Kenneth G. Wilson's views on renormalization group methods

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Context: The larger project

Connections between condensed matter and particle physics in the 20th century

- formal analogies
- exportation of mathematical frameworks from one domain to the other

Examples: quasi-particle/particle concepts; Feynman diagram techniques; spontaneous symmetry breaking; RG methods

Outline

1. Wilson on the place of RG methods in the history of applied mathematics

   Wilson (1975), "The renormalization group: Critical phenomena and the Kondo problem," *Rev. Mod. Phys.*

2. Wilson's (and Kogut's) analogy between CSM and QFT

   Wilson and Kogut (1974), "The renormalization group and the ϵ expansion," *Physics Reports*

3. Conclusions

Wilson on the historical precursors and broad applicability of RG methods

> *In order to be able to specify local equations it is necessary to define continuum limits, namely the limits which define derivatives. The idea of the derivative and the idea of a continuum limit that underlies the derivative is therefore of great importance in all of physics. It is now becoming clear that there is a second form of continuum limit, called the statistical continuum limit, which also has a very broad range of applicability throughout physics.* (Wilson (1975), 773)

continuum limit : derivative :: statistical continuum limit : RG

methods for solving differential equations : RG methods

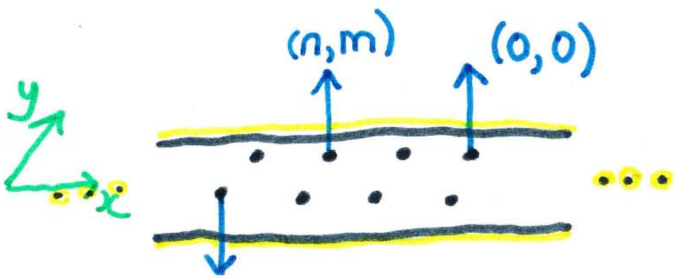
Statistical continuum limit problems

calculated quantities: correlation functions in statistical mechanics; propagators or vacuum expectation values in QFT

statistical continuum limits: limits of functions of a continuous variable which are themselves independent variables (e.g., electromagnetic field $E(\mathbf{x}, t)$, spin $s(n)$)

