Gaseous Detectors

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XIV ICFA School on Instrumentation in Elementary Particle Physics
LA HABANA
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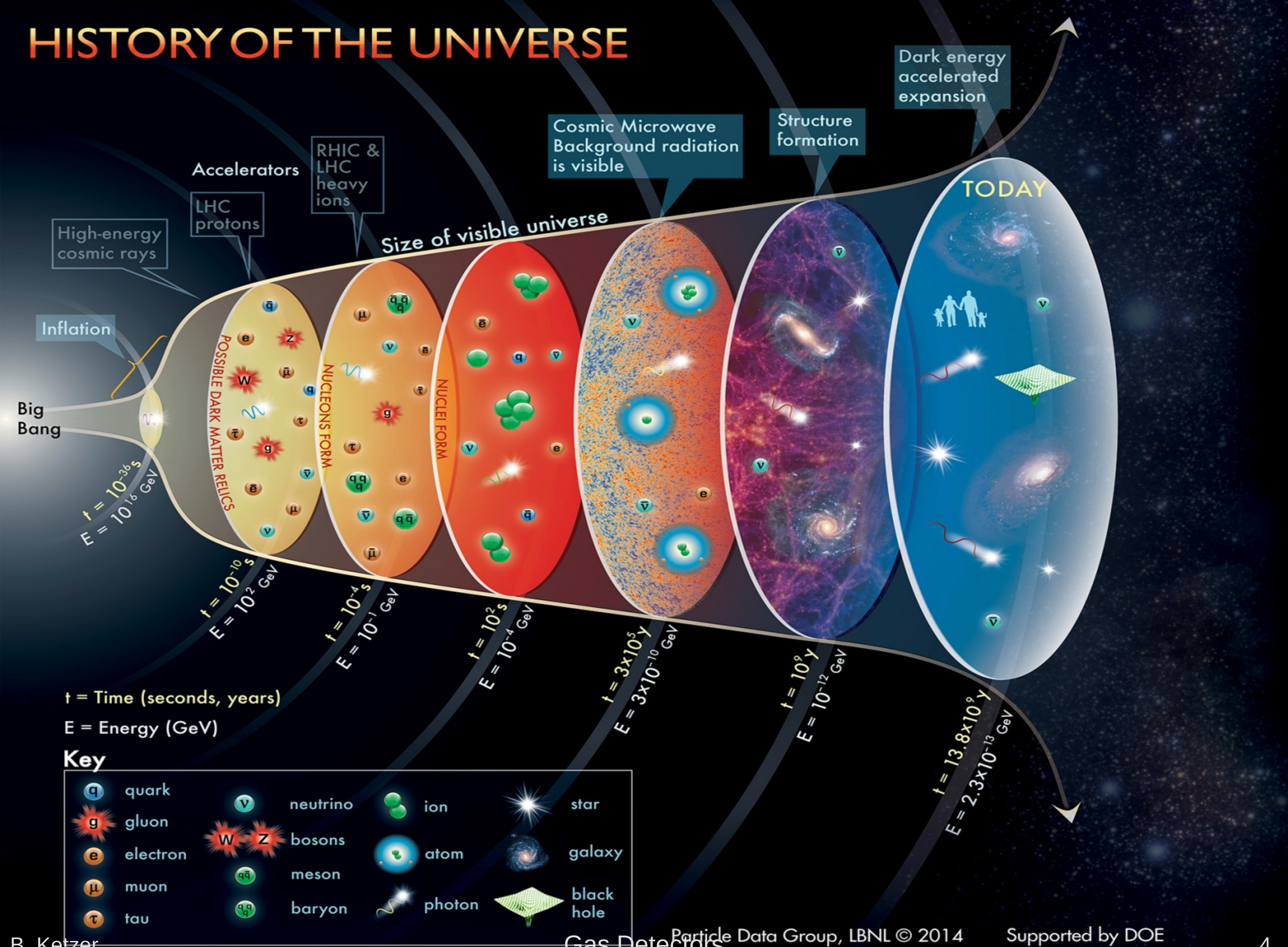
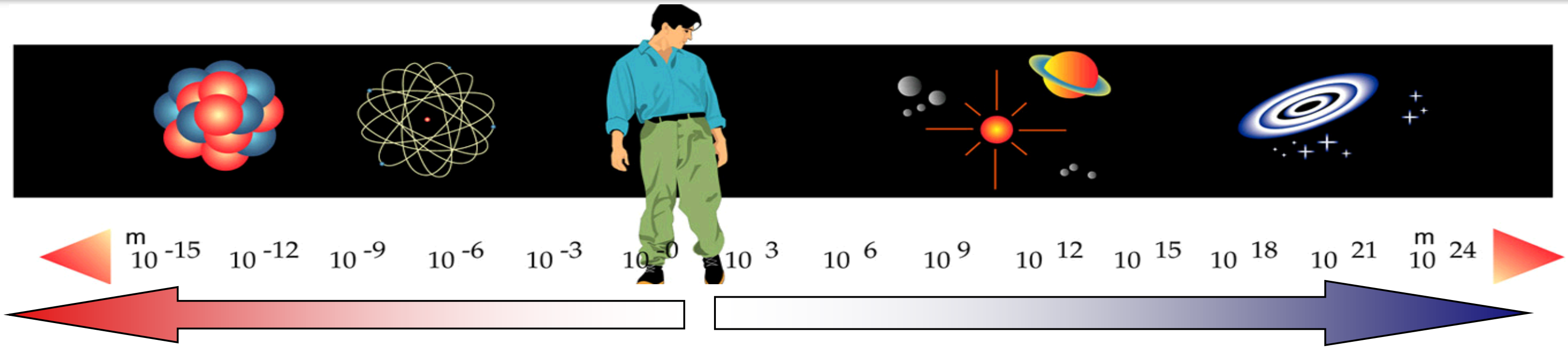
Plan of the Lecture

1. Introduction
2. Interactions of charged particles with matter
3. Drift and diffusion of charges in gases
4. Avalanche multiplication of charge
5. Signal formation and processing
6. Ionization and proportional gaseous detectors
7. Position and momentum measurement / track reconstruction

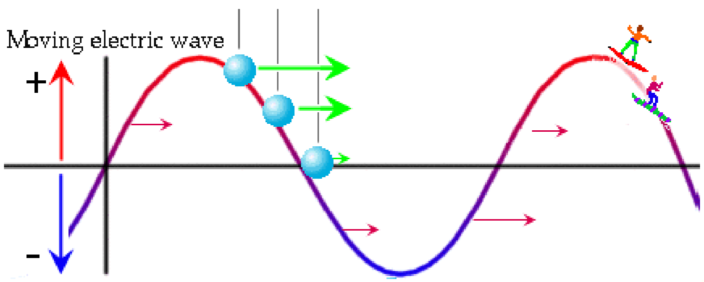
1 Introduction

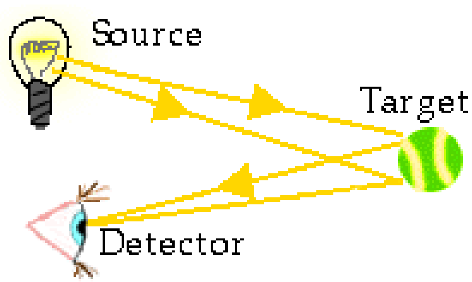

HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

How to observe this?



Accelerators | Microscopes | Binoculars | Optical and radio telescopes



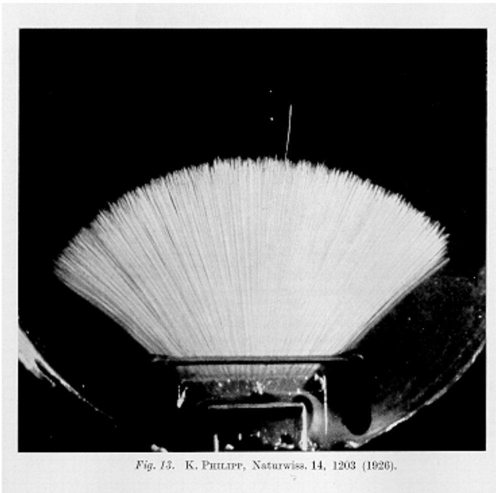


Resolution: $\Delta x \cdot \Delta p \simeq \hbar$

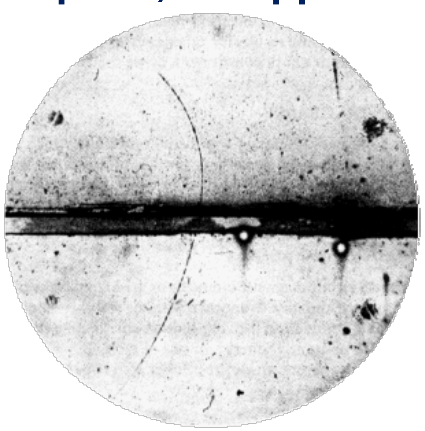
$$\Delta x \sim \frac{\lambda}{\sin\theta}$$

Cloud Chamber

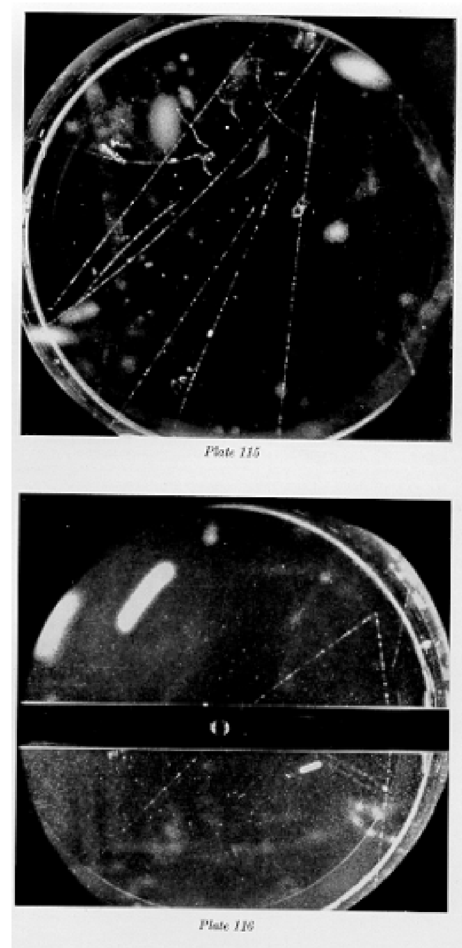
C.T.R. Wilson (1910): Charges act as condensation nuclei in supersaturated water vapor (later: alcohol vapor ⇨ diffusion cloud chamber)



