Naturally light scalar particles: A mechanism involving dangerously irrelevant discrete anisotropies

Bertrand Delamotte, LPTMC, University Paris VI

Aachen, March 2018

In collaboration with
F. Léonard (Paris)
N. Wschebor (Montevideo, Uruguay),

ArXiv 2018

The principle number 0 of physics

“Never make a calculation until you know the answer.”
John A. Wheeler

The principle number 0 of physics

“Never make a calculation until you know the answer. Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle.”
John A. Wheeler

The principle number 0 of physics

“Never make a calculation until you know the answer. Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle.”
John A. Wheeler

“Open any textbook of physics at any page and you will see that all dimensionless numbers are of order one.”

The principle number 0 of physics

"Never make a calculation until you know the answer." Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle.
John A. Wheeler

"Open any textbook of physics at any page and you will see that all dimensionless numbers are of order one."

"In dimensional analysis, all constants are of order one
... if the quantities have been adequately chosen to describe the phenomenon at stake,
... but for exceptions."
J.-M. Lévy-Leblond

Numbers of order 1 are natural...

What are the exceptions?

Example 1: The strength of the gravitational force.
Compute the ratio: gravitational force and electrostatic force between two electrons:

$$\frac{F_{\mathrm{grav}}}{F_{\mathrm{el}}} \sim 10^{-42}.$$

This is incredibly small and has probably a very deep meaning...

What are the exceptions?

Example 2: Second order phase transitions.
The dynamics of the system is able to produce a (correlation) length ξ much larger than the unique scale a of the system which is microscopic (a lattice spacing, an intermolecular distance,...) $\Longrightarrow \xi/a \gg 1$.

The price to pay: a fine-tuning of a parameter of the system (temperature, pressure, magnetic field, ...).

Why is it possible? Because, we can vary at will this kind of parameters.

What are the exceptions?

Example 3: The SM + new physics beyond the SM + “desert hypothesis” .
There is a physics beyond the Standard Model and it occurs at an energy scale Λ much larger than the Higgs scale: $\Lambda/m_H \gg 1$.

The dynamics produces a length scale (Compton wavelength of the Higgs) $\xi \sim m_H^{-1} \gg a \sim \Lambda^{-1} =$ more “fundamental” length scale.

The problem: we cannot fine-tune at will the parameters of the universe $\Longrightarrow$ the situation looks unnatural!

Is the unnaturalness of the SM a problem?

Hypothesis 1: the "desert": Λ exists and $\Lambda \gg m_H$.
Problem if $\Lambda = M_{\rm Planck}$ or $M_{\rm GUT}$.
No problem if $\Lambda/m_H \sim 10$. But the precision tests of the SM seem incompatible with such a "small" Λ. Could we circumvent this? Difficult...

Hypothesis 2: The larger comes from the smaller.
This is reductionism (and emergentism) and it is fully consistent with everything we believe in coming from the renormalization group. Again difficult to avoid...

Hypothesis 3: The problem of fine-tuning in the SM is similar to the fine-tuning occuring in statistical systems close to a second order phase transition.
The formal similarities between the two types of problems are very strong (fluctuations, spontaneous symmetry breaking, field content,...). Again difficult to avoid at least in the simplest models.

Two hierarchy problems

The technical problem: Perturbation theory is unstable with respect to the quantum corrections to the masses in the Higgs sector;

The physical problem: The difference between the Planck (or GUT or...) and gauge breaking scales is unnatural.

Gigantic efforts to solve the first problem while keeping unchanged the predictive power of the SM: supersymmetry, technicolor,...

But, as such, these "solutions" do not solve the second problem (even though there are supersymmetric models that do).

These "solutions" change drastically the SM while leaving unchanged the low-energy world: double the spectrum of particles in susy, introduce new interactions in technicolor, change space-time itself by introducing extra dimensions.

Renormalization: the central point of the problem

All proposed solutions take for granted that the SM cannot be made natural without drastic changes.

