Gaseous Detectors

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Plan of the Lecture

1. Introduction
2. Interactions of charged particles with matter
3. Drift and diffusion of charges in gases
4. Avalanche multiplication of charge
5. Signal formation and processing
6. Ionization and proportional gaseous detectors
7. Position and momentum measurement / track reconstruction

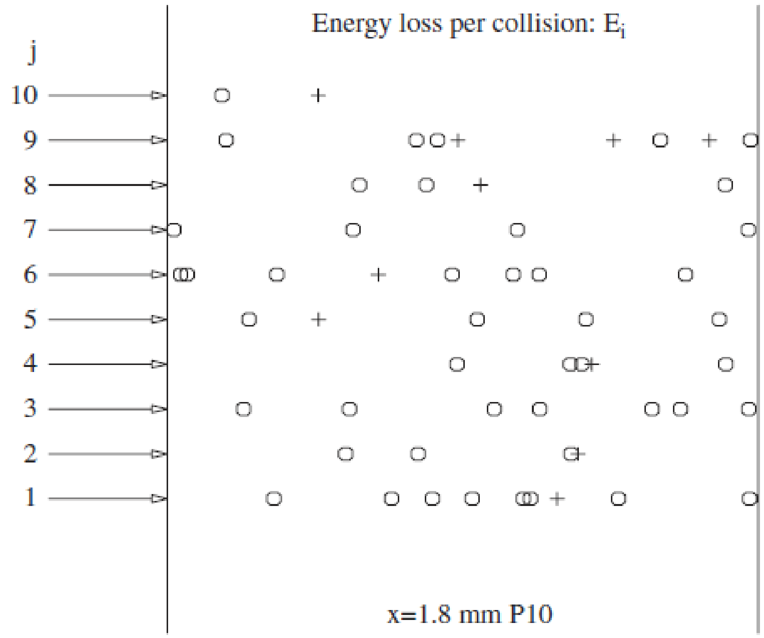
2.3 Fluctuations of Energy Loss

Consider single particle: statistical fluctuations of

- number of collisions
- energy transfer in each collision

⇨ range straggling (if stopped in medium)

⇨ energy straggling (if traversing medium)



n_j	Δ_j(eV)	E_t(eV)
2	56	37
8	703	559
4	126	68
4	82	32
8	419	292
5	565	502
5	95	35
7	146	26
4	105	34
9	930	774

$\Delta = \Sigma E_i$

Fig. 3. Monte Carlo simulation of the passage of 10 particles (index j) with $\beta\gamma = 3.6$ through segments of P10 gas. The thickness of the gas layer (at 1 atm and 25 °C) is $x = 1.8$ mm. The direction of travel is given by the arrows. Inside the gas, the tracks are defined by the symbols showing the location of a collision. The mean free path between collisions is $\lambda = 0.3$ mm (see Fig. 7 or Table 2), thus the *average* number of collisions per track is six. At each collision point a random energy loss E_i is selected from the distribution function $\Phi(E;\beta\gamma)$, Fig. 9. Two symbols are used to represent energy losses: ∘ for $E_i < 33$ eV, + for $E_i > 33$ eV; the mean free path between collisions with $E_i > 33$ eV is 2 mm. Segment statistics are shown to the right: the total number of collisions for each track is given by n_j, with a nominal mean value $\langle n \rangle = x/\lambda = 6$ and the total energy loss is $\Delta_j = \sum E_i$, with the nominal mean value $\langle \Delta \rangle = x\,\mathrm{d}E/\mathrm{d}x = 440$ eV, where $\mathrm{d}E/\mathrm{d}x$ is the Bethe–Bloch *stopping power*, M_1 in Table 2. The largest energy loss E_t on each track is also given. The mean value of the Δ_j is 325 ± 314 eV, much less than $\langle \Delta \rangle$. Note that the largest possible energy loss in a single collision is $E_{max} = 13$ MeV, while the probability for $E > 50{,}000$ eV is 0.002 per cm, Eq. (12) or Figs. 9 and 10.

[H. Bichsel, NIM A 562, 154 (2006)]

Fluctuations of Energy Loss

Important quantity in order to understand response of detector:

$f(\Delta; x)$ probability density function for energy loss Δ in material of thickness x,

determined by

- collision cross section $\mathrm{d}\sigma/\mathrm{d}E$
- $n_e x$

Straggling functions

Calculation of energy loss distribution: two approaches

- Convolution method
- Laplace transform method

[Allison, Cobb, Ann. Rev. Nucl. Part. Sc., 253 (1980)]

[H. Bichsel, NIM A 562, 154 (2006)]

2.3.1 Convolution Method

In each collision, the probability to transfer an energy E is given by

$$F(E) = \lambda n_e \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} = \frac{1}{\sigma}\frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E}$$

Energy loss Δ for exactly N_c collisions ⇨ N_c-fold convolution of $F(E)$

$$\tilde{F}_{N_c}(\Delta) = \int_0^{\Delta} \tilde{F}_1(E) \cdot \tilde{F}_{N_c-1}(\Delta - E)\,\mathrm{d}E$$

with $\tilde{F}_0(\Delta) = \delta(\Delta)$ and $\tilde{F}_1(\Delta) = \frac{1}{\sigma}\frac{\mathrm{d}\sigma(\Delta;\beta)}{\mathrm{d}E} = F(\Delta)$

Convolution Method

Number of collisions N_c in layer of thickness x

$$P(N_c; m_c) = \frac{m_c^{N_c}}{N_c!} \exp\left(-m_c\right) \qquad m_c = \frac{x}{\lambda}$$

⇨ Linked to CCS through mean free path

$$\lambda = \lambda(\beta) = \frac{1}{n_e \sigma} \qquad \sigma = \int_0^\infty \frac{\mathrm{d}\sigma(E';\beta)}{\mathrm{d}E'} \mathrm{d}E'$$

Mean value $\langle P(N_c; m_c) \rangle = m_c$

Standard deviation $s_c = \sqrt{m_c}$

Relative width $s_c / m_c = 1/\sqrt{m_c}$

Convolution Method

⇨ Pdf for total ionization energy loss Δ in material slice of thickness x

= sum of all $\tilde{F}_{N_c}(\Delta)$, weighted by their Poissonian probability for exactly N_c collisions

$$f(\Delta;x) = \sum_{N_c=0}^{\infty} P(N_c;m_c)\,\tilde{F}_{N_c}(\Delta)$$

Straggling functions

- Poissonian contribution dominant for very small number of collisions (very thin absorbers)
- Peak structure vanishing for larger N_c

Convolution Method

Solution for thickness x:

- Iterative application of convolution integral (numerical)
  [Bichsel et al., Phys. Rev. A 11, 1286 (1975)]
- Monte-Carlo method [Cobb et al., Nucl. Instr. Meth. 133, 315 (1976)]
  - calculate mean number of collisions m_c from integrated cross section
  - for each trial (particle penetration) choose actual number of collisions from Poisson distribution with mean m_c
  - total energy loss = sum of energy losses in single collisions, taken from normalized $\mathrm{d}\sigma/\mathrm{d}E$ distribution $F(E)$

