


Inclusive Semi-leptonic Penguin Decays

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Motivation



Error of Branching ratio $\bar{B} \rightarrow X_s \ell^+ \ell^-$

BF (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
≥ 14.4	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

Error of Normalized Forward-Backward-Asymmetry

AFB_n (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
≥ 14.4	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

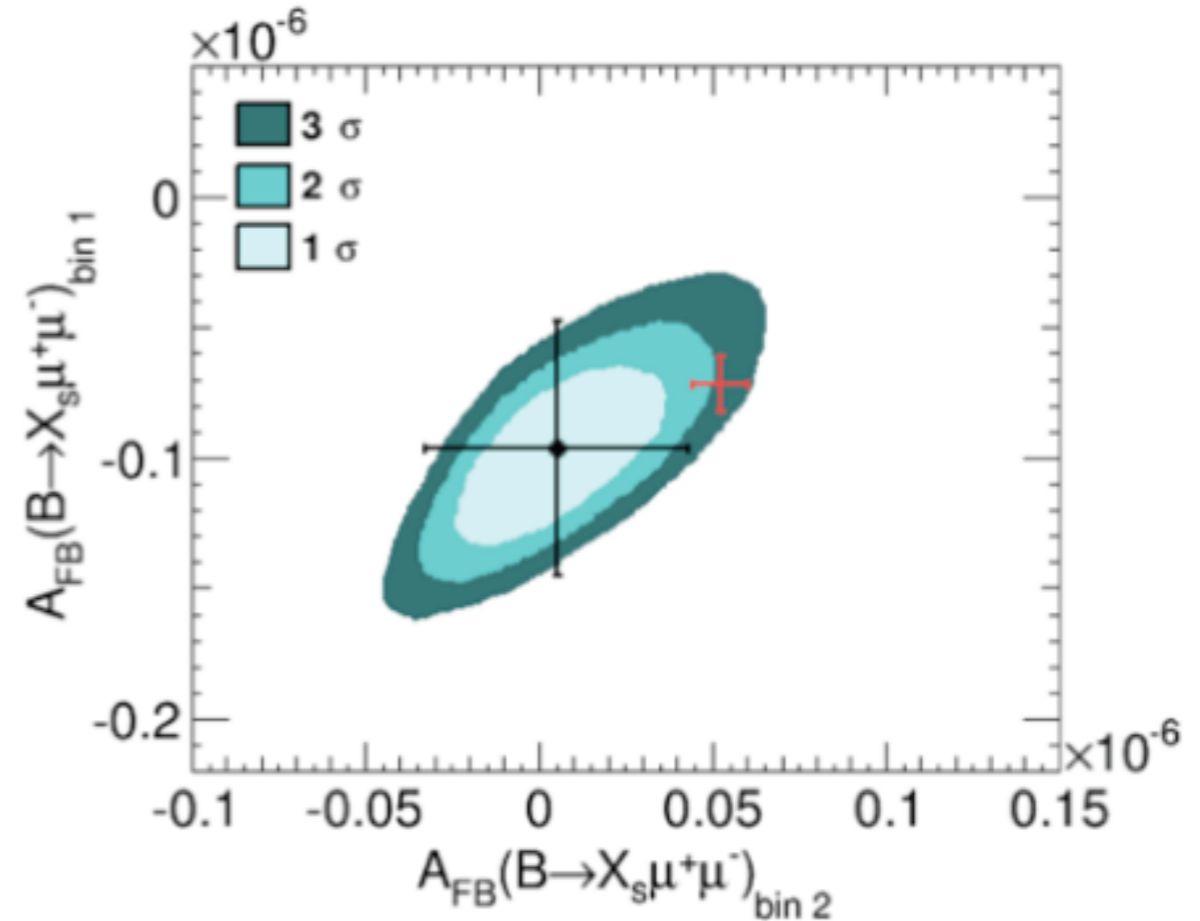
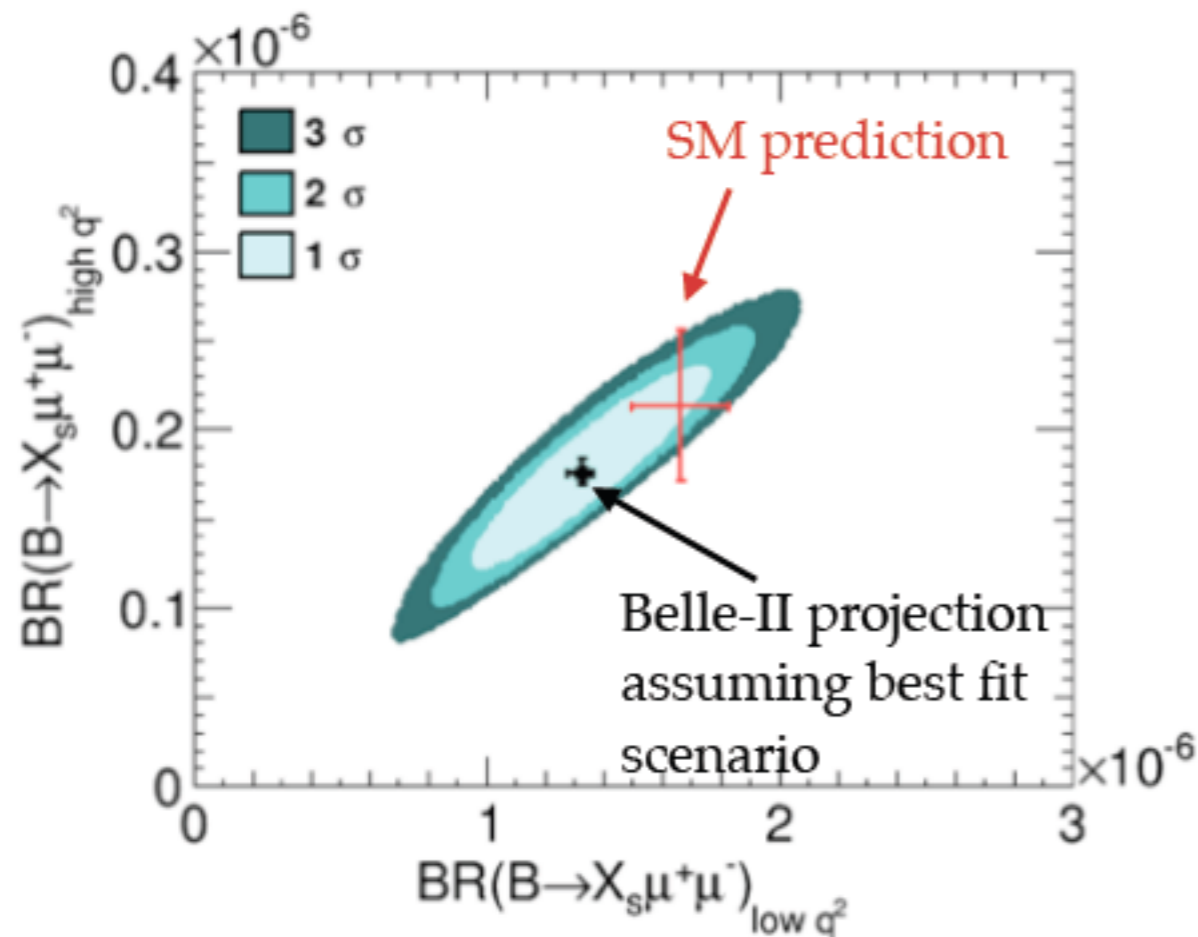
$B \rightarrow (\pi, \rho) \ell^+ \ell^-$, semi-inclusive $\bar{B} \rightarrow X_d \ell^+ \ell^-$ at 50/ab
(uncertainties like $\bar{B} \rightarrow X_s \ell^+ \ell^-$ at 0.7/ab)



Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545

if SM deviations in R_K and P'_5 persist until Belle-II



If NP then the effect of C_9 and C'_9 are large enough to be checked at Belle-II with theoretically clean modes.

Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

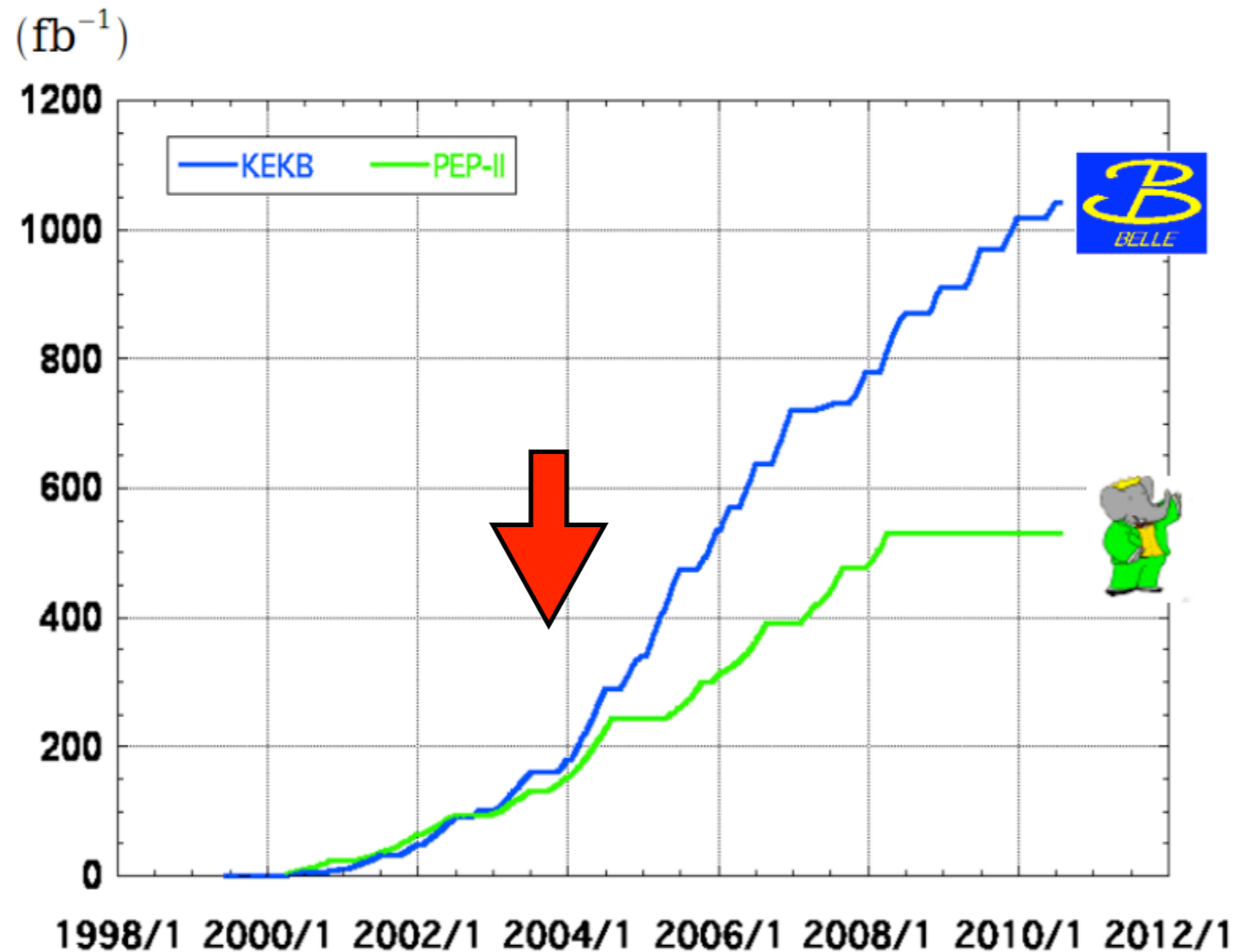


Experiment

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based $152 \times 10^6 B\bar{B}$ events)

Integrated luminosity of B factories



> 1 ab^{-1}

On resonance:

$\Upsilon(5S)$: 121 fb^{-1}

$\Upsilon(4S)$: 711 fb^{-1}

$\Upsilon(3S)$: 3 fb^{-1}

$\Upsilon(2S)$: 25 fb^{-1}

$\Upsilon(1S)$: 6 fb^{-1}

Off reson./scan:

$\sim 100 \text{ fb}^{-1}$

$\sim 550 \text{ fb}^{-1}$

On resonance:

$\Upsilon(4S)$: 433 fb^{-1}

$\Upsilon(3S)$: 30 fb^{-1}

$\Upsilon(2S)$: 14 fb^{-1}

Off resonance:

$\sim 54 \text{ fb}^{-1}$

New Babar analysis on dilepton spectrum arXiv:1312.3664

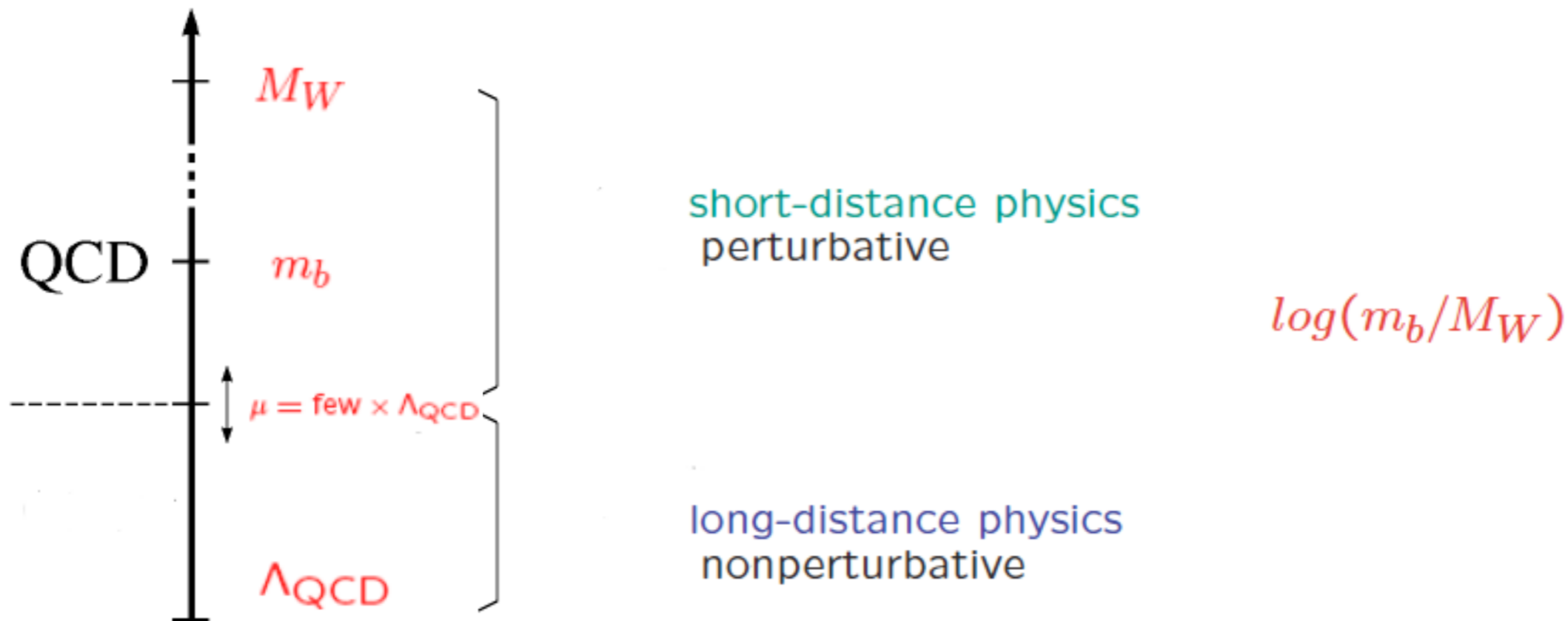
New Belle analysis on AFB arXiv:1402.7134



Theoretical Tools



Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 \gg M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?



Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$

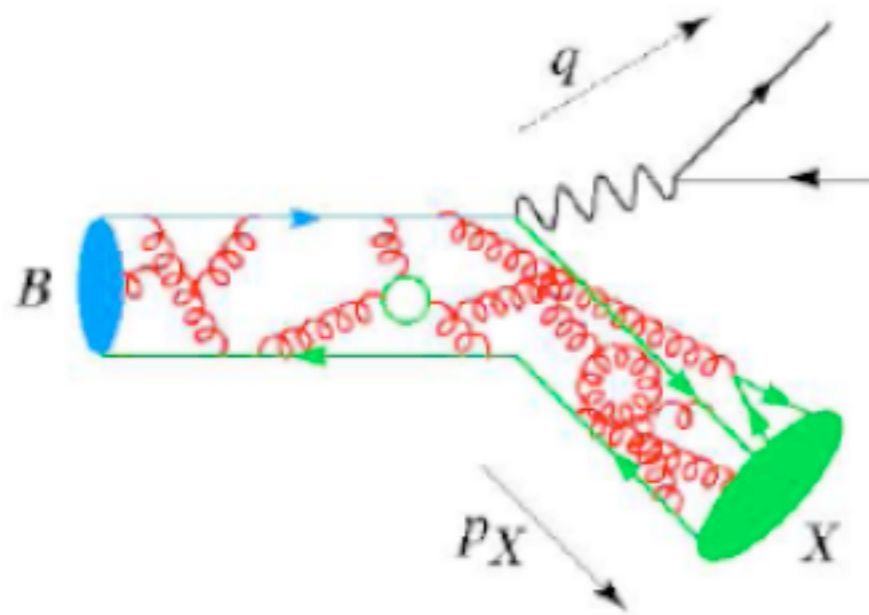
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) \xrightarrow{m_b \rightarrow \infty} \Gamma(b \rightarrow X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \Lambda_{QCD}^2 / m_b^2$$

No linear term Λ_{QCD} / m_b (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$

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An old story:

- If one goes beyond the leading operator ($\mathcal{O}_7, \mathcal{O}_9$):
breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012](#)



Analysis in $B \rightarrow X_s l l$ in this talk; [Benzke, Fickinger, Hurth, Turczyk](#)



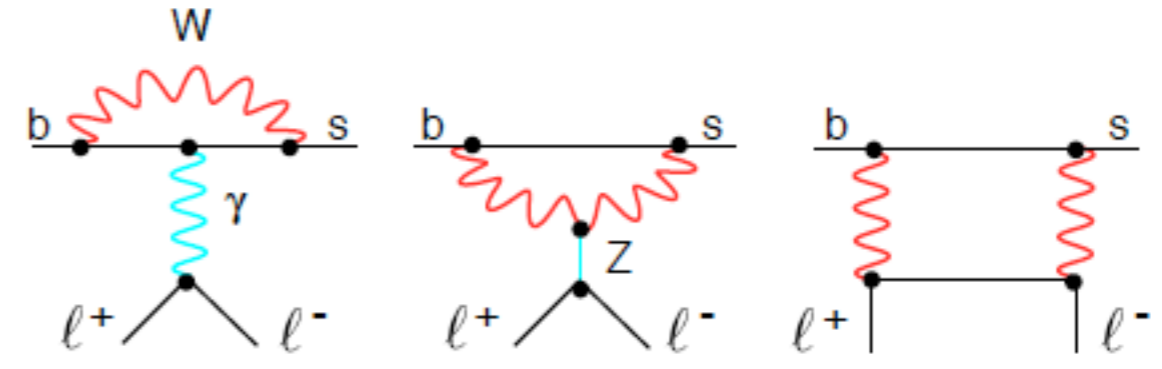
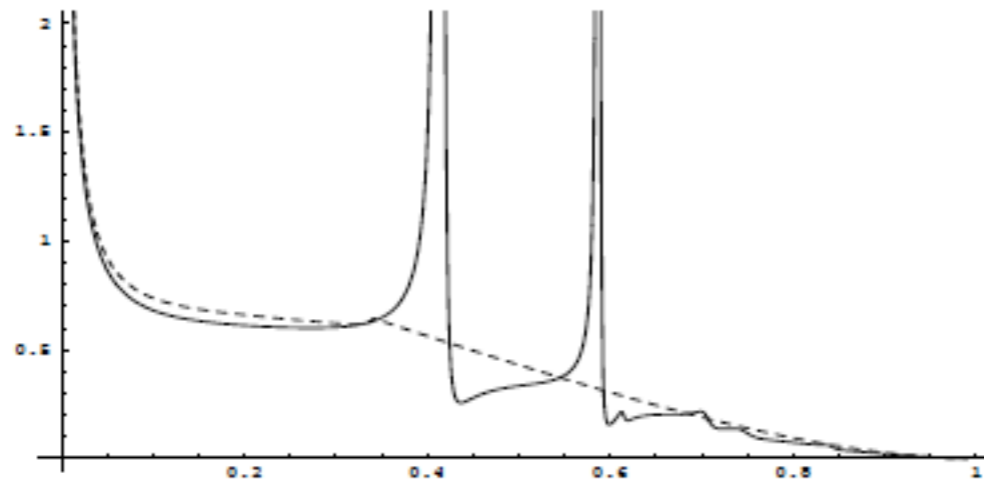
Perturbative contributions



Review of previous calculations for $B \rightarrow X_s ll$

- On-shell- $c\bar{c}$ -resonances \Rightarrow cuts in dilepton mass spectrum necessary :
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2 \Rightarrow$ perturbative contributions dominant

$$\frac{d}{d\hat{s}} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



$$\hat{s} = q^2 / m_b^2$$

- NNLL prediction of $\bar{B} \rightarrow X_s l^+ l^-$: dilepton mass spectrum
 Asatryan, Asatrian, Greub, Walker, hep-ph/0204341
 Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 > 14.4\text{GeV}^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections $q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]$
 central value: -14% , perturbative error: $13\% \rightarrow 6.5\%$



- Further refinements:

- Completing NNLL QCD corrections:

- Mixing into \mathcal{O}_9 (+1%), NNLL matrixelement of \mathcal{O}_9 (-4%)

- NLL QED two-loop corrections to Wilson coefficients

- 1.5% shift for $\alpha_{em}(\mu = m_b)$, -8.5% for $\alpha_{em}(\mu = m_W)$

- Bobeth, Gambino, Gorbahn, Haisch, hep-ph/0312090

- QED two-loop corrections to matrix elements

- Large collinear logarithm $Log(m_b/m_\ell)$ which survive integration if a restricted part of the dilepton mass spectrum is considered

- +2% effect in the low- q^2 region for muons, for the electrons the effect depends on the experimental cut parameters

- Huber, Lunghi, Misiak, Wyler, hep-ph/0512066

- NNLL prediction of $\bar{B} \rightarrow X_s \ell^+ \ell^-$: forward-backward-asymmetry (FBA)

- Asatrian, Bieri, Greub, Hovhannisyan, hep-ph/0209006

- Ghinculov, Hurth, Isidori, Yao, hep-ph/0208088, hep-ph/0312128

$$A_{FB} \equiv \frac{1}{\Gamma_{semilep}} \left(\int_0^1 d(\cos \theta) \frac{d^2\Gamma}{dq^2 d\cos \theta} - \int_{-1}^0 d(\cos \theta) \frac{d^2\Gamma}{dq^2 d\cos \theta} \right)$$

(θ angle between ℓ^+ and B momenta in dilepton CMS)

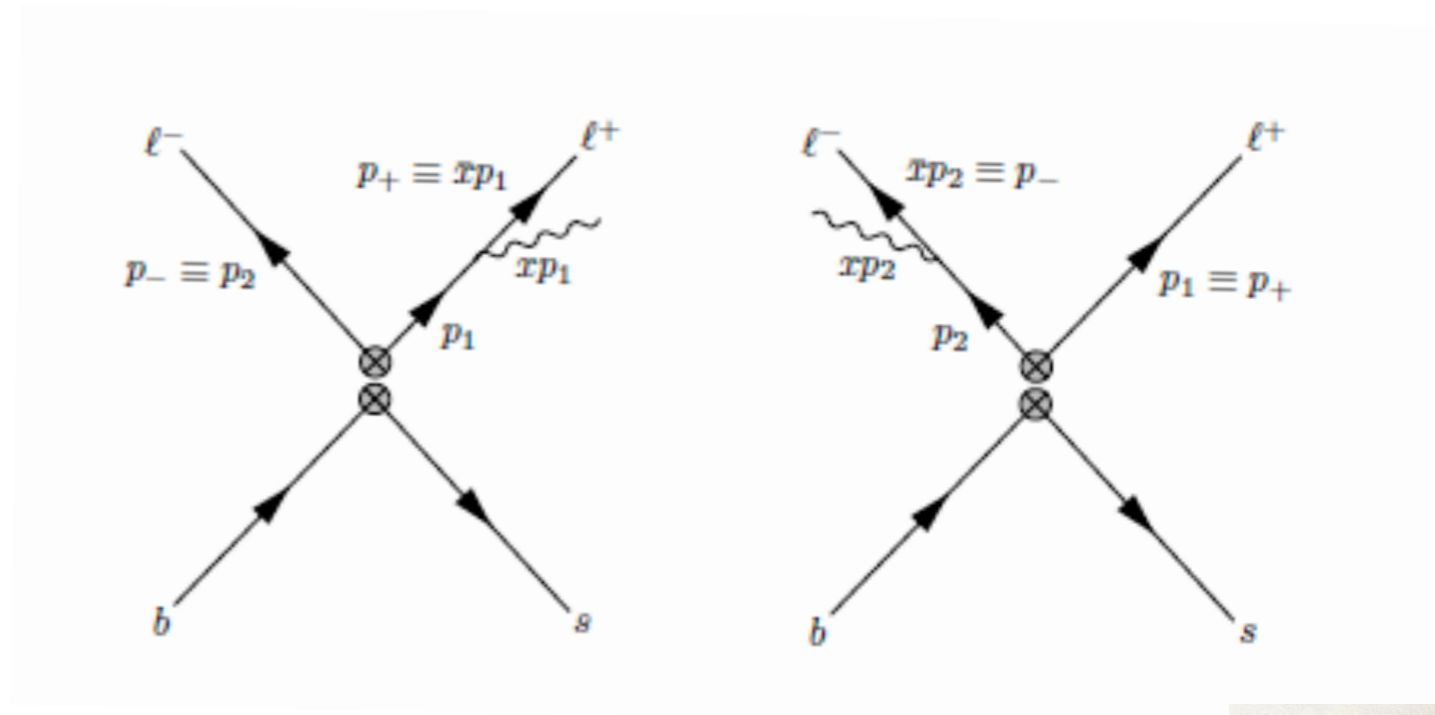
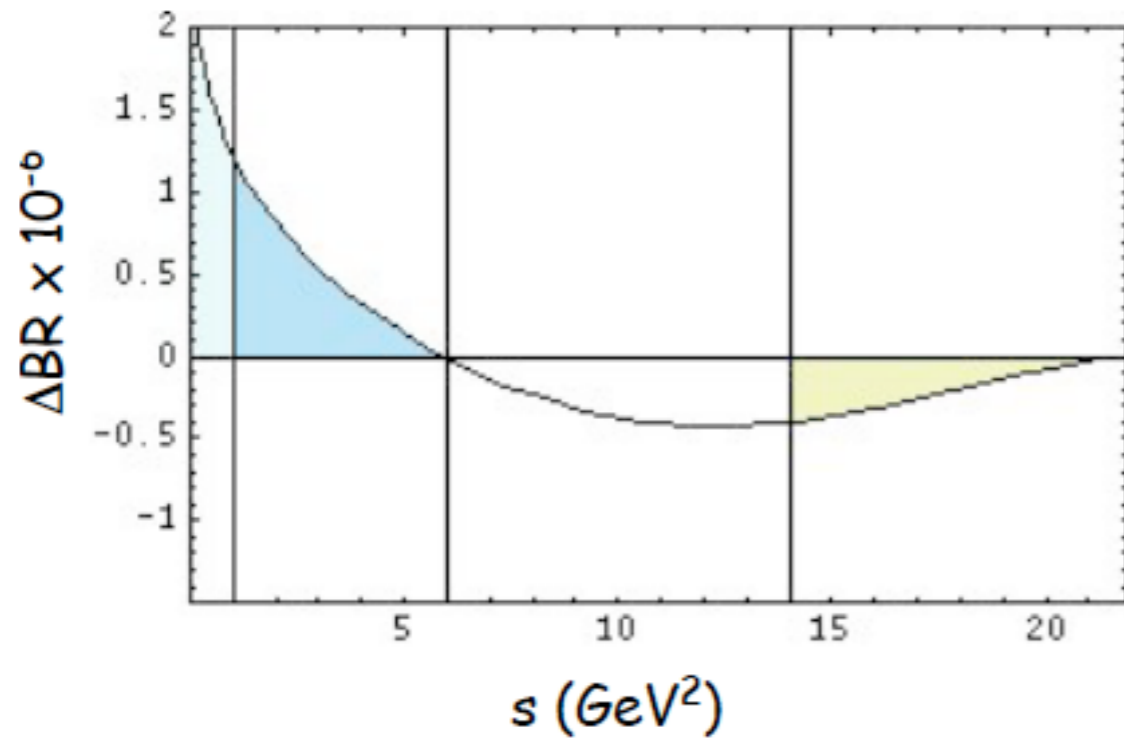
$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) GeV^2$$



