# Scalarization of neutron stars with realistic equations of state

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Zahra Altaha Motahar, Jose Luis Blázquez-Salcedo, Burkhard Kleihaus, Jutta Kunz

CARL VON OSSIETZKY universität OLDENBURG

Research Training Group Graduiertenkolleg Models of Gravity

## The model

### Scalar-tensor theory (STT)

**Action in the physical Jordan frame**

$$S=\frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R}-Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi-2U(\Phi)\right]+ +S_m[\Psi_m;\tilde{g}_{\mu\nu}]$$

$$g_{\mu\nu}=F(\Phi)\tilde{g}_{\mu\nu}$$

$$2F(\Phi)Z(\Phi)+3\left(\frac{d}{d\Phi}F(\Phi)\right)^2\geq 0$$

$$\left(\frac{d\varphi}{d\Phi}\right)^2=\frac{3}{4}\left(\frac{d\ln(F(\Phi))}{d\Phi}\right)^2+\frac{Z(\Phi)}{2F(\Phi)}$$

$$A(\varphi)=F^{-1/2}(\Phi),\ 2V(\varphi)=U(\Phi)F^{-2}(\Phi)$$

**Action in the Einstein frame**

$$S=\frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left[\mathcal{R}-2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-4V(\varphi)\right] +S_m[\Psi_m;A^2(\varphi)g_{\mu\nu}]$$

**The field equations in the Einstein frame**

$$\mathcal{R}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\mathcal{R}=+2\partial_\mu\varphi\partial_\nu\varphi-g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi+8\pi T_{\mu\nu}$$

$$\nabla^\mu\nabla_\mu\varphi=-4\pi k(\varphi)T$$

$$T=T^\mu_\mu,\ k(\varphi)=\frac{d\ln(A(\varphi))}{d\varphi}$$

$$\tilde{T}_{\mu\nu}=(\tilde{\varepsilon}+\tilde{p})\tilde{u}_\mu\tilde{u}_\nu+\tilde{p}\tilde{g}_{\mu\nu}$$

$$T_{\mu\nu}\equiv A^2\tilde{T}_{\mu\nu}$$

**The coupling functions**

$$A_1(\varphi)=e^{\frac{1}{2}\beta\varphi^2},\ k_1(\varphi)=\beta\varphi$$

$$A_2(\varphi)=\frac{1}{\cosh(\sqrt{-\beta}\varphi)},\ k_2(\varphi)=-\sqrt{-\beta}\tanh(\sqrt{-\beta}\varphi)$$

### Slowly rotating neutron stars in STT

**Metric corresponding to the slowly rotating star**

$$ds^2=-e^{f(r)}dt^2+\frac{1}{n(r)}dr^2+r^2d\theta^2+r^2\sin^2\theta(d\phi+x\omega(r)dt)^2$$

### Equation of State

In order to integrate the system we have to provide an equation of state in the form $\tilde{\varepsilon}=\tilde{\varepsilon}(\tilde{p})$.

- Two EOS containing just nuclear matter: SLy and APR4.
- Five EOS containing nucleons+hyperons: BHZBM, GNH3, H4 and WCS1-2.
- Two EOS for pure quark matter: WSPHS1 and 2.
- Four EOS containing hybrid quark+nucleons: ALF2-4, BS4 and WSPHS3.
- Polytropic EOS: $\tilde{\varepsilon}=K\frac{\tilde{\rho}^\Gamma}{\Gamma-1}+\tilde{\rho}$, $\tilde{p}=K\tilde{\rho}^\Gamma$, with $\tilde{\rho}$ being the baryonic mass density. In particular, we have chosen for the polytropic constant $K=1186.0$, and for the adiabatic index $\Gamma=1+\frac{1}{N}$ the polytropic index $N=0.7463$.

### Numerical Method

Using a differential equation solver package. COLSYS. the EOS are implemented using different methods.

- The case of the relativistic polytrope is the simplest one since the relation $\tilde{\varepsilon}=\tilde{\varepsilon}(\tilde{p})$ is known analytically.
- For WCS1-2, WSPHS1-3, BS4 and BHZBM EOS: Tabulated EOS where we use a piecewise monotone cubic Hermite interpolation of the data points.
- For SLy, APR4, GNH3, H4 and ALF2-4 EOS: The piecewise polytropic interpolation is implemented in the code.