Straggling Functions

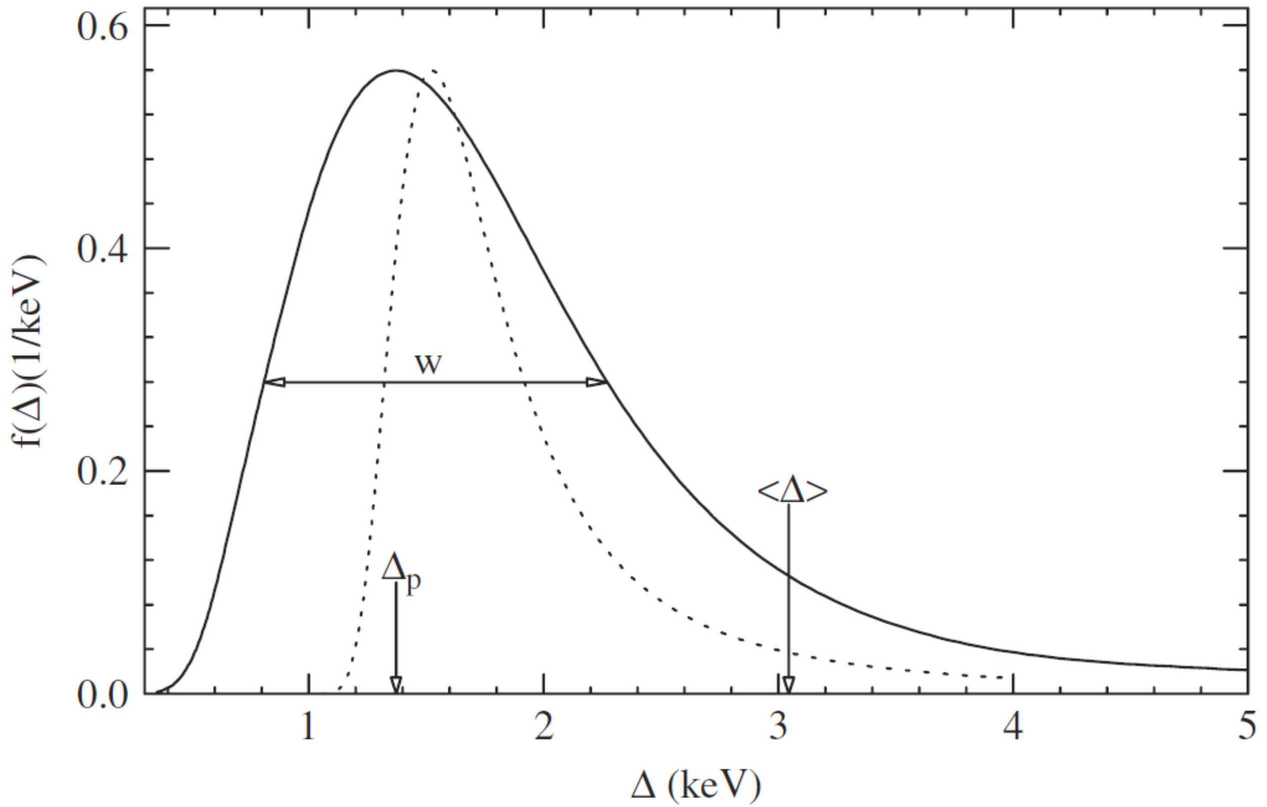
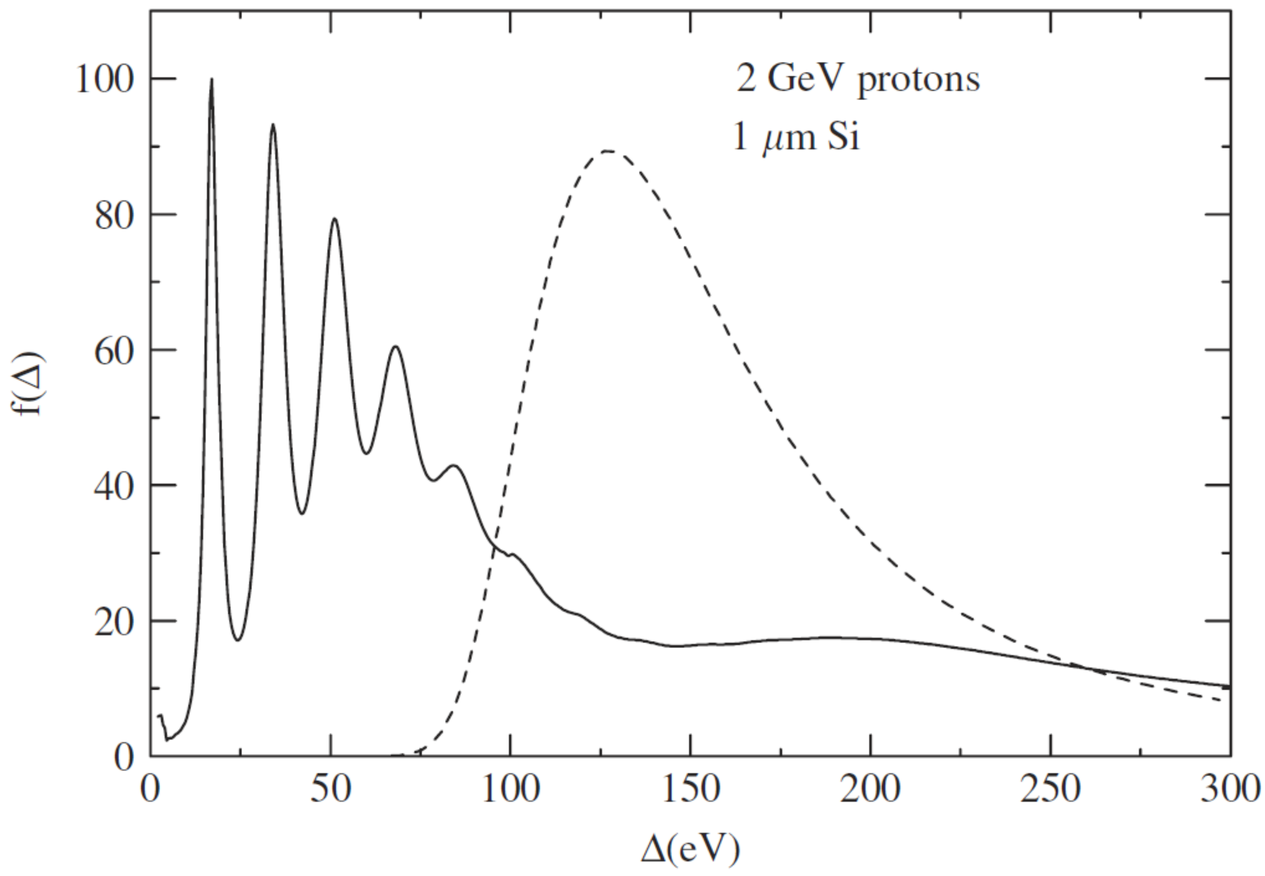


Fig. 1. The straggling function $f(\Delta)$ for particles with $\beta\gamma = 3.6$ traversing 1.2 cm of Ar gas is given by the solid line. It extends beyond $E_{max} \sim 2\,mc^2\beta^2\gamma^2 = 13\,MeV$. The original Landau function [2,3] is given by the dotted line. Parameters describing $f(\Delta)$ are the most probable energy loss $\Delta_p(x;\beta\gamma)$, i.e. the position of the maximum of the straggling function, at 1371 eV, and the full-width-at-half-maximum (FWHM) $w(x;\beta\gamma) = 1463\,eV$. The mean energy loss is $\langle\Delta\rangle = 3044\,eV$.

[H. Bichsel, NIM A 562, 154 (2006)]

Straggling Functions



Bethe-Bloch mean energy loss: $\langle \Delta \rangle = 400 \text{ eV}$

[H. Bichsel, NIM A 562, 154 (2006)]

2.3.2 Laplace Transform Method

[L. Landau, J. Phys. USSR 8, 201 (1944)]

Change of energy-loss distribution $f(\Delta; x)$ as a result of the particle passing through a thin elemental layer δx:

$$\begin{aligned} f(\Delta; x+\delta x) - f(\Delta; x) &= +n_e \delta x \int_0^{\Delta} \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} f(\Delta - E; x)\,\mathrm{d}E \\ &\quad - n_e \delta x \int_0^{\infty} \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} f(\Delta; x)\,\mathrm{d}E \end{aligned}$$

- 1st term: probability that the energy loss in x was $(\Delta - E)$, and a collision with energy transfer E occurred in δx, which makes the total energy loss equal to Δ (particle scattered into Δ)
- 2nd term: probability that the energy loss in x was already equal to Δ before entering δx, where a further collision increased the energy loss beyond Δ (particle scattered out of Δ)

Laplace Transform Method

Put in form of a transport equation:

$$\frac{\partial f(\Delta;x)}{\partial x} = \int_0^\infty n_e \frac{\mathrm{d}\sigma(E)}{\mathrm{d}E} \left[f(\Delta - E;x) - f(\Delta;x)\right] \mathrm{d}E$$

upper integration limit $E \to \infty$ for 1st term ok, since $f(x,\Delta) = 0$ for $\Delta < 0$

Solution: Laplace transform of both sides

$\Delta \quad \circ\!\!-\!\!\bullet \quad s$

+ solve for $\bar{f}(s;x)$

$$\mathcal{L}\{f(\Delta;x)\} = \bar{f}(s;x)$$

+ inverse Laplace transform

$$f(\Delta;x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathrm{d}s \, \exp\left[s\Delta - x\int_0^\infty n_e \frac{\mathrm{d}\sigma(E)}{\mathrm{d}E} \left(1 - e^{-sE}\right) \mathrm{d}E\right]$$

$0 < c \ll 1$

Exact solution, but numerical integration necessary in most cases!

2.3.3 Straggling Functions

Remarks to both methods:

- result determined by $\mathrm{d}\sigma/\mathrm{d}E$
- given the same cross section $\mathrm{d}\sigma/\mathrm{d}E$, both methods are equivalent

Straggling Functions

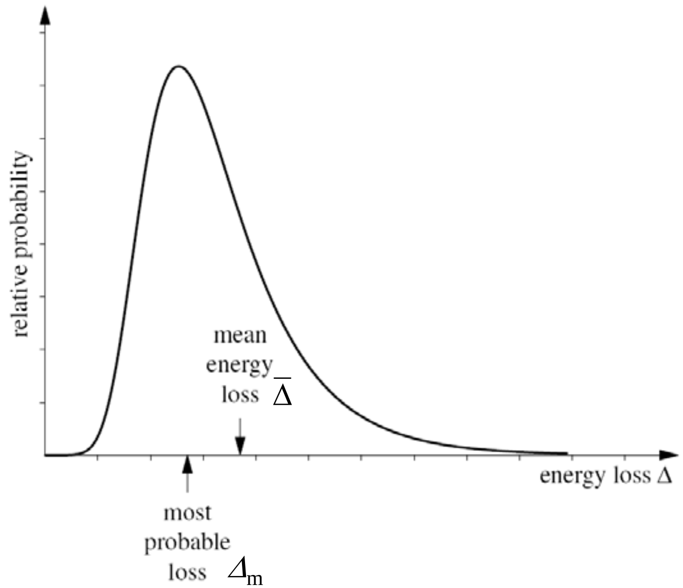
Different approximations, depending on thickness of absorber

Characteristic parameter: $\kappa = \frac{\xi}{T_{\max}}$, $\xi = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{\beta}\right)^2 n_e \cdot x$

ξ = scaling parameter
(1st term of Bethe-Bloch eq.)