Alphas, Philipp 1926



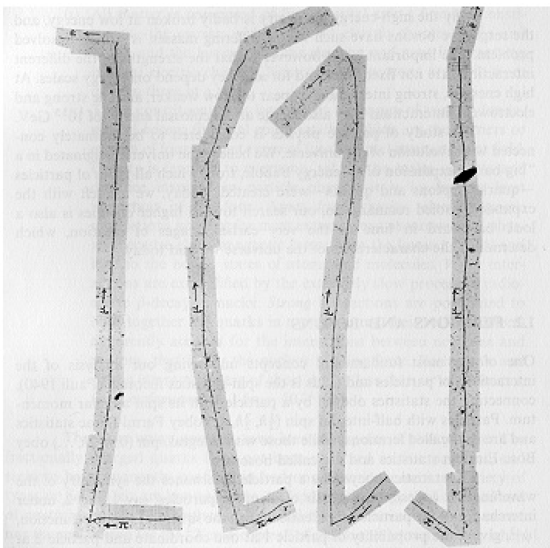
Positron discovery, Carl Andersen 1933



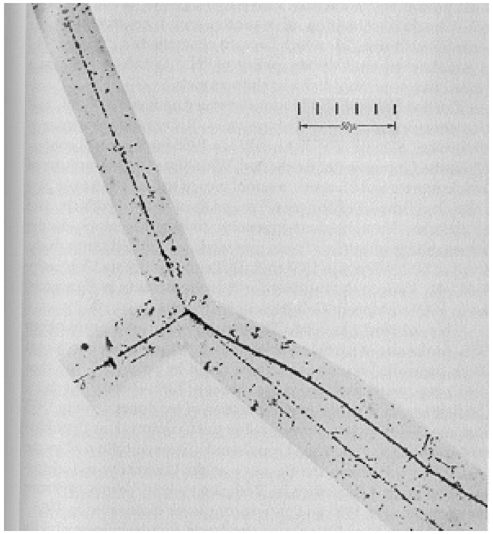
V-particles, Rochester and Wilson, 1940ies

Nuclear Emulsion

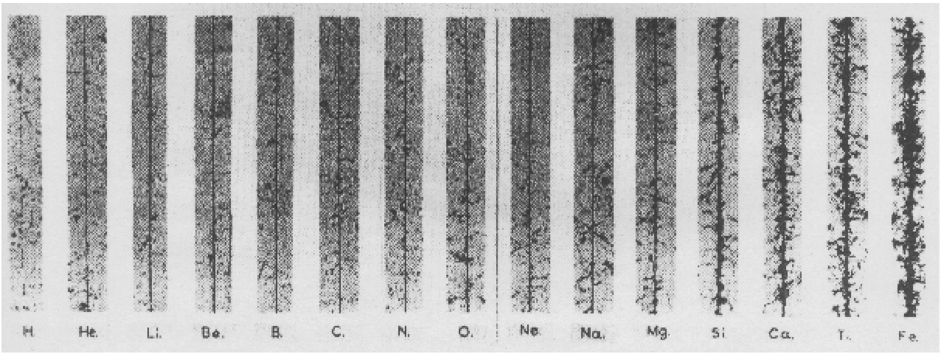
M. Blau (1930s): Charges initiate a chemical reaction that blackens the emulsion (film made of Ag-halide, e.g. AgBr)



C. Powell, Discovery of muon and pion, 1947



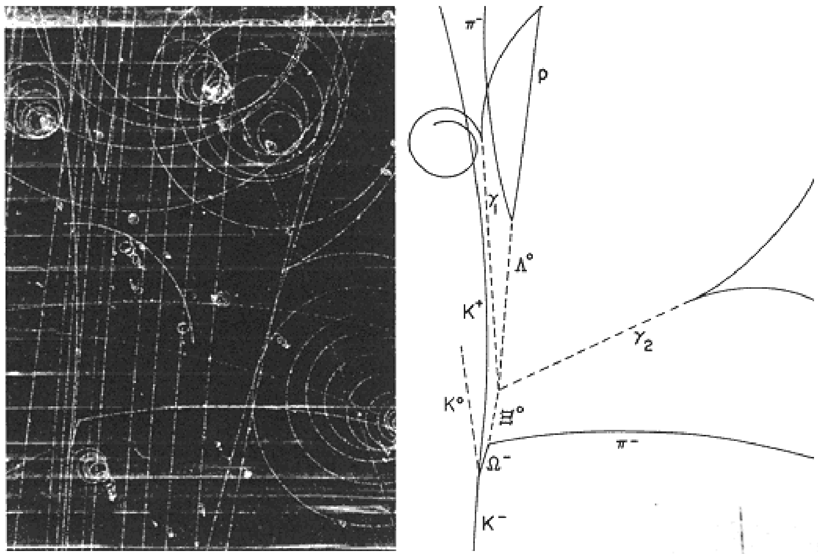
Kaon Decay into 3 pions, 1949



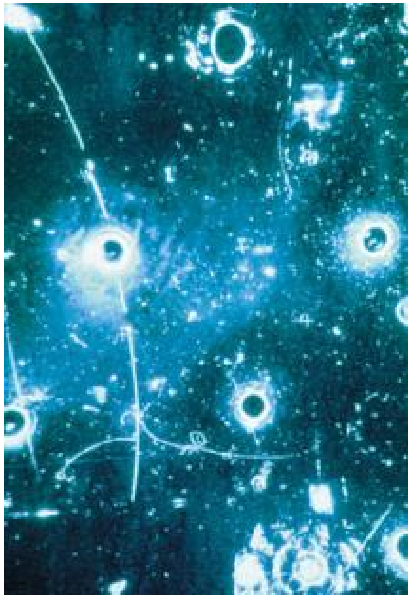
Cosmic Ray Composition

Bubble Chamber

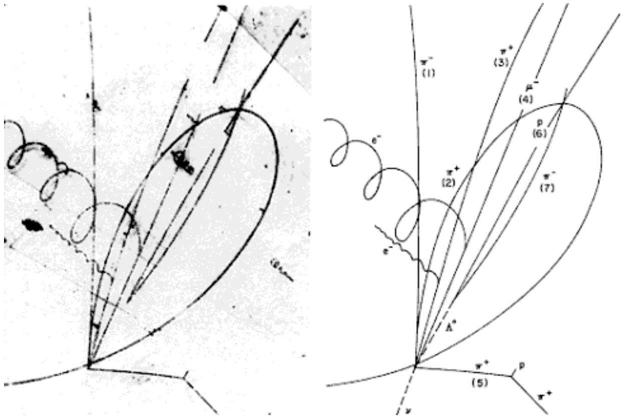
D. Glaser (1952): Charges create bubbles in superheated liquid, e.g. propane or Hydrogen (Alvarez)



