

# W polarization in Vector Boson Scattering at the LHC

Alessandro Ballestrero  
Ezio Maina  
Giovanni Pelliccioli

Physics Department – University of Torino  
INFN – Sez. Torino



UNIVERSITÀ DEGLI STUDI DI TORINO



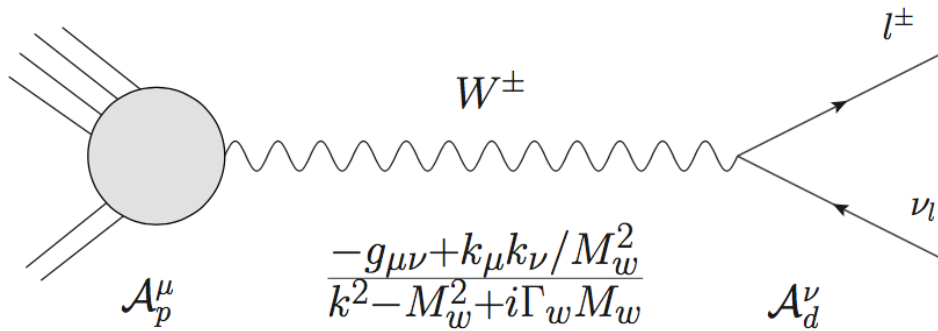
Istituto Nazionale di Fisica Nucleare

Scattering among longitudinally polarized on-shell VB play a central role in testing EWSB

What are the conceptual issues to be addressed when taking into account VB decay in a realistic environment?

Our proposal for defining and measuring VB polarizations in VBS at the LHC

# W polarization vs charged lepton angular distribution



$$\epsilon_{L/R}^\mu = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$\epsilon_0^\mu = (\kappa, 0, 0, E)/\sqrt{Q^2}$$

$$E \gg M_W \quad \epsilon_0^\mu \approx p_W^\mu / M_W$$

$$p_W^\mu = (E, 0, 0, \kappa)$$

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2} = \sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*} \quad \Rightarrow \quad \mathcal{A}_f = \sum_\lambda \frac{\mathcal{A}_p^\mu \epsilon_\mu^\lambda \epsilon_\nu^{\lambda*} \mathcal{A}_d^\nu}{k^2 - M_W^2 + i\Gamma_W M_W} = \sum_\lambda \mathcal{A}_f^\lambda$$

$$\underbrace{|\mathcal{A}_f|^2}_{\text{coherent sum}} = \underbrace{\sum_\lambda |\mathcal{A}_f^\lambda|^2}_{\text{incoherent sum}} + \underbrace{\sum_{\lambda \neq \lambda'} \mathcal{A}_f^{\lambda*} \mathcal{A}_f^{\lambda'}}_{\text{interference term}}$$

No need to be on-shell

The substitution:

$$\sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*} \rightarrow \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*}$$

defines polarized amplitudes

$$\epsilon_0^\mu \mathcal{A}_d^\mu = \mathcal{A}_d^0 = ig \sqrt{2} E \sin \theta$$

$$\epsilon_{R/L}^\mu \mathcal{A}_d^\mu = \mathcal{A}_d^{R/L} = -ig E (1 \mp \cos \theta) e^{\pm i\phi}$$

# Single $W \rightarrow l\nu$ differential cross section

$$\frac{d\sigma}{dX d\cos\theta d\phi} \propto |\mathcal{A}_p^0|^2 \sin^2\theta + |\mathcal{A}_p^R|^2 (1 - \cos\theta)^2 + |\mathcal{A}_p^L|^2 (1 + \cos\theta)^2$$

$$+ 2\text{Re}(\mathcal{A}_p^R \mathcal{A}_p^{L*} e^{2i\phi})(1 - \cos^2\theta) + 2\text{Re}(\mathcal{A}_p^R \mathcal{A}_p^{0*} e^{i\phi})(1 - \cos\theta) \sin\theta$$

$$+ 2\text{Re}(\mathcal{A}_p^L \mathcal{A}_p^{0*} e^{-i\phi})(1 + \cos\theta) \sin\theta$$

**BLUE TERMS** cancel ONLY WHEN INTEGRATED OVER  $\phi$ . In practice **NEVER**.

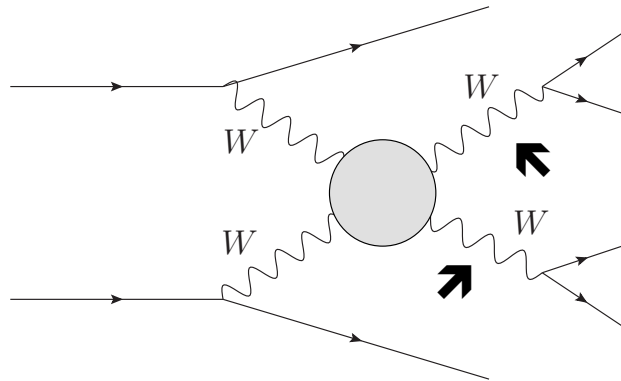
In this case:

$$\frac{1}{\sigma} \frac{d\sigma}{dX d\cos\theta} = \frac{3}{4} f_0(X) \sin^2\theta + \frac{3}{8} f_R(X) (1 - \cos\theta)^2 + \frac{3}{8} f_L(X) (1 + \cos\theta)^2$$

Polarization fractions extracted projecting  $\cos\vartheta$  distribution on first 3 Legendre polynomials  
Does not work with cuts.

**Interference among pols. for any  $W$  production channel**, even in Narrow Width Approx.

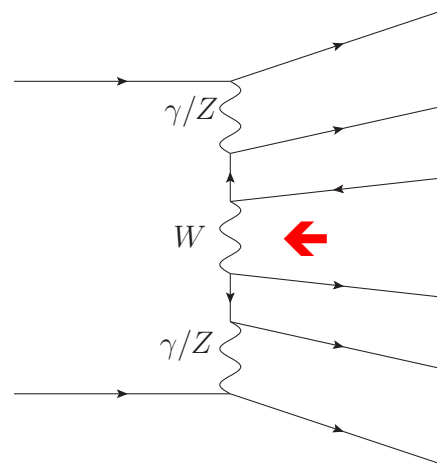
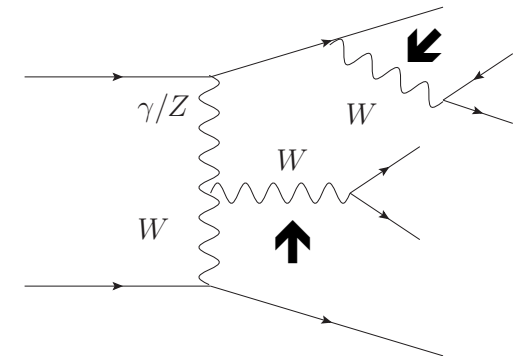
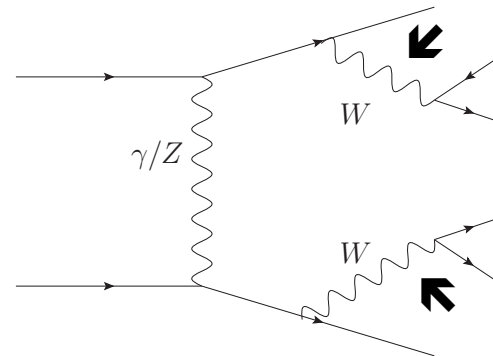
# VBS: more vector bosons, more fun



No resonant propagator.  
Cannot be interpreted  
as W production.

