Naturalness, Wilsonian Renormalization, and “Fundamental Parameters” in Quantum Field Theory

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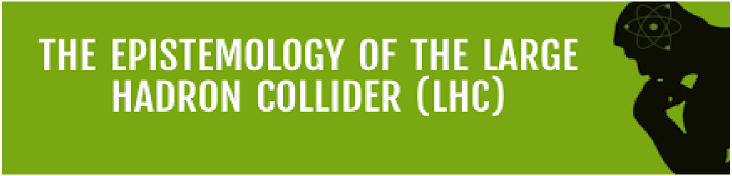


Goal: To trace the origin of one formulation of the naturalness principle to the analogy between high energy physics (HEP) and condensed matter physics (CMP), and show why naturalness in this sense may be rooted in an excessively literal interpretation of this analogy.

What is the Naturalness Principle?

- Extremely influential constraint on high energy physics (HEP) model building over the last ~ 40 years (supersymmetry, large extra dimensions).
- An *Extra-Empirical* Criterion: weaker than empirical adequacy or logical/mathematical consistency
- Unnatural theories thought to be "unattractive" or "contrived".

Origins of the Naturalness Problem

Failure of the naturalness principle in SM arises from the following three facts:

Mathematical Fact:

- **Quadratic Divergences:** The Higgs mass undergoes quantum corrections that are quadratic in the cutoff Λ_{SM}.

Empirical Facts (from LHC):

- **"Light" Pole Mass:** The physical, or "pole" mass of the Higgs boson is 125 GeV.
- **High Cutoff:** Failure to observe new, BSM physics implies that $5 \times 10^3 GeV < \Lambda_{SM} < 10^{19} GeV$.

Combined, these facts entail that the Standard Model violates several formulations of the naturalness principle.

What to Do about Naturalness?

What to make of apparent failure of naturalness in SM?

- Failure of naturalness remains a problem to be solved by BSM physics:

  > *"If the LHC rules out dynamical solutions to Higgs naturalness at the weak scale, it does not eradicate the problem: a doctor who is unable to find the right diagnosis cannot simply declare the patient healed. Even in post-natural times, the concept of naturalness cannot simply be ignored ... One way or another, naturalness will still play a role in the post-naturalness era" (Giudice, 2017).*

- Accept failure of naturalness as a brute fact of nature.
- Modify naturalness - e.g., "stringy naturalness" (Williams)
- Failure of naturalness is unproblematic, and the SM naturalness principle should be abandoned (Hossenfelder, Woit)

Naturalness, Fine Tuning and Bare Parameters

The "bare" fine tuning naturalness (FTN) problem:

$$m_p^2 = m_0^2 - \frac{y_t^2}{8\pi^2}\Lambda_{SM}^2 \tag{1}$$

$$\tag{2}$$

where y_t is the top quark Yukawa coupling, $m_p^2 = (125\,GeV)^2$, $(5 \times 10^3\,GeV)^2 < \Lambda_{SM}^2 < (10^{19}\,GeV)^2$

The bare Higgs mass m_0^2 must be **fine tuned**:

$$\mathcal{O}(10^4) = \mathcal{O}(10^7) - \mathcal{O}(10^7) \quad (\Lambda_{SM} = 5 \times 10^3\,GeV)$$
$$\mathcal{O}(10^4) = \mathcal{O}(10^{38}) - \mathcal{O}(10^{38}) \quad (\Lambda_{SM} = 10^{19}\,GeV)$$

- **Best-case scenario (from LHC):** required cancellation to one part in 10^3.
- **Worst-case scenario:** required cancellation to one part in 10^{34}.

Naturalness, Fine Tuning and Bare Parameters

Clashing perspectives on Higgs fine tuning:

- There is a mysterious “unlikely” cancellation here that must be explained by BSM theories.
- Neither m_0 nor δm^2 is directly measurable. Since they are probably not physical, there is no coincidence to be explained (Wetterich 1984), (Bianchi & Rovelli 2010).

Goals:

1. Show how this tension hinges on the physical interpretation of bare parameters
2. Show how the second view can be grounded in an understanding of Wilsonian RG transformations as re-parametrizations (not coarse grainings).