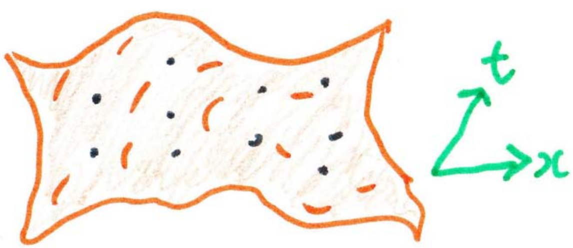
CSM: Ising Model

QFT: scalar quantum field



$s_{n,m}$: spin field

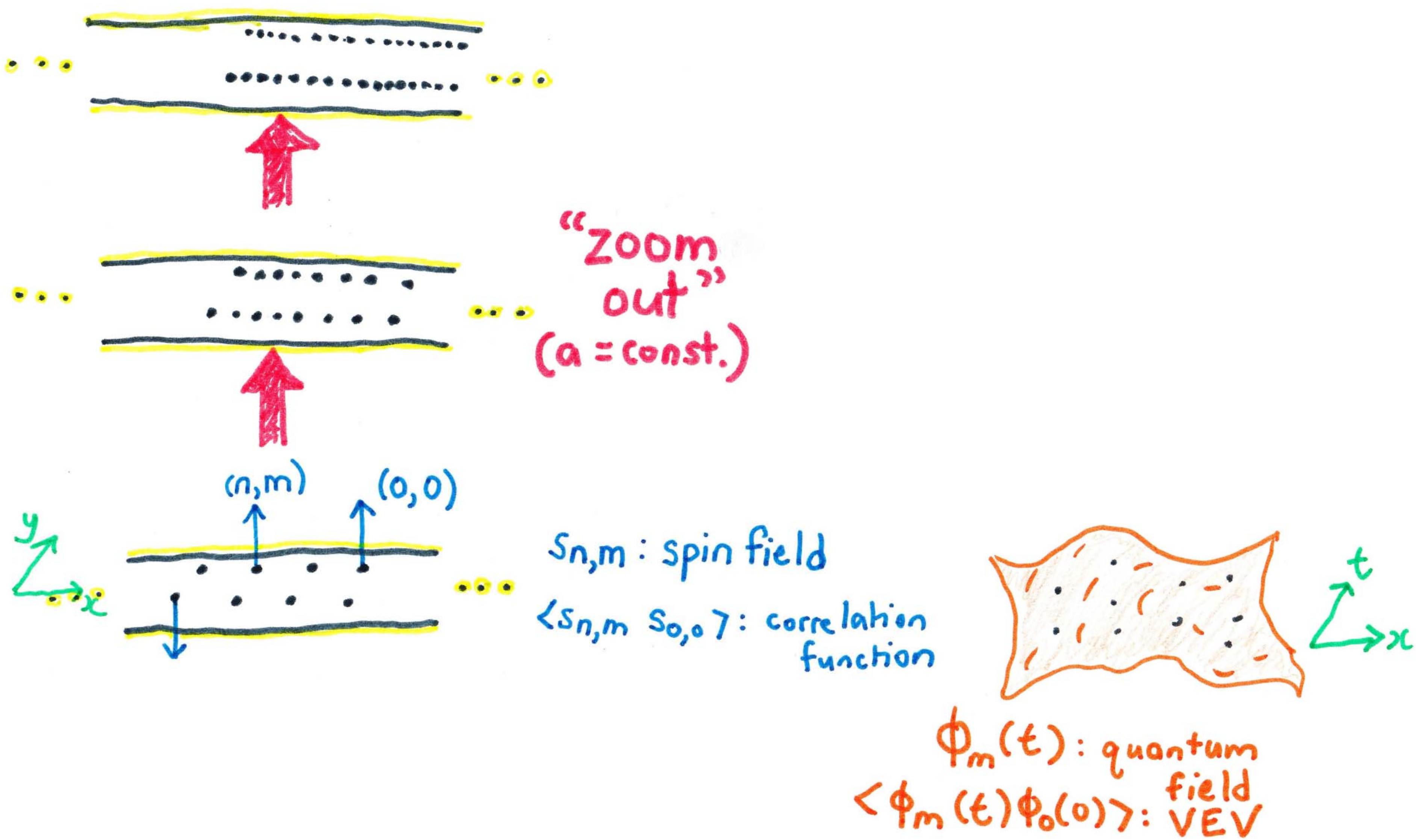
$\langle s_{n,m}\, s_{0,0} \rangle$: correlation function