Thin absorbers: $\kappa \leq 10$

- possibility of large energy transfer in single collisions: δ-electrons
- long tail on high-energy side, strongly asymmetric shape



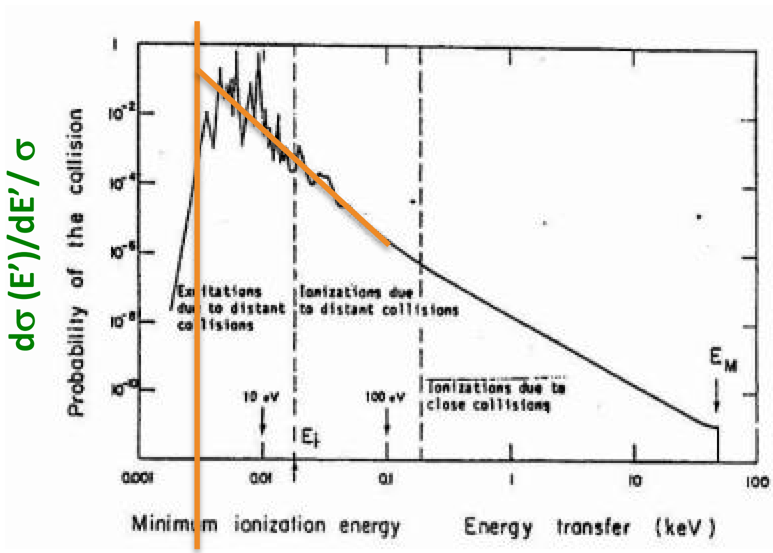
Very thin absorbers: $\kappa \to 0$ (i.e. $T_{max} \to \infty$)

- single energy transfers sufficiently large to consider e^- as free

⇨ Rutherford
$$\left(\frac{d\sigma}{dE}\right)^{\star}_{R} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^{2} = \frac{\xi}{n_e x}\frac{1}{E^2}$$

- particle velocity remains constant

Landau distribution [Landau, J. Phys. USSR 8, 201 (1944)]



Landau Distribution

Very thin absorbers: $\kappa \to 0$ (i.e. $T_{\mathrm{max}} \to \infty$)

- single energy transfers sufficiently large to consider e⁻ as free

⇨ Rutherford
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{R}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2 = \frac{\xi}{n_e x}\frac{1}{E^2}$$

- particle velocity remains constant

Landau distribution [Landau, J. Phys. USSR 8, 201 (1944)]

$$f_{\mathrm{L}}(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

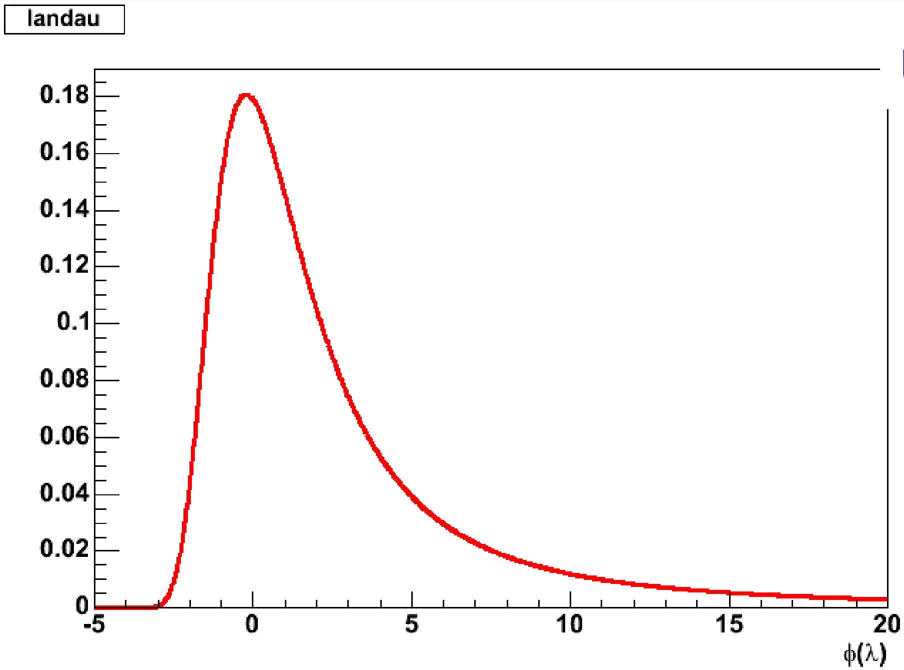
λ = universal parameter, see next page for relation to Δ_m and ξ

Analytical approximation: Moyal distribution [Moyal, Phil. Mag. 46, 263 (1955)]:

$$f_{\mathrm{M}}(x,\Delta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}, \quad \lambda = \frac{\Delta - \Delta_{\mathrm{m}}}{\xi}$$

Note: λ different from parameter in Landau distr. above

Landau Distribution



Universal Landau distribution:

$$\phi(\lambda) = \frac{1}{\pi}\int_0^\infty e^{-\pi u/2}\cos(u\ln u + \lambda u)\,du \quad ,$$

mit $\lambda = \frac{\Delta - \overline{\Delta}}{\xi} - (1+\beta^2 - C) - \ln\kappa \quad ,$

$$\xi = \frac{2\pi}{(4\pi\varepsilon_0)^2}\cdot\frac{z^2 e^4}{m v^2}\cdot n_e x \approx \overline{\Delta} \quad , \kappa = \frac{\xi}{T_{\max}} \quad ,$$

$C = 0.5772\ldots$ (Euler-Konstante) ,

$$\overline{\Delta} = 2\xi\left[\frac{1}{2}\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{\max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right]$$

Landau distribution in ROOT:

$$f(\Delta|p_1,p_2,p_3) = p_1 \times \phi\left(\frac{\Delta - p_2}{p_3}\right) \quad ,$$

mit p_1 Normierung (Integral)

$$\Delta_m = p_2 - 0.22278 \times p_3 \quad ,$$

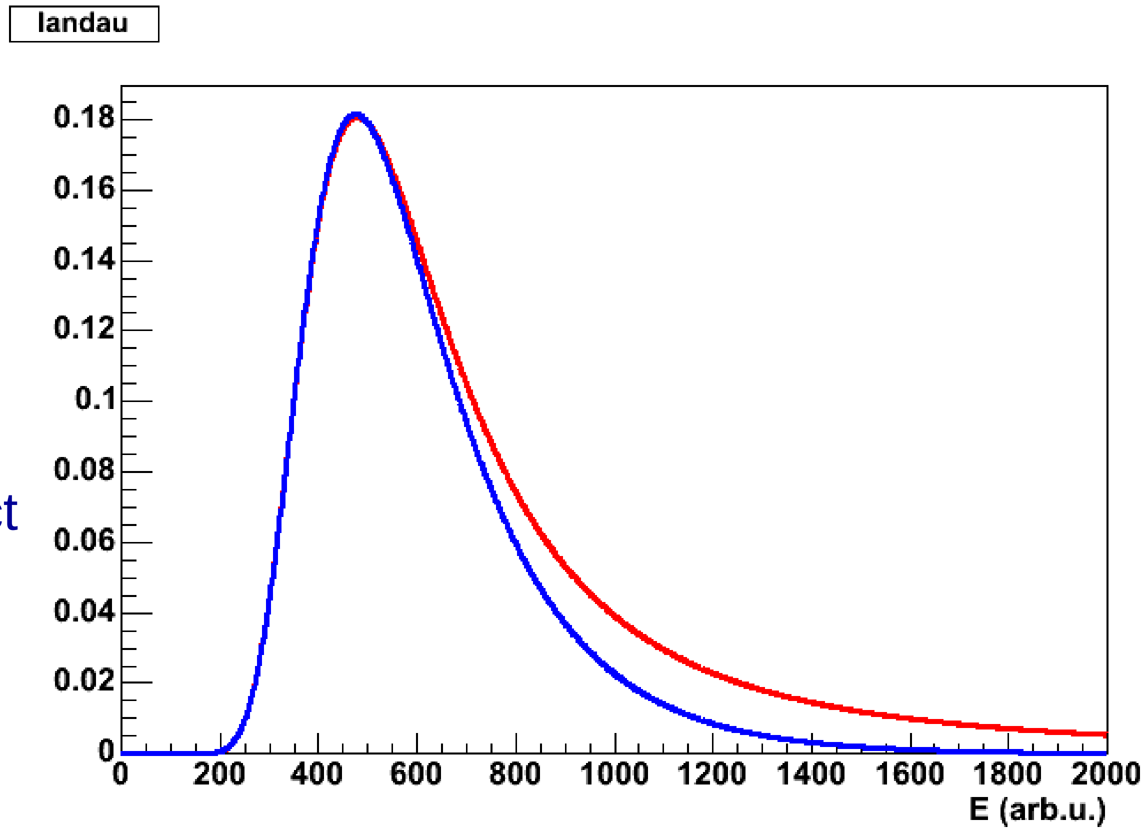
$$\text{FWHM} = 2\sqrt{2\ln 2}\times\frac{p_3}{0.5860} \quad .$$

Properties of $\phi(\lambda)$:

- asymmetric: tail up to $T_{max} \to \infty$
- Maximum at λ=-0.223
- FWHM=4.02•λ
- numerical evaluation

Comparison Landau - Moyal

Landau:
p_1=1.
p_2=500.
p_3=100.