Central Claims

- FTN problem supposes that bare SM parameters are physically interpreted as "fundamental parameters," analogous to microscopic lattice parameters of a condensed matter system.
- "Fundamental" interpretation of bare SM parameters extends "formal" analogies between high-energy physics (HEP) and condensed matter physics (CMP) to "physical" analogies - cf. (Fraser & Koberinski 2016), (Fraser 2013).
- There is an alternative interpretation of bare SM parameters as unphysical "auxiliary parameters": Wilsonian RG transformations interpreted as invertible re-parametrizations (not coarse grainings); bare parameters in HEP and CMP only formally, not physically, analogous.
- Motivation for FTN substantially weakened on auxiliary view of bare SM parameters.

Outline

I Origins of Naturalness in Analogies with Condensed Matter Theory

- Bare parameters as "Fundamental Parameters"
- Arguments in Favor of the Fundamental Interpretation of Bare SM Parameters
- Fraser on "Formal" vs. "Physical" Analogies

II Naturalness on the View of Bare Parameters as "Auxiliary Parameters"

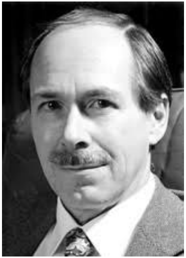
- Wilsonian Renormalization as Re-parametrization
- Deflating Naturalness: Bare Parameters as "Auxiliary Parameters"
- Arguments in Favor of the Auxiliary Interpretation

I. Origins of Naturalness in Analogies with Condensed Matter Theory

Origins of Fine Tuning Naturalness

'tHooft, 1979 - "Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking"

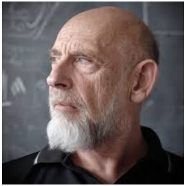
> *The concept of causality requires that macroscopic phenomena follow from microscopic equations. Thus the properties of liquids and solids follow from the microscopic properties of molecules and atoms. ...* **it is unlikely that the microscopic equations contain various free parameters that are carefully adjusted by Nature to give cancelling effects such that the macroscopic systems have some special properties**. *This is a philosophy which we would like to apply to unified gauge theories ...*



Origins of Fine Tuning Naturalness

Susskind, 1979 - "Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Model"

> *... there exists a real difficulty connected with the quadratic mass divergences which always accompany scalar fields. These divergences violate a concept of naturalness which requires* **the observable properties of a theory to be stable against minute variations of the fundamental parameters ...**



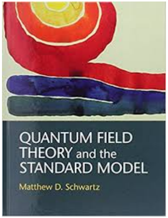
> *The basic underlying framework of discussion of naturalness assumes the existence of a fundamental length scale κ^{-1}, which serves as a real cutoff ...* **The basic parameters of such a theory are some set of dimensionless bare couplings g_0 and masses ...** $\mu_0 = \frac{m_0}{\kappa}$.

"Fundamental parameters" also invoked by (Barbieri & Giudice 1988) in discussion of fine tuning measures.

Origins of Fine Tuning Naturalness

Schwartz, *Quantum Field Theory and the Standard Model*

> *Suppose the theory were finite, for example if it were UV completed into string theory, or more simply if it were the effective description of some condensed matter system (in which case Λ might represent some parameter of the microscopic description, such as the inverse atomic spacing).* **Then the bare mass m and cutoff Λ would be physical.** *In this situation, we could take the Λ^2 divergence ... literally. ... If the scalar were the Higgs whose pole mass is $m_p \approx 125$ GeV, and Λ were of the order of the Planck scale, $\Lambda \sim M_{pl} \sim 10^{19}$ GeV, we would need $m^2 = (1 + 10^{-34})\Lambda^2$. This is called fine-tuning. Fine-tuning is a sensitivity of physical observables (the pole mass) to variation of parameters in the theory ...*



Much of our intuition for fine-tuning and naturalness comes from condensed matter physics ...

“Fundamental Parameters” and the Analogy with Condensed Matter Physics

Bare parameters in CMP:

- Condensed matter system = atoms organized into a regular lattice
- Vibrations of atoms around equilibrium represented mathematically as quantum field
- Bare lattice parameters **independently ascertainable, directly physical**
- Bare parameters **fundamental** - i.e., collective features derivable from bare lattice parameters
- There exists a **single, physically preferred bare parametrization**.

The fundamental view of bare parameters in HEP attributes many of these features to SM bare parameters.

In Defense of the Fundamental Interpretation of Bare SM Parameters

The fundamental view of bare parameters in HEP provides a **concrete, intuitive physical picture** of physics as described by an EFT:

- Particles emerge as transient excitations in more fundamental dof's of the quantum field.
- Underlying physical dof's clearly defined: Quantum field degrees of freedom are concretely physical, and exist at some finite cutoff scale.
- Fundamental bare parameters are those that describe the interactions among fundamental dof's of the quantum field
- ...

