Why Be Natural?

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1. How to Construct an EFT.

2. Why Be Natural?
   - Modest empirical success.
   - Quantifiable.
   - Consistent with "Central Dogma".

3. Naturalness and Emergence.

4. Conclusion.

Less than compelling!

More interesting...
1. How to Construct an EFT.

(Wilson Version):

Given a theory described by $S[\phi, \partial \phi], \quad$ (Polchinski 1994)
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2. Integrate out $\phi_H$ to obtain the Wilsonian effective action $S_\Lambda[\phi_L]$. 

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3. Expand the effective action in a set of local operators $O_i[\phi_L, \partial \phi_L]$.

$$S_\Lambda = S_0 + \int d^Dx \sum_i g_i O_i$$

*encode high-energy DOF*
1. How to Construct an EFT.

4. Perform dimensional analysis on $S_\Lambda$. For $E \ll \Lambda$:

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- Use $S_0$ to determine $\delta_i$.

(i) Define dimensionless $\lambda_i \equiv \Lambda^{\delta_i-D} g_i$

(ii) The order of the $i$th term is $\lambda_i (E/\Lambda)^{\delta_i-D}$.

- Irrelevant term: $\delta_i > D$. Falls as $E \to 0$.
- Relevant term: $\delta_i < D$. Grows as $E \to 0$.
- Marginal term: $\delta_i = D$. Constant as $E \to 0$. 
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Ideal? Insensitive to high-energy DOF.

Worrisome? Indicates sensitivity to high-energy DOF.
1. How to Construct an EFT.

**Ex.** Scalar field theory in 4-dim ($\Phi \rightarrow -\Phi$ symmetry).

$$S_\Lambda[\Phi_L] = \frac{1}{2} \int d^4 x (\partial_\mu \Phi_L)^2 + \int d^4 x \left[ \lambda_2 \Lambda^4 + \lambda_0 \Lambda^2 \Phi_L^2 + \lambda_2 \Phi_L^4 + \lambda_4 \Lambda^{-2} \Phi_L^6 + \cdots \right]$$

$$+ \int d^4 x \left[ \sum_{n>0} \lambda_n' \Lambda^{-n} (\partial_\mu \Phi_L)^2 \Phi_L^n + \sum_{n\geq 0} \lambda_n'' \Lambda^{-(n+4)} (\partial_\mu \Phi_L)^4 \Phi_L^n + \cdots \right]$$
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- \(\Phi_L\) must have units \(E^\delta\) satisfying \(E^{-4}E^2E^{2\delta} = E^0\), thus \(\delta = 1\).
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**Relevant terms:**

- **Additive term:** \( \lambda_{-2} \Lambda^4 \)
  - *quartic dependence on cut-off.*

- **Mass term:** \( \lambda_0 \Lambda^2 \Phi_L^2 \)
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**Relevant terms:**
- Additive term: $\lambda_{-2}\Lambda^4$
  - quartic dependence on cut-off.
- Mass term: $\lambda_0\Lambda^2\Phi_L^2$
  - quadratic dependence on cut-off.

**Worrisome?**

\[
m^2_{\text{phys}} = \lambda_0\Lambda^2 \\
m^2_{\text{phys}} = m^2_{\text{bare}} + \kappa\Lambda^2
\]
2. Why be Natural?

*Naturalness* (Williams 2015)
No sensitive correlations between low- and high-energy phenomena.
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**Naturalness**  (Williams 2015)
No sensitive correlations between low- and high-energy phenomena.

*Common to other formulations:*  
- No parameters with quadratic (or higher) dependence on cutoff/heavy fields.  
- No dimensionless parameters that are not order 1, unless protected by a symmetry.  
- No bare parameters that require fine-tuning.

*Intuition:* Apparent sensitivity is due to presence of new physics.
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- General Claim: Where naturalness fails, seek new physics.
  - Prediction of charm quark.
  - Postdiction of positron, $\rho$-meson.
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But: Spectacular failures:

• Hierarchy Problem: \( \lambda_0 = \frac{m_{\text{Higgs}}^2}{M_{\text{Pl}}^2} \sim 10^{-34} \).
• Cosmological constant Problem: \( \lambda_{-2} = \frac{\Lambda_C^4}{M_{\text{Pl}}^4} \sim 10^{-120} \).
• Strong CP Problem: \( \theta_{\text{QCD}} < 10^{-10} \).
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*Where's the new physics?*
2. Why be Natural?

(ii) Quantifiable.

(a) Measures of sensitivity of low-energy parameters to high-energy parameters.