## Abstract

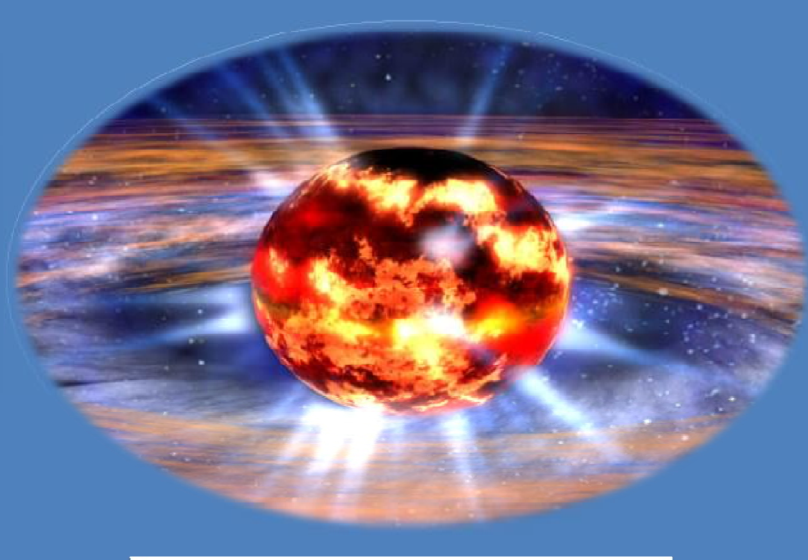
Neutron stars are some of the most fascinating objects in the universe. Due to their compactness and high density, they represent an ideal laboratory to test alternative theories of gravity. In addition, studying these compact objects will expand our limited knowledge of the properties and the physics of nuclear matter at the very high density found inside neutron stars.

We demonstrate the effect of scalarization on static and slowly rotating neutron stars in scalar-tensor theories of gravity, implementing various realistic Equations Of State (EOSs). Beside a polytropic EOS and some EOSs for pure nuclear matter and pure quark matter, we include several EOSs describing nuclear matter with hyperons and hybrid matter for the first time in this context.

We investigate the onset of scalarization for these different EOSs, presenting a universal (independent of the EOS) relation for the critical coupling parameter versus the compactness. We then recognize that the most significant universal feature of value the onset and the magnitude of the scalarization is the correlation with the of the the gravitational potential at the center of the star. We also analyze the moment-of-inertia--compactness relations and confirm universality for the nuclear matter, hyperon and hybrid equations of state.

NEUTRON STARS

TESTING GRAVITY



A neutron star (Art by NASA/Dana Berry)

## Conclusions

- Extending earlier investigations of scalarization for neutron star models with realistic EOSs by considering also the classes of hyperon and hybrid stars.
- Restricting to static and slowly rotating models, we have confirmed and extended the results on the universal *I-C* relations.
- The most striking universal feature found relates the gravitational potential at the center of the star, as embodied in $g_{tt}(0)$, to the properties of the scalar field.
- Considering the effect of scalarization with an alternative coupling function $A_2$ based on the hyperbolic cosine. Clearly, the onset of the scalarization is only determined by β, while the magnitude of scalarization is also governed by the coupling function, leading to less scalarization for this new coupling function.