$\phi_m(t)$: quantum field

$\langle \phi_m(t)\phi_0(0) \rangle$: VEV

Setting up Statistical Continuum Limits



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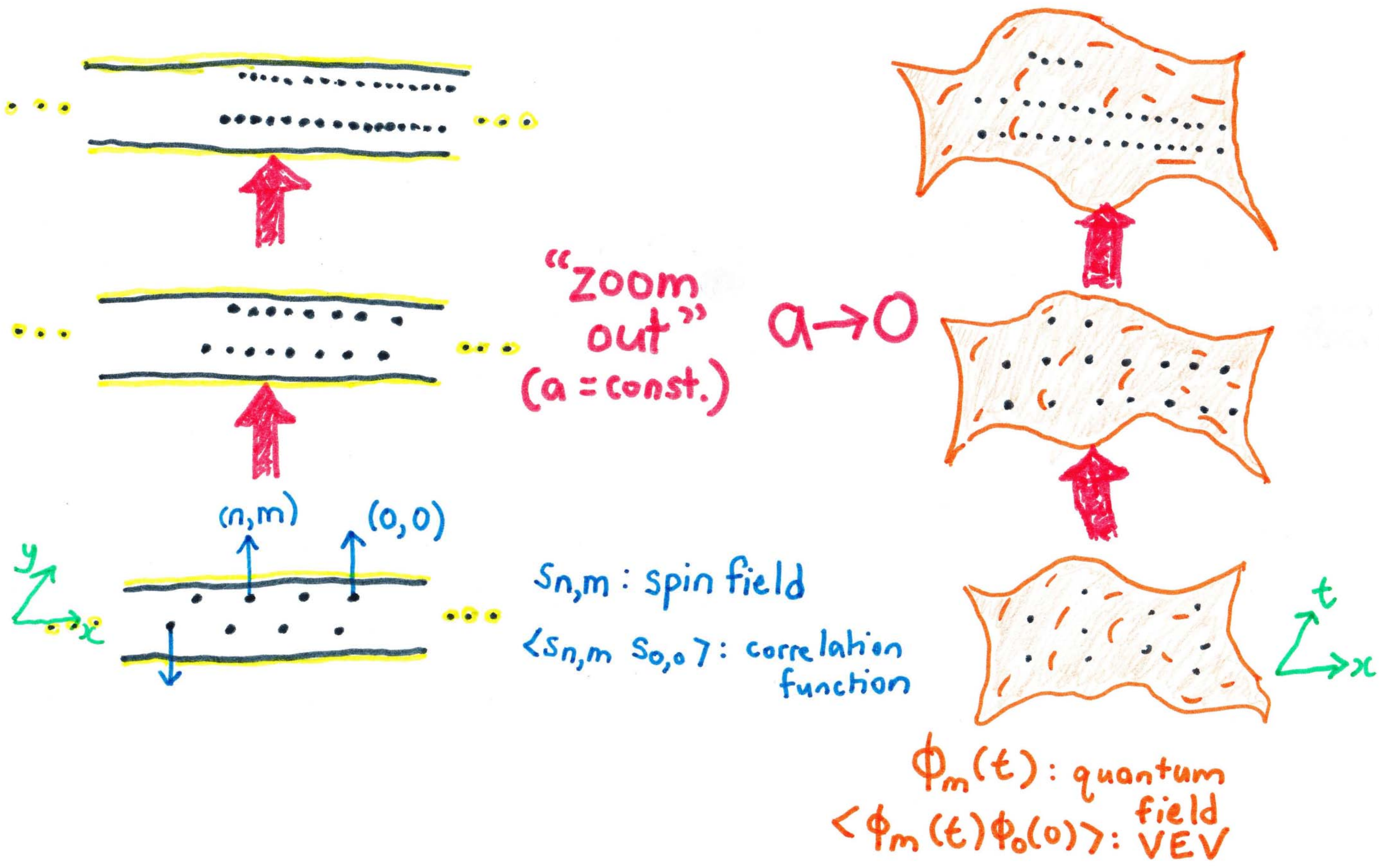


RG methods are a valuable technique for solving "very difficult" computational problems

"Problems with infinitely many variables can be very difficult to solve." - Wilson

2. Wilson's (and Kogut's) analogy between CSM and QFT

- Wilson and Kogut (1974) presents "construction" of continuum, renormalized, effective QFT from CSM
- Goals: (1) μ_R (and λ_R) finite, independent of Λ_0 and (2) renormalized propagators (or VEVs) well-defined

"Performing a renormalization conventionally means giving μ_0 and λ_0 a Λ_0 dependence such that the renormalized mass μ_R and renormalized coupling constant are cutoff independent"

Construction of renormalized QFT from CSM

QFT model: scalar ϕ^4 interaction on $d-1$ dimensional spatial lattice plus continuous time