Discovery of the Ω^- in 1964

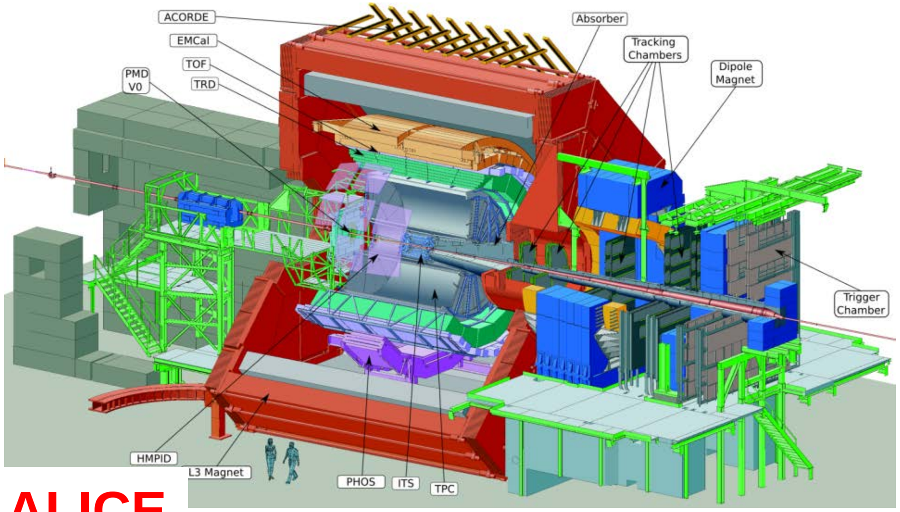


Neutral Currents 1973



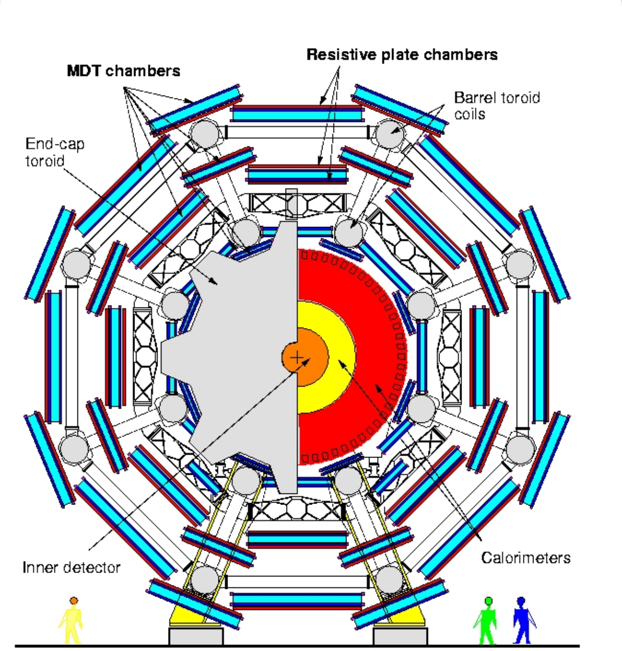
Charmed Baryon, 1975

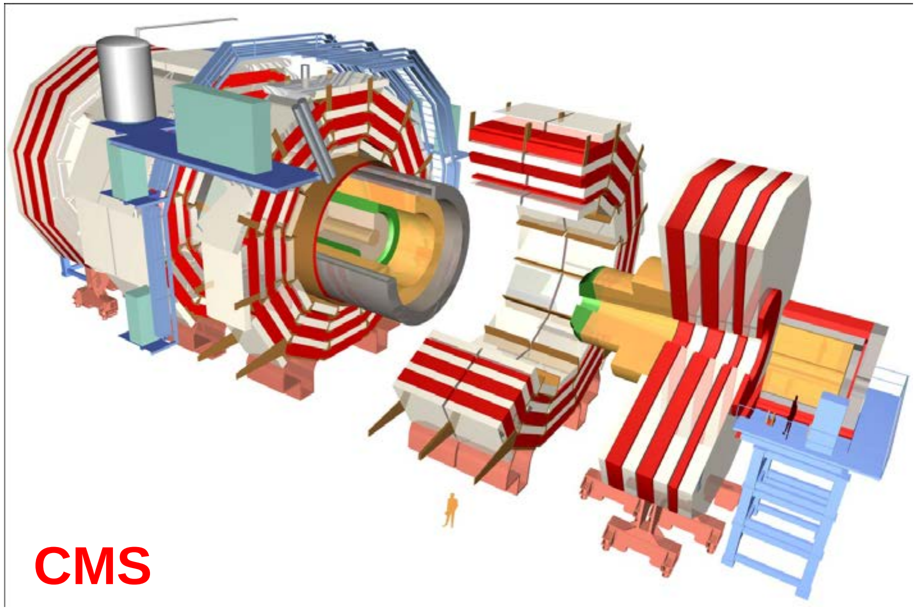
The Giants



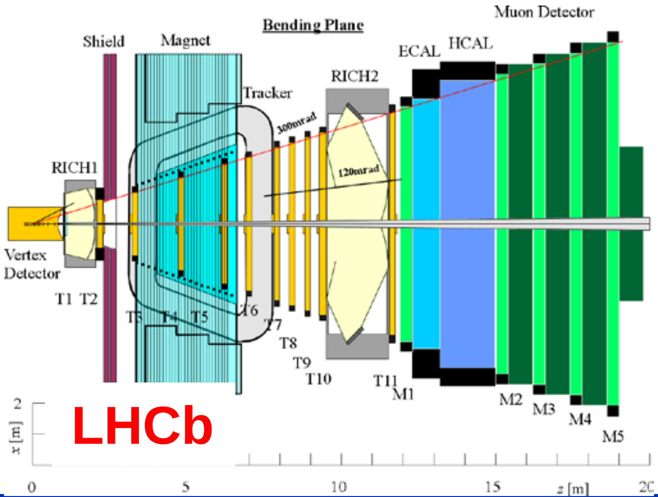
ALICE

ATLAS





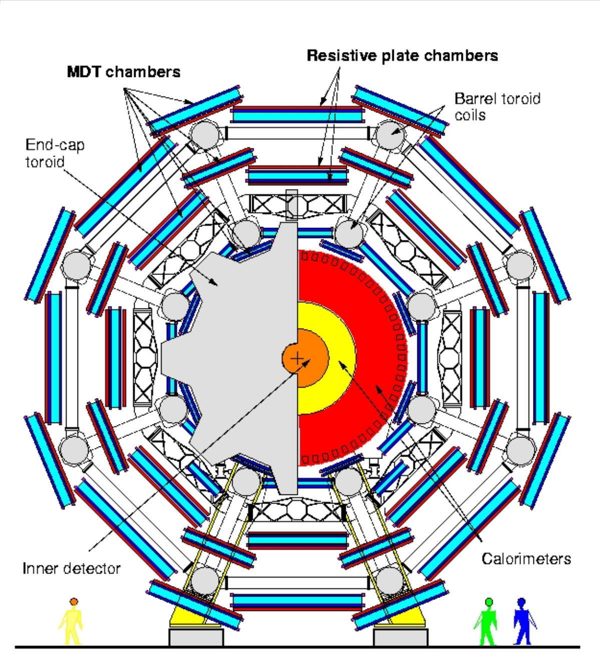
CMS

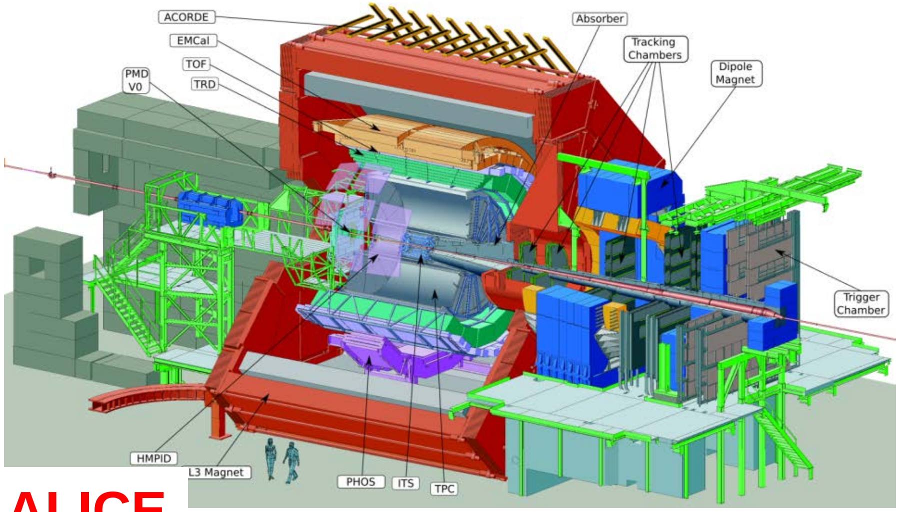


LHCb

The Giants

ATLAS





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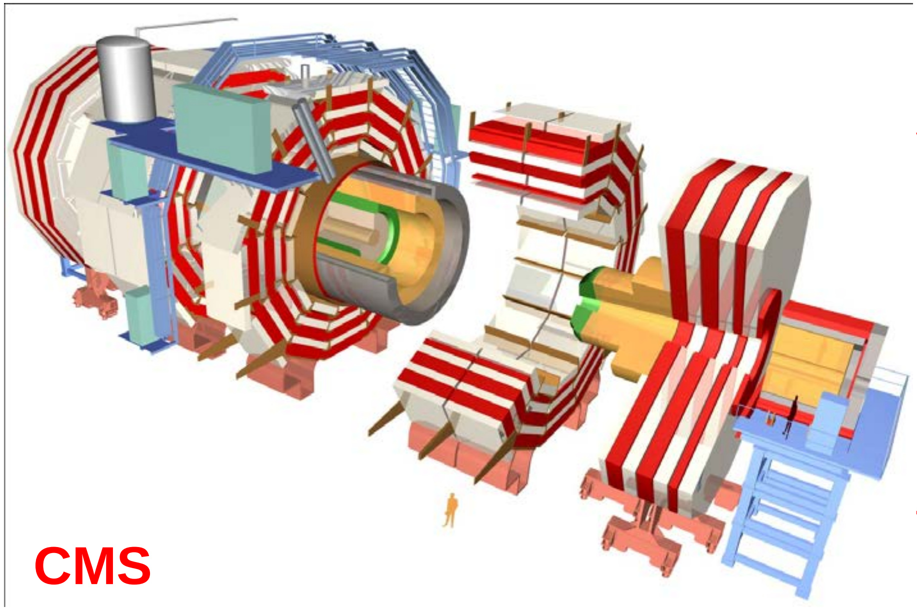


CMS

Very Large Structures

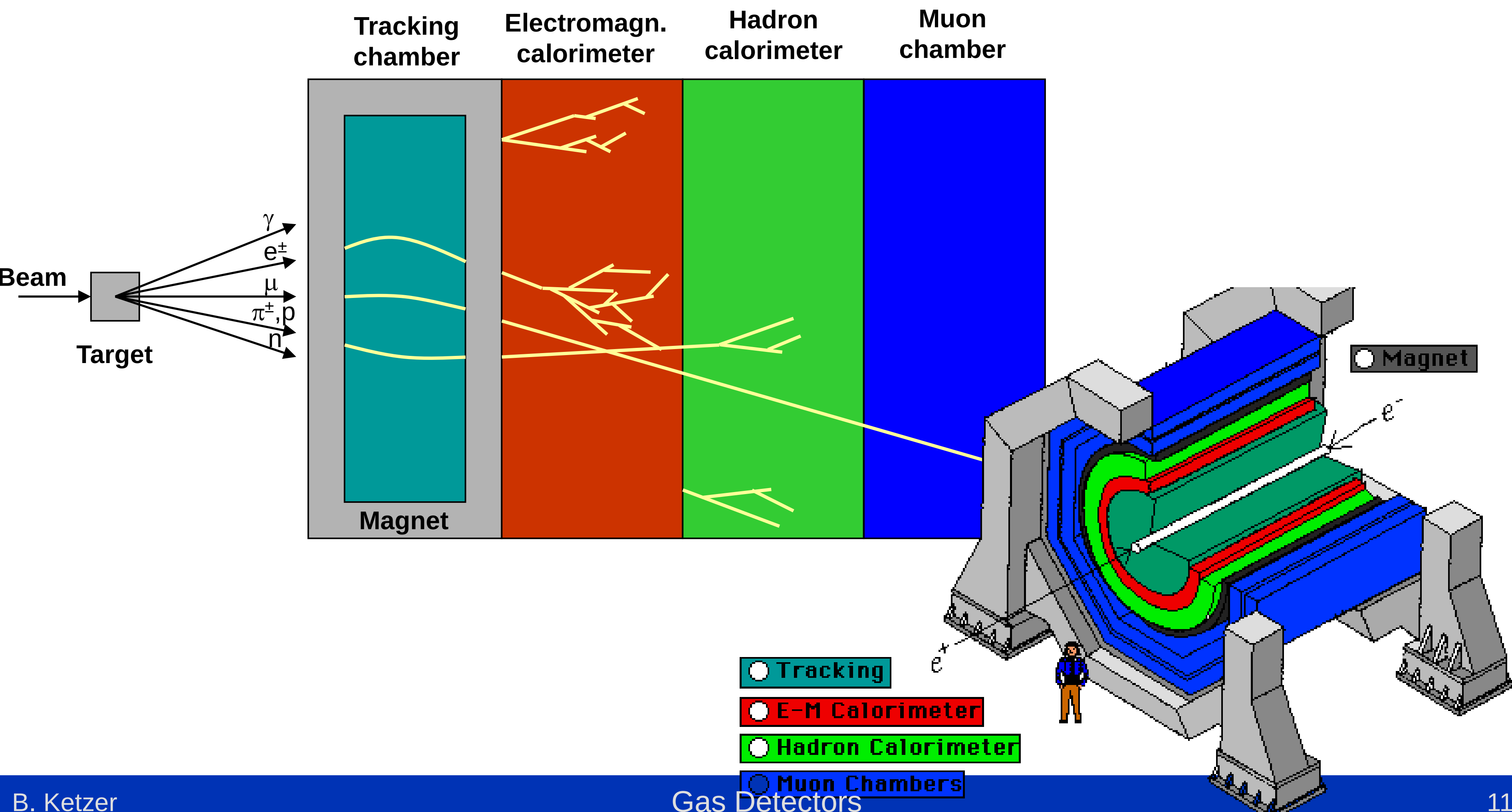
- Engineering, Services, Cooling
- Electronics

But in the end:

resolution limits are still defined by the fundamental detector physics processes ..

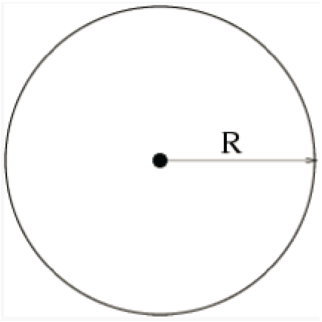
A Typical Detector Setup

Different **components**, measuring different **aspects** of reaction products: track, charge, energy, momentum, particle type, ...

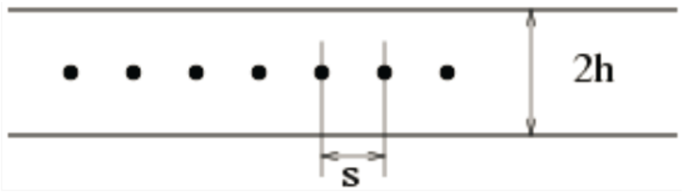


Gaseous Detectors at LHC

Geiger-Müller (1908), 1928
Drift Tube (1968)