### References:


- K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016), 1602.00504
- .S. S. Yazadjiev, D. D. Doneva, K. D. Kokkotas, and K. V. Staykov, JCAP 1406, 003 (2014).
- D. D. Doneva, S. S. Yazadjiev, and K. D. Kokkotas, Phys. Rev. D 92, 064015 (2015), 1507.00378.
- T. Damour and G. Esposito-Farese, Physical Review Letters 70, 2220 (1993).
- T. Damour and G. Esposito-Farese, ` Phys. Rev. D 54, 1474 (1996).
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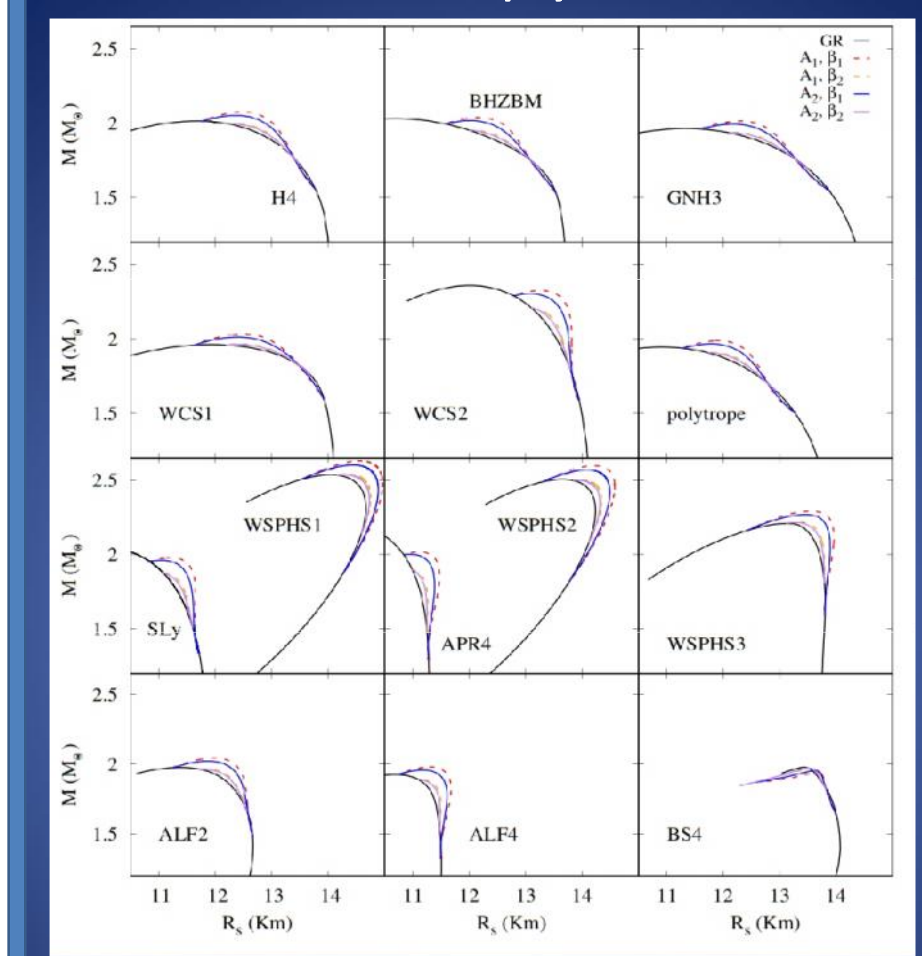
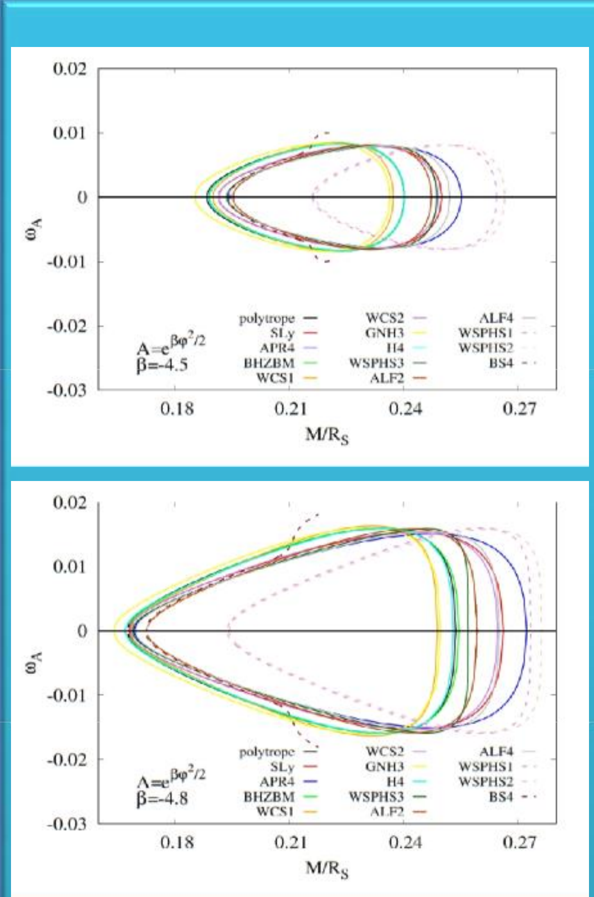
### Total mass versus physical radius