Moyal:
- coarse shape correct
- tail too low!

But: tails are important for detector resolution!

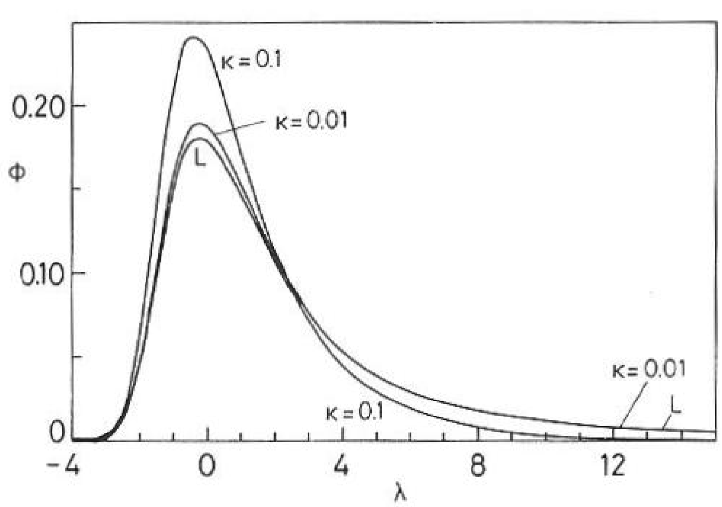
Landau Distribution

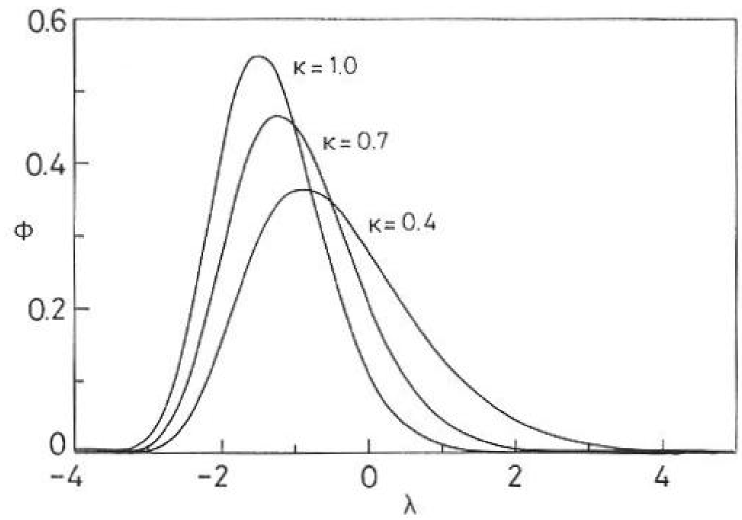
Landau distribution:

- Uses Rutherford cross section
- Does not reproduce straggling functions based on more realistic models for CCS for very small N_c
- Narrower width also for higher N_c ⇨ related to mean free path λ
  - Rutherford CCS underestimates λ (overestimates N_c)
  - Poisson contribution to straggling function leads to broadening

Thin absorbers: $0.01 < \kappa < 10$

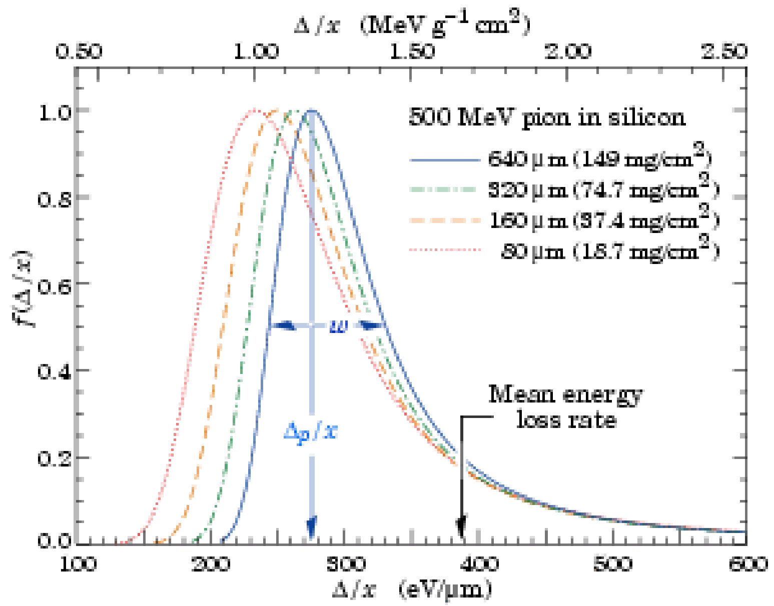
- use correct expression for $T_{\max}$
- use Mott cross section instead of Rutherford
- reduces to Landau distribution for very small κ
- less asymmetric shape for larger κ





[S.M. Seltzer, M.J. Berger,
Nucl. Sc. Ser. Rep. No. 39 (1964)]

Realistic Straggling Functions



Iterative Solution of convolution integral

[H. Bichsel, Rev. Mod. Phys. 60, 663 (1988)]

PAI Model vs Landau

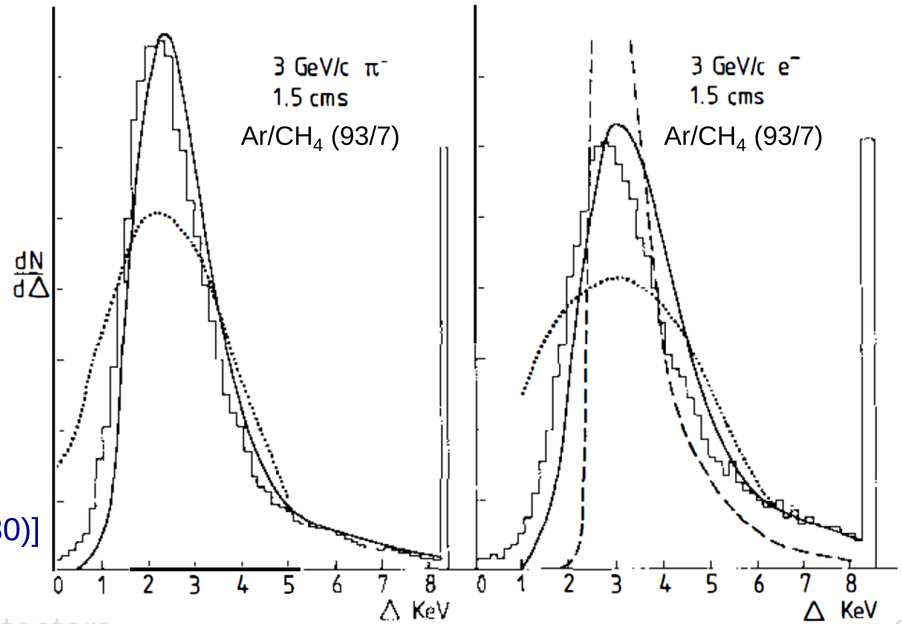
- Histogram: data
- Landau + corrections

– – – – [Maccabee, Papworth, Phys. Lett. A 30, 241 (1969)]

.... [Blunck, Leisegang, Z. Phys, 128, 500 (1950)]

- PAI model

——— [Allison, Cobb, Ann. Rev. Nucl. Part. Sci. 30, 253 (1980)]



Restricted energy loss:

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T<T_{\mathrm{cut}}} = \frac{4\pi}{(4\pi\varepsilon_0)^2} \frac{z^2 e^4 n_\mathrm{e}}{mc^2\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_{\mathrm{cut}}}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_{\mathrm{cut}}}{T_{\mathrm{max}}}\right) - \frac{\delta}{2} \right]$$

approaches normal Bethe-Bloch equation for $T_{\mathrm{cut}} \to T_{\mathrm{max}}$

Transforms into average total number of e⁻ ion pairs n_T along path length x:

$$x\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_\mathrm{T} W$$

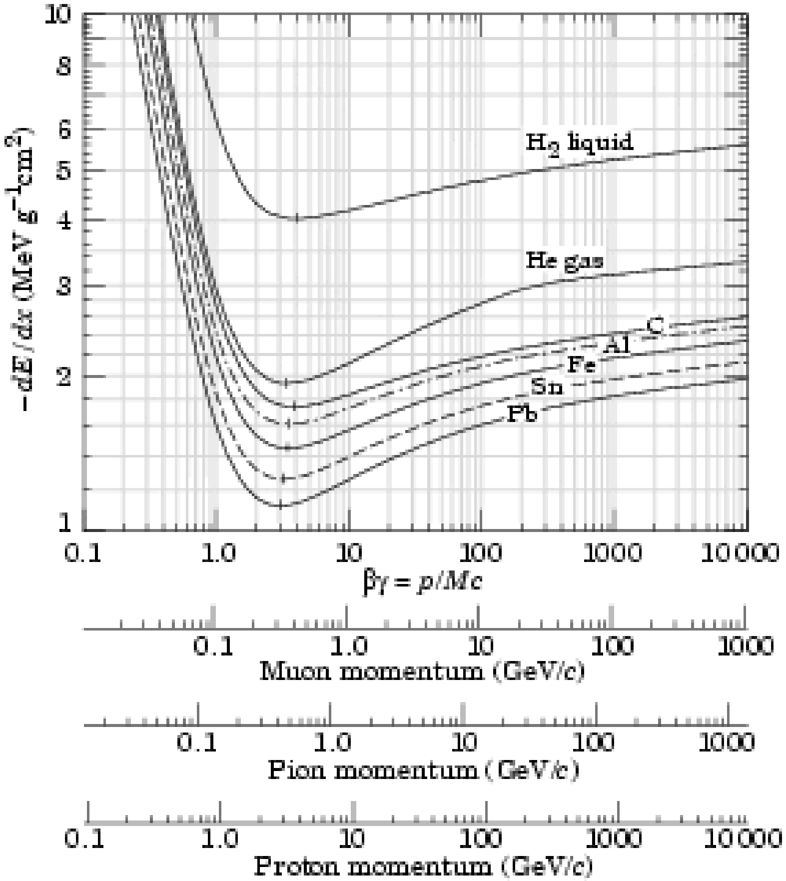
But: actual energy loss fluctuates with a long tail (Landau distribution)