Origin of the problem: the renormalization of the square mass of the Higgs is quadratic in Λ:

$$m_H^2 = m_0^2 - \alpha\, g_0\, \Lambda^2 + O(g_0^2) \qquad (1)$$

$\Longrightarrow$ fine cancellation between m_0^2 and Λ^2 to get $m_H^2 \ll \Lambda^2$.

This is what tries to do supersymmetry: eliminating the Λ^2 corrections for logarithmic corrections. Possible but... unseen in experiments. Same for technicolor and extra-dimensions.

But does renormalization go the way we think it does?

A bit of history about gauge theories

Enormous success of gauge theories because in $d = 4$, the only (perturbatively) renormalizable theories involving spin one particles are gauge theories (and they are asymptotically free in the UV).

$\Longrightarrow$ very difficult to think at gauge theories outside the framework of renormalizable theories.

A bit of history about renormalization

Perturbative and Wilson renormalization yield the same results.
But interpretation drastically different:
$\Longrightarrow$ Perturbatively renormalized (and UV free) theories were interpreted as fundamental theories.
$\Longrightarrow$ Wilsonian point of view: The leading couplings at low energy are the renormalized ones,

$\Longrightarrow$ a renormalized theory = effective theory valid at low energy (compared to Λ).

$\Longrightarrow$ no need to consider only theories valid up to asymptotically high energies (metaphysical option),

$\Longrightarrow$ possible to consider nonrenormalizable couplings if Λ is finite,

$\Longrightarrow$ renormalization group: the leading effect at low energy of nonrenormalizable couplings is to modify the values of the renormalizable couplings.

Proposition for a “new type” of modifications of the Higgs sector

We can marry these two ideas: the SM does not need to involve only renormalizable couplings.

$\Longrightarrow$ the nonrenormalizable couplings will not change the low energy world,

$\Longrightarrow$ and they can modify the masses in the Higgs sector.

A less ambitious program

A remark: very often the two problems of generating small masses in the Higgs and in the gauge sectors are considered as being one and the same. Not necessarily true.

$\Longrightarrow$ We propose a solution for generating "at will" light (compared to Λ) scalar particles but not gauge particles (unfortunately),

$\Longrightarrow$ in which both the technical and physical hierarchy problems are solved.

In fact, the quantum corrections are no longer a problem, they are the solution!

An obvious candidate...

... the Goldstone modes:

$\Longrightarrow$ they are generated by the spontaneous breaking of a continuous symmetry: OK for the SM,

$\Longrightarrow$ they have a vanishing mass: almost OK, but overshoots our goal,

$\Longrightarrow$ Recipe: modify the symmetry breaking pattern so as to have pseudo-Goldstone bosons with a small correction to their mass (an old idea).

But

in general, this modification spoils the good properties of the Golsdtone bosons: they become massive and tend to suffer from the naturalness problem.

Idea: modify the spontaneous symmetry breaking pattern by nonrenormalizable couplings.

The role of discrete symmetries

Spontaneous breaking of a discrete symmetry $\Longrightarrow$ no Goldstone modes.

Idea: Consider a toy model:

- invariant under the continuous symmetry O(2) (for simplicity),
- to which is added a term that explicitly breaks O(2) down to $\mathbb{Z}_q$,
- with q sufficiently large for the explicit breaking term to be nonrenormalizable: $q > 4$.

$\Longrightarrow$ The spontaneous breaking of $\mathbb{Z}_q$ will produce pseudo-Goldstone modes with naturally (extremely) small masses, that is, without fine-tuning.

Easy to understand: $\mathbb{Z}_q$ for $q \to \infty = \mathrm{O}(2) \Longrightarrow$ and for O(2): true Goldstone bosons.