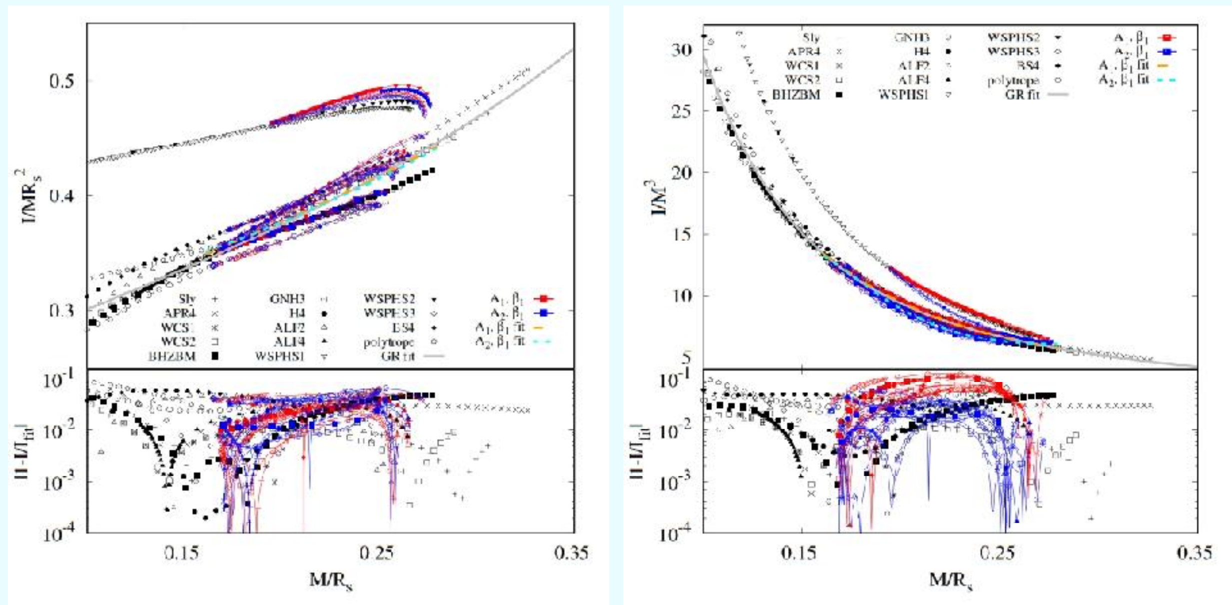



Total mass **M** (in solar masses $M_\odot$) versus the physical radius $R_s$ (in km) of the neutron star models for all EOSs considered: The first two rows show the 5 hyperon EOSs (H4, BHZBM, GNH3, WCS1, WCS2) and the polytropic EOS, the last two rows contain the 2 nuclear EOSs (SLy, APR4), the 2 quark EOSs (WSPHS1, WSPHS2) and the 4 hybrid EOSs (WSPHS3, ALF2, ALF4, BS4)

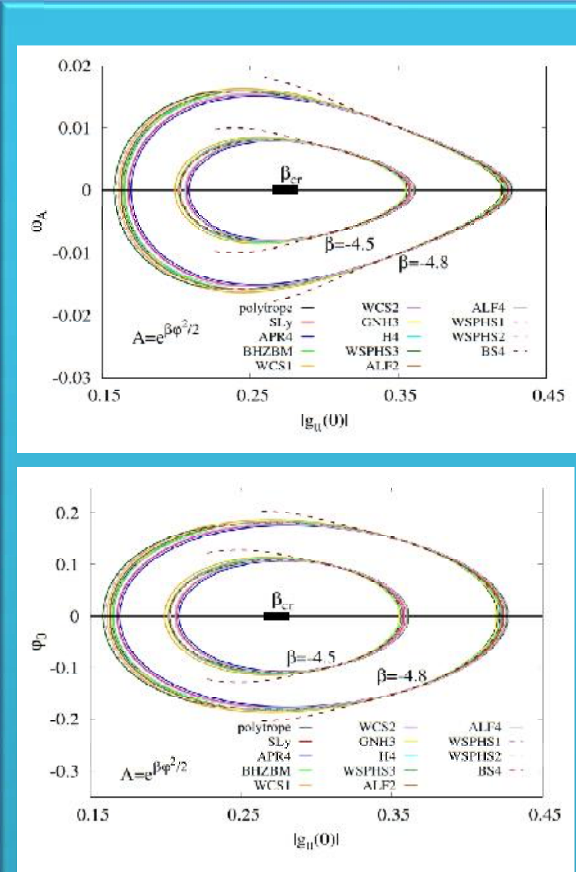



Scalar field charge $\omega_A$ versus the the compactness $C=M/R_s$ (in units of $c=G_*=1$) of the neutron star models for all EOSs considered: The 5 hyperon EOSs (H4, BHZBM, GNH3, WCS1, WCS2), the polytropic EOS, the 2 nuclear EOSs (SLy, APR4), the 2 quark EOSs (WSPHS1, WSPHS2) and the 4 hybrid EOSs (WSPHS3, ALF2, ALF4, BS4). Note that the scalar charge can be positive and negative.