Necessary for gauge  
invariance!  
Numerically relevant in  
some phase space regions

Two resonant W propagators



$$\mathcal{A}_f = \mathcal{A}_{RES} + \mathcal{A}_{NONRES}$$

To define an amplitude  
in which a W is polarized  
an approximation which  
considers only resonant  
diagrams is NECESSARY

# On Shell Projection (OSP)

Is it possible to devise an APPROXIMATION which

- uses only **doubly** resonant diagrams
- reproduces well the exact results over most of phase space?

$$\mathcal{A}_f = \sum_{\lambda} \frac{\mathcal{A}_{p,RES}^{\mu}(p, k) \varepsilon_{\mu}^{\lambda} \varepsilon_{\nu}^{\lambda*} \mathcal{A}_d^{\nu}(k, q)}{k^2 - M_W^2 + i\Gamma_W M_W} + \mathcal{A}_{NONRES} \quad \Rightarrow \quad \sum_{\lambda} \frac{\mathcal{A}_{p,RES}^{\mu}(p, k_{OSP}) \varepsilon_{\mu,OSP}^{\lambda} \varepsilon_{\nu,OSP}^{\lambda*} \mathcal{A}_d^{\nu}(k_{OSP}, q_{OSP})}{k^2 - M_W^2 + i\Gamma_W M_W}$$

Gauge invariant provided  $\Gamma_W, \Gamma_Z \rightarrow 0$  in  $\mathcal{A}_{p,RES}^{\mu}$  and  $\cos \theta_W, \sin \theta_W$

Similar to DPA Denner, Dittmaier, Roth, Wakeroth NP B587(2000)67

Not uniquely defined. To fully specify conserve:

1. the total four-momentum of the  $WW$  system;
2. the direction of the two  $W$  bosons in the  $WW$  center of mass frame;
3. the direction of each charged lepton in his  $W$  center of mass frame.

Applicable only for  $M_{WW} > 2 M_W$

# PHANTOM

Unitary gauge, Complex Mass scheme

Can compute singly or doubly polarized amplitudes

Public version in preparation

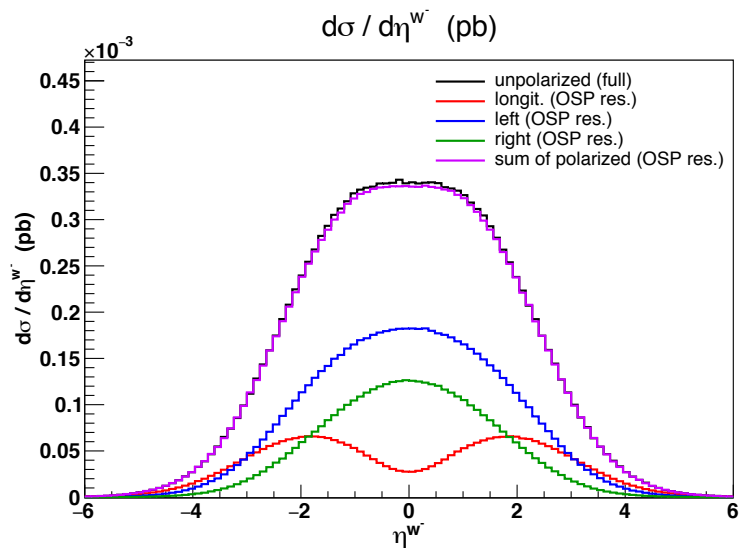
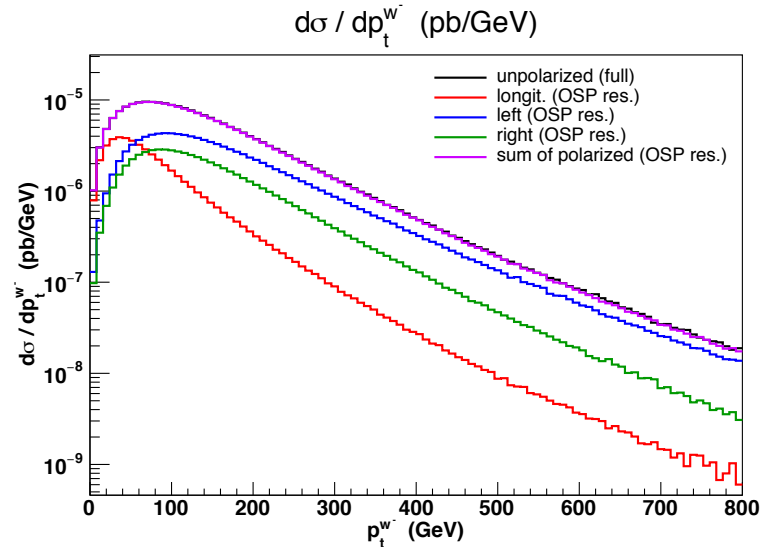
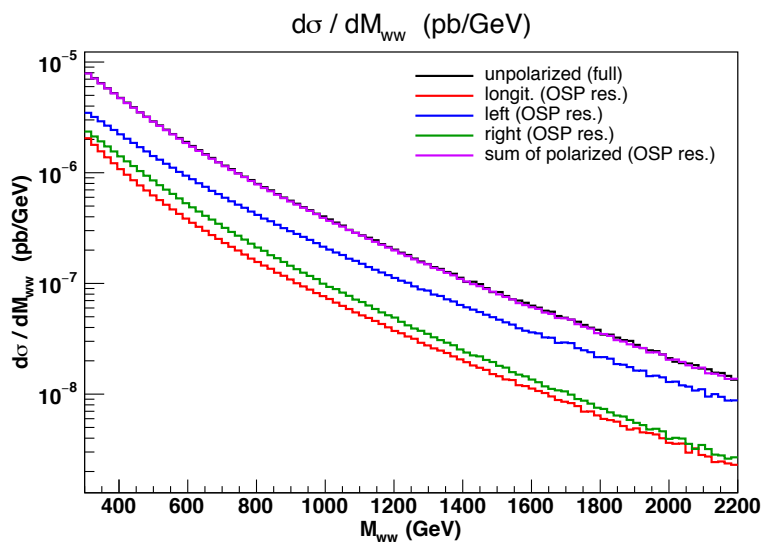
$$pp \rightarrow jj e^- \bar{\nu}_e \mu^+ \nu_\mu \mathcal{O}(\alpha_{EM}^6)$$