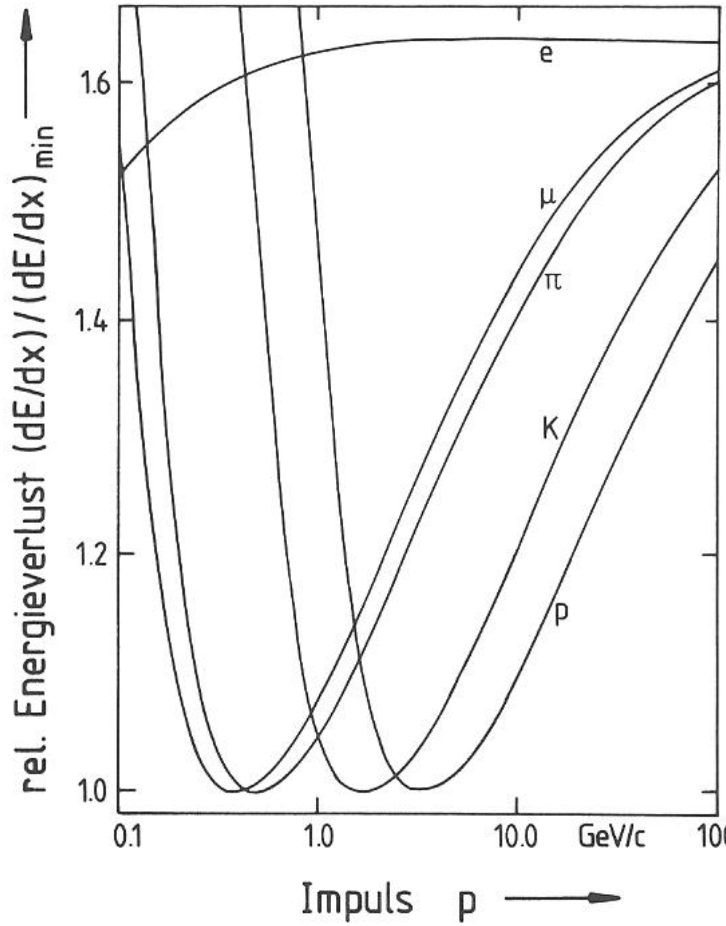
⇨mean value of energy loss is a bad estimator

⇨ use truncated mean of N pulse height measurements along the track:

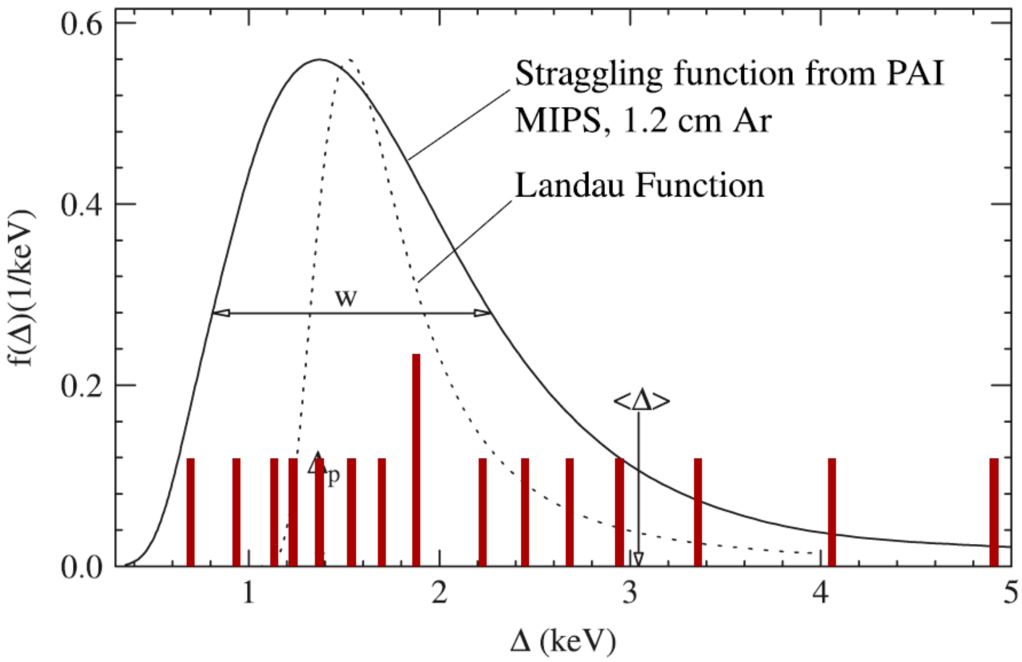
$$\langle A \rangle_t = \frac{1}{N_t} \sum_{i=1}^{N_t} A_i \qquad \begin{array}{l} A_i \le A_{i+1} \quad \text{for} \quad i = 1, \dots, N \\ N_t = t \cdot N, \quad t \in [0,1] \end{array}$$

Measurement of Energy Loss





Measurement of Energy Loss



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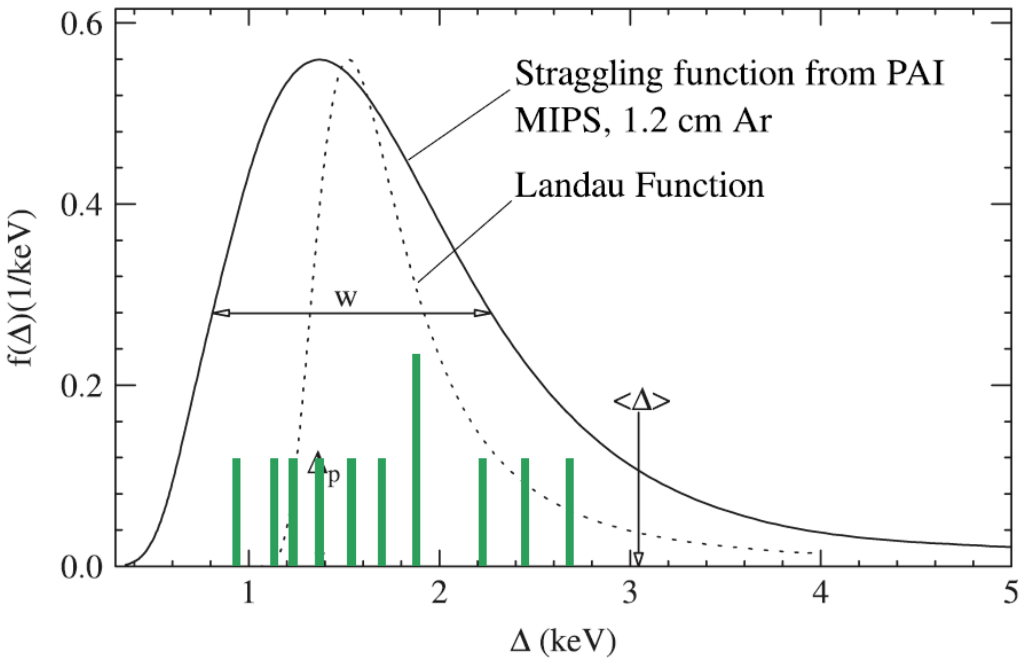
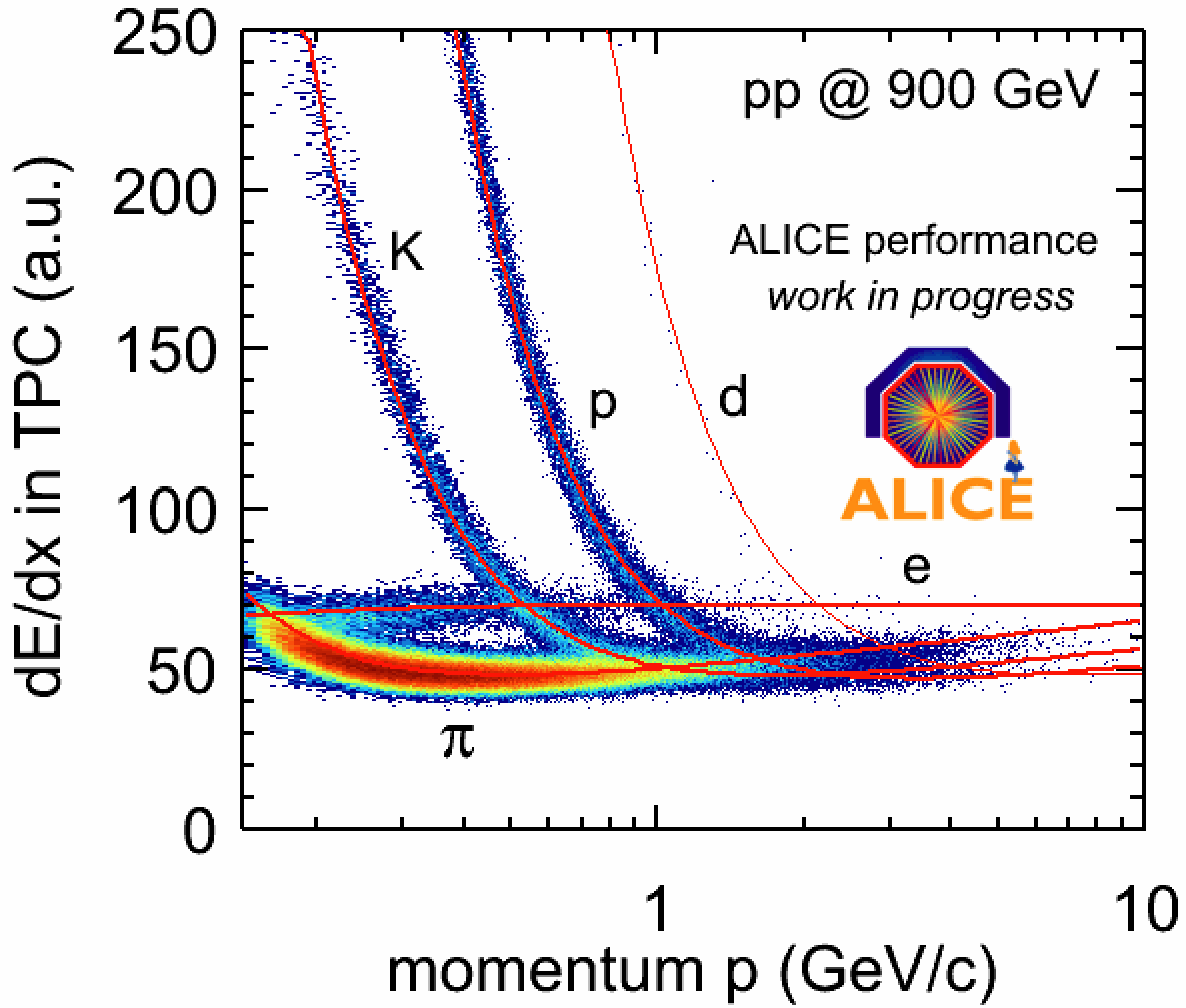