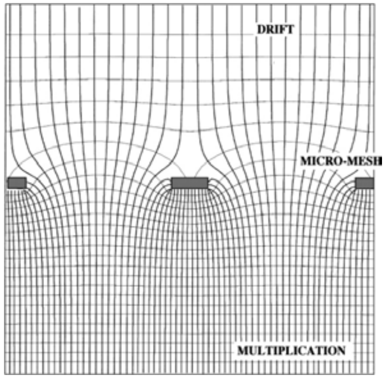


G. Charpak, 1968
Multiwire Proportional Chamber

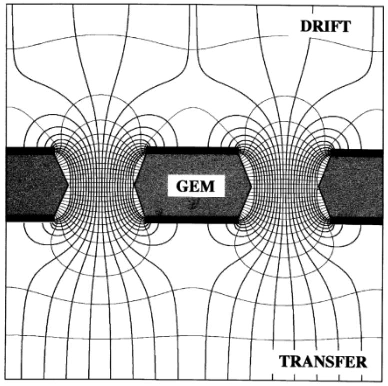


R. Santonico, 1980
Resistive Plate Chamber

G. Giomataris, 1996
Micromesh Gaseous Structure



F. Sauli, 1997
Gas Electron Multiplier

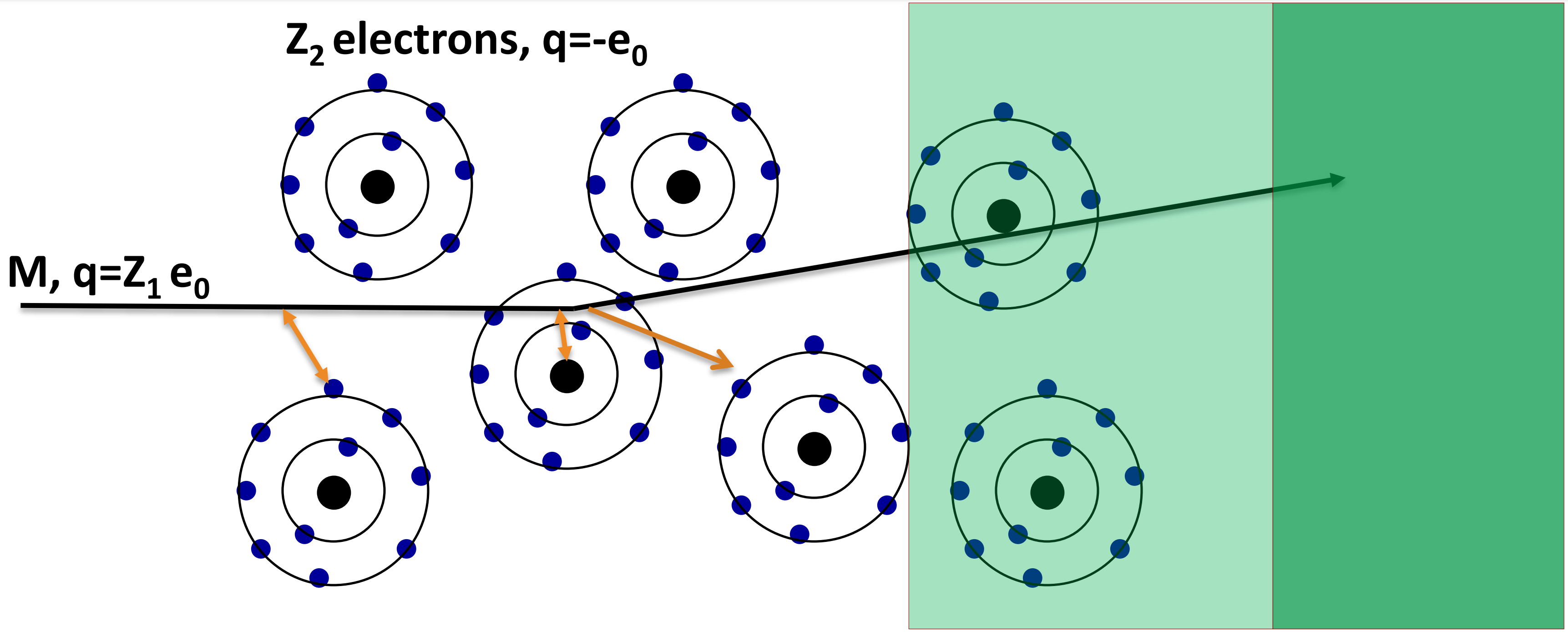


Warm Up

1. What is the general relation between energy and momentum?
2. Which approximations can be used?
3. What are β and γ? How are they calculated from E, p, m?
4. How large are the fluctuations in radioactive decay?
5. What is a cross section?
6. What are typical values of cross sections?
7. How is the cross section related to luminosity?
8. How do charged particles interact with matter?

2 Electromagnetic Interactions of Charged Particles with Matter

Electromagnetic Interaction of Particles with Matter



Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called Transition radiation.

Slide courtesy of W. Riegler

2.1 Ionizing Collisions

Interactions of a fast charged particle with speed $\beta = v/c$ and momentum $p = Mc\beta\gamma$ with matter

⇨ Occurrence of random individual collisions

⇨ In each collision the particle loses a random amount of energy E

Characterization by mean free path λ and collision cross section σ:

$$\lambda = \frac{1}{n_e \sigma} = \frac{1}{n_p}$$

n_e number density of electrons
n_p number of (primary) collisions per unit length

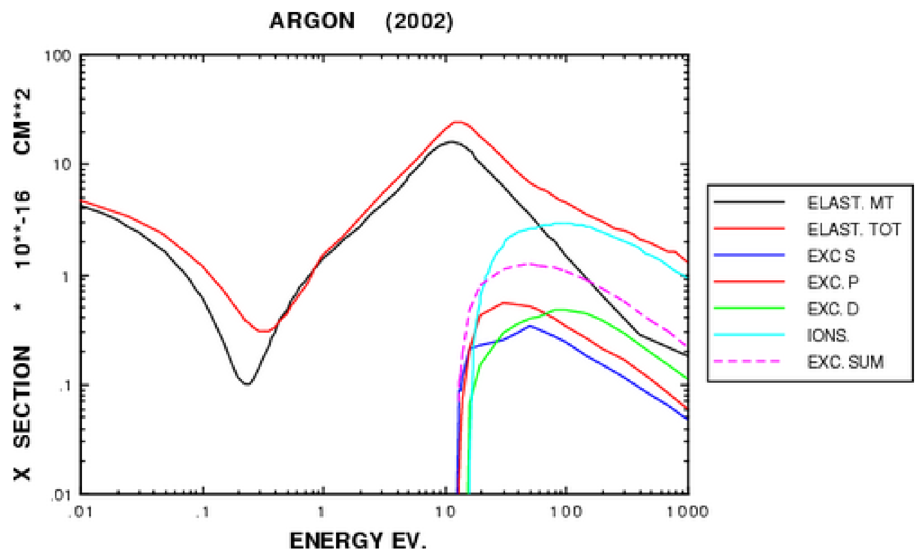
Number of encounters in length L described by Poisson distribution

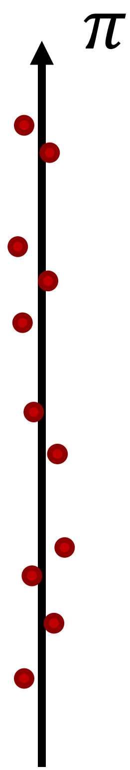
$$P(k;\mu) = \frac{\mu^k}{k!}\, e^{-\mu} \qquad \mu = \frac{L}{\lambda} = L\, n_p$$

Ionizing Collisions

Distinguish between primary and secondary ionization:

1. Primary ionization: created by incident fast particle, e.g. π
   - Cross section ~ 10^{-17} cm^2
   - Energy threshold for ionization



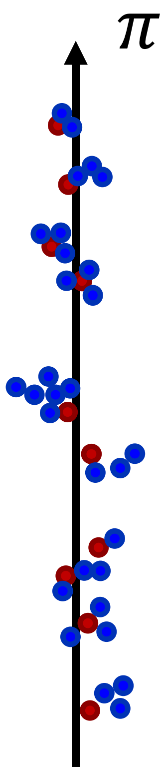


Ionizing Collisions

Distinguish between primary and secondary ionization:

1. Primary ionization: created by incident fast particle, e.g. π
   - Cross section ~ 10^{-17} cm^2
   - Energy threshold for ionization

2. Secondary ionization: electrons are ejected from atoms not encountered by fast particle
   - collision of ionization e^- with atoms (δ electrons)
   - through intermediate excited states, e.g. Penning effect, Jesse effect
   - through creation of excimers



$$\frac{n_t}{n_p} \approx 2-3$$ "ionization cluster" ⇨ most of the total charge!

Ionizing Collisions

- Only a certain fraction of all the energy lost is spent in ionization
- Total amount of ionization (number of electrons) n_t characterized by energy W which is spent on average on the creation of one free electron

$$W\, n_t = \langle \Delta E \rangle \cong L \left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle$$

$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle$ average energy loss per unit pathlength
"stopping power" or "specific energy loss"

Caution:

- Collisions are a statistical process, i.e. there will be fluctuations
- The definition of an average requires that the fluctuations are not too large!

2.2 Mean energy loss

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = \frac{4\pi}{(4\pi\varepsilon_0)^2} \frac{z^2 e^4 n_\mathrm{e}}{mc^2\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_\mathrm{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

"Bethe equation"

with

ze	charge of incoming particle
n_e	electron number density of material
m	electron mass
$\beta = v/c$	velocity of incoming particle
γ	relativistic factor
T_max	maximum kinetic energy imparted to electron in single collision
I	mean excitation energy
δ	density effect correction

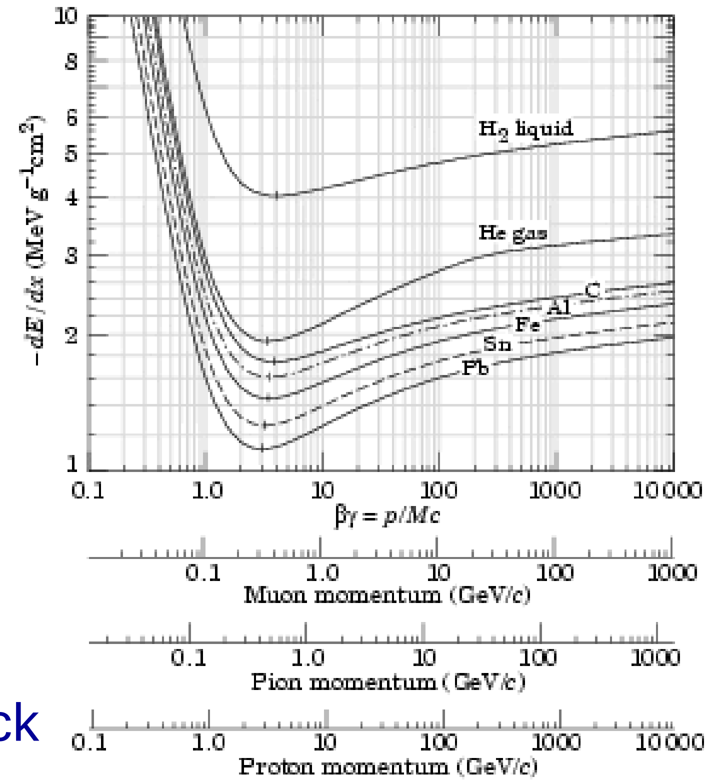
$$n_\mathrm{e} = \frac{Z}{A} N_\mathrm{A} \rho$$

$$T_\mathrm{max} = \frac{2\gamma^2\beta^2 mc^2}{1 + 2\gamma(m/M) + (m/M)^2} \underset{M \gg 2m\gamma}{\simeq} 2mv^2\gamma^2$$

Mean Energy Loss

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{4\pi}{(4\pi\varepsilon_0)^2}\frac{z^2 e^4 n_e}{mc^2\beta^2}\left[\frac{1}{2}\ln\frac{2mc^2\beta^2\gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$

- independent of mass of incident particle
- depends only on velocity of inc. particle and on I ⇨ main parameter
- low energies ⇨ $\left\langle -\mathrm{d}E/\mathrm{d}x \right\rangle \propto 1/\beta^2$
- minimum at $\beta\gamma \approx 3$: "MIP"
- high energies ⇨ $\left\langle -\mathrm{d}E/\mathrm{d}x \right\rangle \propto \ln \beta^2\gamma^2$: relativistic rise
- mass stopping power: $\left\langle -\mathrm{d}E/\rho\mathrm{d}x \right\rangle \propto z^2 \left(Z/A\right) \cdot f(\beta, I)$
  ⇨ almost independent of material
- density effect: polarization of atoms along track
  ⇨ partly compensates relativistic rise



Calculation of Energy Loss

Quantum picture: energy loss caused by a number of discrete collisions per unit length, each with energy transfer E

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = \int_0^\infty E'\, f(E')\mathrm{d}E'$$

$f(E)\,\mathrm{d}E$ probability of energy loss per unit path length between E and E+dE

and with $f(E) = n_e\, \mathrm{d}\sigma(E,\beta)/\mathrm{d}E$

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_e \int_0^\infty E' \frac{\mathrm{d}\sigma}{\mathrm{d}E'}\mathrm{d}E'$$

n_e electron density

E energy transfer in single collision

$\mathrm{d}\sigma/\mathrm{d}E$ collision cross section differential in transferred energy

Mean free path: $n_p = \frac{1}{\lambda} = \int_0^\infty f(E')\mathrm{d}E'$

n_p number of primary collisions per unit path length

Spectrum of energy transfer $F(E)\mathrm{d}E = \frac{f(E)\mathrm{d}E}{n_p}$

probability of energy loss in $[E, E+\mathrm{d}E]$ per collision

⇨ need a model for collision cross section!