W+W- leptonic W CM not reconstructable

$$|\eta_j| < 5, p_t^j > 20 \text{ GeV}, M_{jj} > 600 \text{ GeV}, |\Delta\eta_{jj}| > 3.6$$

$$M_{WW} > 300 \text{ GeV}$$

# Validation and Results



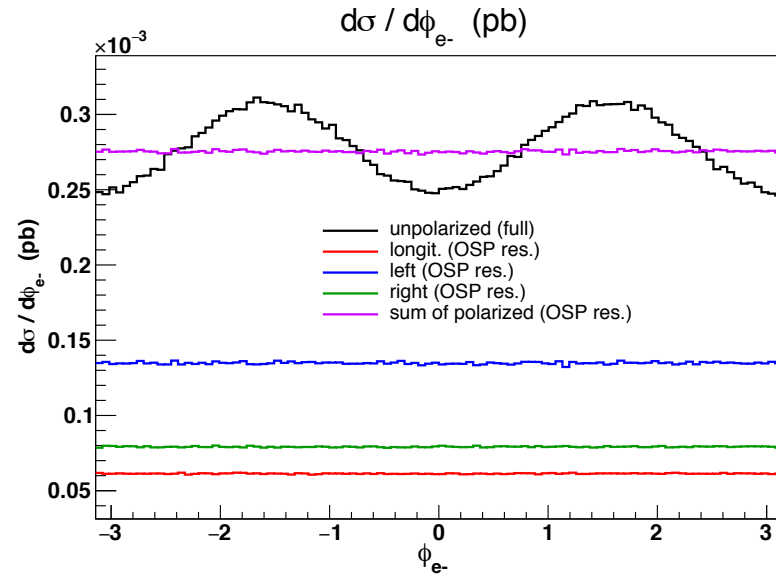
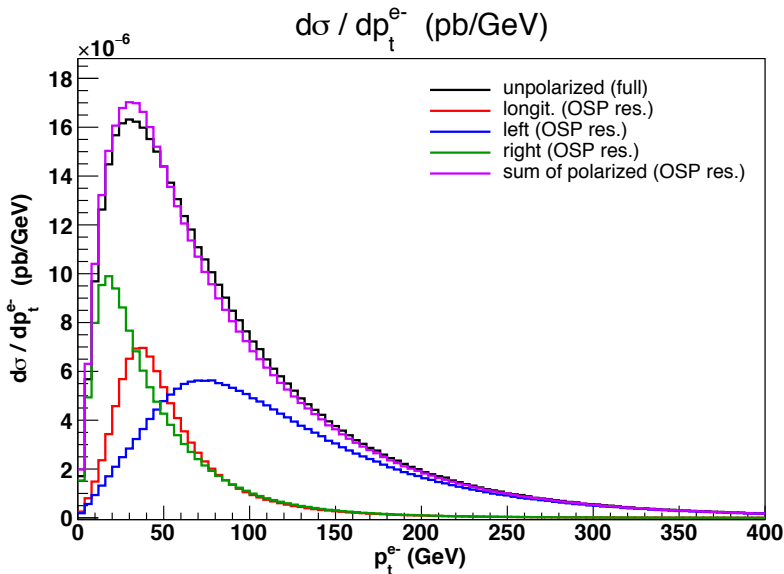
Full
  L
  R
  0
  Sum

W- polarized, W+ unpolarized.  
 Variables which do not limit the range of  $\varphi$ .  
 No interference.

Exact cross section	1.748 fb
Sum of polarized OSP	1.731 fb
Diff $\approx$ 1%	



# Validation and Results 2

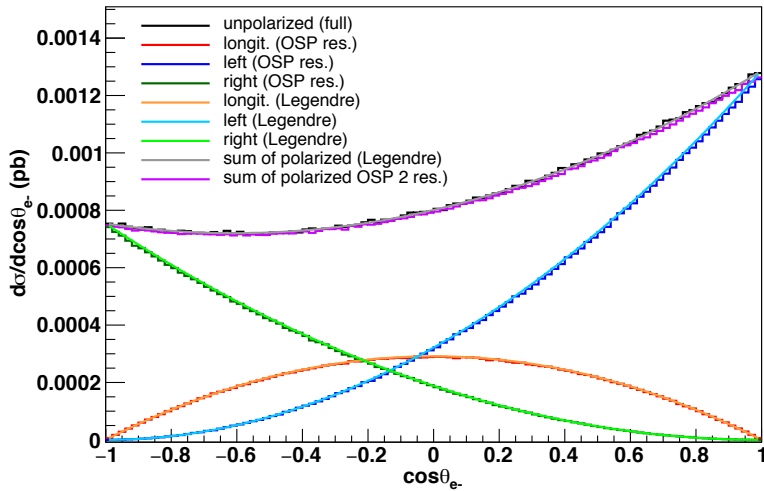


Variables which do limit the range of  $\varphi$ . Interference among polarizations.

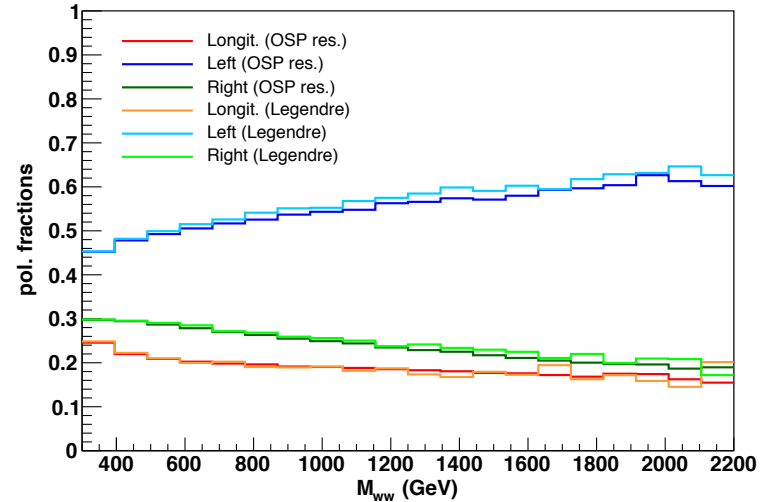
Different polarizations have different kinematical distributions

# Polarization fractions

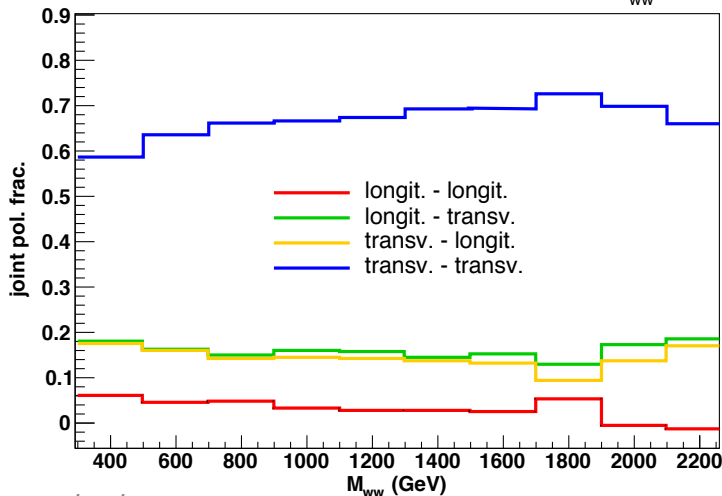
$d\sigma / d\cos\theta_{e^-}$  (pb),  $M_{\nu\nu} > 300$  GeV