Resolution (empirical):

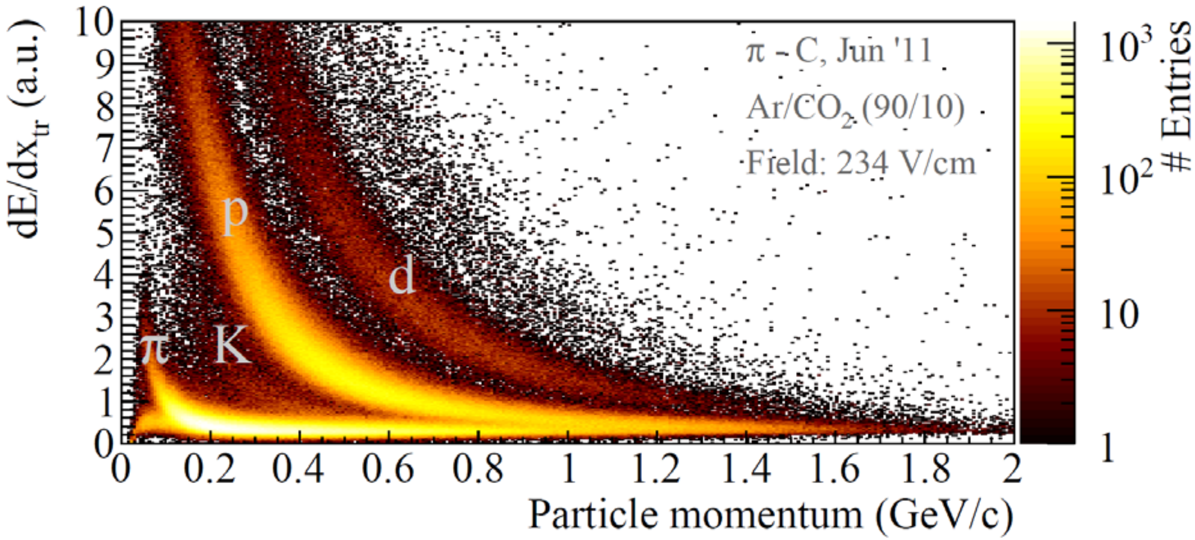
$$\Delta E/E = 0.96 \cdot N^{-0.46} \cdot (\Delta x \cdot p)^{-0.32}$$

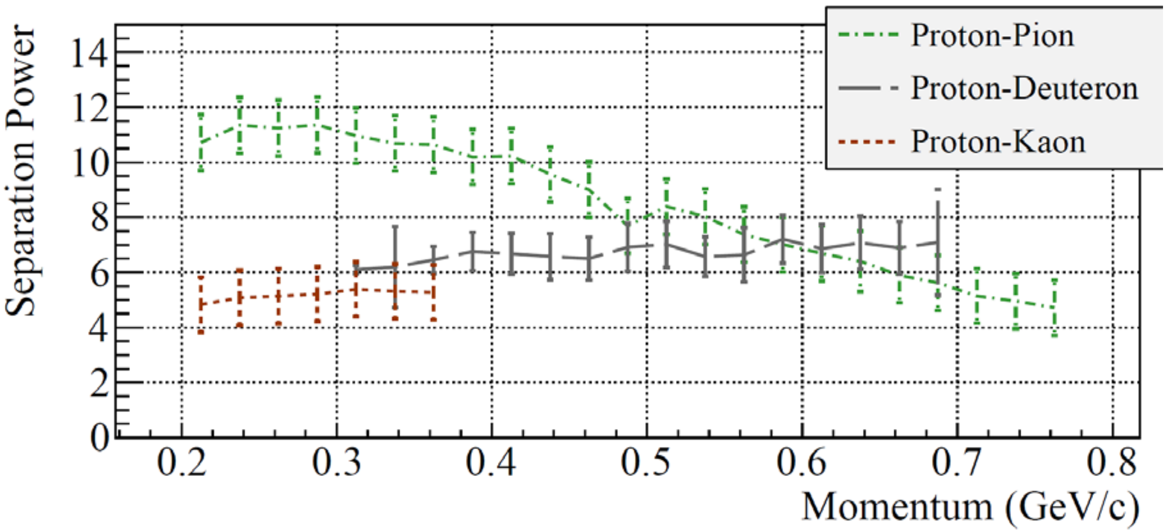
N = number of samples
Δx = sample length (cm)
p = gas pressure (atm)

Example: ALICE TPC



Example: FOPI GEM-TPC





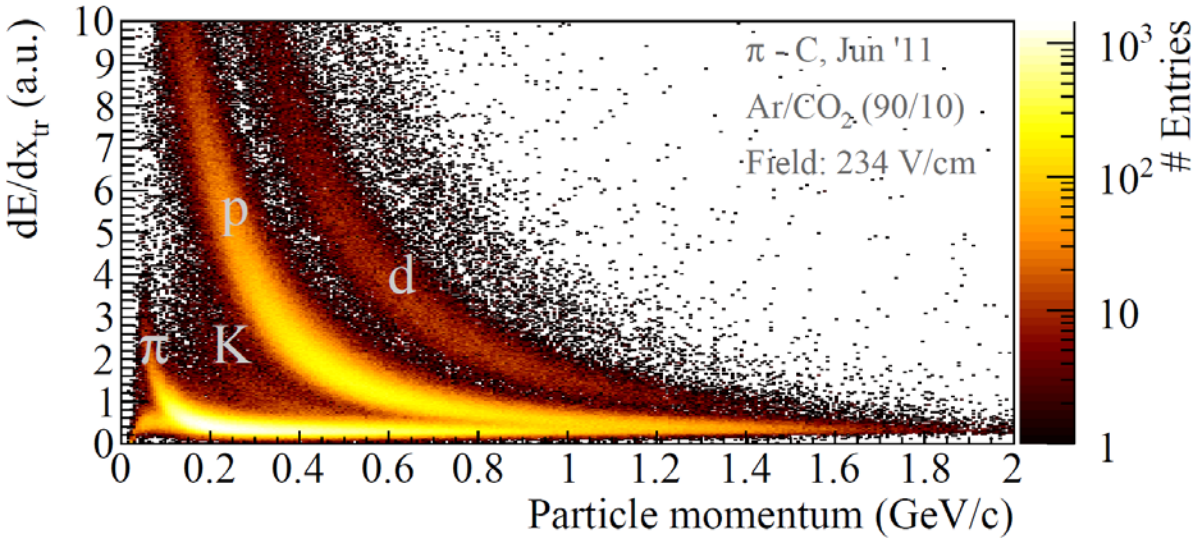
- Sum up charge in 5 mm steps
- Truncated mean: 0-70%
- Momentum from TPC + CDC