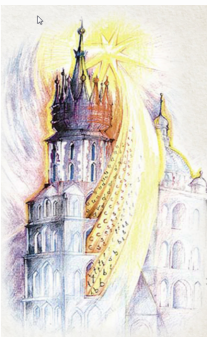
- Electromagnetic corrections in high- q^2 and for A_{FB}
 Huber, Hurth, Lunghi, Nucl. Phys. B802(2008)40

Corrections to matrix elements lead to large collinear log $\text{Log}(m_b/m_\ell)$

$$\delta\text{BR}(B \rightarrow X_s \mu^+ \mu^-) = \begin{cases} (+2.0\%) & \text{low } q^2 \\ (-6.8\%) & \text{high } q^2 \end{cases} \quad \delta\text{BR}(B \rightarrow X_s e^+ e^-) = \begin{cases} (+5.2\%) & \text{low } q^2 \\ (-17.6\%) & \text{high } q^2 \end{cases}$$



:



Complete angular analysis of inclusive $B \rightarrow X_{sll}$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} [(1+z^2)H_T(q^2) + 2zH_A(q^2) + 2(1-z^2)H_L(q^2)] \quad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)$$

$$\frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

- Dependence on Wilson coefficients

H_T suppressed in low- q^2 window

$$H_T(q^2) \propto 2s(1-s)^2 \left[\left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$

$$H_A(q^2) \propto -4s(1-s)^2 \text{Re} \left[C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right]$$

$$H_L(q^2) \propto (1-s)^2 \left[|C_9 + 2C_7|^2 + |C_{10}|^2 \right]$$

- Devide low- q^2 bin in two bins (zero of H_A in low- q^2)

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156



- Most important input parameters

$$m_b^{1S} = (4.691 \pm 0.037)\text{GeV}, \quad \bar{m}_c(\bar{m}_c) = (1.275 \pm 0.025)\text{GeV}$$

$$|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027, \quad BR_{b \rightarrow c e \nu}^{\text{exp.}} = (10.51 \pm 0.13)\%$$

- Perturbative expansion (NNLO QCD + NLO QED) α_s $\kappa = \alpha_{\text{em}}/\alpha_s$

$$A = \kappa [A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3)]$$

$$+ \kappa^2 [A_{LO}^{\text{em}} + \alpha_s A_{NLO}^{\text{em}} + \alpha_s^2 A_{NNLO}^{\text{em}} + \mathcal{O}(\alpha_s^3)] + \mathcal{O}(\kappa^3)$$

$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$



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$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Size of logs depend on experimental set-up

We assume no photons are included in the definition of q^2 (di-muon channel at Babar/Belle, di-electron at Belle)

Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in q^2

Monte Carlo techniques needed to estimate this effect !

$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma, \text{coll}})^2$$

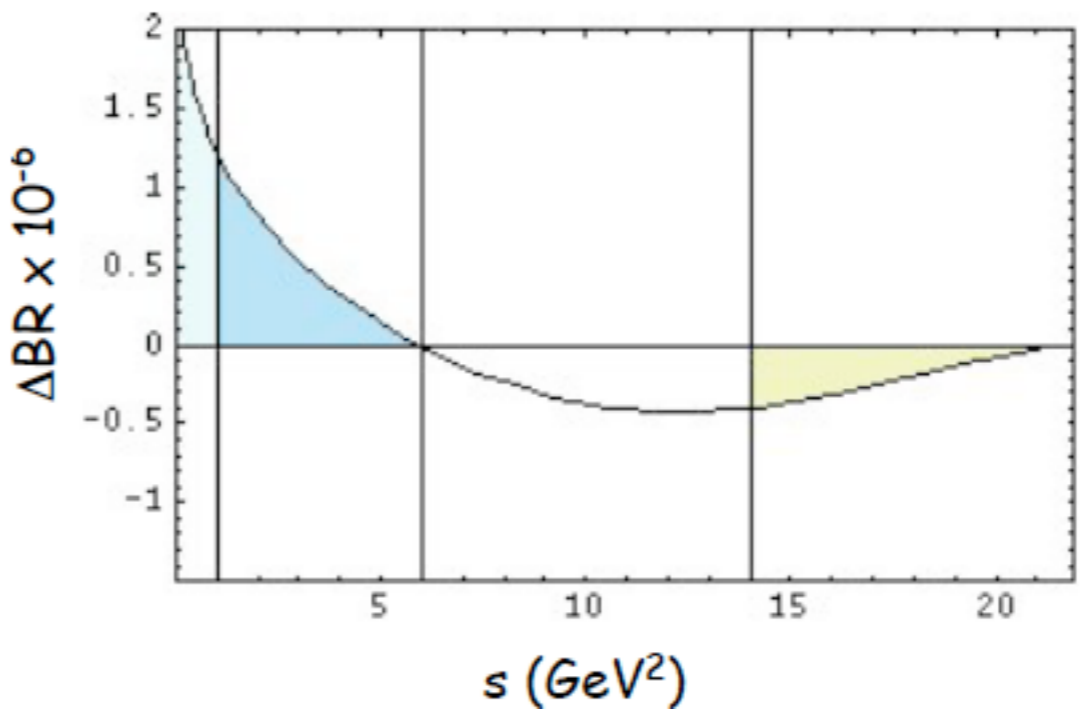
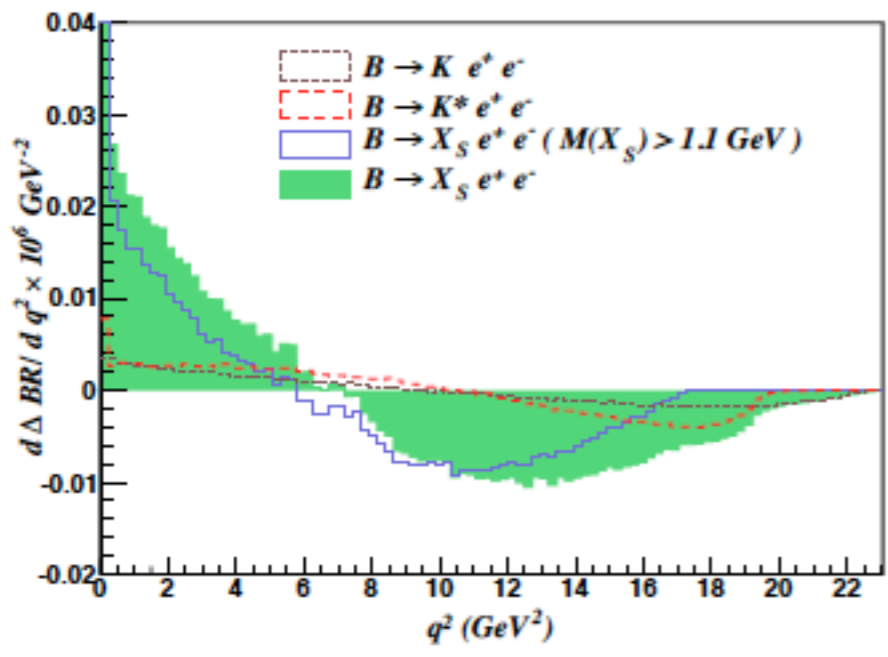
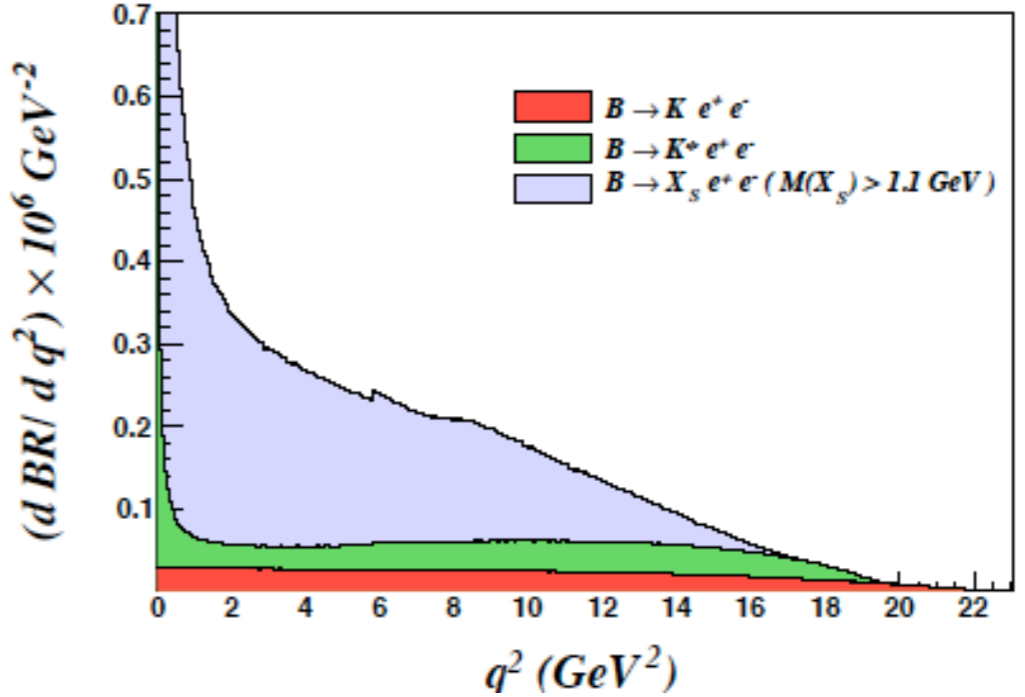
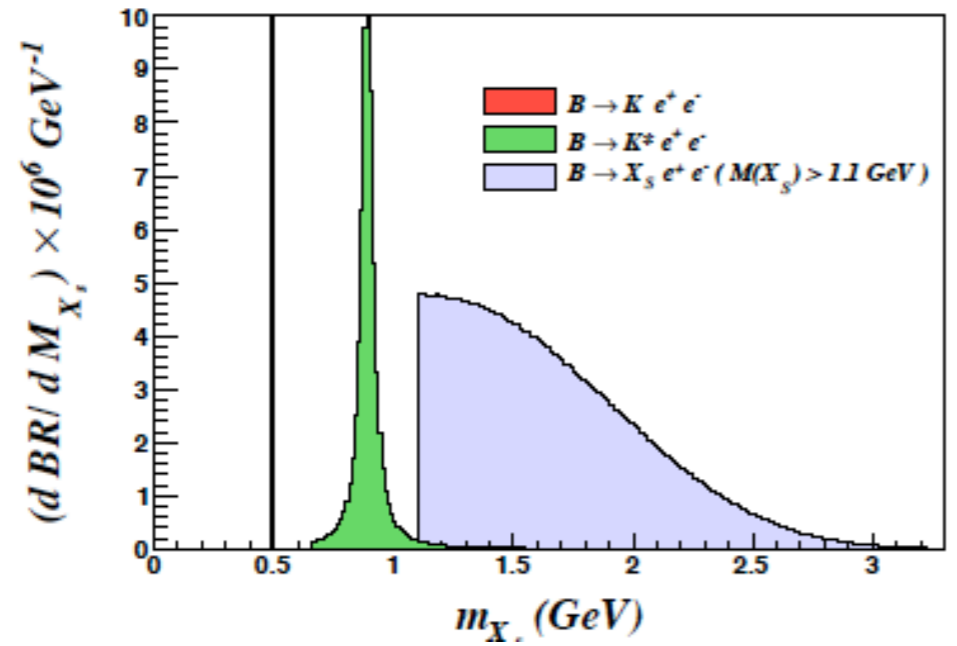