Polarization fractions as functions of  $M_{\nu\nu}$

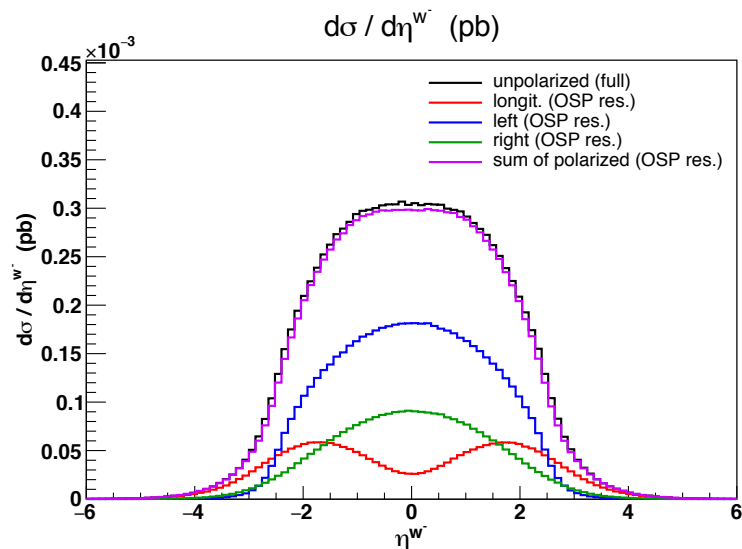
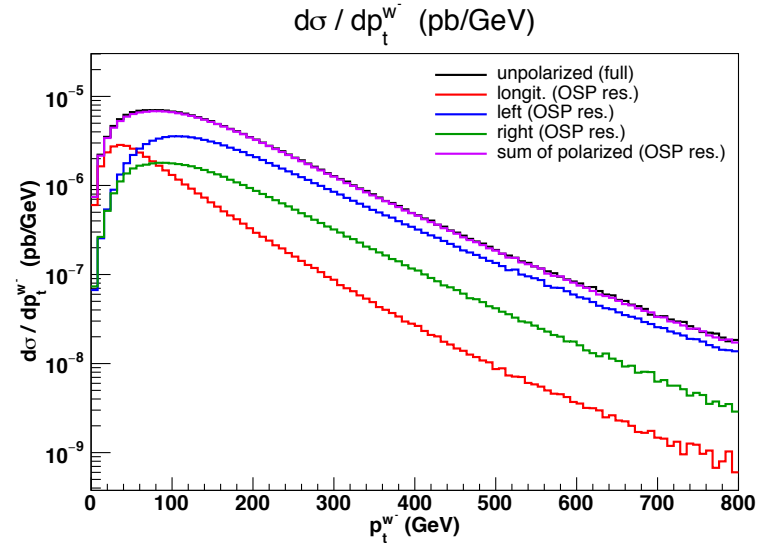
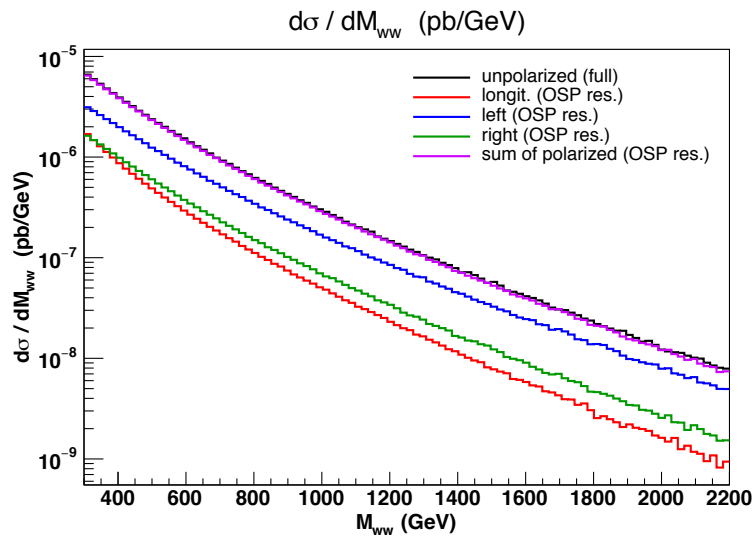


Joint polarization fractions as functions of  $M_{\nu\nu}$



Polarization content through Legendre analysis  
and through direct computation of polarized  
cross section agree!  
Consistent prediction!

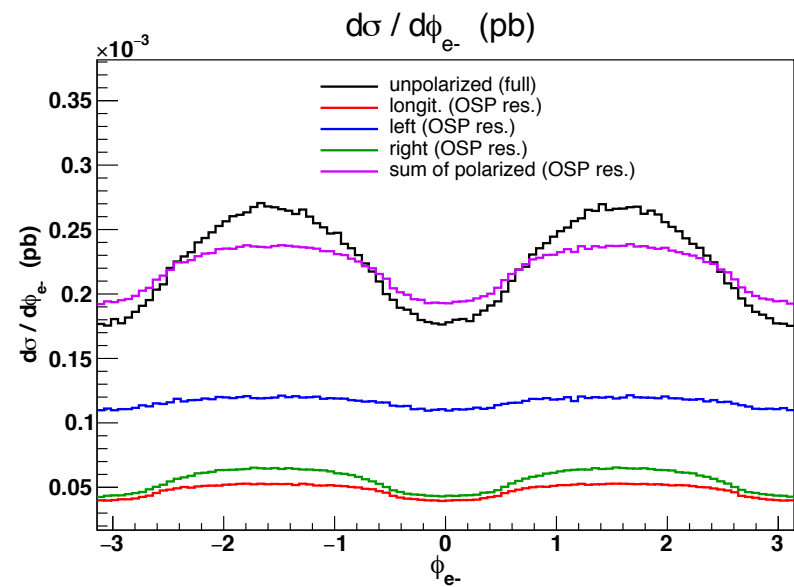
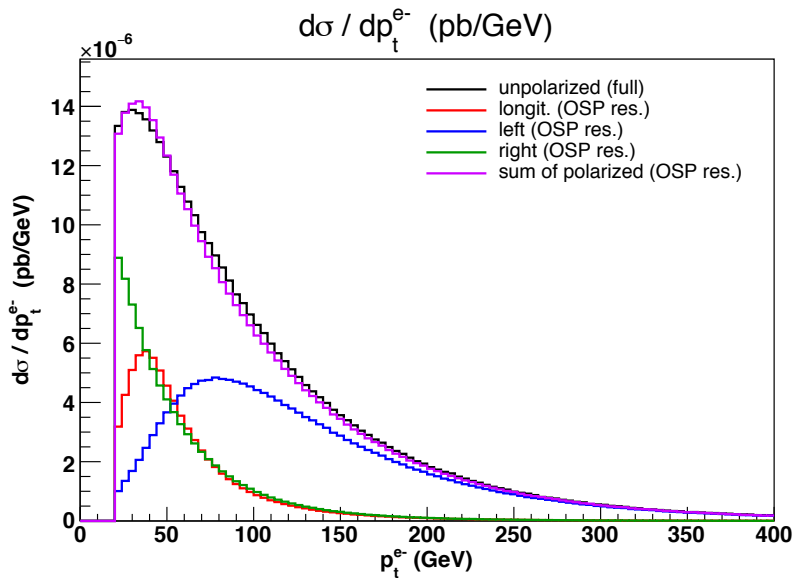
# Introducing Lepton Cuts: Results