The model

We consider two scalar fields: $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)$
and a O(2)-invariant action:

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \boldsymbol{\varphi})^2 + \frac{1}{2} m_0^2 \boldsymbol{\varphi}^2 + \frac{u_0}{8} (\boldsymbol{\varphi}^2)^2 \right]. \qquad (2)$$

to which is added a $\mathbb{Z}_q$-invariant breaking term:

$$\Delta S_{\mathrm{disc.}} = \lambda_q \int d^4x \, \sigma_q(x) \qquad (3)$$

For instance: $\sigma_6 = (\varphi_1 - \varphi_2)^2 (\varphi_1^2 + 4\varphi_1\varphi_2 + \varphi_2^2)^2/8$
The overall symmetry is $\mathbb{Z}_q \Longrightarrow$ its spontaneous sym. breaking does not generate Goldstone modes.

There are two masses (m_L, m_T) in the problems: the masses of the longitudinal and transverse (= pseudo-Goldstone) modes.

The RG flow eaquations

In the simplest (one-loop - like) approximation:

$$\partial_t \kappa = \left[1 + 4\frac{m_T^2}{m_L^2}\right] k^2 l_2(m_T^2) + 3k^2 l_2(m_L^2) \tag{4a}$$

$$\begin{aligned}\partial_t u =& -36\lambda_6 k^2 l_2(m_T^2) + 18u^2 l_3(m_L^2) \\ & +2(u + 36\kappa\lambda_6)^2 l_3(m_T^2)\end{aligned} \tag{4b}$$

$$\partial_t \lambda_6 = 30\lambda_6 k^2 (u + 6\kappa\lambda_6) \frac{l_2(m_T^2) - l_2(m_L^2)}{m_L^2 - m_T^2} \tag{4c}$$

with $l_n(m^2) = (1 + m^2/k^2)^{-n}$

$\kappa =$ the running minimum of the potential,

$k =$ the RG scale and $t = \log(k/\Lambda)$.

Turn the crank

Take natural initial conditions of the RG flow:

$$u(k=\Lambda)=1 \quad , \quad \frac{\lambda_q(k=\Lambda)}{\Lambda^2}=1$$

and try two values of $\kappa(k=\Lambda)$ such that:

1. The system is in its broken phase,
2. $\kappa(k=\Lambda)$ such that the bare mass m_0 is either 10% or 1% below the value where the model is right at the symmetry breaking point: this is tuning but not fine-tuning.

The flow of the masses

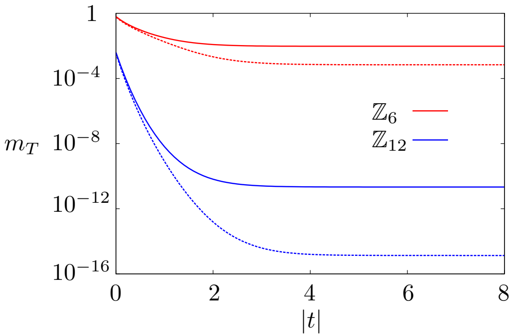

Figure: Flows of m_T for $\mathbb{Z}_6$ and $\mathbb{Z}_{12}$ models with two initial conditions. Upper (resp. lower) curves are obtained for a bare mass which is 10% (resp. 1%) below its critical value.

$\Longrightarrow$ The fluctuations coming from the nonrenormalizable term drive m_T to very small values with a small amount of tuning.

Conclusions

- Nonrenormalizable terms do not perturb the low energy physics,
- the model looks like O(2) at low energy...
- ... which becomes an emergent symmetry,
- minor changes in the microscopic model (bare action) yield naturally light scalars,
- no stability problem w.r.t. quantum corrections.
- In stat mech: nonrenormalizable couplings that trigger new phenomena are dubbed dangerously irrelevant operators.
- In the present case they make the critical exponents to be different on the two sides of the O(2) phase transition.

Problem for the SM: not so easy to export this idea in the gauge sector!

Maybe a lot remains to be explored about the role of nonrenormalizable terms...