Monte Carlo analysis

(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

$$\frac{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma\text{coll}}} - 1}{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}}} = 1.65\%$$

$$\frac{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma\text{coll}}} - 1}{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}}} = 6.8\%$$



Theory predictions



Results

Low- q^2 ($1\text{GeV}^2 < q^2 < 6\text{GeV}^2$)

$$BR(B \rightarrow X_s ee) = (1.67 \pm 0.10) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (1.62 \pm 0.09) 10^{-6}$$

Babar: $BR(B \rightarrow X_s ll) =$

$$= (1.60 (+0.41 - 0.39)_{stat} (+0.17 - 0.13)_{syst} (\pm 0.18)_{mod}) 10^{-6}$$

good agreement with SM



Results

High- q^2 , Theory: $q^2 > 14.4\text{GeV}^2$, Babar: $q^2 > 14.2\text{GeV}^2$

$$BR(B \rightarrow X_s ee) = (0.220 \pm 0.070) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (0.253 \pm 0.070) 10^{-6}$$

Babar: $BR(B \rightarrow X_s ll) =$

$$(0.57 (+0.16 - 0.15)_{stat} (+0.03 - 0.02)_{syst}) 10^{-6}$$

2σ higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077

(but perfect agreement if we use their prescriptions)



Further refinement

Normalization to semileptonic $B \rightarrow X_u l \nu$ decay rate **with the same cut** reduces the impact of $1/m_b$ corrections in the high- q^2 region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31) 10^{-3}$$

$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30) 10^{-3}$$

Largest source of error are CKM elements (V_{ub})

Note: Additional $O(5\%)$ uncertainty due to nonlocal power corrections $O(\alpha_s \Lambda/m_b)$



Further results in units of 10^{-6}

$$H_L[1, 3.5]_{ee} = 0.64 \pm 0.03$$

$$H_L[1, 3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5, 6]_{ee} = 0.50 \pm 0.03$$

$$H_L[3.5, 6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1, 6]_{ee} = 1.13 \pm 0.06$$

$$H_L[1, 6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1, 3.5]_{ee} = 0.29 \pm 0.02$$

$$H_T[1, 3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5, 6]_{ee} = 0.24 \pm 0.02$$

$$H_T[3.5, 6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1, 6]_{ee} = 0.53 \pm 0.04$$

$$H_T[1, 6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1, 3.5]_{ee} = -0.103 \pm 0.005$$

$$H_A[1, 3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5, 6]_{ee} = +0.073 \pm 0.012$$

$$H_A[3.5, 6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1, 6]_{ee} = -0.029 \pm 0.016$$

$$H_A[1, 6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error $\mathcal{O}(5 - 8\%)$. Still dominated by scale uncertainty.



New physics sensitivity

Huber, Hurth, Lunghi, arXiv:1503.04849

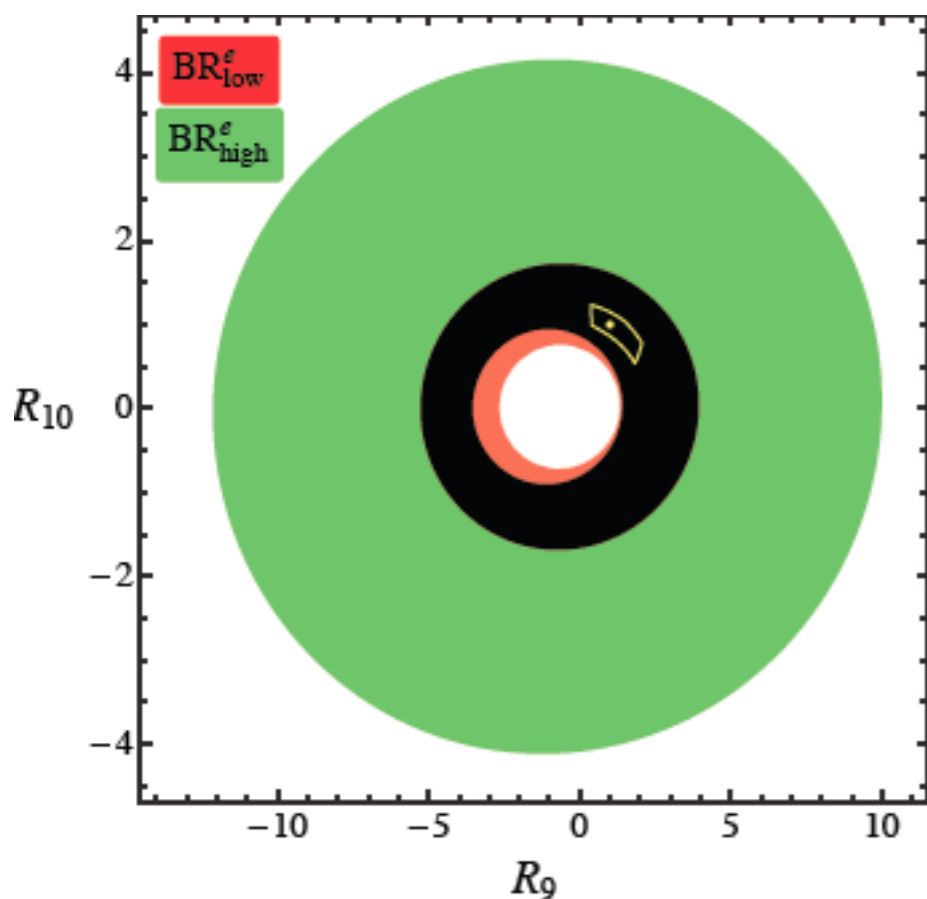
Constraints on Wilson coefficients C_9/C_9^{SM} and $C_{10}/C_{10}^{\text{SM}}$

$$R_i = \frac{C_i(\mu_0)}{C_i^{\text{SM}}(\mu_0)}$$

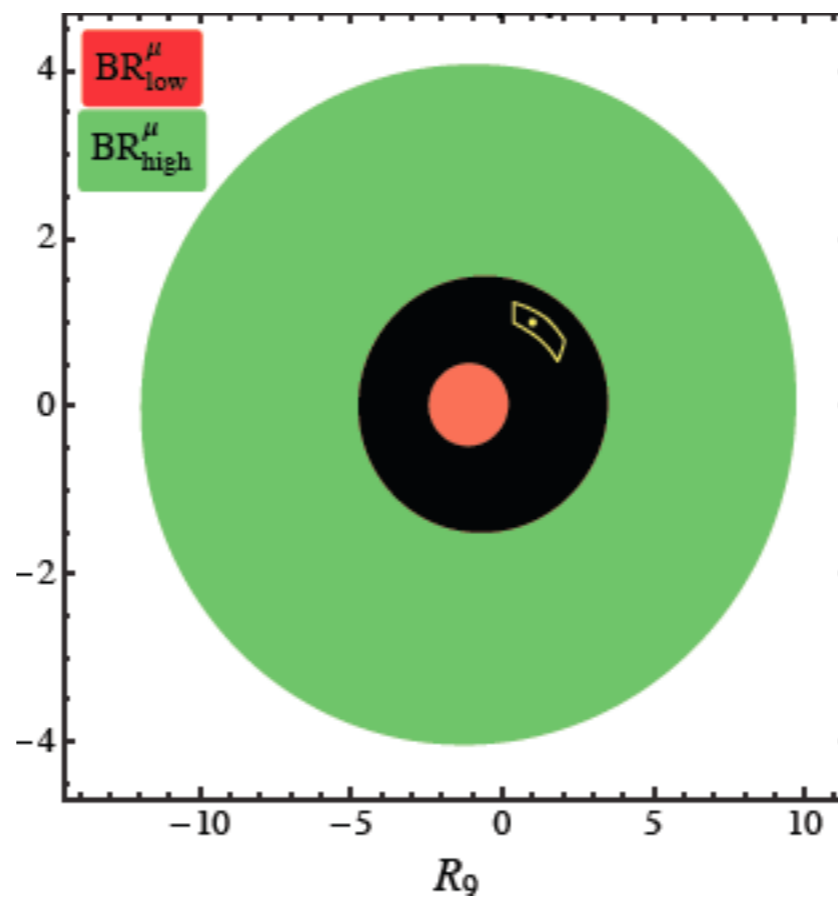
that we obtain at 95% C.L. from present experimental data
(red low q^2 , green high q^2)

that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II
(yellow)

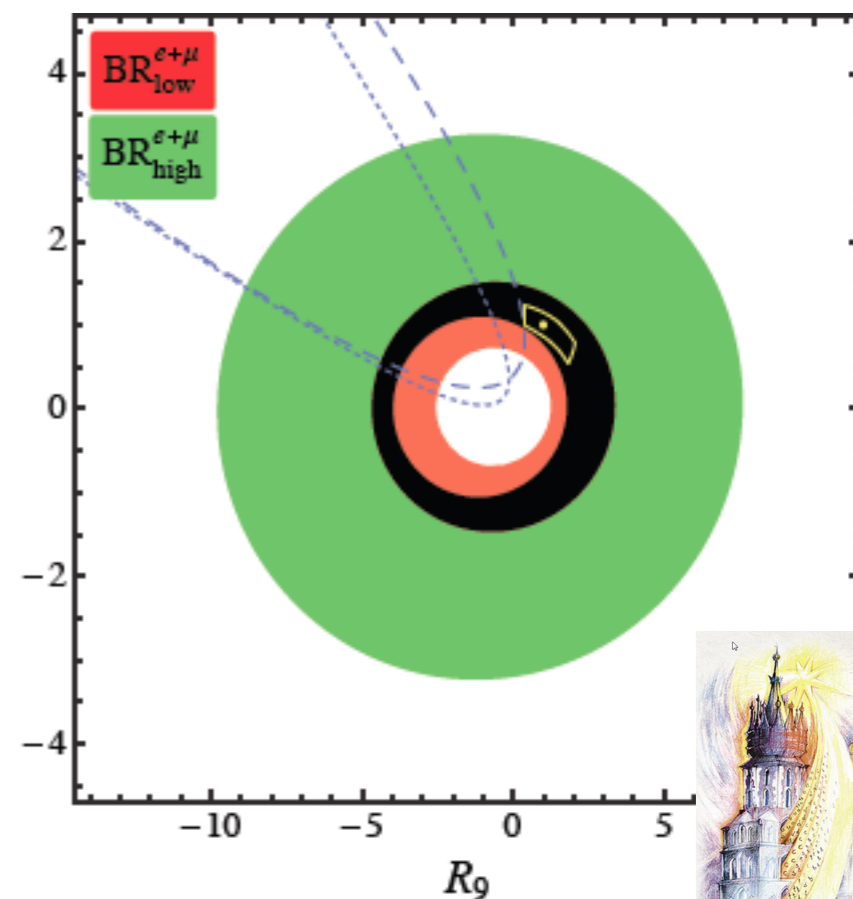
$B \rightarrow X_s e e$



$B \rightarrow X_s \mu \mu$



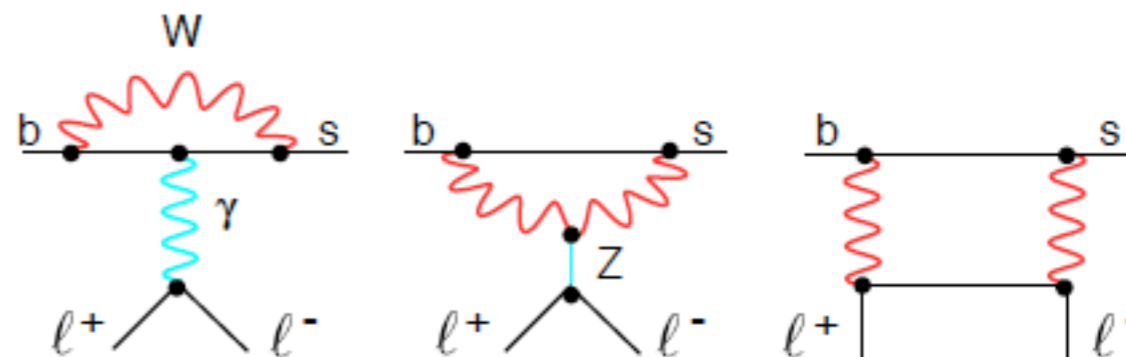
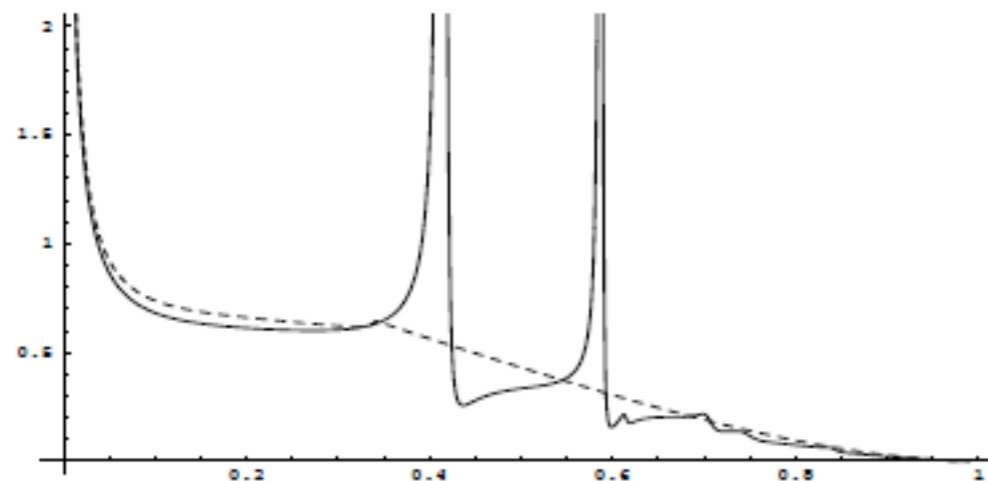
$B \rightarrow X_s l l$