CSM model: variant of classical Ising model on d-dimensional space

Step 1: Wick rotate and identify

Wick rotation: $t \to -it$

After Wick rotating the QFT,

$$\Gamma_{n,m} = \zeta^2 D_m(-in\tau) \tag{1}$$

where ζ, τ are constants

Correspondences

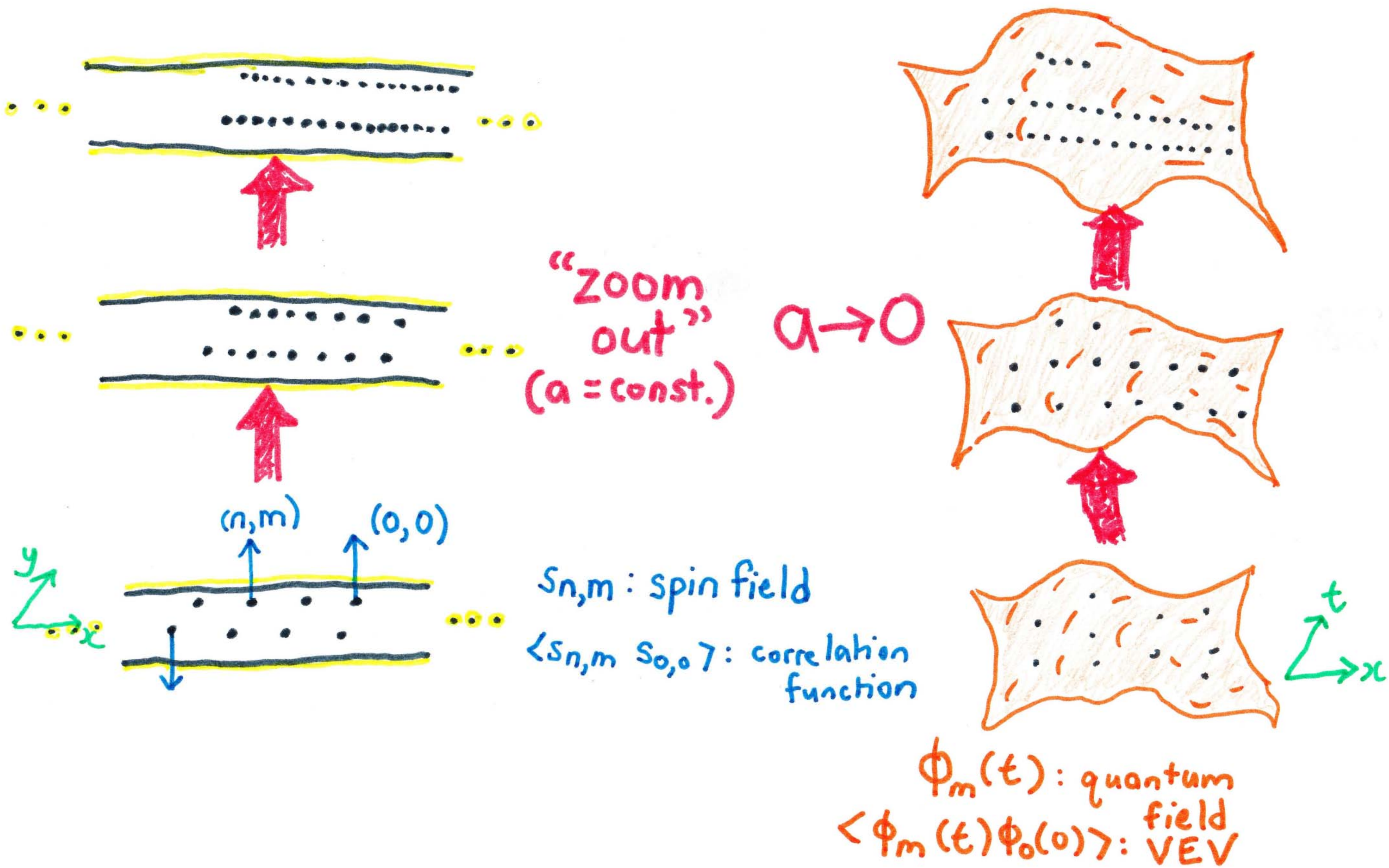
CSM	**QFT**
spin field s_m	quantum field ϕ_m
space x_d	space*time* $(x_{d-1}, -it)$
corr function $\Gamma_{n,m}$	VEV $D_m(t)$