[H. Bichsel, NIM A 562, 154 (2006)]

Rutherford - Mott Model

Simplest ansatz: hard collisions

- Coulomb scattering of projectile with charge ze off free electrons
- only valid for energy transfers $\gg$ typical atomic binding energies I
- in rest frame of projectile: electron scattering off heavy particle at rest

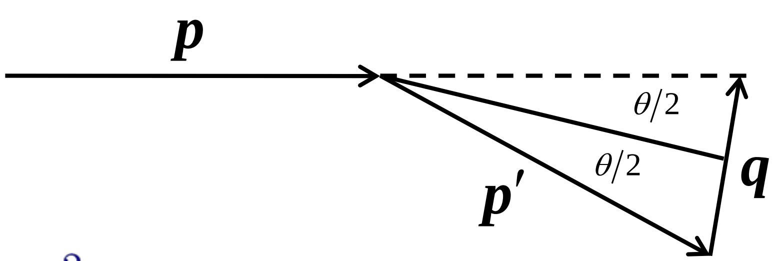
⇨ Mott cross section:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\star}_{\mathrm{Mott}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} \cdot \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} = \left(\frac{z\alpha\hbar c}{2\,|\boldsymbol{p}|\,|\boldsymbol{v}|}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$ for static potential (no recoil)

Rutherford - Mott

With $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$, $\sin\frac{\theta}{2} = \frac{|\boldsymbol{q}|}{2\,|\boldsymbol{p}|}$, $\boldsymbol{p} = \gamma m \boldsymbol{v}$



follows the cross section
differential in transferred energy $E = \frac{|\boldsymbol{q}|^2}{2m}$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2\left[1 - \frac{E}{2mc^2}\left(1-\beta^2\right)\right]$$

Exercise: show this…

Rutherford - Mott

Evaluation of integral $$\int_{T_{\min}}^{T_{\max}} E' \left(\frac{\mathrm{d}\sigma(E',\beta)}{\mathrm{d}E'}\right)_{\mathrm{Mott}} \mathrm{d}E'$$

Validity range of Mott CCS: $T_{\min} < E < T_{\max}$

$$T_{\max} = \frac{2\gamma^2\beta^2 mc^2}{1 + 2\gamma(m/M) + (m/M)^2} \underset{M \gg 2m\gamma}{\simeq} 2mv^2\gamma^2$$

$$T_{\min} = \epsilon \gg I$$

I: mean excitation energy

Therefore we arrive at

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{\mathrm{R}} = n_e \cdot \frac{2\pi}{m} \cdot \left(\frac{z\alpha\hbar}{\beta}\right)^2 \left[\ln\frac{2mv^2\gamma^2}{\epsilon} - \beta^2\right]$$

Yields Bethe equation, except

- Factor 2
- ϵ instead of I

Contribution from hard scattering!

Bethe – Fano Model

Bethe, 1930: [H. Bethe, Ann. Phys. 5, 325 (1930)]

- drop assumption of free electrons
- derive expression for cross section double-differential in energy loss E and momentum transfer $\boldsymbol{q}$ for inelastic scattering on free atoms
- use first Born approximation

$$\frac{\mathrm{d}\sigma(E,Q)}{\mathrm{d}E\mathrm{d}Q} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{*}_{\mathrm{Mott}} \cdot \frac{E^2}{Q^2} \cdot |F(\mathbf{q})|^2 \qquad Q = \frac{q^2}{2m}$$

with $$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2 \left[1 - \beta^2 \frac{E}{T_{\max}}\right]$$

Fano, 1963: [U. Fano, Ann. Rev. Nucl. Sci. 13, 1 (1963)]

- extend method for solids
- no calculations exist for gases

Bethe – Fano Model

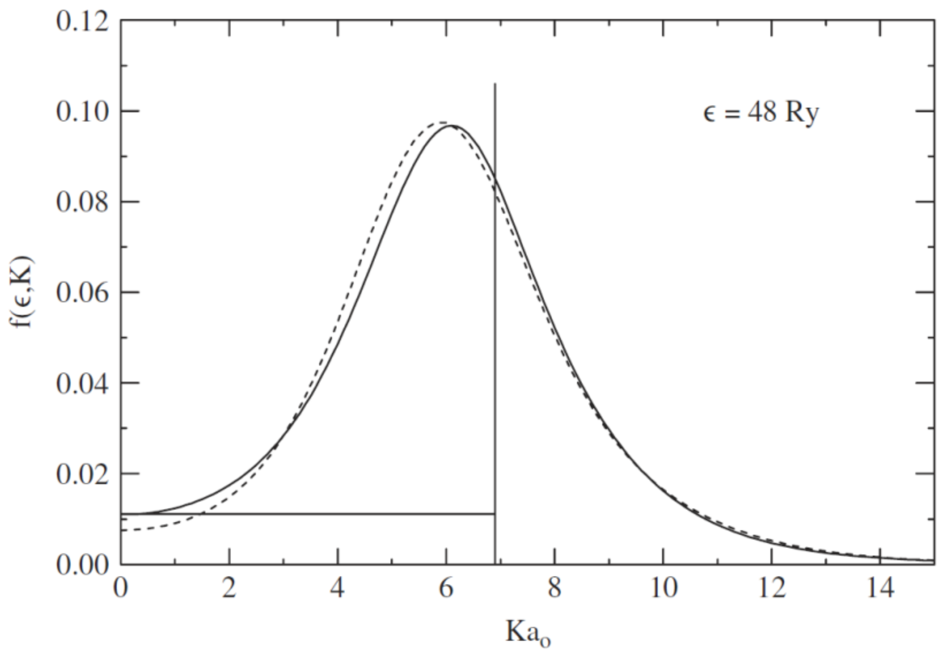


Fig. 4. Generalized oscillator strength (GOS) for Si for an energy transfer $\varepsilon = 48Ry$ ($Ry = 13.6$ eV) to the 2p-shell electrons [18]. Solid line: calculated with Herman–Skilman potential, dashed line: hydrogenic approximation. The horizontal and vertical line define the FVP approximation (Section 2.3).

$$f_n(E,k) = \frac{E}{Q} \cdot |F_n(k)|^2$$

Generalized oscillator strength

Form factor for excitation of state $|n\rangle$

[H. Bichsel, NIM A 562, 154 (2006)]

Bethe – Fano Model

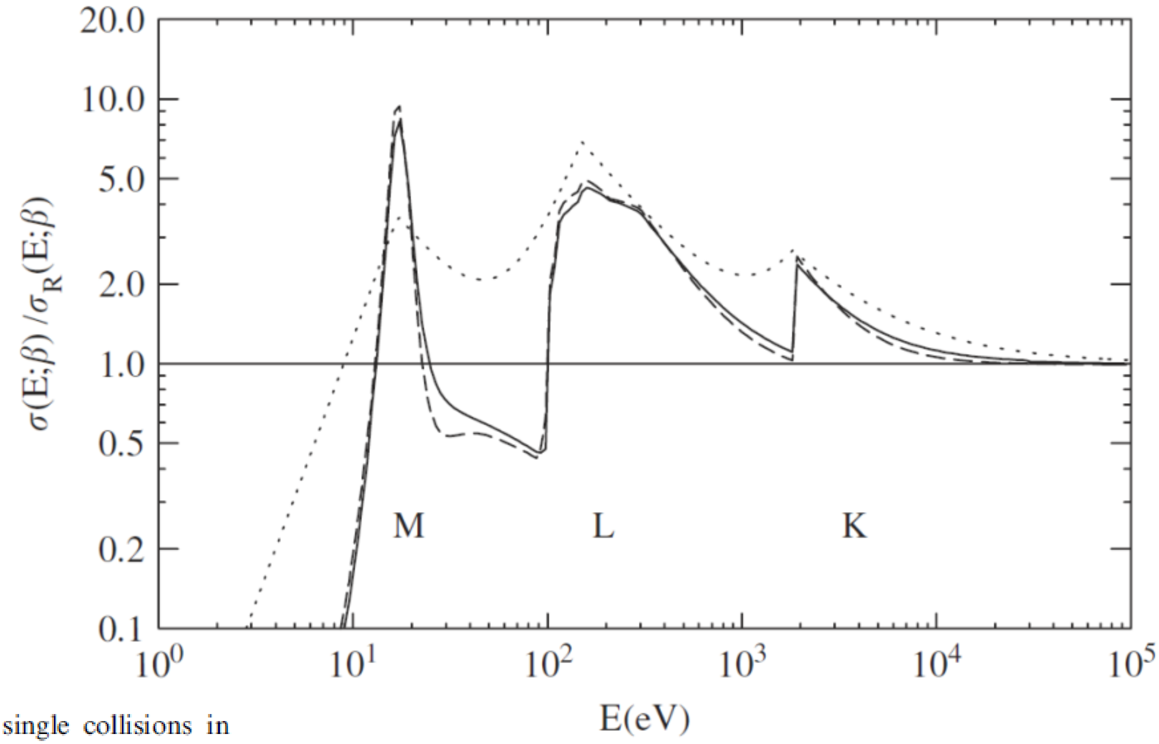


Fig. 5. Inelastic collision cross-sections $\sigma(E;\beta)$ for single collisions in silicon by particles with $\beta\gamma = 4$, calculated with different theories. In order to show the structure of the functions clearly, the ordinate is $\sigma(E;\beta)/\sigma_R(E;\beta)$. The abscissa is the energy loss E in a single collision. The Rutherford cross-section Eq. (1) is represented by the horizontal line at 1.0. The solid line was obtained with the relativistic version of Eq. (5) of the Bethe–Fano theory [18]. The cross-section calculated with FVP (Eq. (7)) is shown by the dotted line. The dashed line is calculated with a binary encounter approximation [35,36]. The functions all extend to $E_{max} \sim 16\,MeV$; see Eq. (1). The moments (Section 3) are $M_0 = 4\,collisions/\mu m$ and $M_1 = 386\,eV/\mu m$. The atomic shells are indicated by the letters M, L, K.