$$p_t^\ell > 20 \text{ GeV}, |\eta^\ell| < 2.5$$

Exact cross section	1.412 fb
Sum of polarized OSP	1.378 fb
Diff $\approx$ 2%	

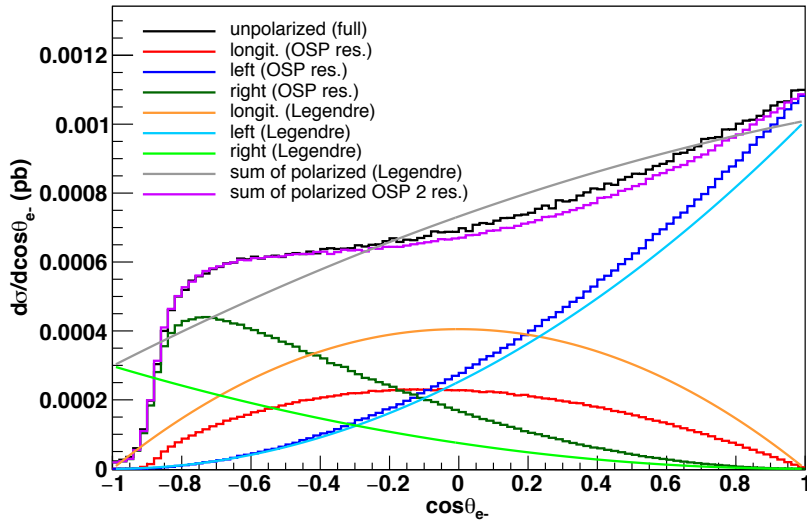
# Introducing Lepton Cuts: Results 2



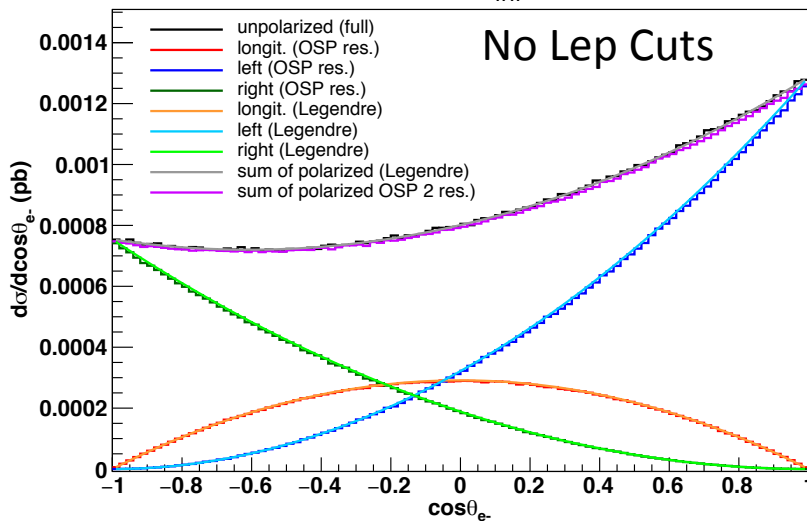
$$p_t^\ell > 20 \text{ GeV}, |\eta^\ell| < 2.5$$

# Polarization fractions with cuts

$d\sigma / d\cos\theta_{e^-}$  (pb),  $M_{\nu\nu} > 300$  GeV



$d\sigma / d\cos\theta_{e^-}$  (pb),  $M_{\nu\nu} > 300$  GeV



Pols are affected differently by cuts  
Mainly at  $\vartheta = \pi$

Legendre expansion fails

Interference among pols is small

Sum of singly polarized distributions  
reproduces within few % the exact result

Interference terms can be extracted from MC

Different kinematical distributions can be  
exploited for fit

# Conclusions

For the first time we have a consistent framework to describe VB polarizations in VBS

OSP provides a gauge invariant, good approximation to exact VBS which allows to predict Vector Bosons polarization fractions

Works very well for variables which do not restrict the lepton decay angles

Singly polarized distributions provide templates for measuring the polarization fractions of the  $W$ 's in the presence of reasonable acceptance cuts

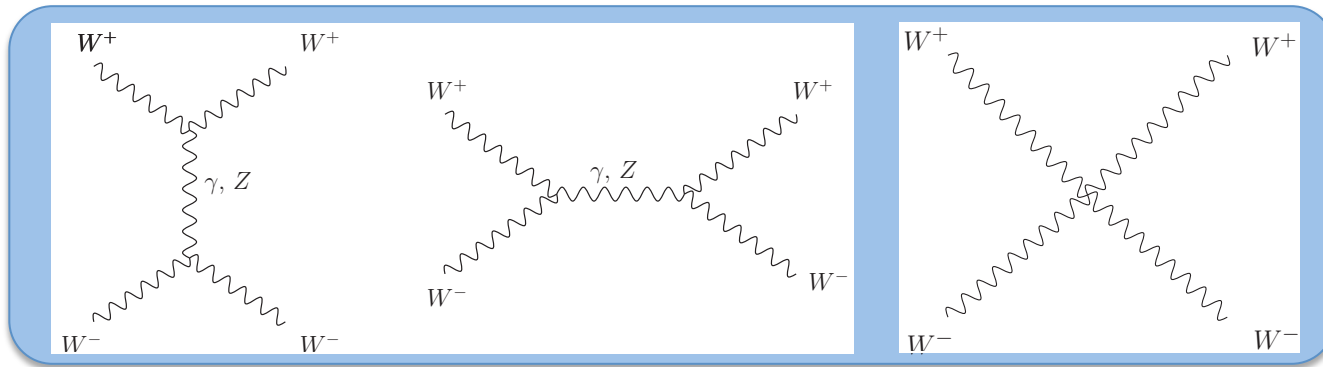
# Spares

# Longitudinal polarization, gauge invariance, unitarity

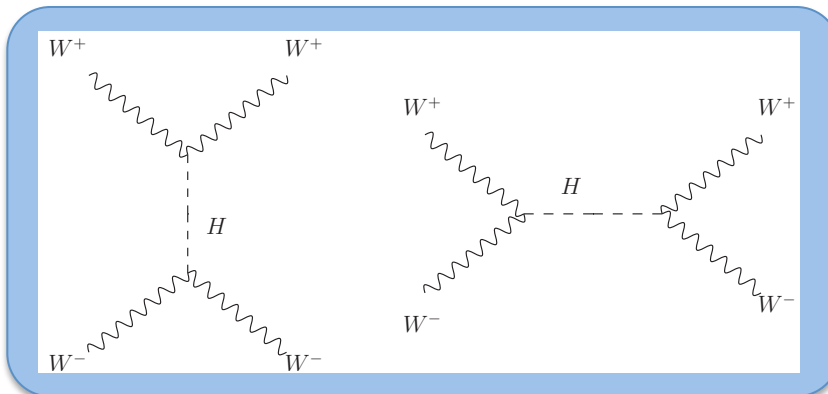
EWSB gives mass to W, Z. Massive vector bosons have three physical polarization states.