See, e.g., (Wallace 2006) for an elaboration of this viewpoint.

Fundamental Parameters and Fine Tuning Naturalness

Following Susskind, assume $\tilde{m}_0^2 = \frac{m_0^2}{\Lambda_{SM}^2}$ and y_t are fundamental parameters. Then $\tilde{m}_0^2$ **and y_t are mutually independent**. Quantities such as the Higgs pole mass are **derived**:

$$\begin{aligned} m_p^2 &= m_0^2 + \delta m^2 \\ &= m_0^2 - \frac{y_t^2}{8\pi^2}\Lambda_{SM}^2 + \dots \\ &= \Lambda_{SM}^2(\tilde{m}_0^2 - \frac{y_t^2}{8\pi^2}) + \dots \end{aligned} \tag{3}$$

Smooth probability measure over fundamental parameter space $\rightarrow$ very likely that $m_p^2 \sim \mathcal{O}(\Lambda_{SM}^2)$. "Light" 125 GeV Higgs pole mass appears "unnatural."

"Unlikely" cancellation between $\tilde{m}_0^2$ and $\frac{y_t^2}{8\pi^2}$ **urgently demands explanation** by a deeper theory.

(Hossenfelder 2018): How do we justify our choice of probability measure over the fundamental, bare parameter space?

Fraser on "Formal" vs. "Physical" Analogies

(Fraser 2013), (Fraser & Koberinski 2016):

- **"Formal" Analogy:** Applicability of the same mathematical structure (e.g., QFT) across different physical contexts (e.g., many-body systems vs. elementary particles)
- **"Physical" Analogy:** Analogous elements of the mathematical formalism also have analogous physical interpretations, as understood in terms of their causal, reductive and other physical relations.

Example of a formal, but not physical, analogy (Fraser & Koberinski 2016):

- SSB in the SM Higgs mechanism is NOT a temporal process (assumes zero-temperature field theory)
- SSB in models of superconductivity (BCS, Ginzburg-Landau) *is* a temporal process

Fraser on "Formal" vs. "Physical" Analogies

Formal vs. Physical Analogies and the Interpretation of Bare Parameters in HEP

- Fundamental view of bare SM parameters extends formal analogy between HEP and CMP to physical analogy - particularly vis-a-vis reductive relationship of collective behavior of quantum field to "microscopic" bare parameters
- Auxiliary view of bare SM parameters (see below) restricts HEP/CMP analogy to a purely formal one

“Formal” vs. “Physical” Analogies

Many interpretations of Wilson’s approach in HEP take the HEP/CMP analogy as a physical analogy.

For, example, Peskin and Schroeder Ch. 12, p. 402:

> *Wilson’s analysis takes just the opposite point of view, that any quantum feld theory is defined fundamentally with a cutoff that has some physical significance. In statistical mechanical applications, this momentum scale is the inverse atomic spacing. In QED and other quantum field theories appropriate to elementary particle physics, the cutoff would have to be associated with some fundamental graininess of spacetime, perhaps a result of quantum fluctuations in gravity*

Physical cutoff Λ_{phys} of an EFT interpreted as **intrinsic** to the definition of the EFT.

Fraser on "Formal" vs. "Physical" Analogies

In HEP, one need not import the lessons of Wilsonian renormalization in this manner ...

II. Naturalness and the "Auxiliary" View of Bare Parameters

Bare Parameters as "Auxiliary Parameters"

A popular textbook on condensed matter field theory (Altland & Simons 2010, Ch. 8, p. 432):

> *in high energy physics ... there is actually no reason to believe in the existence of a well-defined "bare" action with finite coupling constants. (Contrary to the situation in condensed matter physics, the bare action of quantum electrodynamics, say, is in principle inaccessible), p. 432.*

On the "old-fashioned" view of renormalization, bare parameters were "auxiliary" and unphysical because they turned out to be infinite.

Claim: Even adopting the Wilsonian approach of defining QFT with finite values for the cutoff and bare parameters, one need not take these quantities as physical.

Renormalization as Re-parametrization

Auxiliary view grounded in interpretation of Wilsonian RG transformations as relating physically equivalent parameterizations of the same EFT.

Precedent for viewing renormalization as mere re-parametrization:

> the strong dependence of a regularized scattering amplitude on an ultraviolet cutoff can be dramatically weakened by a reparametrization of the theory in terms of "renormalized" parameters ... The intimate connection *reparametrization* $\Longleftrightarrow$ *subtractions* ... is the essence of the proof of cutoff-insensitivity for perturbatively renormalizable theories ... (Duncan 2012, Ch. 17).