[H. Bichsel, NIM A 562, 154 (2006)]

Total Energy Loss

Total energy loss: Bethe-Bloch formula

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = \underbrace{\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T>\varepsilon}}_{\text{Hard: Mott}} + \underbrace{\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T<\varepsilon}}_{\text{Soft}} = n_e \frac{4\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_{\max}}{I^2} - \beta^2 \right]$$

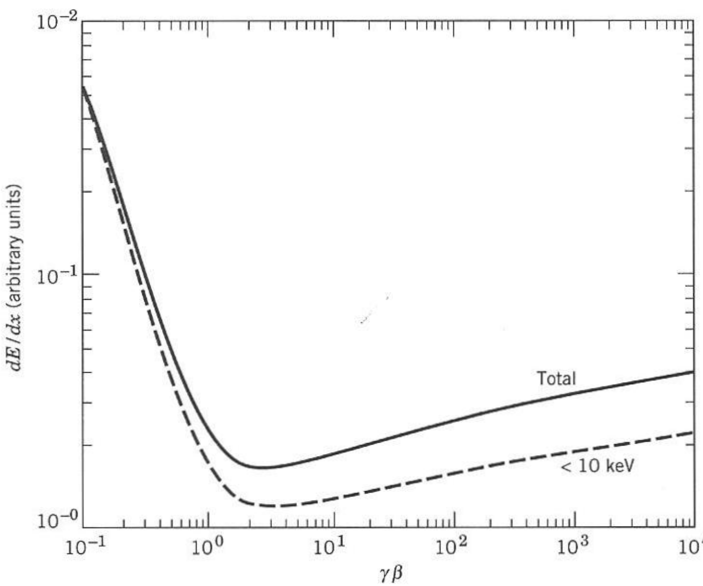
$$n_e = \frac{Z}{A} \cdot N_A \cdot \rho$$

with

independent of ε

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T>\varepsilon} = n_e \frac{2\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\ln \frac{T_{\max}}{\varepsilon} - \beta^2 \right]$$

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T<\varepsilon} = n_e \frac{2\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\ln \frac{2mc^2\beta^2\gamma^2\varepsilon}{I^2} - \beta^2 \right]$$



Total Energy Loss

In principle, mean excitation energy I can be calculated from atomic theory:

$$Z \cdot \ln(I) \propto \sum_n f_n \ln(\hbar\omega_n)$$

⇨ models needed for all but lightest atoms

⇨ often used in practice: I as phenomenological constant

Goal: Simplify cross section expression based on measured photo-absorption cross sections

⇨ Photoabsorption Ionization Model

… also called Fermi virtual photon (FVP) model

PAI Model

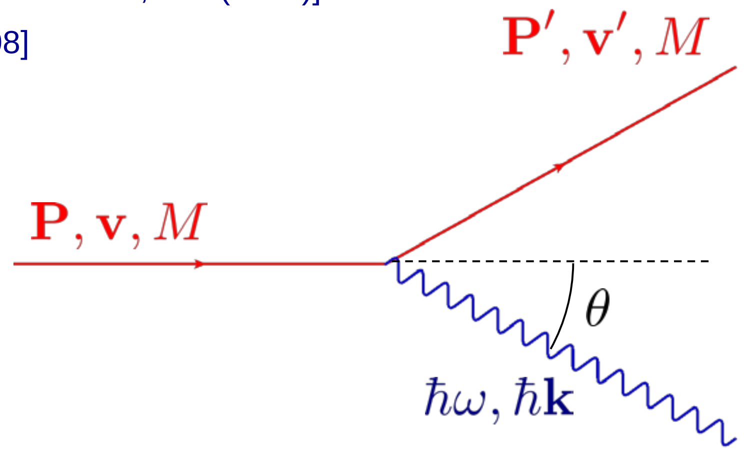
Idea: Calculate $\langle \mathrm{d}E/\mathrm{d}x \rangle$ of a moving charged particle (other than $e^{\pm}$) in a polarizable medium

- ⇨ classical calculation: medium treated as continuum with $\varepsilon = \varepsilon_1 + i\varepsilon_2$
- ⇨ later: quantum mechanical interpretation

$\langle \mathrm{d}E/\mathrm{d}x \rangle \Leftrightarrow$ longitudinal component of electric field $E(\mathbf{r}, t)$ generated by the moving particle in the medium at its own position $\mathbf{r} = \mathbf{v}t$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = eE_{\mathrm{long}}$$

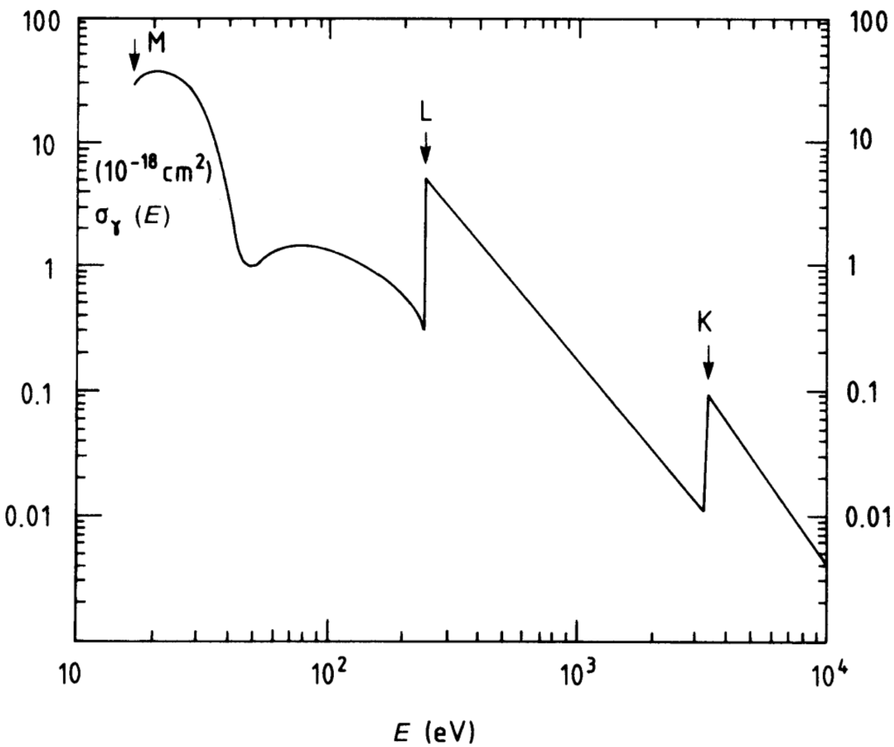
[L. Landau, E.M. Lifshitz, Electrodynamics of continuous media, 1960]
[W.W.M Allison, J.H. Cobb, Ann. Rev. Nucl. Part. Sci. 30, 253 (1980)]
[W. Blum, W. Riegler, L. Rolandi, Springer 2008]



- ⇨ $\varepsilon(k, \omega)$ needed
- ⇨ only known for real photons: $\sigma_\gamma(\omega)$
- ⇨ model for virtual photons needed: Photo-absorption ionization model

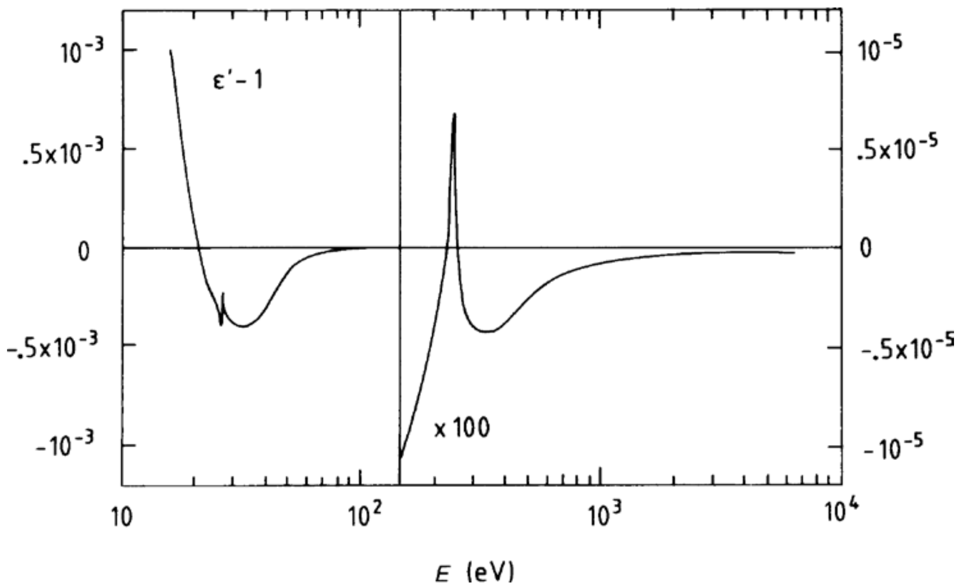
PAI Model

Example: Argon



Total photo-absorption cross section

[G.V. Marr, J.B. West, At. Data and Nucl. Data Tables 18, 497 (1976)]



Real part of ε, calculated from σ_γ using Kramers-Kronig relation

[F. Lapique et al., Nucl. Instr. Meth. 175, 297 (1978)]

PAI Model

Experiment: $\varepsilon = \varepsilon_1 + i\varepsilon_2$ known only for free photons, i.e. on $q_{\mathrm{f}\gamma}$ line

PAI model: extend into the kinematic domain of virtual photons

- Below free-electron line q_{fe} (resonance region): dipole approximation

  $\varepsilon(k,\omega) = \varepsilon(\omega)$ independent of k, as for free photons

- On free-electron line q_{fe} :

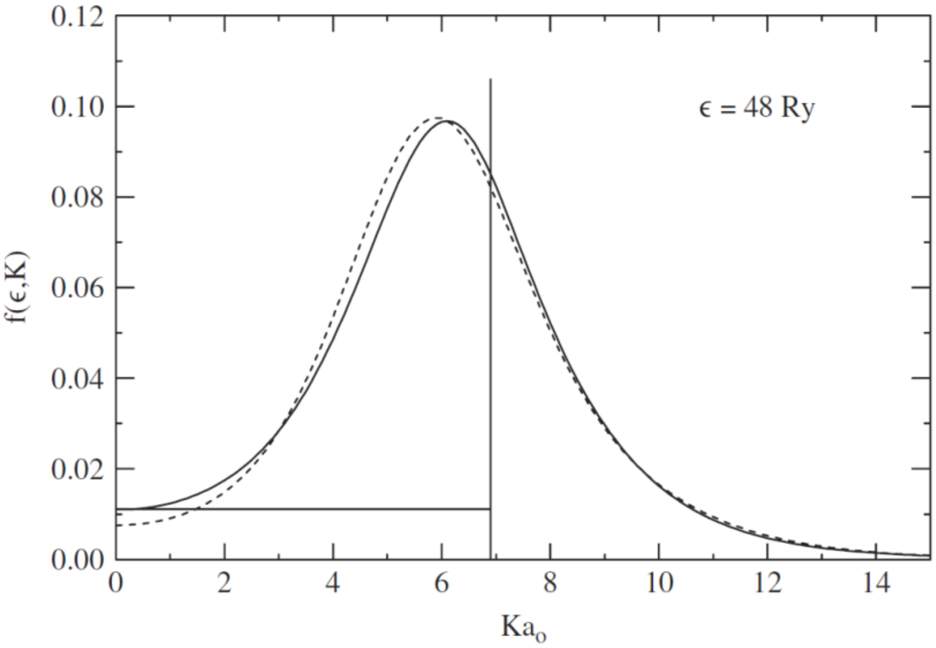
  $$\varepsilon_2(k,\omega) = C\,\delta\left(\omega - \hbar k^2/(2m)\right), \quad \varepsilon_1 = 1$$

