# Kenneth G. Wilson's views on renormalization group methods

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**Outline**

1. Wilson on the place of RG methods in the history of applied mathematics

   Wilson (1975), "The renormalization group: Critical phenomena and the Kondo problem," *Rev. Mod. Phys.*

2. Wilson's (and Kogut's) analogy between CSM and QFT

   Wilson and Kogut (1974), "The renormalization group and the $\epsilon$ expansion," *Physics Reports*

3. Conclusions

## 1. Wilson on the place of RG methods in the history of applied mathematics

calculated quantities: correlation functions in statistical mechanics; propagators or vacuum expectation values in QFT

statistical continuum limits: limits of functions of a continuous variable which are themselves independent variables (e.g., electromagnetic field $E(\mathbf{x}, t)$, spin $s(n)$)

Wilson on statistical continuum limit problems:

> ***There are two ways in which a statistical continuum limit can arise.*** The obvious way is when the independent field variables are defined on a continuous space; the case of statistical or quantum fluctuations of the electromagnetic field is an example. If one were to replace the continuum by a discrete lattice of points, the field averages would consist of integrals over the value of the field $E$ at each lattice site $n$. Thus for the discrete lattice case one has a multiple integration, $\prod_n \int dE_n$, the variables of integration being the fields $E_n$. In the continuum limit, one has infinitely many integration variables $E_n$. ...
>
> The second source of statistical continuum limits is the situation where one has a lattice with a fixed lattice spacing, usually an atomic lattice. The number of independent variables (i.e., independent degrees of freedom) at each lattice site is fixed and finite. The continuum limit arises when one considers large size regions containing very many lattice sites. When the lattice is viewed on a macroscopic scale one normally expects the lattice structure to be invisible. That is, large scale effects should be describable by a continuum picture making no reference to the lattice spacing. (Wilson 1975, 773, emphasis added)

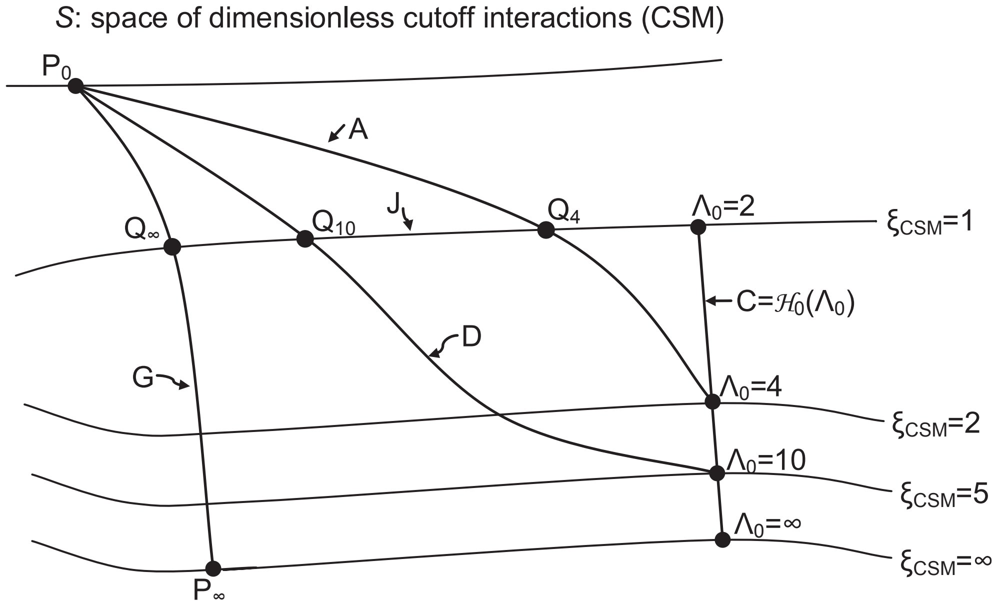
## 2. Wilson's (and Kogut's) analogy between CSM and QFT

STEP 1: WICK ROTATE AND IDENTIFY

STEP 2: IMPOSE CONSTRAINT

STEP 3: TAKE THE CONTINUUM LIMIT $\Lambda_0 \to \infty$ OF THE QFT

Figure adapted from Wilson and Kogut's Figure 12.7




**3. Conclusions**

Analysis of quotation from Peskin and Schroeder:

- **From Part 1:** There is a misunderstanding about the relationship between condensed matter and particle physics. The statistical continuum limits arising in critical phenomena and the renormalization problem for particle physics have different physical interpretations. $\xi_{CSM} \to \infty$ is a physical process that can be controlled by taking $T \to T_c$. The corresponding particle physics limit is $\Lambda_0 \to \infty$, which is not a physical process. Therefore, we should not expect $\Lambda_0 \to \infty$ to be parametrized by a physical parameter.

- **From Part 2:** The sensitive adjustment of bare mass is artificial because it is imposed as an artificial assumption to solve the renormalization problem! By construction, the delicate adjustment of the bare mass *is determined by* the sensitive adjustment of $T \to T_c$ (curve C). Curve C is fixed by the constraint imposed to solve the renormalization problem.

- **What about finite $\Lambda_0$?** Wilson and Kogut take the $\Lambda_0 \to \infty$ limit. If a specific value of $\Lambda_0$ (e.g., $10^{19}$ GeV) is taken instead, it does not make much difference to the interpretive conclusions. $\mu_0(\Lambda_0)$ on curve C is still determined by the constraint (i.e., by the requirement that the corresponding CSM system approach the critical point).