Construction of renormalized QFT from CSM

Step 2: Impose constraint

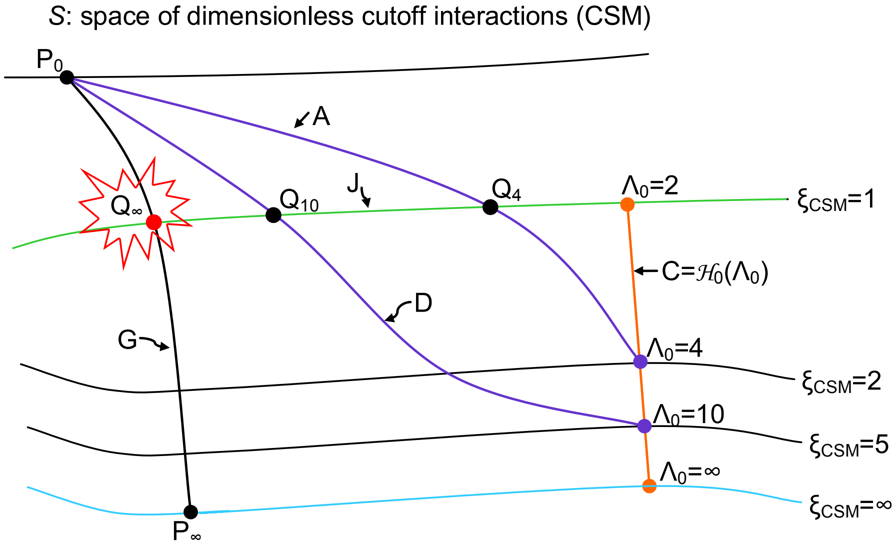
$$\mu_R^{-1} = \frac{\xi_{CSM}}{\Lambda_0} \qquad (2)$$

Setting up Statistical Continuum Limits



Construction of renormalized QFT from CSM

Step 3: Take the continuum limit $\Lambda_0 \to \infty$ of the QFT



3. Conclusions

Peskin and Schroeder on the *gauge hierarchy problem*:

> *One more aspect of ϕ^4 theory deserves comment. Since the mass term, $m^2\phi^2$, is a relevant operator, its coefficient diverges rapidly under the renormalization group flow. We have seen above that, in order to end up with the desired value of m^2 at low momentum, we must imagine that the value of m^2 in the original Lagrangian has been adjusted very delicately. This adjustment has a natural interpretation in a magnetic system as the need to sensitively adjust the temperature to be very close to the critical point. However, it seems quite artificial when applied to the quantum field theory of elementary particles, which purports to be a fundamental theory of Nature. ... Perhaps this is the reason why there seem to be no elementary scalar fields in Nature. (p.406)*

Conclusions

Analysis of Peskin and Schroeder:

- **From Part 1:** There is a misunderstanding about the relationship between condensed matter and particle physics. The statistical continuum limits arising in critical phenomena and the renormalization problem for particle physics have different physical interpretations. $\xi_{CSM} \to \infty$ is a physical process that can be controlled by taking $T \to T_c$. The corresponding particle physics limit is $\Lambda_0 \to \infty$, which is not a physical process. Therefore, we should not expect $\Lambda_0 \to \infty$ to be parametrized by a physical parameter.

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- **From Part 2:** The sensitive adjustment of bare mass is artificial because it is imposed as an artificial assumption to solve the renormalization problem! By construction, the delicate adjustment of the bare mass *is determined by* the sensitive adjustment of $T \to T_c$ (curve C). Curve C is fixed by the constraint imposed to solve the renormalization problem.

Conclusions

Analysis of Peskin and Schroeder:

- **What about finite Λ_0?** Wilson and Kogut take the $\Lambda_0 \to \infty$ limit. If a specific value of Λ_0 (e.g., 10^{19} GeV) is taken instead, it does not make much difference to the interpretive conclusions. $\mu_0(\Lambda_0)$ on curve C is still determined by the constraint (i.e., by the requirement that the corresponding CSM system approach the critical point).

Conditions of applicability of RG methods

- functions of a continuous variable are themselves independent variables (e.g., $\lim_{n\to\infty}\prod_n \int dE_n$)
- fields fluctuate $\rightarrow$ computations are of average over ensemble of fields (e.g., $\langle E(x,t), E(y,t')\rangle$), VEVs)
- fluctuations over a large range of scales contribute to the average
- no characteristic scales that dominate the calculation
- locality: scales are locally coupled (e.g., 1000-2000 $\mathring{A}$ wavelengths primarily affected by nearby wavelengths 500-1000 $\mathring{A}$ and 2000-4000 $\mathring{A}$)