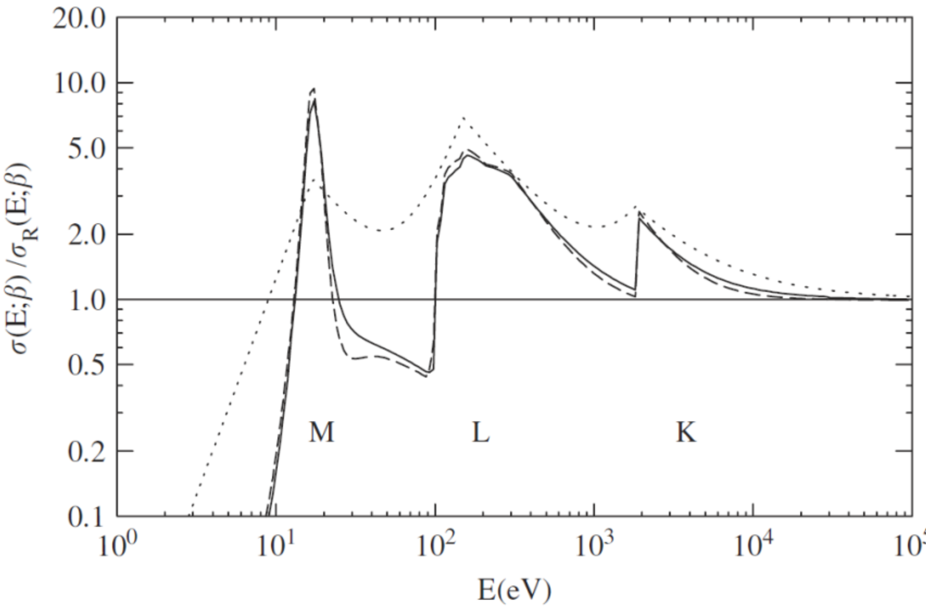
  Normalization C chosen such that total coupling strength satisfies

  $$\int_0^\infty f(k,\omega)\,\mathrm{d}\omega = 1, \quad \varepsilon_2(k,\omega) = \frac{\pi N e^2}{2\varepsilon_0 m\omega} f(k,\omega)$$

  Bethe sum rule

PAI Model





[H. Bichsel, NIM A 562, 154 (2006)]

Optical dipole oscillator strength

$$\lim_{\boldsymbol{q}\to 0} f_n(E,k) = f_n(E)$$

$$f_n(E) = \frac{E}{Q}\left|\langle n|\sum_{i=1}^{Z}\boldsymbol{r_i}|0\rangle\right|$$

PAI Model

Result of classical calculation:

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\frac{e^2}{4\pi\varepsilon_0\beta^2c^2\pi}\int_0^\infty \mathrm{d}\omega \left[\frac{Nc}{Z}\sigma_\gamma(\omega)\ln\left[\left(1-\beta^2\varepsilon_1\right)^2+\beta^4\varepsilon_2^2\right]^{-1/2}\right.$$

$$\left. + \frac{Nc}{Z}\sigma_\gamma(\omega)\ln\left(\frac{2mc^2\beta^2}{\hbar\omega}\right) + \frac{Nc}{Z\omega}\int_0^\omega \sigma_\gamma(\omega')\mathrm{d}\omega' + \omega\left(\beta^2-\frac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta\right]$$

N electron density

$E = \hbar\omega$ energy transfer in single collision

$q = \hbar k$

$\Theta = \arg\left(1-\varepsilon_1\beta^2 + i\varepsilon_2\beta^2\right)$

Energy loss per unit path length obtained in the framework of electrodynamics of a continuous medium, using a model for $\varepsilon(k,\omega)$ inspired by a picture of photon collision and absorption.

PAI Model

Quantum picture: energy loss caused by a number of discrete collisions per unit length, each with energy transfer $E = \hbar\omega$ (single photon exchange)

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\int_0^\infty E\, f(E)\,\mathrm{d}E$$

$f(E)\,\mathrm{d}E$ probability of energy transfer per unit path between E and E+dE

and with $f(E) = N\,\mathrm{d}\sigma/\mathrm{d}E$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\int_0^\infty E N \frac{\mathrm{d}\sigma}{\mathrm{d}E}\hbar\,\mathrm{d}\omega$$

N electron density

$E = \hbar\omega$ energy transfer in single collision

$q = \hbar k$

PAI Model

Therefore: differential cross section per electron

$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left[(1-\beta^2\varepsilon_1)^2+\beta^4\varepsilon_2^2\right]^{-1/2}$$
$$+\frac{\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left(\frac{2mc^2\beta^2}{E}\right)$$
$$+\frac{\alpha}{\beta^2\pi}\frac{1}{E^2Z}\int_0^E\sigma_\gamma(E')dE'$$
$$+\frac{\alpha}{\beta^2\pi}\frac{1}{N\hbar c}\left(\beta^2-\frac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta$$

Energy loss by ionization

Rutherford scattering (for $E >> E_K$)
⇨ δ electrons

- Optical region: $\sigma_\gamma = 0$
  ⇨ Cherenkov radiation
- Transition radiation for thin radiators

with $\varepsilon_1, \varepsilon_2$: real and imaginary part of dielectric constant (for real photons)

$\Theta = \arg\left(1-\varepsilon_1\beta^2+i\varepsilon_2\beta^2\right)$ angle in pointer representation of complex number

σ_γ: atomic cross section of medium for absorption of photon with energy E

N: electron density in the medium

Energy Loss by Ionization

Described by first three terms of $\frac{\mathrm{d}\sigma}{\mathrm{d}E}$

- Large energy transfers $E \gg E_K$ ⇨ only third term survives ($\sigma_\gamma(E)$ small)

$$\frac{\alpha}{\beta^2\pi}\frac{1}{E^2}\int_0^E \frac{\sigma_\gamma(E')}{Z}\mathrm{d}E' \xrightarrow[\text{Bethe sum rule}]{E \gg E_K} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)_\mathrm{R} = \frac{2\pi r_e^2 mc^2}{\beta^2 E^2}, \quad r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2}$$

Rutherford cross section: elastic scattering on free electron

⇨ extremely long tail of energy loss distribution due to δ electrons

⇨ ill-defined average energy loss! (log. divergence)

⇨ better: most probable value

⇨ in practice: upper limit for E depending on detector: restricted energy loss

Energy Loss by Ionization

- Remaining two terms:

$$\frac{d\sigma}{dE} = \frac{a}{\beta^2}\left[b + \ln \frac{\beta^2}{\left[\left(1-\beta^2\varepsilon_1\right)^2 + \beta^4\varepsilon_2^2\right]^{1/2}} \right]$$

$a, b = f(E, \sigma_\gamma)$

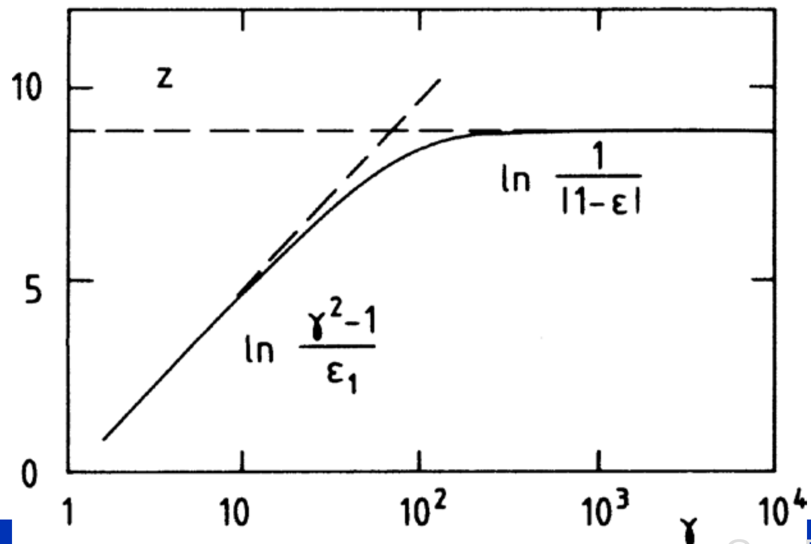
– small β: factor $\frac{1}{\beta^2}$ dominates

– $\beta \to 1$: logarithmic term dominates $\rightarrow$

$$\begin{cases} \ln \frac{\gamma^2 - 1}{\varepsilon_1} & \text{for} \quad \gamma^2 \ll 1/|1-\varepsilon| \\ \ln \frac{1}{|1-\varepsilon|} & \text{for} \quad \gamma^2 \gg 1/|1-\varepsilon| \end{cases}$$

relativistic rise

plateau



Plateau due to density of medium!

$\varepsilon_1 - 1, \varepsilon_2 \propto N$ e$^-$ density

$\Rightarrow \varepsilon_1 = 1, \varepsilon_2 = 0 \quad \text{for} \quad N \to 0$

$\Rightarrow \frac{d\sigma}{dE}$ continues to rise for $N \to 0$!

Cherenkov Radiation

Term $\dfrac{\alpha}{\beta^2\pi}\dfrac{1}{N\hbar c}\left(\beta^2-\dfrac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta\,,\quad \Theta=\arg\left(1-\varepsilon_1\beta^2+i\varepsilon_2\beta^2\right)$

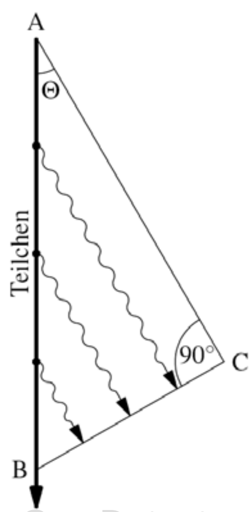
- Only remaining term for photon energies below excitation energy of atom (optical region), where $\sigma_\gamma=0\,,\varepsilon_2=0\,,\varepsilon=\varepsilon_1$
- $\Theta=\arg\left(1-\varepsilon_1\beta^2\right)$ jumps from 0 to π at $\beta_0^2=\dfrac{1}{\varepsilon_1}$ Cherenkov threshold

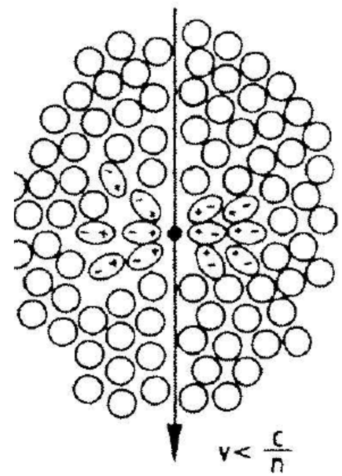
⇨ Emission of radiation if

$$\varepsilon\beta^2>1\Leftrightarrow v>\frac{c}{n}\,,\quad n=\sqrt{\varepsilon}$$

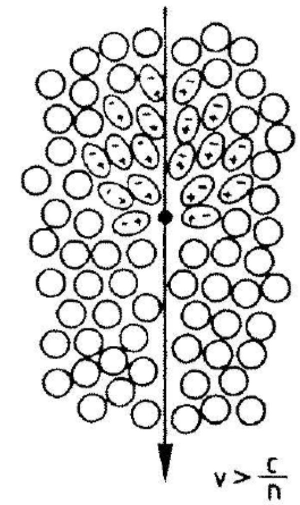
Emission angle:

$$\cos\theta=\frac{1}{\beta n}$$





destructive



constructive

interference

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_e \int_0^{\infty} E' \frac{\mathrm{d}\sigma}{\mathrm{d}E'} \mathrm{d}E'$$

Models:

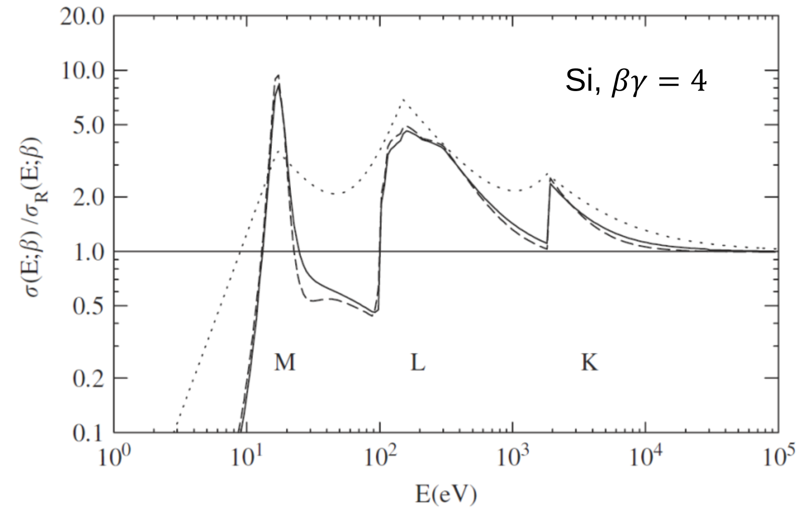
- Rutherford – Mott

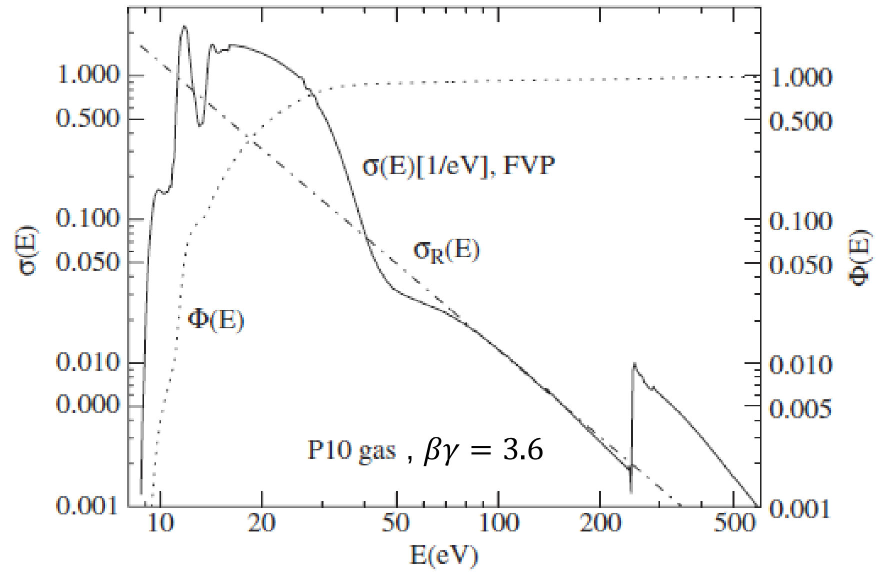
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^{2}\left[1 - \frac{E}{2mc^2}\left(1-\beta^2\right)\right]$$

- Bethe – Fano

$$\frac{\mathrm{d}\sigma(E,Q)}{\mathrm{d}E\mathrm{d}Q} = \left(\frac{\mathrm{d}\sigma(E;v)}{\mathrm{d}E}\right)^{*}_{\mathrm{Mott}} \cdot \frac{E}{Q} \cdot f(k,\omega)$$

- PAI (FVP)





[H. Bichsel, NIM A 562, 154 (2006)]

Collision Cross Section

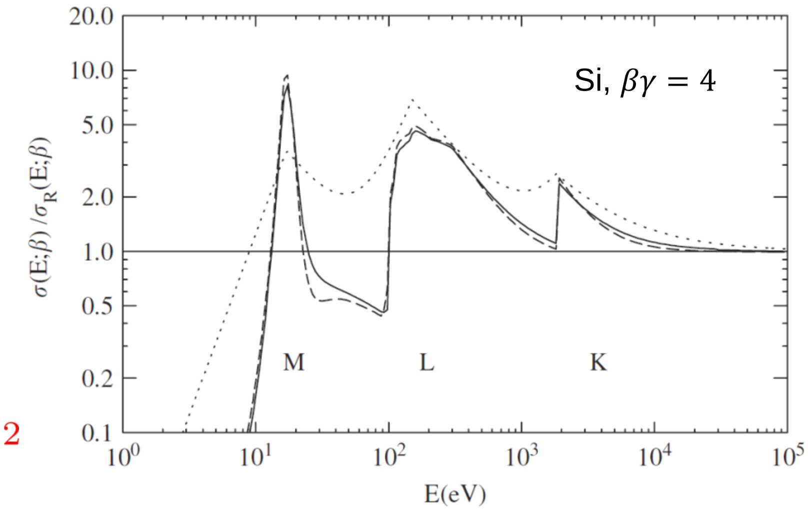
$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_e \int_0^\infty E' \, \frac{\mathrm{d}\sigma}{\mathrm{d}E'} \mathrm{d}E'$$

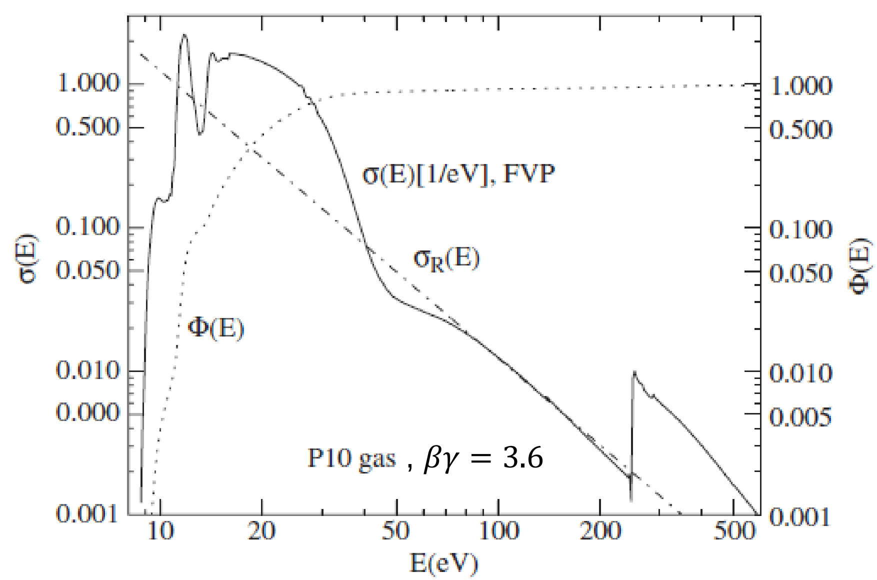
Models:

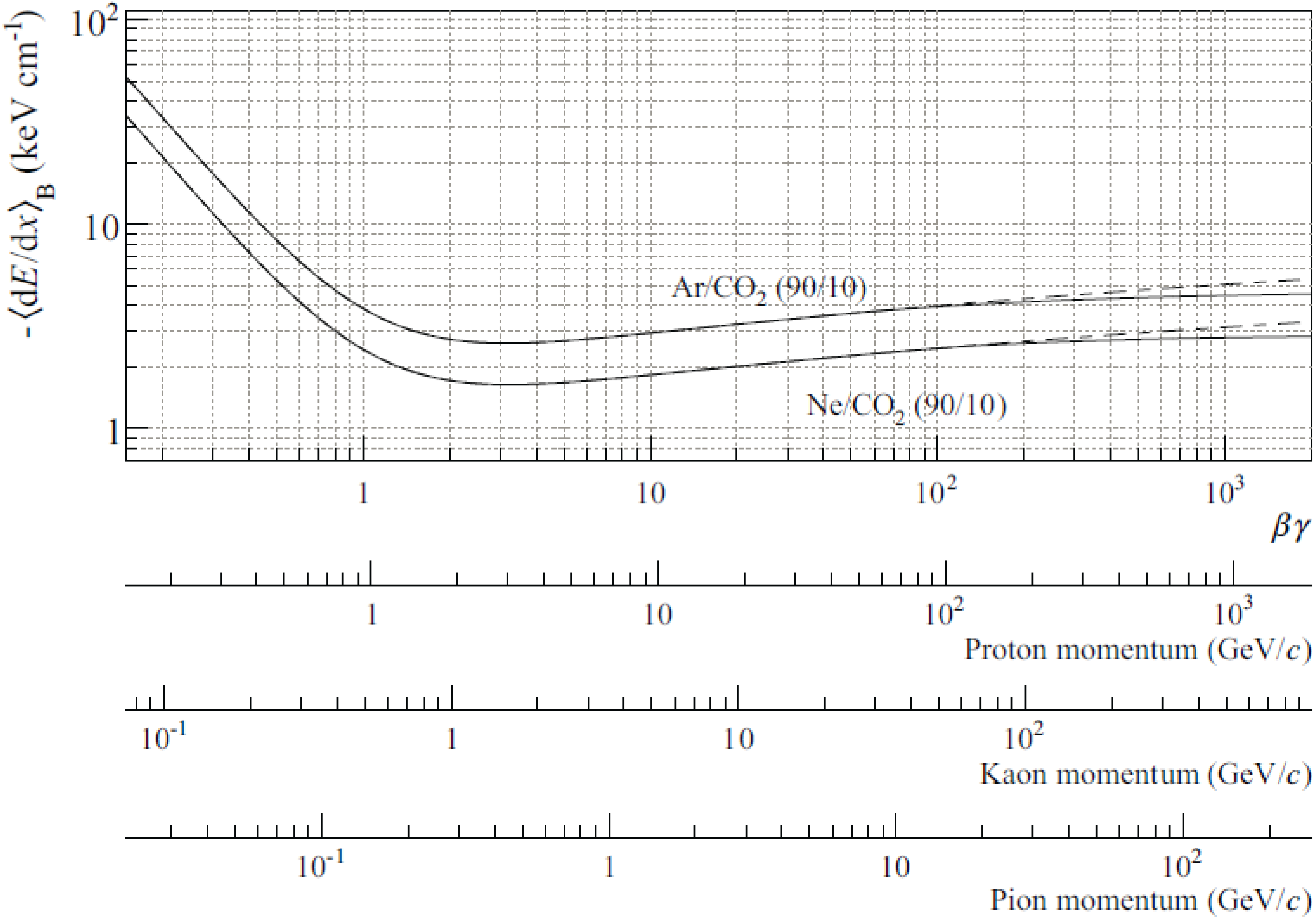
- PAI (FVP)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{\alpha}{\beta^2 \pi} \frac{\sigma_\gamma(E)}{EZ} \ln \left[\left(1 - \beta^2 \varepsilon_1\right)^2 + \beta^4 \varepsilon_2^2 \right]^{-1/2}$$
$$+ \frac{\alpha}{\beta^2 \pi} \frac{\sigma_\gamma(E)}{EZ} \ln \left(\frac{2mc^2 \beta^2}{E} \right)$$
$$+ \frac{\alpha}{\beta^2 \pi} \frac{1}{E^2 Z} \int_0^E \sigma_\gamma(E') \mathrm{d}E'$$
$$+ \frac{\alpha}{\beta^2 \pi} \frac{1}{N \hbar c} \left(\beta^2 - \frac{\varepsilon_1}{|\varepsilon|^2} \right) \Theta$$

[H. Bichsel, NIM A 562, 154 (2006)]







[F. Böhmer, PhD thesis, TUM]