$$\varepsilon_{L/R}^\mu = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \quad \varepsilon_0^\mu = (\kappa, 0, 0, E)/\sqrt{Q^2} \quad E \gg M_W \quad \varepsilon_0^\mu \approx p_W^\mu/M_W \quad p_W^\mu = (E, 0, 0, \kappa)$$

$$\varepsilon_0^{W^+} \cdot \varepsilon_0^{W^-} \propto p^{W^+} \cdot p^{W^-} = s \quad \Rightarrow \quad D_i \propto s^2$$



$$\Sigma \propto s^1$$



$$\Sigma \propto s^1$$

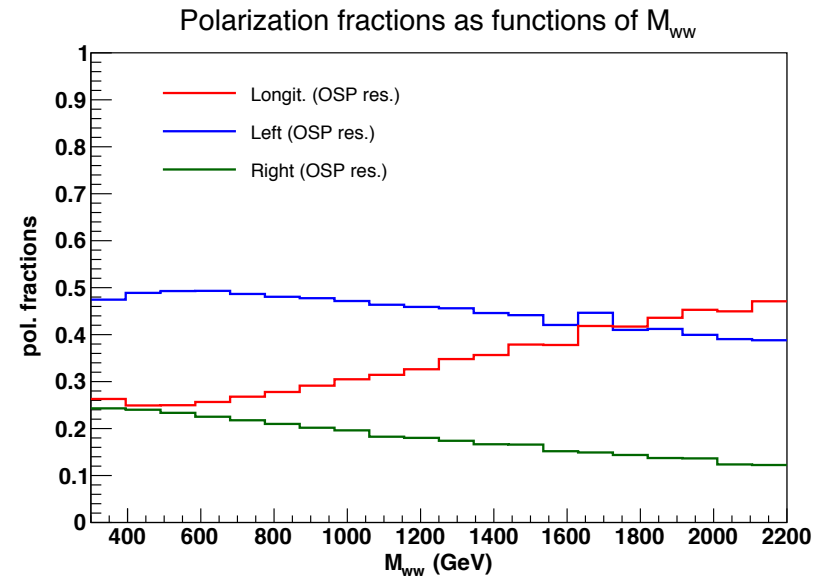
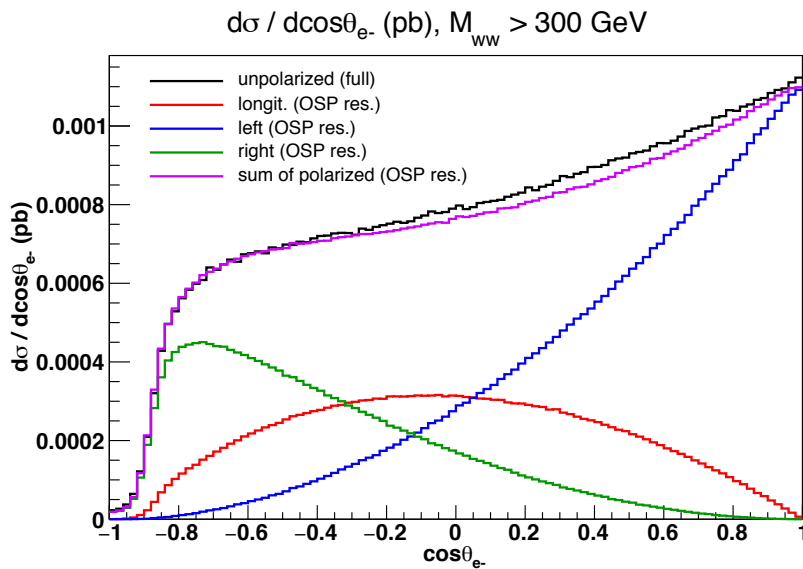
$$\Sigma \propto s^0$$



# Model (in)dependence

Do we have to repeat the analysis for each model separately?

Higgsless model: SM with  $m_h \rightarrow \infty$ , no cancellation of terms  $\propto s$  in VBS  
 Unphysical but maximizes differences compared to SM

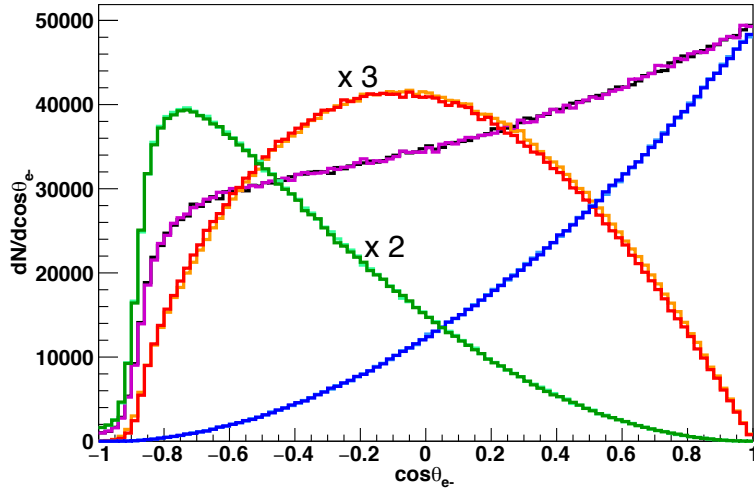


Exact cross section	1.543 fb
Sum of polarized	1.501 fb

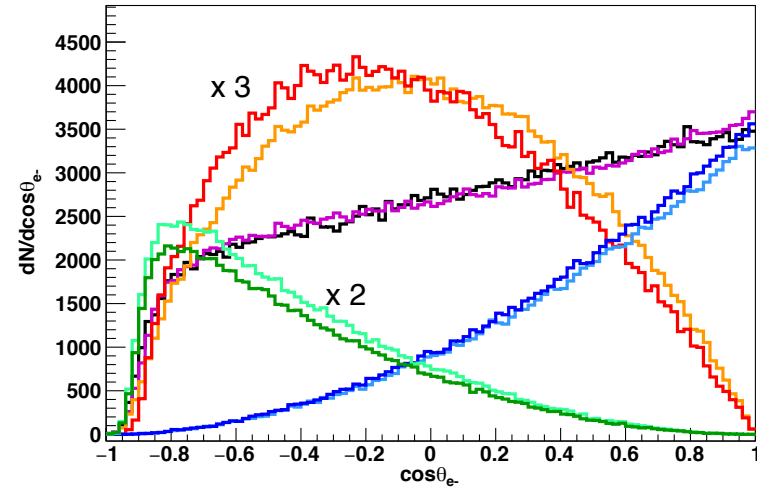
$p_t^\ell > 20$  GeV,  $|\eta^\ell| < 2.5$