### Universal $I\_C$ relations

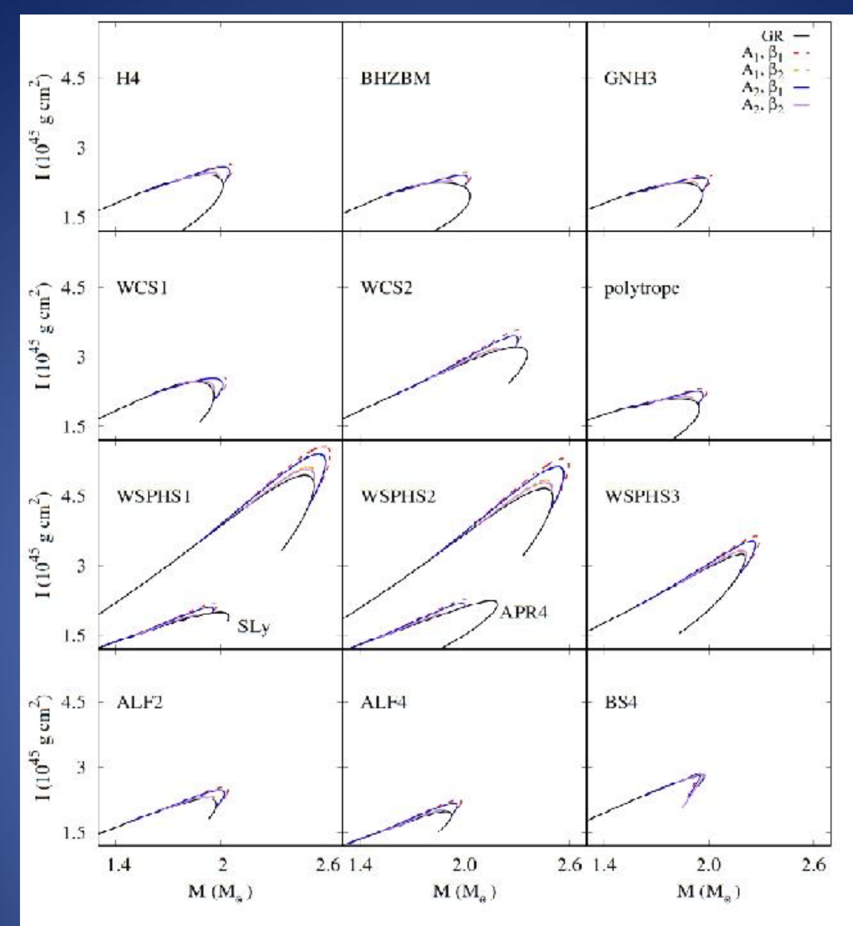



Moment of inertia *I* versus the compactness $C=M/R_s$ in the slow rotation approximation for two different normalizations, $I/MR_s^2$ (a) and $I/M^3$ (b), for all 14 EOSs, for both coupling functions $A_i$ with both values of β, as well as for GR. The upper panels show the scaled values of *I* (symbols) together with the fitted curves (lines) of the universal relations (excluding the two quark EOSs). The lower panels exhibit the deviations from the fitted values, $|1-I/I_{fit}|$.




Scalar field charge $\omega_A$ and scalar field at the center $\varphi_0$ versus the metric function $g_{tt}(0)$ at the center of the neutron star models for all EOSs considered: The 5 hyperon EOSs (H4, BHZBM, GNH3, WCS1, WCS2), the polytropic EOS, the 2 nuclear EOSs (SLy, APR4), the 2 quark EOSs (WSPHS1, WSPHS2) and the 4 hybrid EOSs (WSPHS3, ALF2, ALF4, BS4). The critical values $\beta_{cr}$ are also indicated. Note that the scalar charge and the scalar field can be positive and negative.

### Moment of inertia versus total mass




Moment of inertia *I* (in $10^{45}$ g cm$^2$) versus the total mass M (in solar masses $M_\odot$) of the neutron star models for all EOSs considered: The first two rows show the 5 hyperon EOSs (H4, BHZBM, GNH3, WCS1, WCS2) and the polytropic EOS, the last two rows contain the 2 nuclear EOSs (SLy, APR4), the 2 quark EOSs (WSPHS1, WSPHS2) and the 4 hybrid EOSs (WSPHS3, ALF2, ALF4, BS4). The solid black lines represent the GR configurations. The scalarized solutions for $A_1=e^{1/2\beta\varphi^2}$ and for $A_2=1/\cosh\sqrt{(-\beta\varphi)}$ with $\beta_1=-4.5$ and $\beta_2=-4.8$ are presented as well.