Renormalization understood as akin to coordinate transformation.

Renormalization as Re-parametrization

Renormalization: Transforms from a regulator-dependent to a regulator-independent parametrization.

w/ Cutoff Regulator

$$S(p_1, ..., p_n) = S(p_1, ..., p_n; g, \Lambda) = S(p_1, ..., p_n; g(g_r, \mu, \Lambda), \Lambda) = S(p_1, ..., p_n; g_r, \mu) + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

$$m_p^2 = m_p(g, \Lambda) = m(g(g_r, \mu, \Lambda), \Lambda) = m_p(g_r, \mu) + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

Renormalization as Re-parametrization

Here, we extend this attitude to also to RG transformations:

- **Wilsonian Renormalization Group (WRG) transformations**

$$S(p_1, ..., p_n) = S(p_1, ..., p_n; g(\Lambda), \Lambda) = S(p_1, ..., p_n; g(\Lambda'), \Lambda')$$
$$m_p^2 = m_p(g(\Lambda), \Lambda) = m_p(g(\Lambda'), \Lambda')$$

- **Continuum renormalization group (RG) transformations:**

$$S(p_1, ..., p_n) = S(p_1, ..., p_n; g_r(\mu), \mu) = S(p_1, ..., p_n; g_r(\mu'), \mu')$$
$$m_p^2 = m_p(g_r(\mu), \mu) = m_p(g_r(\mu'), \mu').$$

Renormalization as Re-parametrization

The WRG equations,

$$\Lambda \frac{dg_i(\Lambda)}{d\Lambda} = \beta_i(g(\Lambda), \Lambda) \tag{4}$$

are first-order, so **invertible**.

- Bare parametrization $g(\Lambda_h)$ at high cutoff scale NOT more fundamental than $g(\Lambda_l)$ at low scale.
- Interpret WRG transformations as **relating physically equivalent parametrizations of one EFT**.
- **Precedent in continuum RG**: running up a renormalized parameter $g_r(\mu)$ is not usually interpreted as a move to a more fundamental theory.
- Λ interpreted as an unphysical **reference scale.** Different choices Λ **akin to different choices of spatial origin/axes** in Cartesian geometry.
- Bare params $g(\Lambda)$ therefore also unphysical, "auxiliary."
- An EFT has a physical UV cutoff, Λ_{phys}, but this is **not intrinsic** to the definition of the EFT. Emerges only through comparison with empirical data.

Coarse Graining, Fundamentality, and Wilsonian Renormalization

Wilson RG:

$$Z = \int^{\Lambda} \mathcal{D}\phi_\Lambda \; e^{i\int d^4x \; \mathcal{L}(\phi_\Lambda; g(\Lambda))} \tag{5}$$

$$= \int^{\Lambda'} \mathcal{D}\phi_{\Lambda'} \, e^{i\int d^4x \; \mathcal{L}(\phi_{\Lambda'}; g(\Lambda'))} \tag{6}$$

with $e^{i\int d^4x \; \mathcal{L}(\phi_{\Lambda'}; g(\Lambda'))} \equiv \int \mathcal{D}\phi_{\delta\Lambda} e^{i\int d^4x \; \mathcal{L}(\phi_\Lambda + \phi_{\delta\Lambda}; g(\Lambda))}$. **Invertible**, not a coarse graining. $\mathcal{L}(\phi_\Lambda; g(\Lambda))$ representation is **not more fundamental** than $\mathcal{L}(\phi_{\Lambda'}; g(\Lambda'))$ representation.

Integrating out heavy fields:

$$Z = \int^{\Lambda} \mathcal{D}\phi\mathcal{D}\psi \; e^{i\int d^4x \; \mathcal{L}(\phi,\psi)}$$

$$= \int^{\Lambda} \mathcal{D}\phi \; e^{i\int d^4x \; \mathcal{L}_{eff}(\phi)}$$

with $e^{i\int d^4x \; \mathcal{L}_{eff}(\phi)} \equiv \int^{\Lambda} \mathcal{D}\psi \; e^{i\int d^4x \; \mathcal{L}(\phi,\psi)}$. **Non-invertible**. Constitutes coarse graining - $\mathcal{L}(\phi,\psi)$ theory **is more fundamental** than $\mathcal{L}_{eff}(\phi)$ theory.