# Fitting the noH model using SM shapes

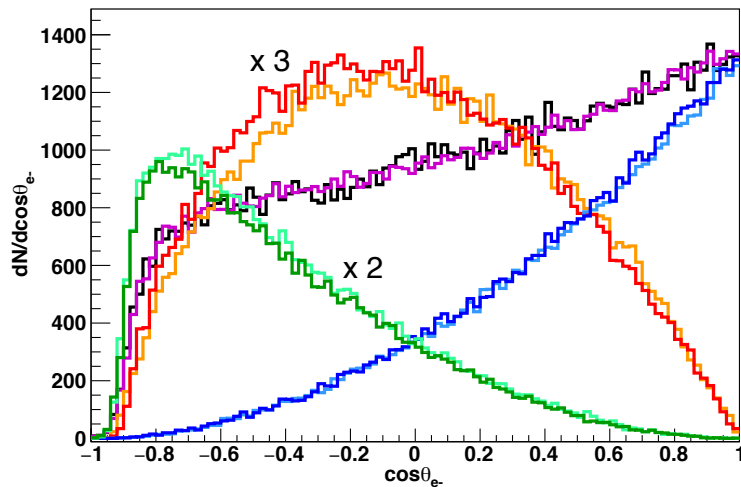
SM fit of  $\cos\theta_e$  distributions,  $M_{\text{ww}} > 300$  GeV



SM fit of  $\cos\theta_e$  distributions,  $M_{\text{ww}} > 1000$  GeV



SM fit of  $\cos\theta_e$  distributions,  $1000 \text{ GeV} < M_{\text{ww}} < 1100$  GeV



Black: exact noH

Light colours: singly polarized noH

Dark colours: fit of exact noH using SM shapes

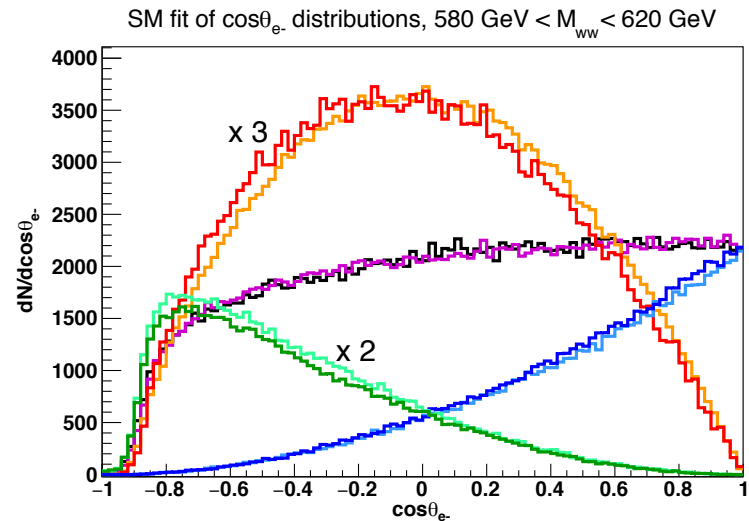
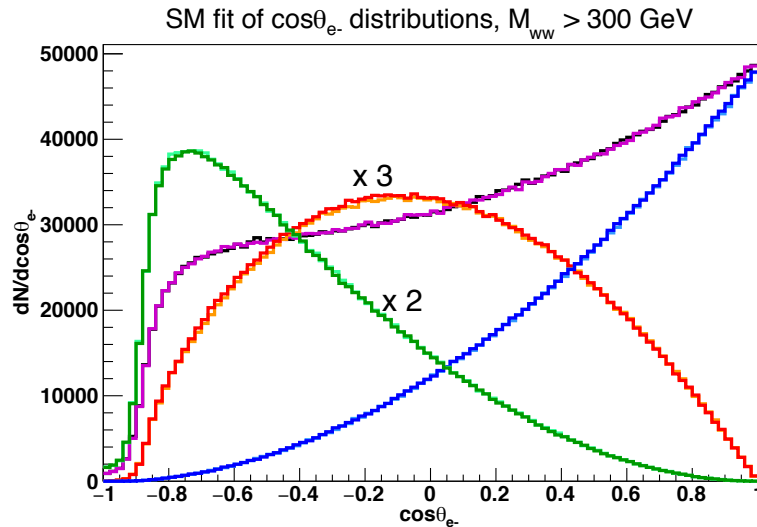
	Long.	L	R	Int.
$M_{WW} > 300 \text{ GeV}$				
SM	21	52	25	2
no Higgs	27	48	23	2
Fit	26	48	23	2
$M_{WW} > 1000 \text{ GeV}$				
SM	15	58	22	4
no Higgs	35	45	17	3
Fit	35	47	15	2

**Table 1.** Polarization fractions in percent.

Fit uses SM shapes

# Fitting the Singlet model using SM shapes

Singlet model: one extra Higgs,  $m_H = 600$  GeV,  $\Gamma_H = 6.5$  GeV,  $\sin\alpha = 0.2$



Black: exact Singlet

Light colours: singly polarized Singlet

Dark colours: fit of exact Singlet using SM shapes

# W polarization in other processes

Bern et al., PhysRevD.84.034008; arXiv:1103.5445

W+jets, no lepton cuts

Stirling, Vryonidou, JHEP07(2012)124; arXiv:1204.6427 W+jets, top  $\rightarrow$  W, WW, WZ, WH

both with and without lepton cuts

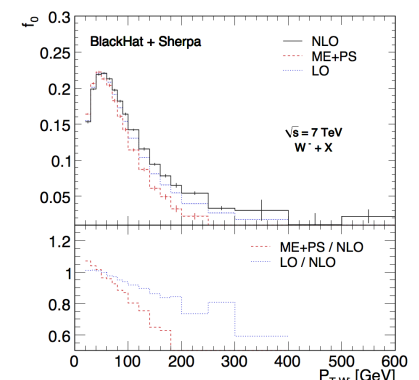
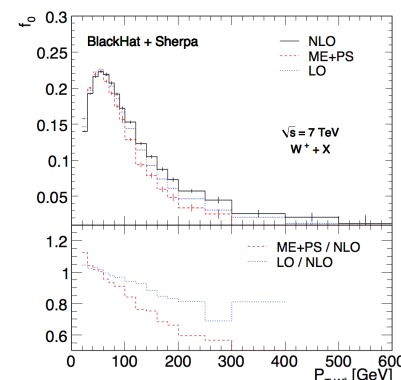
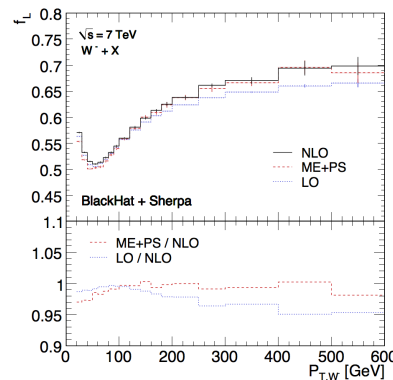
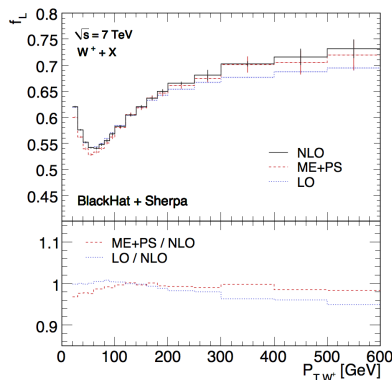
$$\frac{1}{\sigma} \frac{d\sigma}{dX d\cos\theta} = \frac{3}{4} f_0(X) \sin^2\theta + \frac{3}{8} f_R(X) (1 - \cos\theta)^2 + \frac{3}{8} f_L(X) (1 + \cos\theta)^2$$

$$f_0 = 2 - 5\langle\cos\theta^{*2}\rangle,$$

$$f_L = -\frac{1}{2} - \langle\cos\theta^*\rangle + \frac{5}{2}\langle\cos\theta^{*2}\rangle,$$