Cuts in the dilepton and hadronic mass spectra

- On-shell- $c\bar{c}$ -resonances \Rightarrow cuts in dilepton mass spectrum necessary :
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2 \Rightarrow$ perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \rightarrow c (\rightarrow se^+ \nu) e^- \bar{\nu} = b \rightarrow se^+ e^- +$ missing energy
 - * Babar, Belle: $m_X < 1.8$ or 2.0GeV
 - * high- q^2 region not affected by this cut
 - * kinematics: X_s is jetlike and $m_X^2 \leq m_b \Lambda_{QCD} \Rightarrow$ shape function region
 - * SCET analysis: universality of jet and shape functions found:
 the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \rightarrow X_s \gamma$ shape function
 5% additional uncertainty for 2.0GeV cut due to subleading shape functions

Lee, Stewart hep-ph/0511334

Lee, Ligeti, Stewart, Tackmann hep-ph/0512191

Lee, Tackmann arXiv:0812.0001 (effect of subleading shape functions)

Bell, Beneke, Huber, Li arXiv:1007.3758 (NNLO matching QCD \rightarrow SCET)



Nonlocal subleading contributions



Subleading power factorization in $B \rightarrow X_s l^+ l^-$

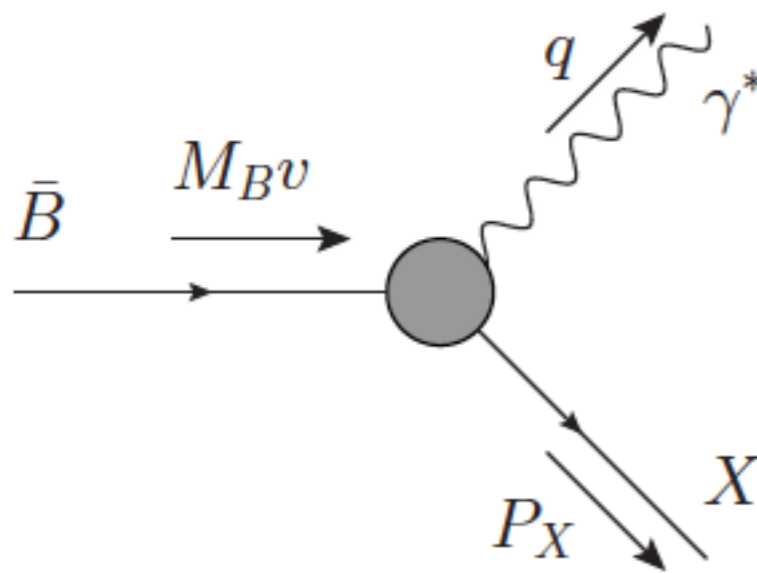
Benzke, Hurth, Turczyk, arXiv:1705.10366

Hadronic cut

Additional cut in X_s necessary to reduce background affects only low- q^2 region.

Hadronic invariant $m_X^2 < 1.8(2.0) \text{ GeV}^2$, jet-like X_s $E_X \sim \mathcal{O}(m_b)$

Multiscale problem \rightarrow SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\text{QCD}} m_b \gg \Lambda_{\text{QCD}}^2$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

Scaling

$$\lambda = \Lambda_{\text{QCD}} / m_b$$



Kinematics

B meson rest frame

$$q = p_B - p_X \quad 2 m_B E_X = m_B^2 + M_X^2 - q^2$$

X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

two light-cone components $p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

$$q^+ = n q = m_B - p_X^+ \quad q^- = \bar{n} q = m_B - p_X^-$$



Scaling

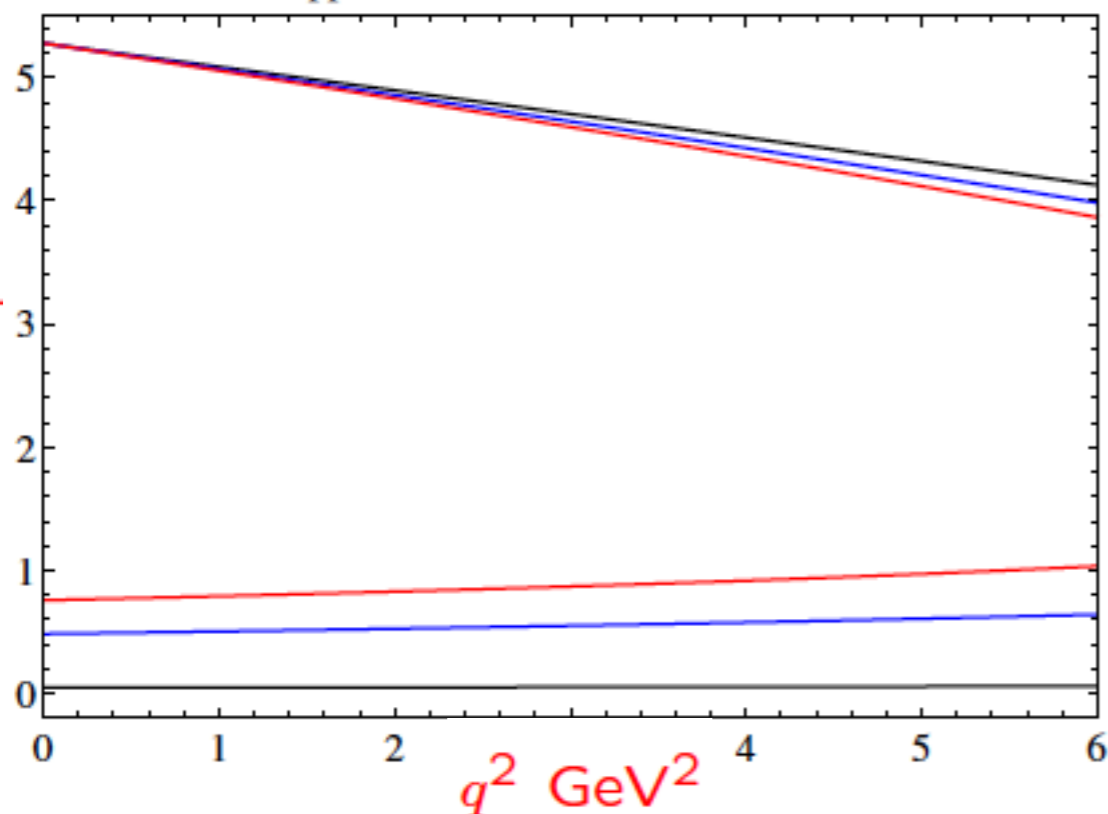
$$\lambda = \Lambda_{\text{QCD}}/m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

$M_X = [0.5, 1.6, 2]$ GeV [Black, Blue, Red]

Upper lines : P_X^- , lower lines : P_X^+

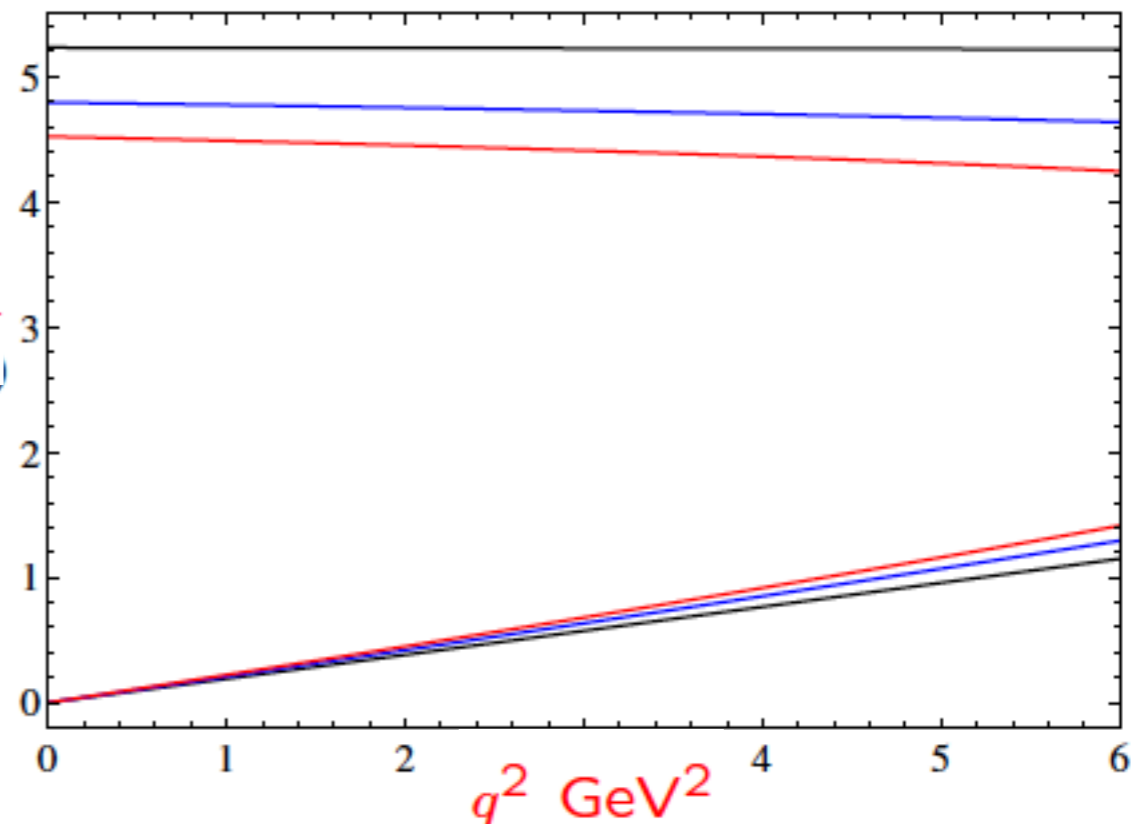
$P_X^-/+$
GeV



$M_X = [0.5, 1.6, 2]$ GeV [Black, Blue, Red]

Upper lines : q^+ , lower lines : q^-

$q^+/-$
GeV



For $q^2 < 6 \text{ GeV}^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ , means q anti-hard-collinear (just kinematics).

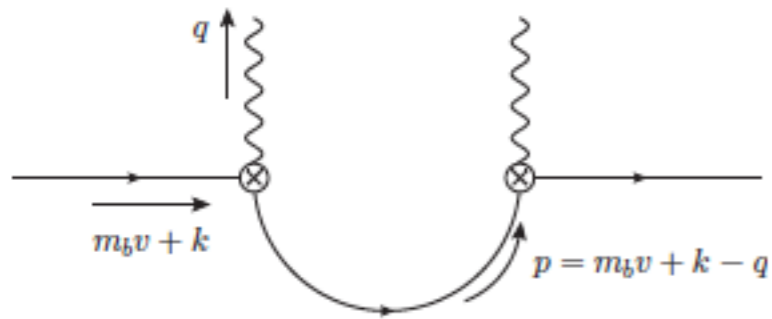
Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.

This problematic assumption implies a different matching of SCET/QCD.



Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left(1 - \frac{n \cdot k}{m_b - n \cdot q} + \dots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \rightarrow X_s \gamma$)

Factorization theorem $d\Gamma \sim H \cdot J \otimes S$

The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

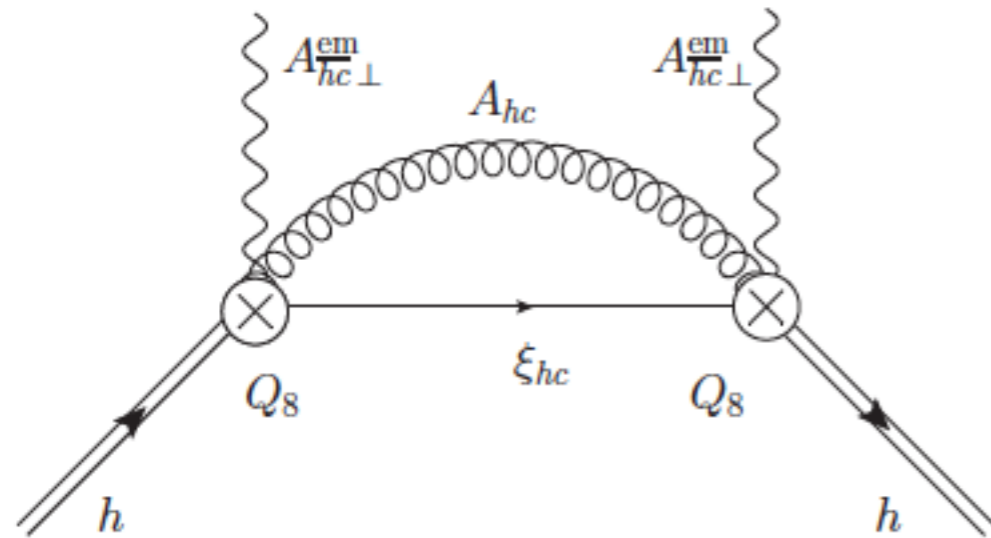
(universality of the shape function, uncertainties due to subleading shape functions)



Calculation at subleading power

Example of **direct** photon contribution which factorizes

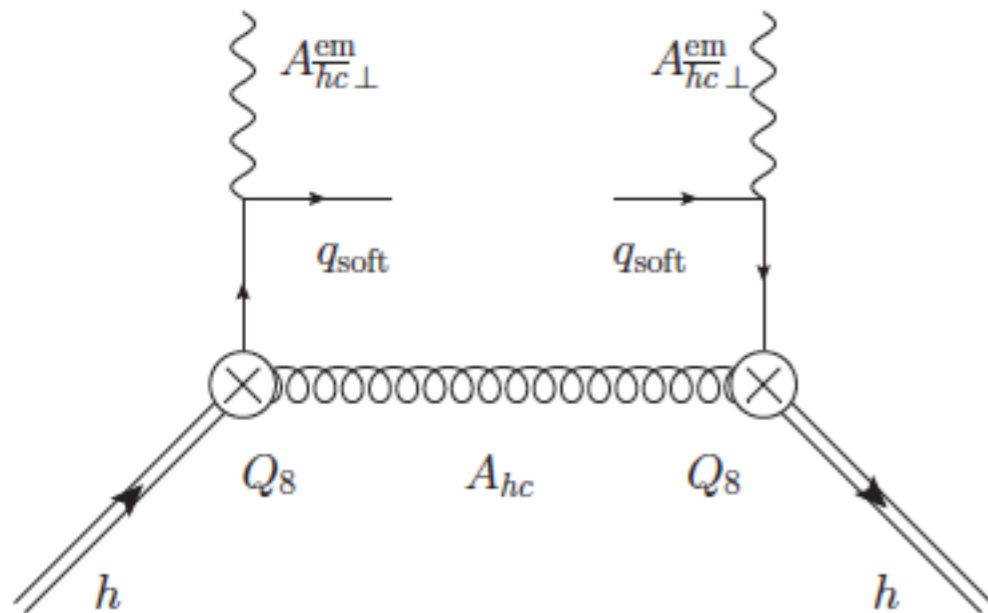
$$d\Gamma \sim H \cdot j \otimes S$$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_χ^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

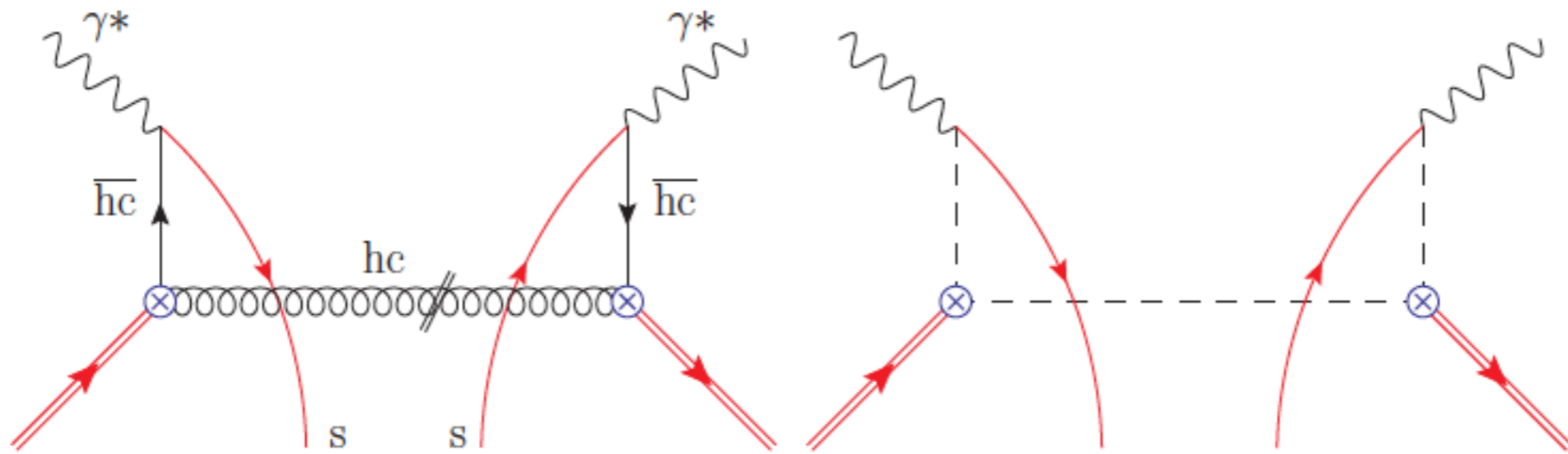
$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_8 and \bar{Q}_8



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{\mathbf{n}}) \bar{s}(\mathbf{r}\bar{\mathbf{n}}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

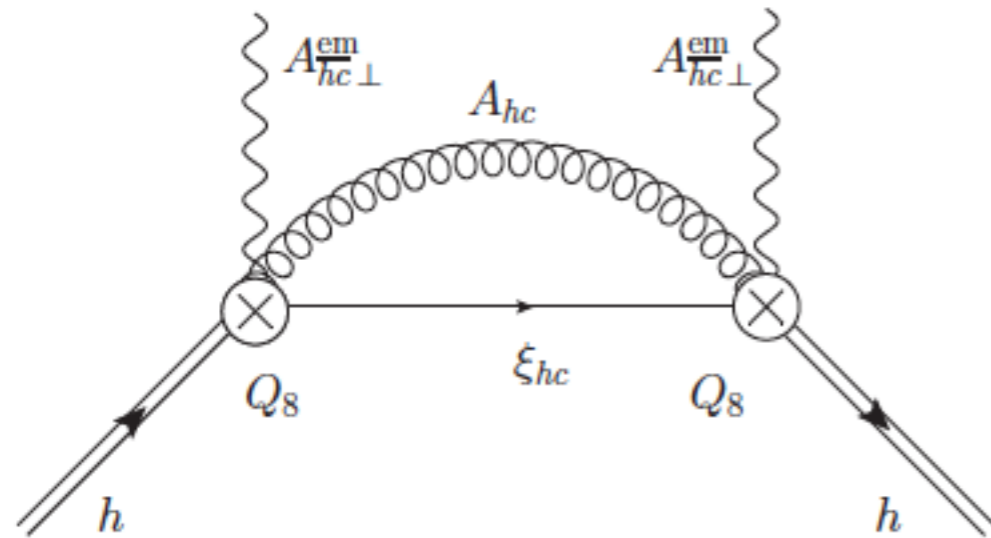
Shape function is non-local in two light-cone directions.

It survives $M_X \rightarrow 1$ limit (irreducible uncertainty).

Calculation at subleading power

Example of **direct** photon contribution which factorizes

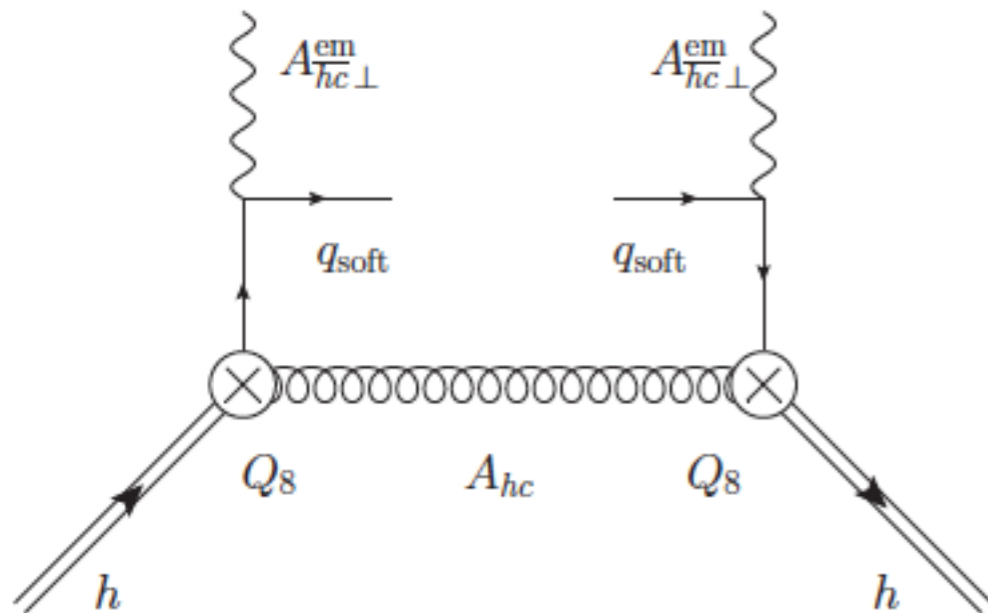
$$d\Gamma \sim H \cdot j \otimes S$$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_χ^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

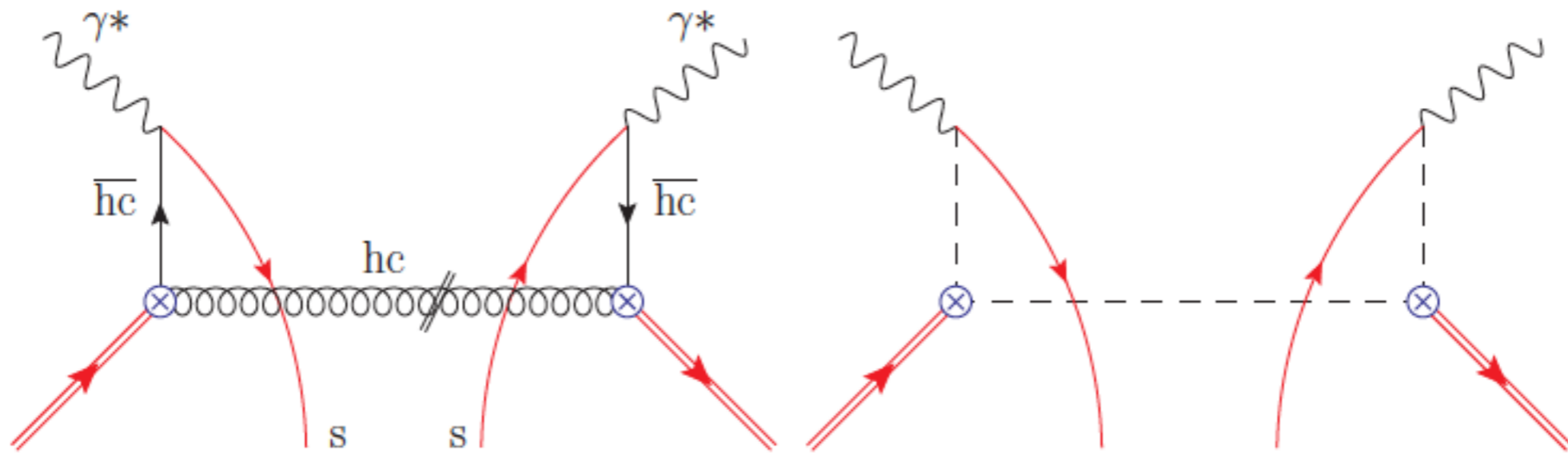
$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_8 and Q_8



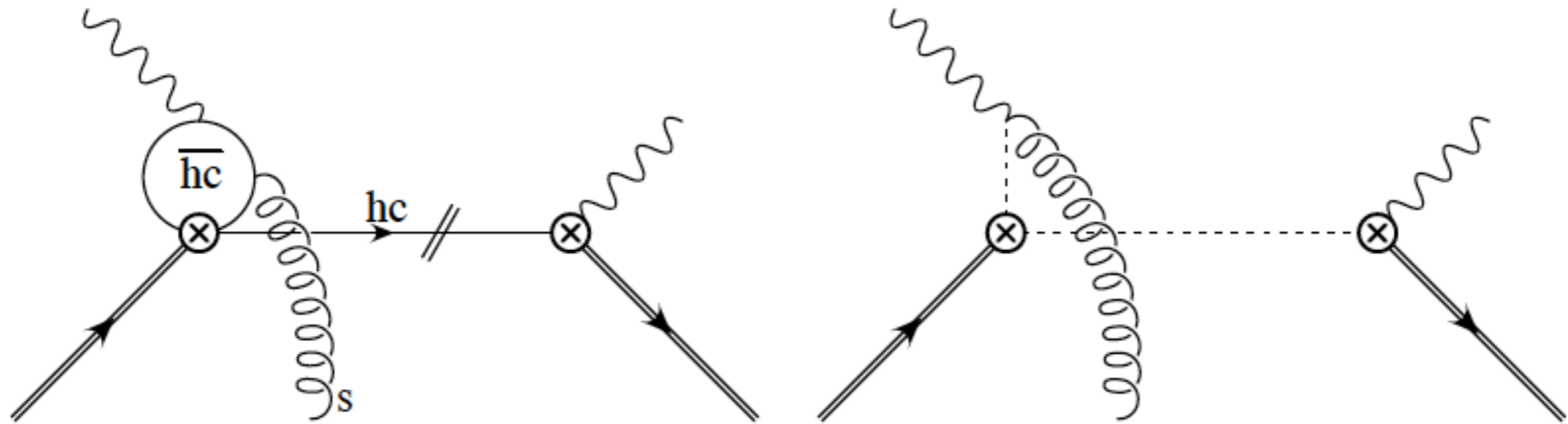
$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{\mathbf{n}}) \bar{s}(\mathbf{r}\bar{\mathbf{n}}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

Shape function is non-local in two light-cone directions.