(b) Measures of likeliness of fine-tuned values of bare parameters.
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But: (Hossenfelder 2018)

- Problems with (a):
  - Different results
  - Different tolerance levels
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But: (Hossenfelder 2018)

- Problems with (a):
  - Different results
  - Different tolerance levels

- Problems with (b):
  - Requires a probability distribution.
  - Risk of begging the question that fine-tuned parameters are unlikely.
2. Why be Natural?

(iii) Consistent with "spirit" of EFTs.

*The Central Dogma of EFTs* (Williams 2015)
Phenomena at widely separated scales should decouple.
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**But:**

- A failure of naturalness does not signify a failure of decoupling.
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**But:**

- A failure of naturalness does not signify a failure of decoupling.
- While decoupling may be EFT dogma, naturalness seems dogmatic only for Wilsonian EFTs.

*What about "continuum" EFTs?* (Georgi 1993)
2. Why be Natural?

Mass-dependent renormalization and Wilsonian EFTs

- Use the cut-off $\Lambda$ to regulate divergent integrals.
  
  - Replace $\int_0^\infty \kappa(p) d^D p$ with $\int_0^\Lambda \kappa(p) d^D p + \int_\Lambda^\infty \kappa(p) d^D p$.
  
  - Absorb second piece into renormalized parameters.
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- Absorb second piece into renormalized parameters.

- Requires a subtraction scheme that is "mass-dependent": renormalized parameters are dependent on the masses of the heavy fields.
2. Why be Natural?

Mass-dependent renormalization and Wilsonian EFTs

*Advantages:*

(a) Consistent with image of an EFT as a low-energy approximation to a high-energy theory based on a restriction of the latter to a particular energy scale \( \Lambda \).
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(b) Necessary for proof of the Decoupling Theorem...
2. Why be Natural?

Mass-dependent renormalization and Wilsonian EFTs

**Decoupling Theorem** (Appelquist & Carazzone 1975)

In a perturbatively renormalizable theory with two widely separated mass scales, there is always a mass-dependent renormalization scheme by means of which the effects of the heavy mass can be encoded in the parameters of an effective theory in which only the light mass appears.
Disadvantages:

(a) Momentum cut-off regularization violates Poincaré and gauge invariance.

(b) Dependence of irrelevant terms on orders of $E/\Lambda$ breaks down for higher-order loop calculations: *Power* dependence of terms on $\Lambda$.

- Higher-order loop calculations cannot ignore irrelevant terms.
2. Why be Natural?

Mass-independent renormalization and continuum EFTs

- Use mass-independent subtraction scheme: Energy scale parameter $\mu$ appears in loop corrections in logarithms.
  - Irrelevant terms can be ignored at both tree- and high-order loop levels.
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Mass-independent renormalization and continuum EFTs

- Use mass-independent subtraction scheme: Energy scale parameter $\mu$ appears in loop corrections in logarithms.
  - Irrelevant terms can be ignored at both tree- and high-order loop levels.

- Use dimensional regularization:
  - Replace $\int_0^\infty \kappa(p)d^Dp$ with $\int_0^\infty \kappa(p)d^{D-\epsilon}p$.
  - Analytically continue $D - \epsilon$ to $D$.
  - Absorb poles into (mass-independent) renormalization constants.
2. Why be Natural?

Mass-independent renormalization and continuum EFTs

*Advantages:*

(a) Dimensional regularization respects Poincaré and gauge invariance.

(b) Mass-independent subtraction allows truncation of the effective action to a finite number of terms for both tree-level calculations and higher-order loop corrections.
Mass-independent renormalization and continuum EFTs

Disadvantages:

(a) Violates the "spirit" of an EFT: heavy field terms are present in a dim-regularized action.

(b) Decoupling Theorem does not hold.
2. Why be Natural?

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What about the Central Dogma?
2. Why be Natural?

How to construct a continuum EFT

\[ S[\phi_L] + S_H[\phi_L, \phi_H] \]

\[ \mu = m_H \]

\[ S[\phi_L] + \delta S[\phi_L] \]

High energy \( \phi_L, \phi_H \)

Matching

RG

low energy \( \phi_L \)
2. Why be Natural?

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1. Start with $S = S[\phi_L] + S_H[\phi_L, \phi_H]$ at energy scale $\mu$.
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   at energy scale \( \mu \).

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   \textit{via} RG: \( \mu \rightarrow \mu - d\mu \).