Naturalness on the "Auxiliary" View of Bare Parameters

On the auxiliary view, **bare parameters are NOT mutually independent**. Instead, we have a **one-parameter equivalence class of physically equivalent parametrizations** of the Higgs pole mass:

$$\begin{aligned} m_p^2 &= m_0^2(\Lambda) + \delta m^2(\Lambda) \\ &= m_0^2(\Lambda) - \frac{y_t^2(\Lambda)}{8\pi^2}\Lambda^2 + \ldots \\ &= \Lambda^2\left(\tilde{m}_0^2(\Lambda) - \frac{y_t^2(\Lambda)}{8\pi^2}\right) + \ldots \end{aligned} \tag{7}$$

where $\tilde{m}_0^2 = \frac{m_0^2}{\Lambda^2}$. Can choose as a matter of convention to set $\Lambda = \Lambda_{SM}$, but does not provide more accuracy or scope than parametrization in terms of any Λ above or below Λ_{SM}.

Parameterization by $(\tilde{m}_0^2(m_p), y_t^2(m_p))$ **no less fundamental** than $(\tilde{m}_0^2(\Lambda_{SM}), y_t^2(\Lambda_{SM}))$.

Naturalness on the "Auxiliary" View of Bare Parameters

On the auxiliary view, regarding the delicate cancellations between $\tilde{m}_0^2(\Lambda_{SM})$ and $\frac{y_t^2(\Lambda_{SM})}{8\pi^2}$ as coincidence is akin to finding coincidence in the following:

- The mass of the system (ant $+$ earth) agrees with the mass of the system (earth) to one part in 10^{30}.
- Relative to a coordinate system with origin at the center of the Milky Way, vectors describing the locations of the Super C and Aachener Dom agree to one part in 10^{18}.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Naturalness on the "Auxiliary" View of Bare Parameters

"Delicate" cancellation due to an **inconvenient, conventional choice of reference point** - i.e. Λ_{SM}.

In Defense of the Auxiliary Interpretation

An Argument from Ad Hoc 'ness:

- Points of **physical disanalogy** between HEP and CMP.
  - In a condensed matter system, one can directly measure the values of bare, microscopic lattice parameters. Physical cutoff sharply defined.
  - In HEP, canNOT directly measure bare parameters. Physical cutoff NOT sharply defined.
- To assume that there is a sharply defined matter of fact about the values of bare parameters in the Standard Model, even though it appears they cannot be directly measured even in principle, is *ad hoc*, in the sense that there is no independent evidence for this (Friederich, Harlander & Karaca 2014).

In Defense of the Auxiliary Interpretation

An Argument from the Continuum:

- Possible to imagine scenarios where bare parameters could not possibly be fundamental: e.g., world that described by the SM up to Λ_{SM}; beyond this scale, a more encompassing QFT, theory X, with new, heavy fields that applies up to arbitrarily high energies.
- Higgs pole mass cannot be derived from fundamental bare parameters of X, since X has no cutoff, so m_p is fundamental in X. Makes little sense to regard m_p as fundamental in X but derived in SMEFT. SM bare parameters must be auxiliary.

Caveat: Need to incorporate gravity at the Planck scale, as well as mathematical problems with QFT in the continuum limit, may ultimately block this argument.

Conclusions

- **"Fundamental" view of bare parameters:** 1) extends "formal" analogy between HEP and CMP to "physical" analogy. 2) assumes a single, physically preferred bare parametrization.
- **"Auxiliary" view of bare parameters:** 1) limits the analogy to one that is merely "formal." 2) assumes a **"democracy" of bare parametrizations**. 3) Physical quantities are Wilson RG invariant.
- **Naturalness on the fundamental view:** need for fine tuning of bare parameters may be genuinely problematic - still subject to worries such as Hossenfelder's.
- **Naturalness on the auxiliary view:** delicate cancellations understood as formal artefacts resulting from conventional choice of Λ.

Conclusions (ct'd)

- **Pro Fundamental:** preserves concrete, intuitive physical picture of QFT from CMP. Underlying physical degrees of freedom clearly defined, localized at a particular scale.
- **Pro Auxiliary:** avoids the charge of *ad hoc*'ness. No sharp cutoff. Forced in the case of continuum BSM physics.
- **Fundamentality on the Auxiliary View:** relinquishes intuition that bare parametrizations at high cutoff scales are more fundamental. Preserves intuition that EFT's describing both light and heavy fields are more fundamental than EFT's describing only light fields.

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