It survives $M_X \rightarrow 1$ limit (irreducible uncertainty).

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\epsilon}$$

$$\frac{1}{\omega_1} \left[\bar{n} \cdot q \left(F \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left(F \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right.$$

$$\left. + \bar{n} \cdot q \left(G \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) \right] g_{17}(\omega, \omega_1)$$

$$g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B} | \bar{h}(tn) \dots G_s^{\alpha\beta}(r\bar{n}) \dots h(0) | \bar{B} \rangle$$

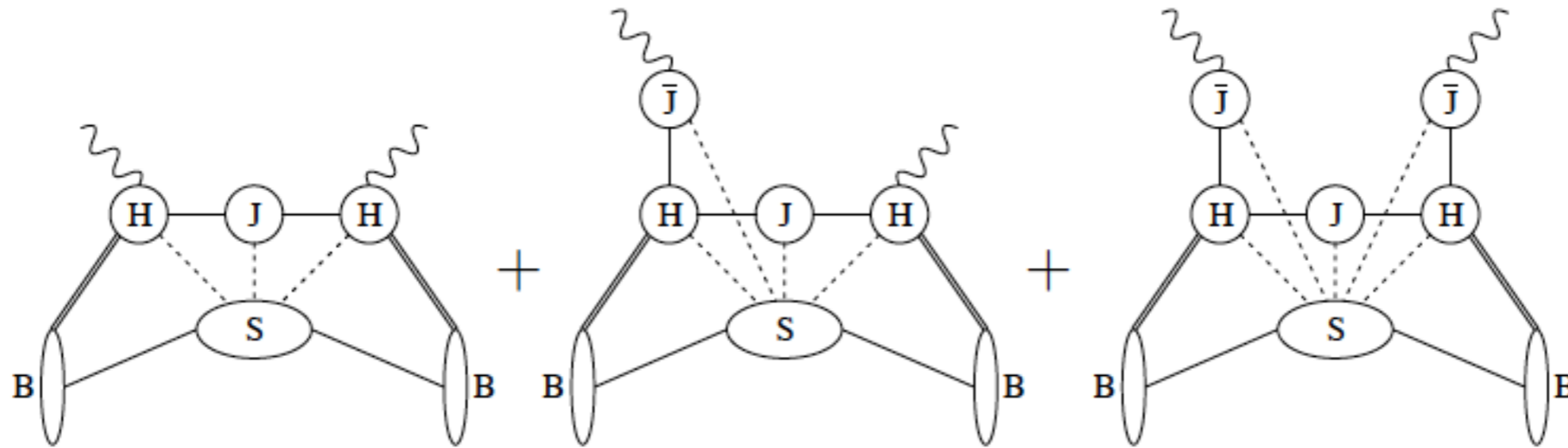
Expansion for $m_c \sim m_b$ leads to Voloshin term in the total rate ($-\lambda_2/m_c^2$), the terms stays non-local for $m_c < m_b$.



Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

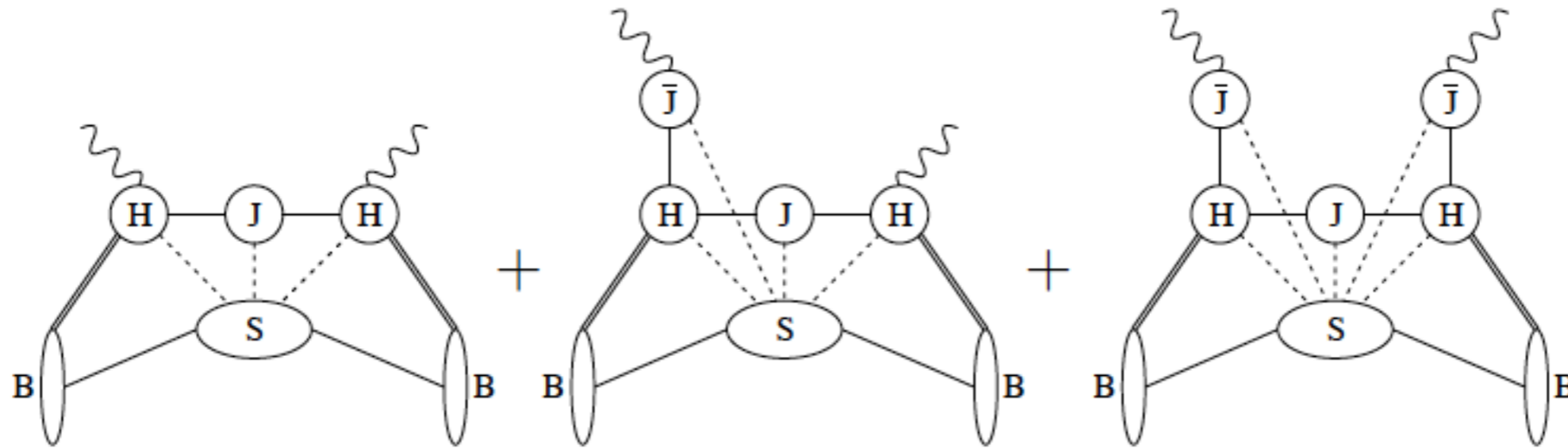
$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Subtlety in the Q_8 and \bar{Q}_8 contribution: convolution integral is UV divergent

- This subtlety implies that there is no complete proof of the factorization formula.
- Nevertheless one shows that scale dependence of direct and resolved contribution cancel.
- No direct analogy to the problem of IR divergent convolution integrals in power-suppressed contributions to exclusive B decays.



- Subleading shape functions of resolved contributions similar to $b \rightarrow s\gamma$
- Use explicit definition to determine properties:
 - * PT invariance: soft functions are real
 - * Moments of g_{17} related to HQET parameters
 - * Vacuum insertion approximation relates g_{78} to the B meson LCDA
- Perform convolution integrals with model functions



Numerical evaluation

- Subleading shape functions of resolved contributions similar to $b \rightarrow s\gamma$
- Use explicit definition to determine properties:
 - * PT invariance: soft functions are real
 - * Moments of g_{17} related to HQET parameters
 - * Vacuum insertion approximation relates g_{78} to the B meson LCDA
- Perform convolution integrals with model functions

Final result

$$\mathcal{F}_{17} \in [-0.5, +3.4] \%, \quad \mathcal{F}_{78} \in [-0.2, -0.1] \%, \quad \mathcal{F}_{88} \in [0, 0.5] \%$$

(normalized to OPE result)

$$\mathcal{F}_{1/m_b} \in [-0.7, +3.8]$$

$$\mathcal{F}_{19}: \quad O(1/m_b^2) \text{ but } |C_{9/10}| \sim 13|C_{7\gamma}|$$



Angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \left[(1 + z^2) H_T(q^2) + 2(1 - z^2) H_L(q^2) + 2z H_A(q^2) \right]$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

$$d\Gamma(\bar{B} \rightarrow X_s \ell^+ \ell^-) \equiv d\Lambda_{\alpha\beta} W^{\alpha\beta}(v, q),$$

$$d\Lambda_{\alpha\beta; 1/m_b} = dn \cdot q d\bar{n} \cdot q dz \frac{\alpha}{128\pi^3} (1 + z^2) \frac{n \cdot q}{\bar{n} \cdot q} g_{\perp, \alpha\beta}.$$

At $O(1/m_b)$ nonlocal powercorrections only to $H_T(q^2)$.



Power corrections in the inclusive mode

- For q anti-hard-collinear we have identified a new type of subleading power corrections.
- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.
- They constitute an irreducible uncertainty because they survive the $M_X \rightarrow 1$ limit.
- If q was hard then these resolved contributions would not exist

Nonlocal power corrections of $O(1/m_b^2)$ numerically relevant

M_X cut effects in the low- q^2 region with q^2 anti-hard-collinear

(work in progress)



Summary

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables available
- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events
- Nonlocal power corrections are under control and calculated to $O(1/m_b)$
- Theory predictions for inclusive $\bar{B} \rightarrow X_d \ell^+ \ell^-$ (including QED and power corrections) will be soon available (work in progress)
- Inclusive $\bar{B} \rightarrow X_s \ell \ell$ has a complementary role in new physics search to $\bar{B} \rightarrow X_s \gamma$ and $B \rightarrow K^{(*)} \ell \ell$





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Tilman Plehn Heidelberg U
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Bridging the Standard Model to New Physics with the Parity Violation Program at MESA
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The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment
Carlo Carloni Calame INFN Pavia, Massimo Passera INFN Padua,
Luca Trentadue U Parma, Graziano Venanzoni INFN Pisa
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Eric Bergshoeff U Groningen, Niels Obers NBI Copenhagen,
Dam Thanh Son U Chicago
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TOPICAL WORKSHOPS

The Evaluation of the Lepton
to the Muon Anomalous
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Feb

call for proposals 2019:
deadline 31.1.2018

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Extra

Subtlety in the high- q^2 region

Locally: breakdown of OPE in Λ_{QCD}/m_b in the high- s (q^2) endpoint
Partonic contribution vanishes in the limit $s \rightarrow 1$, while the $1/m_b^2$
corrections in $R(s)$ tend towards a nonzero value.

Theoretically: s -quark propagator in the correlator of OPE:

$$S_s(k) = \frac{\not{k} + i \not{D}}{k^2 + 2ik \cdot D - \not{D} \not{D} + i\varepsilon}.$$

Endpoint region of the q^2 spectrum in $\bar{B} \rightarrow X_s l^+ l^-$ different from endpoint
region of the photon spectrum of $\bar{B} \rightarrow X_s \gamma$:

$q^2 \approx m_b^2 \approx M_B^2 \Rightarrow k \sim \Lambda, \quad k^2 \sim \Lambda^2 \Rightarrow$ complete breakdown of OPE

no partial all-orders resummation possible, shape-function irrelevant
Buchalla, isidori

Practically: for integrated high- s (q^2) spectrum one finds an effective
expansion ($s_{\min} \approx 0.6$): Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

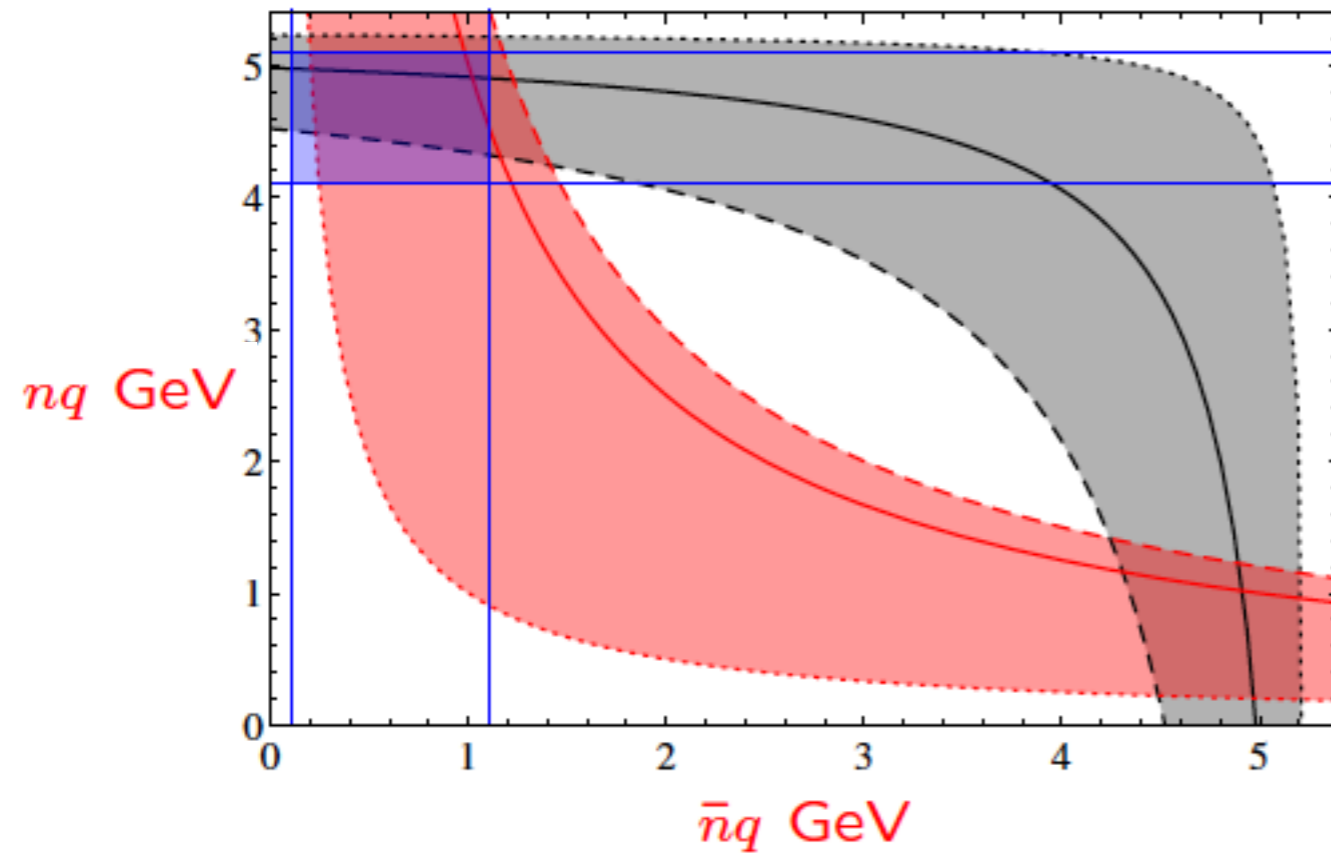
$$\int_{s_{\min}}^1 ds R(s) = \left[1 - \frac{1.6\lambda_2}{m_b^2(1 - \sqrt{s_{\min}})^2} + \frac{1.8\rho_1 + 1.7f_1}{m_b^3(1 - \sqrt{s_{\min}})^3} \right] \times \int_{s_{\min}}^1 ds R(s)|_{m_b \rightarrow \infty}$$