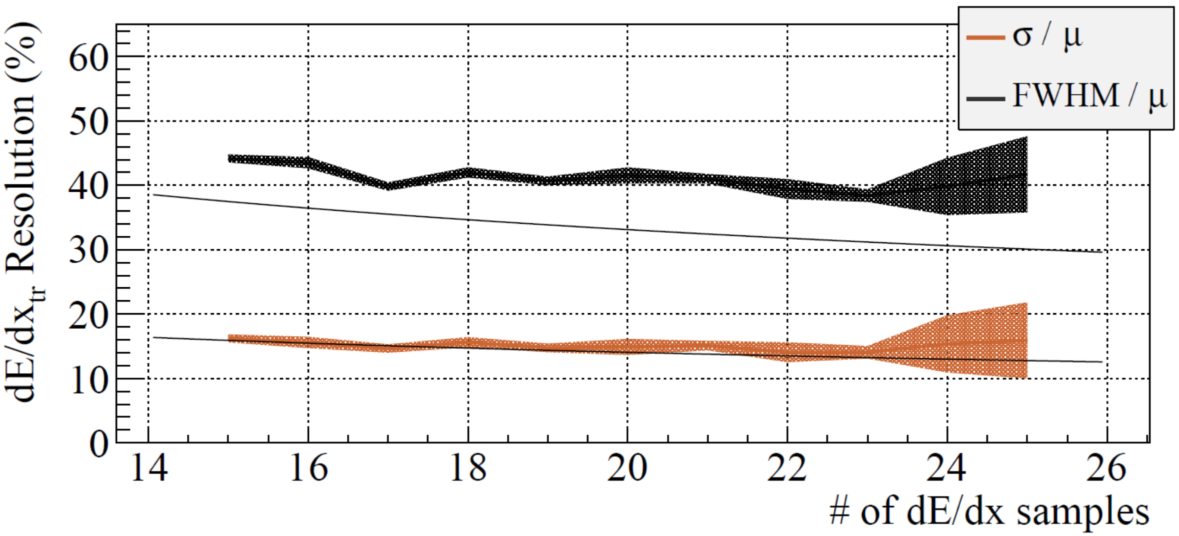
- PID using dE/dx:
  - Resolution ~ 14-17%
  - No density correction

First dE/dx measurement with GEM-TPC!

[F. Böhmer et al., NIM A 737, 214 (2014)]

Example: FOPI GEM-TPC





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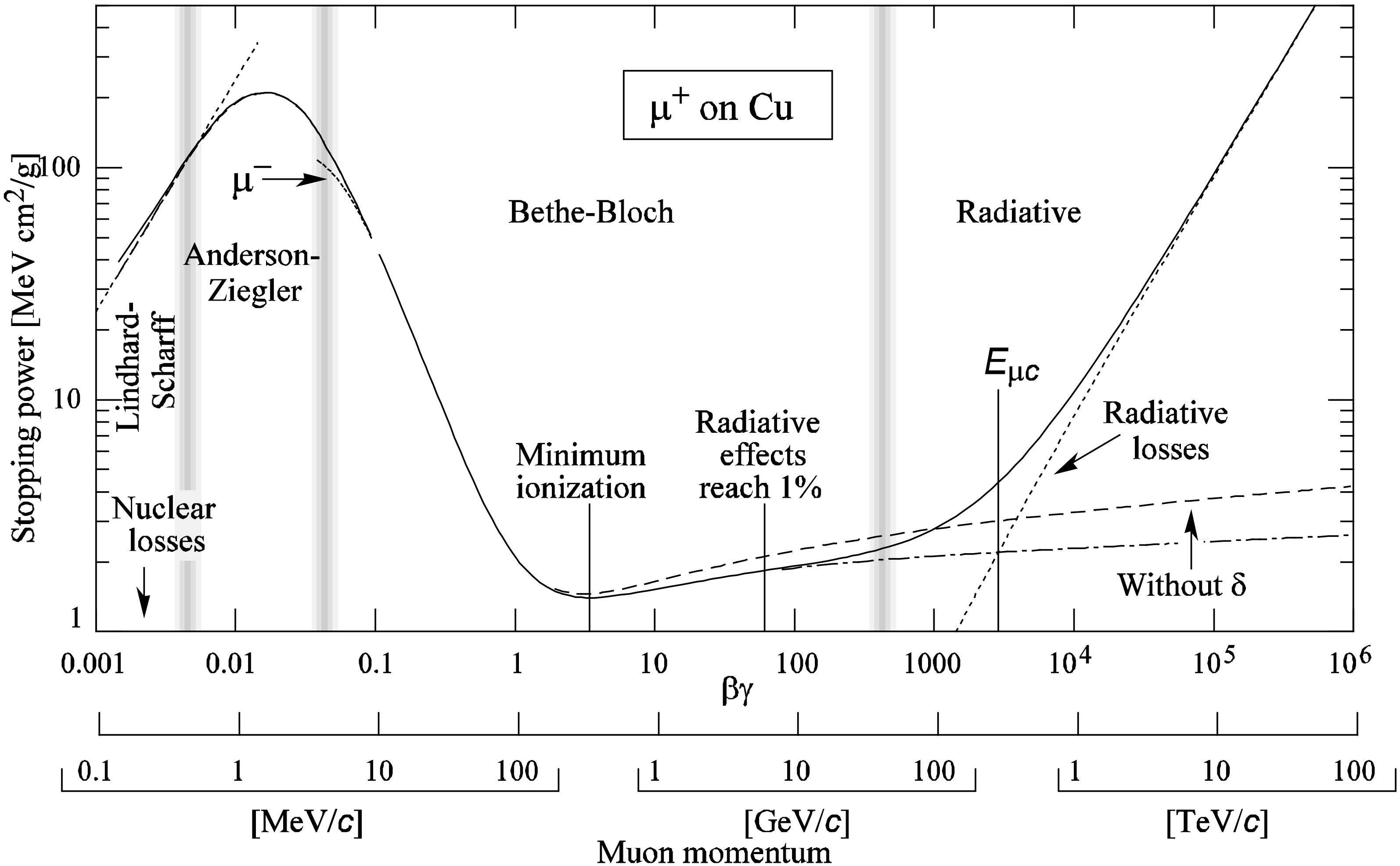
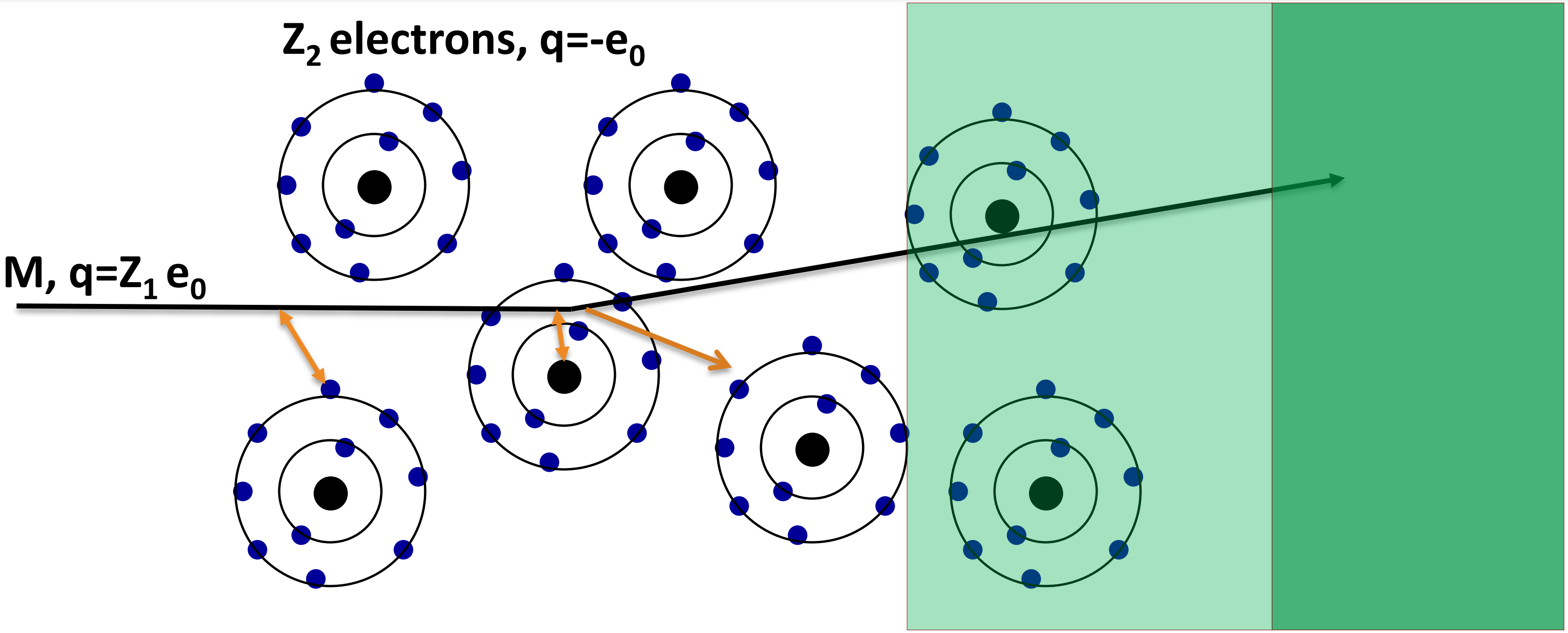
Energy Loss of Charged Particles



[Review of Particle Physics, S. Eidelmann et al., Phys. Lett. B 592, 1 (2004)]

Electromagnetic Interaction of Particles with Matter



Interaction with the atomic electrons. The incoming particle loses energy and the atoms are <u>excited</u> or <u>ionized.</u>

Interaction with the atomic nucleus. The particle is deflected (scattered) causing <u>multiple scattering</u> of the particle in the material. During this scattering a <u>Bremsstrahlung</u> photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as <u>Cherenkov Radiation</u>. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called <u>Transition radiation</u>.