3. \textit{Matching}: When \( \mu \) gets below \( m_H \),
   replace \( S \) with \( S_{\text{eff}} = S[\phi_L] + \delta S[\phi_L] \),
   where \( \delta S[\phi_L] \) encodes a "matching condition" that guarantees \( S \) and \( S_{\text{eff}} \)
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"Decoupling by hand" as a guarantee of empirical adequacy
2. Why be Natural?

Wilsonian EFTs: Naturally biased?
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Wilsonian EFTs: Naturally biased?

Physical cut-off $\Lambda$ plays double role:

(a) Demarcates low-energy physics from high-energy physics.

(b) Regulates divergent integrals.

- $\Lambda$ imposes implicit assumptions about the order of effective couplings $g_i$. 
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**Wilsonian EFTs: Naturally biased?**

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**Continuum EFTs: Naturally agnostic?**
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$\Lambda$ imposes implicit assumptions about the order of effective couplings $g_i$.

Continuum EFTs: Naturally agnostic?

- Renormalization scale $\mu$ plays Role (a).
- Role (b) replaced by dimensional regularization.
- No implicit assumptions about the order of effective couplings.
  - Fine-tuning: What me worry?
3. Naturalness and Emergence.

Wilsonian EFTs:

- Motivated by condensed matter physics...
  ...which is enthralled by "emergence":


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Wilsonian EFTs:

- Motivated by condensed matter physics...
- ...which is enthralled by "emergence":

\[
\text{Informal references to emergence:}
\]

- "emergent gravitational features in condensed matter systems";
  "emergent spacetime symmetries".  (Barcelo et al. 2005)
- "...an effective electrodynamics emerges from an underlying fermionic condensed matter system."  (Dziarmaga 2002)
- "emergent relativistic quantum field theory and gravity";
  "emergent nontrivial spacetimes".  (Volovik 2003)
- "emergence of relativity".  (Zhang & Hu 2001)
3. Naturalness and Emergence.

**Emergence** = a characteristic of the ontology associated with a physical system (the emergent system), with respect to another physical system (the fundamental system).
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**Crieteria for Emergence**  Crowther (2015)

(i) *Dependence*. Emergent system is "ontologically determined" by the fundamental system.

(ii) *Independence*. Emergent system is "robustly novel" with respect to fundamental system.
Emergence = a characteristic of the ontology associated with a physical system (the emergent system), with respect to another physical system (the fundamental system).

Criteria for Emergence  Crowther (2015)
(i) Dependence. Emergent system is "ontologically determined" by the fundamental system.
(ii) Independence. Emergent system is "robustly novel" with respect to fundamental system.

Task: Resolve tension between Dependence and Independence.

Suggestion: Natural EFTs accomplish this task.
3. Naturalness and Emergence.

How an EFT satisfies *Dependence*

- Low-energy phenomena decouple from high-energy phenomena.
  - *Low-energy phenomena are low-energy DOF of high-energy phenomena.*
  - *High-energy effects encoded in low-energy dynamics.*
- **Interpretation:** Low-energy phenomena are ontologically determined by high-energy phenomena.
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How an EFT satisfies *Dependence*

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  - Low-energy phenomena are low-energy DOF of high-energy phenomena.
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- *Interpretation*: Low-energy phenomena are ontologically determined by high-energy phenomena.

How a natural EFT satisfies *Independence*

- *Naturalness*: No sensitive correlations between low- and high-energy phenomena.
- *Interpretation*: Low-energy phenomena are robustly dynamically independent of high-energy phenomena.
Conclusion.

Why be natural?
Conclusion.

Why be natural?

*Not* because:

- It's empirically warranted.
- It's quantifiable.
- It underwrites the EFT Central Dogma.
Conclusion.

Why be natural?

_Not_ because:

- It's empirically warranted.
- It's quantifiable.
- It underwrites the EFT Central Dogma.

_Perhaps_ because:

- It helps to underwrite a non-trivial notion of emergence associated with EFTs.
Conclusion.

Why be natural?

General Morals:

(a) Naturalness is an *empirical* hypothesis with *ontological* implications.

(b) As an empirical hypothesis with limited empirical support, one should be cautious in using it as a guiding principle; and one should be cognizant of where it occurs as an assumption in theoretical frameworks (*viz.*, Wilsonian EFTs).

(c) As an ontological hypothesis, there is nothing wrong with the project of examining what the world would be like if it were true, or how current theories might be extended if it were true, as long as one is cognizant of Moral (b).
References.


Williams, P. (2015) "Naturalness, the autonomy of scales, and the 125 GeV Higgs", *SHPMP 51*, 82.