Allowed regions

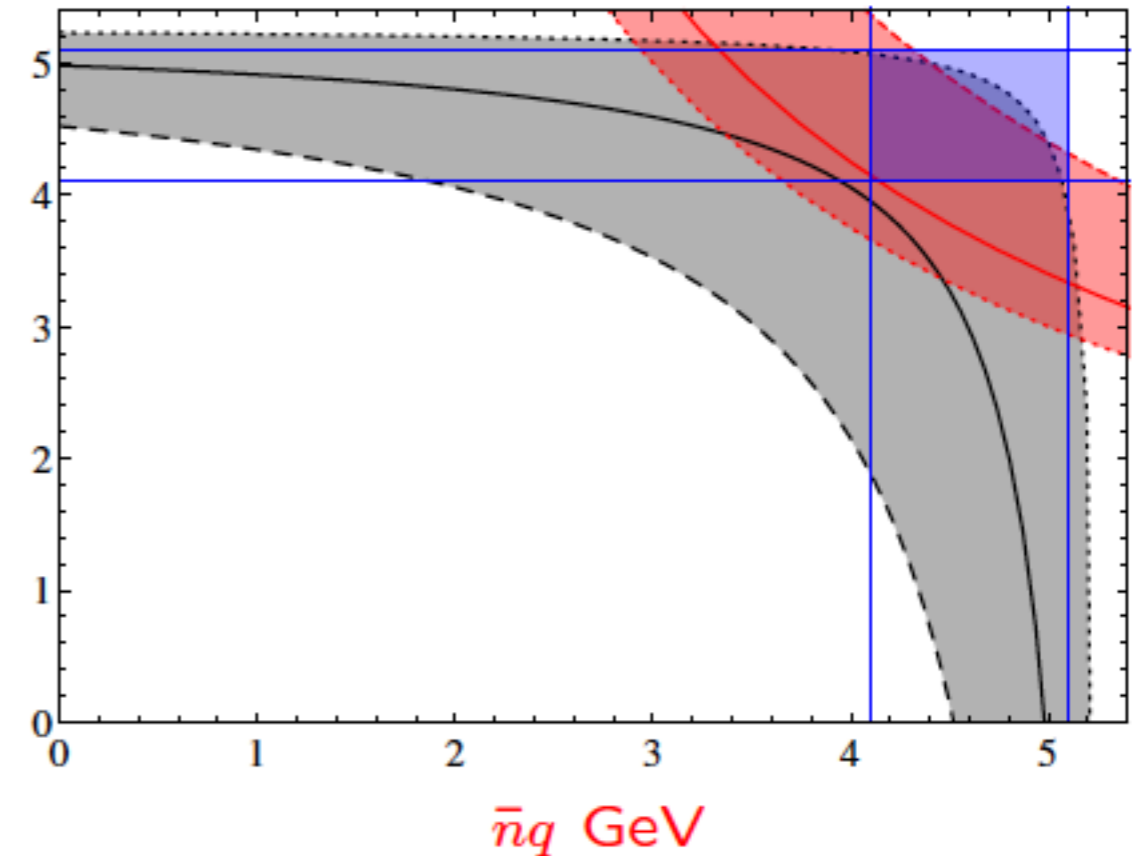
low- q^2

Red: $q^2 = [1, 5, 6] \text{ GeV}^2$ [Dotted, Solid, Dashed]
 Black: $M_x = [0.495, 1.25, 2] \text{ GeV}$ [Dotted, Solid, Dashed]
 Blue: anti-hard-collinear component scaling



high- q^2

Red: $q^2 = [15, 17, 22] \text{ GeV}^2$ [Dotted, Solid, Dashed]
 Black: $M_x = [0.495, 1.25, 2] \text{ GeV}$ [Dotted, Solid, Dashed]
 Blue: hard component scaling



Scaling

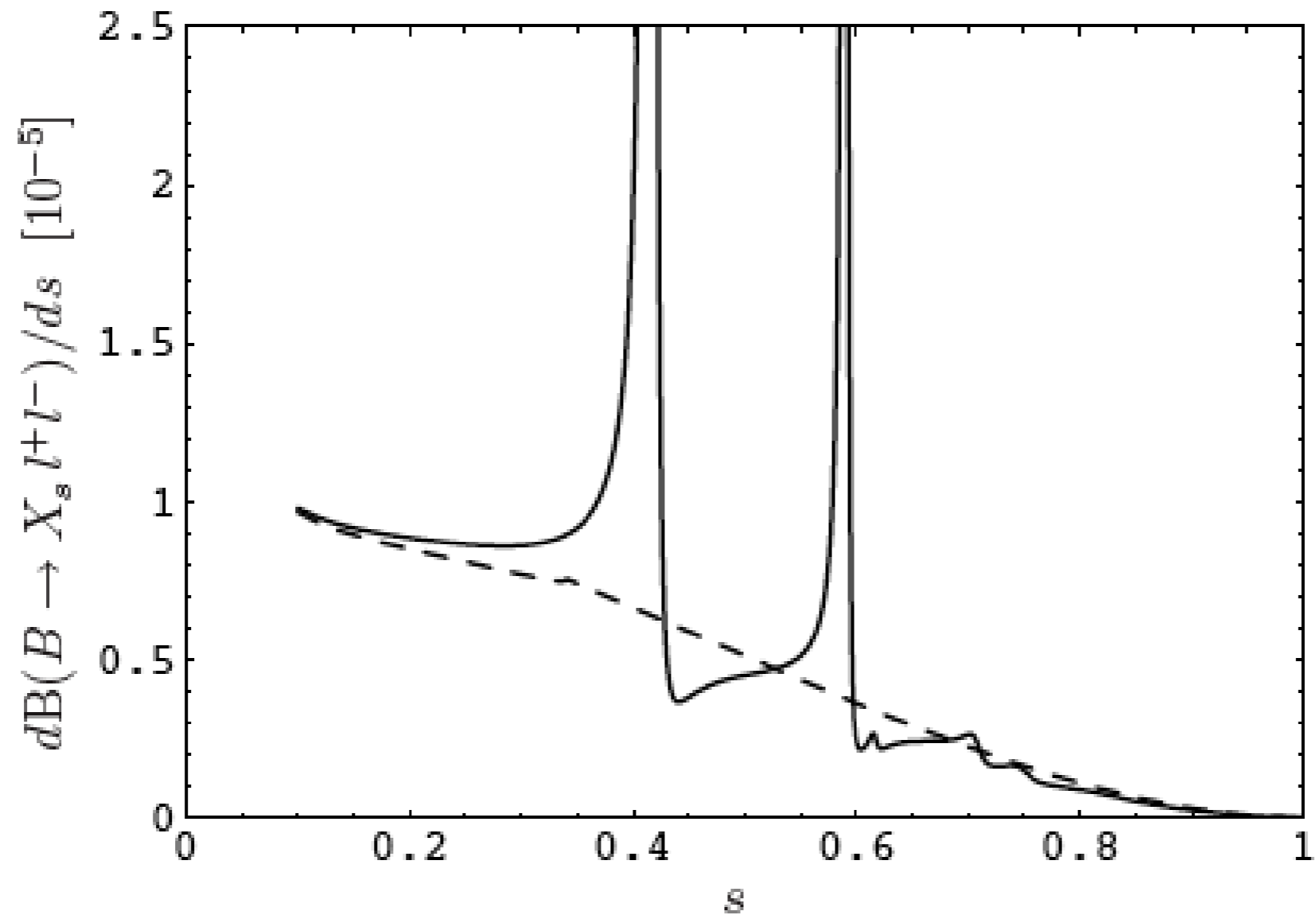
$$\lambda = \Lambda_{\text{QCD}}/m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$



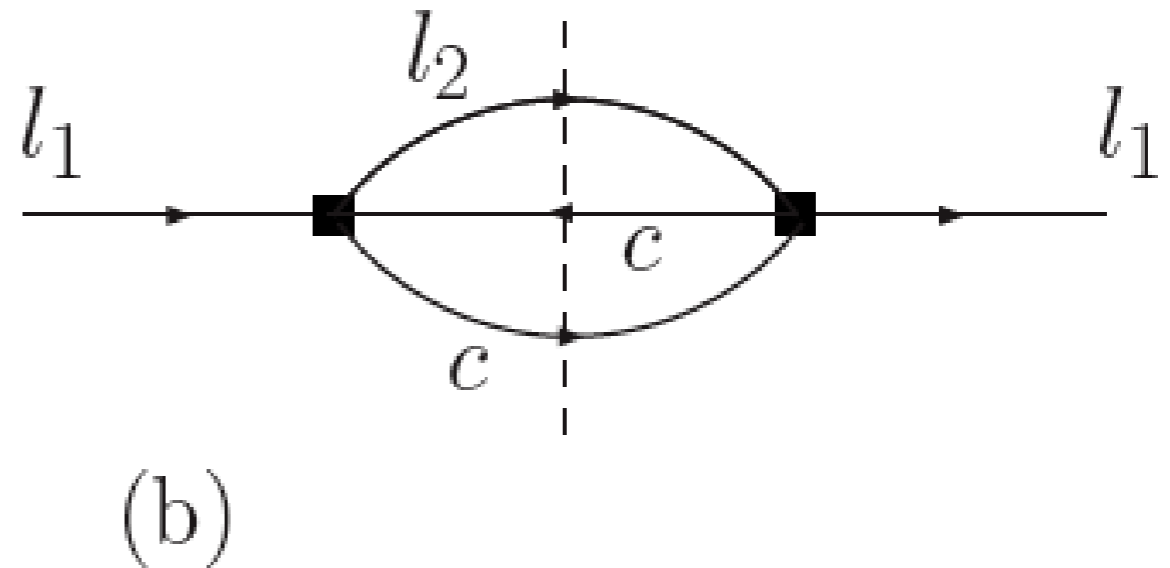
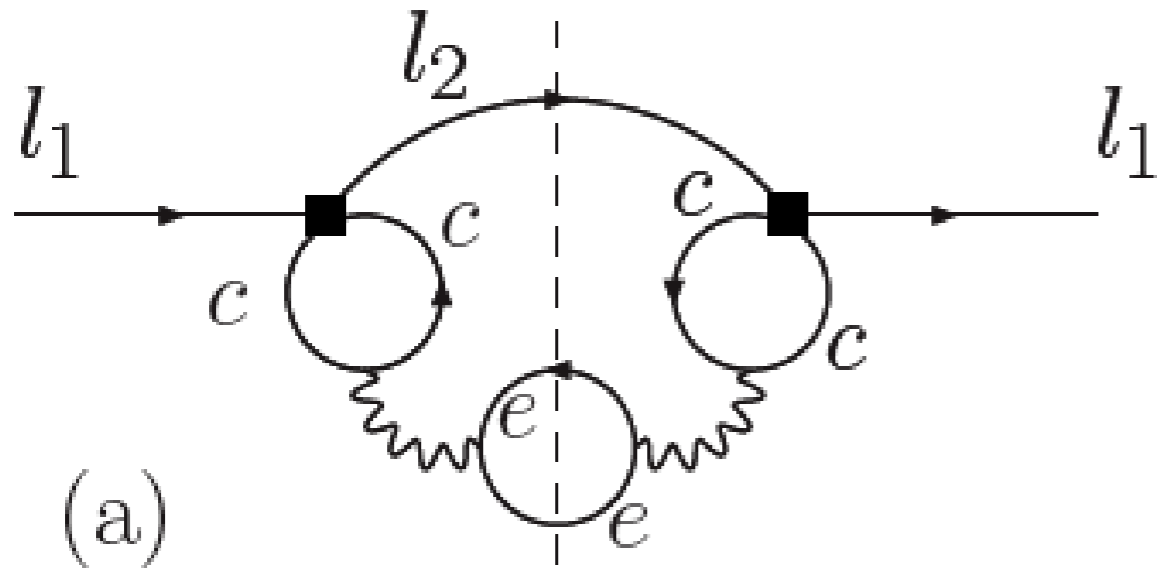
Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions **by two orders** of magnitude.



Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions **by two orders** of magnitude.



The rate $l_1 \rightarrow l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is **NOT** expected to hold.

In contrast the inclusive hadronic rate $l_1 \rightarrow l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