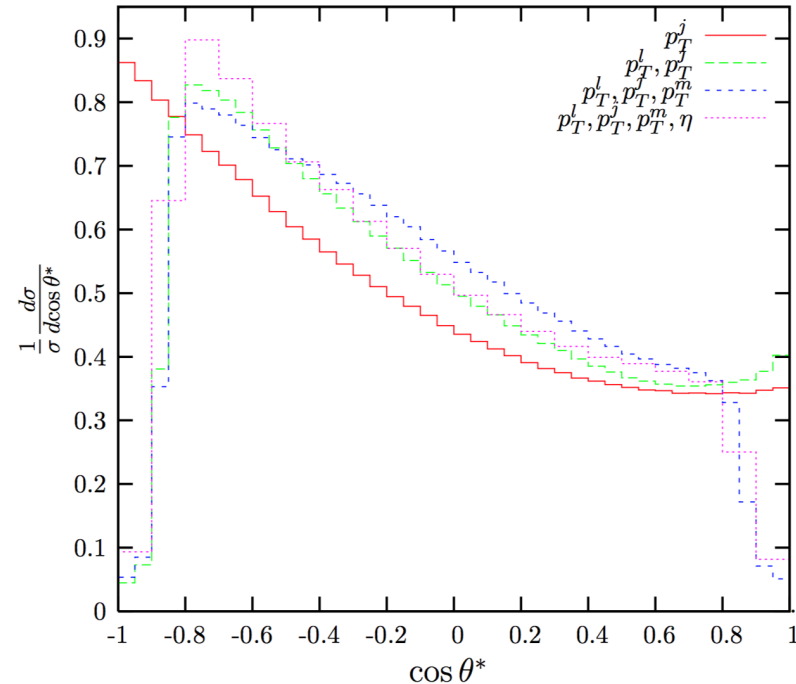
$$f_R = -\frac{1}{2} + \langle\cos\theta^*\rangle + \frac{5}{2}\langle\cos\theta^{*2}\rangle.$$

Bern et al.



# W pol with selection cuts in W+Jets

Stirling, Vryonidou



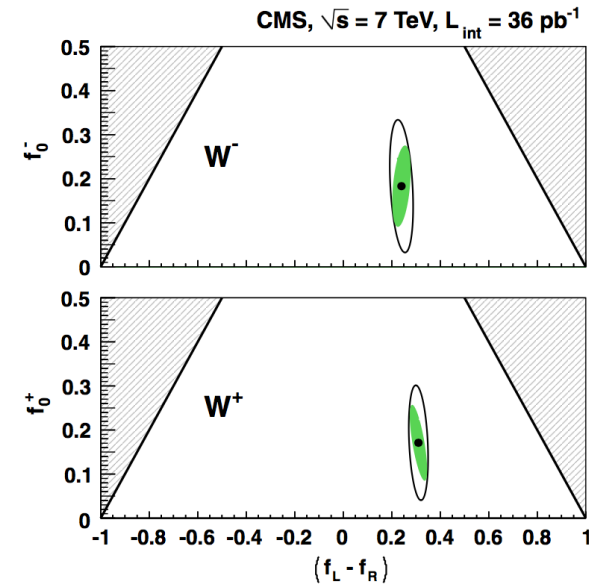
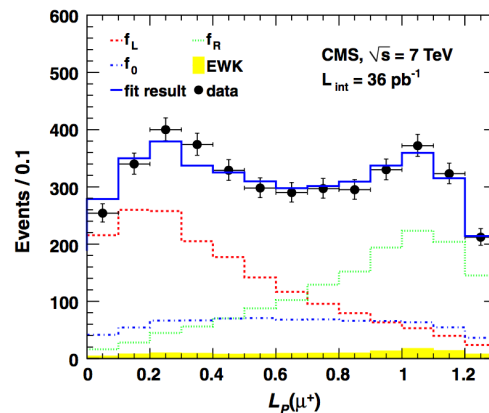
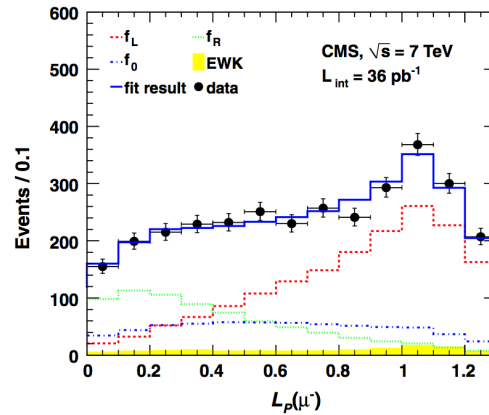
Cuts	" $f_0$ "	" $f_L$ "	" $f_R$ "
$p_T^j > 30 \text{ GeV}$	0.20	0.56	0.23
$p_T^j > 20 \text{ GeV}$	0.18	0.59	0.23
$p_T^j > 20 \text{ GeV}, p_T^l > 20 \text{ GeV}$	0.50	0.35	0.15
$p_T^j > 20 \text{ GeV}, p_T^l > 20 \text{ GeV}, p_T^m > 20 \text{ GeV}$	0.68	0.29	0.03
$p_T^j > 20 \text{ GeV}, p_T^l > 20 \text{ GeV}, p_T^m > 20 \text{ GeV},  \eta_{l,j}  < 2.5$	0.59	0.36	0.05

# Experimental results CMS

CMS, PhysRevLett.107.021802; arXiv:1104.3829 W+jets, 7 TeV, 35 pb<sup>-1</sup>

CMS

$$L_P = \frac{\vec{p}_T(\ell) \cdot \vec{p}_T(W)}{|\vec{p}_T(W)|^2}$$



# Experimental results ATLAS

ATLAS, Eur.Phys. J. C72(2012)2001; arXiv:1203.2165  
 W+jets, 7 TeV, 35 pb<sup>-1</sup>

$$\cos \theta_{2D} = \frac{\vec{p}_T^{\ell*} \cdot \vec{p}_T^W}{|\vec{p}_T^{\ell*}| |\vec{p}_T^W|}$$

