ). When fitting pulse profiles, we use  $a_{bb} = 0$  (isotropic intensity) for blackbody component of the spectrum and varying  $a_{bb}$  for Comptonized component of the spectrum (see Figure 2 for the energy-dependency of intensity).

## Introduction

The aim of this work is to constrain masses and radii for neutron stars by fitting waveform models to the X-ray oscillations of accretion-powered millisecond pulsars (AMP). These oscillations are produced when the neutron star is accreting matter, via an accretion disc, from a non-collapsed low-mass companion star in a binary system. Gas from the disc is channeled onto the magnetic poles of the rapidly rotating neutron star due to the strong magnetic field. The result is a pair of "hot spots" on the pulsar surface. This gives rise to the X-ray pulsations with typical periods of a few milliseconds corresponding to the spin frequency of the neutron star. The pulses carry information about the mass and radius of a neutron star since e.g., the light bending and thus the pulse shape strongly depends on the compactness of the star.

### In Figure 3, we see that our simulation is able to reproduce the original synthetic pulse profile closely. In Figure 5, are presented the posterior probability distributions using synthetic data for equatorial radius $R_{eq}$ , mass M, observer inclination *i*, spot colatitude $\theta$ , spot angular radius $\rho$ , distance *D*, linear beaming factor $a_{bb}$ , photon spectral index $\Gamma$ , scattered fraction of photons from blackbody to Compton spectrum $X_{sc}$ , temperature of the seed spectrum of the spot T, intrinsic scatter of the model $\sigma_i$ , and neutral hydrogen column density for interstellar absorption $N_{\rm H}$ . To improve the performance of our sampling, we have used $i + \theta$ , $i - \theta$ , and $M/R_{eq}$ as our parameters instead of i, $\theta$ , and $R_{eq}$ . As expected, we find degeneracies between some of the parameters and bimodality in i and $\theta$ . However, the results show that we are able to get meaningful constraints for the mass and radius, as well as for the binary system parameters like *i*, and for the radiative parameters like beaming factor $a_{bb}$ .

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## Methods

We trace the photons from the hot spot to the observer by using "oblate Schwarzschild-Doppler" approximation (see Figure 1). In addition to the shape of the light curve, we also model both the energy and angular distribution of the radiation (see Figures 2 and 4). We then apply Bayesian analysis and ensemble sampling method (Goodman & Weare 2010) to obtain probability distributions for all parameters in our model. As a first step, we have created synthetic data resembling the observations of SAX J1808.4 - 3658 observed with *RXTE* satellite. Simulations are currently being performed also to the real observations. Previously, Bayesian analysis for synthetic data on X-ray burst oscillations has been applied by e.g., Lo et al. (2013) and Miller & Lamb (2015), but with less physical flexibility in the model.

#### Results 3

Figure 5: The dark orange color shows a 68% and the light orange color a 95% highest posterior density credible interval. In the 2D posterior distributions the solid contour shows a 95% and the dashed contour a 68% highest posterior density credible region. The blue crosses show the correct solution.

## References

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