Slide courtesy of W. Riegler

2.5 Multiple Scattering

In addition to inelastic collisions with atomic electrons

⇨ elastic Coulomb scattering from nuclei

⇨ Rutherford cross section for single scattering (no spin, recoil)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} = \frac{\left(zZe^2\right)^2}{\left(4\pi\varepsilon_0\right)^2 \cdot 4E^2 \sin^4\frac{\theta}{2}} \propto \frac{1}{\sin^4\frac{\theta}{2}}$$

⇨ small angular deflection of incident particle

⇨ random zigzag path, $\langle\theta\rangle = 0$

For hadronic projectiles also the strong interaction contributes to multiple scattering

⇨ contribution to momentum resolution!

Multiple Scattering

Theory:

- Single scattering: very thin absorber such that probability for more than one Coulomb scattering small
  - ⇨ angular distribution given by Rutherford formula
- Plural scattering: average number of scatterings $N<20$
  - ⇨ difficult!
- Multiple scattering: $N>20$, energy loss negligible
  - ⇨ statistical treatment
  - ⇨ Gaussian distribution via Central Limit Theorem
  - ⇨ non-Gaussian tails due to less frequent hard scatterings
  - ⇨ probability distribution for net angle of deflection as function of thickness of material traversed: Molière distribution (up to $\theta \approx 30^\circ$)

[G. Molière, Z. Naturforsch. 3a, 78 (1948), H.A. Bethe, Phys. Rev. 89, 1256 (1953)]

Multiple Scattering

Gaussian approximation: ignore large-angle (>10°) single scatterings

$$P(\theta_{\text{plane}})d\theta_{\text{plane}} = \frac{1}{\sqrt{2\pi}\theta_0}\exp\left(-\frac{\theta_{\text{plane}}^2}{2\theta_0^2}\right)d\theta_{\text{plane}}$$

central limit theorem

$$\theta_0 = \theta_{\text{plane}}^{\text{rms}} = \sqrt{\langle\theta_{\text{plane}}^2\rangle} = \frac{1}{\sqrt{2}}\sqrt{\langle\theta^2\rangle}$$

$$\theta_0 = \frac{13.6\,\text{MeV}}{\beta c p} z \sqrt{\frac{L_T}{X_0}}\left[1 + 0.038\ln\left(\frac{L_T}{X_0}\right)\right]$$

from fit to Molière distribution

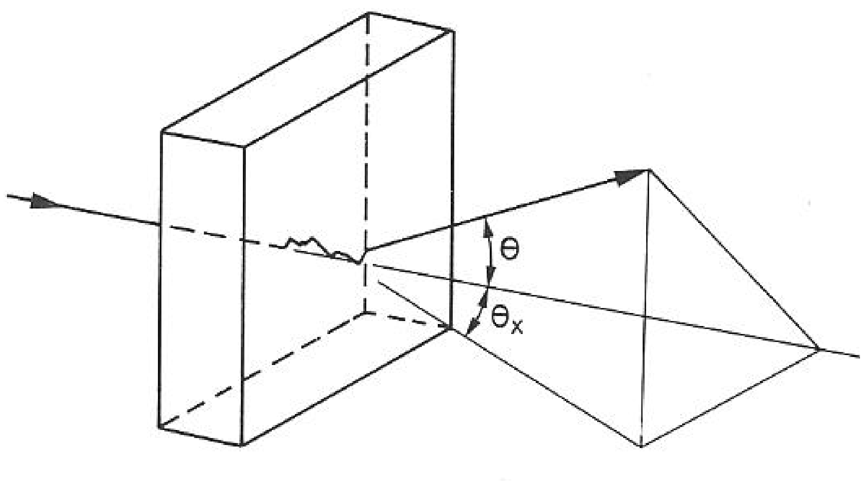
X_0 = radiation length of absorber

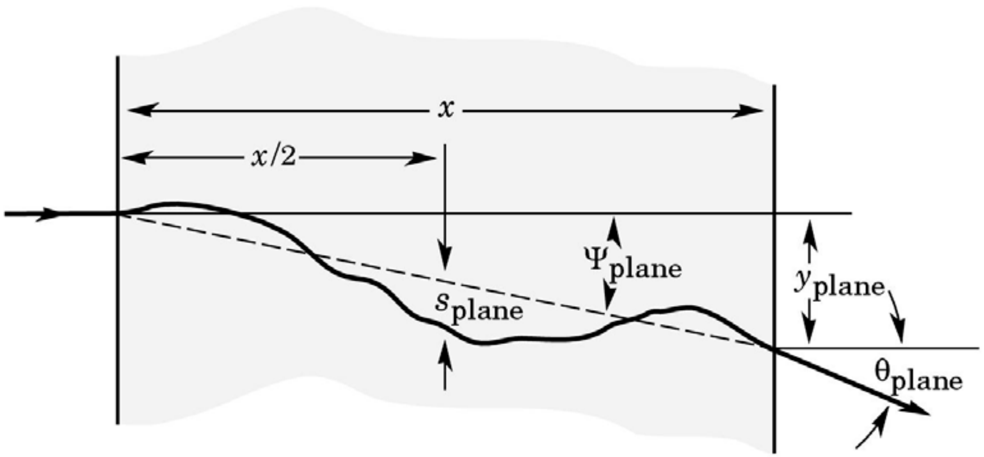
L_T = track length in medium (w/o multiple scattering),

i.e. straight line or helix

Accurate to < 11% for $10^{-3} < \frac{L_T}{X_0} < 100$

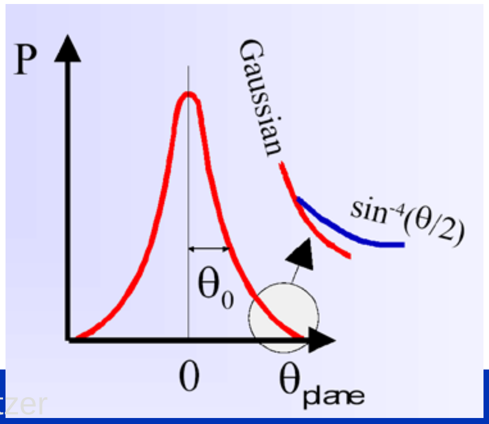
[V.L. Highland, NIM 129, 497 (1975)]
[G.R. Lynch et al., NIM B 58, 6 (1991)]

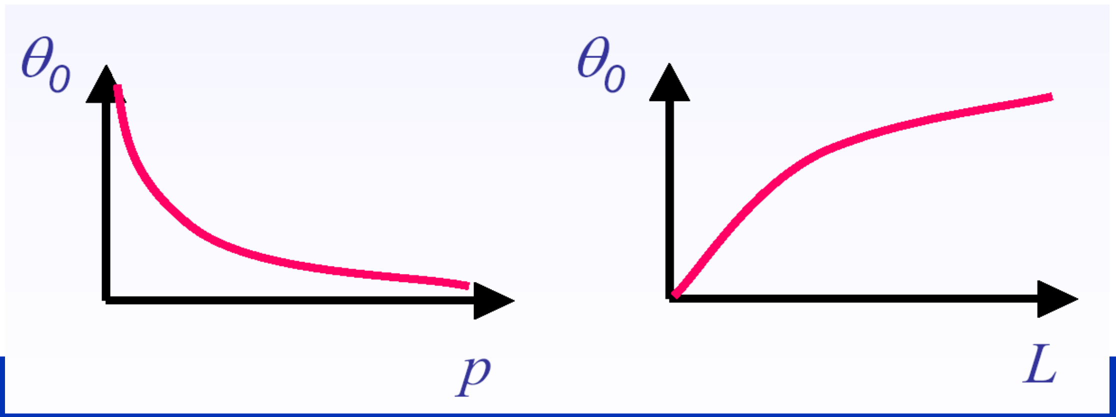




$$P(\theta_{\text{plane}})d\theta_{\text{plane}} = \frac{1}{\sqrt{2\pi}\theta_0}\exp\left(-\frac{\theta_{\text{plane}}^2}{2\theta_0^2}\right)d\theta_{\text{plane}}$$

$$\theta_0 = \frac{13.6\,\text{MeV}}{\beta c p} z \sqrt{\frac{L_T}{X_0}}\left[1 + 0.038\ln\left(\frac{L_T}{X_0}\right)\right]$$





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